Tax Progression under Collective Wage Bargaining and Individual Effort Determination

Erkki Koskela
University of Helsinki, RUeSG and HECER

and

Ronnie Schöb
Otto-von-Guericke-University Magdeburg

Discussion Paper No. 109
June 2006

ISSN 1795-0562
Tax Progression under Collective Wage Bargaining and Individual Effort Determination*

Abstract

In this paper we study the impact of tax policy on wage negotiation, workers’ effort and employment when effort is only imperfectly observable. We show that the different wage setting motives – rent sharing and effort incentives – reinforce the effects of partial tax policy measures but not necessarily those of more fundamental tax reforms. We show that a higher degree of tax progression always leads to wage moderation but the well-established result from the wage bargaining literature that a revenue-neutral increase in the degree of tax progression is good for employment does not carry over to the case with wage negotiations and imperfectly observable effort. While it remains true that introducing tax progression increases employment, we cannot rule out negative employment effects from an increase in tax progression when tax progression is already very high.

JEL Classification: J41, J51, H22

Keywords: Wage bargaining, effort determination, tax progression

Erkki Koskela
Department of Economics
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014 University of Helsinki
FINLAND
e-mail: erkki.koskela@helsinki.fi

Ronnie Schöb
Department of Economics and Management
Otto-von-Guericke-University Magdeburg
P.O. Box 4120
D-39016 Magdeburg
GERMANY
e-mail: ronnie.schoeb@ww.uni-magdeburg.de

* Koskela thanks Otto-von-Guericke-University Magdeburg for great hospitality as well as the Research Unit of Economic Structures and Growth (RUESG) in the University of Helsinki, the Yrjö Jahnsson Foundation and the Academy of Finland (grant No.1109089) for financial support. Schöb thanks WZB in Berlin for great hospitality.
1. Introduction

Tax progression leads to wage moderation and is thus good for employment. This result has been derived for different assumptions about the wage setting motives such as rent sharing in wage bargaining models (see e.g. Holm and Koskela 1996, Koskela and Vilmunen 1996, Koskela and Schöb 1999) or effort incentives in efficiency wage models, where firms unilaterally decide both upon the wage rate and the employment level (see e.g. Pisauro 1991, Rasmussen 2002).

The effect of tax progression, however, has not yet been analyzed in a framework that combines these different wage setting motives in a uniform framework. So far only very few papers combine wage bargaining and effort considerations at all. Early contributions by Lindbeck and Snower (1991), Sanfey (1993) do not provide a uniform answer to the question in how far different wage setting motives analyzed in efficiency wage and union bargaining models reinforce or weaken each other. Later on, Bulkley and Myles (1996) show that with imperfect monitoring of workers’ effort, a monopoly trade unions will set a higher wage than the pure efficiency wage set by the firms. This provides a higher bonus for non-shirking and results in a higher level of effort than we would observe in a competitive labor market. Garino and Martin (2000), on the other side, show that efficiency wages offset the cost of higher wages and thus induce firms to make more concessions in wage negotiations. Thus there is theoretical evidence that the different wage setting motives reinforce each other.

Within such a framework, Altenburg and Straub (1998) analyzes variations of the benefit-replacement ratio. They find that in contrast to the standard result in both efficiency wage and union bargaining models, the effect of a higher reservation utility on wages, employment and effort is ambiguous when benefits are financed through lump-sum taxes. A higher replacement ratio may then reduce the wage rate and raise employment. A higher reservation utility of workers will induce firms to reduce their
demand for effective labor. If, as a consequence, the labor share decreases firms experience a higher relative reduction in profits from a wage increase. This explains why the wage may actually fall and – in the end – employment will rise.

To our knowledge, only one paper analyzes the impact of taxes in this framework. Garcia and Rios (2004) adopt the Altenburg and Straub (2002) model to analyze revenue-neutral tax reforms numerically. There numerical calculations suggest that a revenue-neutral increase in the tax exemption financed by an increase in the wage tax increases employment. That indicates that the result by Koskela and Schöb (1999) according to which a revenue-neutral shift from payroll taxes to wage taxes raises employment when there is a higher tax exemption for wage taxes also applies when effort is unobservable. Furthermore, they argue that it is better for employment in the case of constant fiscal revenues to compensate higher tax exemption through increases in wage taxes rather than payroll taxes. Since Garcia and Rios (2004) only provide numerical rather than analytical results, we present an analytical framework to elaborate the way in which tax policy affects wage negotiations and employment when effort is only imperfectly observable and trade unions and firms negotiate on wages.

Our comparative statics analysis indicates that the standard results from the trade union literature must be modified in the case of imperfect monitoring of individual effort determination. In these standard models, tax policy only affects wages by altering the size of the labor surplus. When both wage setting motives are present, however, tax policy also affects the strength by which tax policy parameters affect the negotiated wage and employment. When effort is not observable, tax policy affects the wage elasticity of effort, which in turn affects the wage elasticity of labor demand. Since these alter the scope by which workers can attract labor rents, this constitutes an additional channel by which tax policy can influence the wage negotiation. As it turns out, this additional impact reinforces the effects of partial tax policy measures that we observe in the standard bargaining and efficiency models.
### Table 1: Labour taxation in the OECD countries

<table>
<thead>
<tr>
<th>Country</th>
<th>(1) Average wage tax</th>
<th>(2) Marginal wage tax</th>
<th>(3) Average wage tax rate progression</th>
<th>(4) Calculated relative tax exemption a/w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>28.6</td>
<td>35.4</td>
<td>6.8</td>
<td>22.9</td>
</tr>
<tr>
<td>Austria</td>
<td>44.9</td>
<td>55.5</td>
<td>10.6</td>
<td>56.1</td>
</tr>
<tr>
<td>Belgium</td>
<td>54.2</td>
<td>66.4</td>
<td>12.2</td>
<td>34.8</td>
</tr>
<tr>
<td>Canada</td>
<td>32.3</td>
<td>33.9</td>
<td>1.6</td>
<td>26.4</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>43.6</td>
<td>48.1</td>
<td>4.5</td>
<td>34.9</td>
</tr>
<tr>
<td>Denmark</td>
<td>41.5</td>
<td>49.2</td>
<td>7.7</td>
<td>20.7</td>
</tr>
<tr>
<td>Finland</td>
<td>43.8</td>
<td>55.1</td>
<td>11.3</td>
<td>36.6</td>
</tr>
<tr>
<td>France</td>
<td>47.4</td>
<td>66.6</td>
<td>19.2</td>
<td>30.3</td>
</tr>
<tr>
<td>Germany</td>
<td>50.7</td>
<td>64.0</td>
<td>13.3</td>
<td>44.9</td>
</tr>
<tr>
<td>Greece</td>
<td>34.9</td>
<td>44.2</td>
<td>9.3</td>
<td>95.2</td>
</tr>
<tr>
<td>Hungary</td>
<td>45.8</td>
<td>54.7</td>
<td>8.9</td>
<td>52.3</td>
</tr>
<tr>
<td>Iceland</td>
<td>29.7</td>
<td>40.4</td>
<td>10.7</td>
<td>30.7</td>
</tr>
<tr>
<td>Ireland</td>
<td>23.8</td>
<td>33.2</td>
<td>9.4</td>
<td>49.5</td>
</tr>
<tr>
<td>Italy</td>
<td>45.7</td>
<td>58.0</td>
<td>12.3</td>
<td>46.7</td>
</tr>
<tr>
<td>Japan</td>
<td>26.6</td>
<td>31.5</td>
<td>4.9</td>
<td>47.8</td>
</tr>
<tr>
<td>Korea</td>
<td>16.6</td>
<td>24.8</td>
<td>8.2</td>
<td>80.0</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>31.9</td>
<td>45.9</td>
<td>14.0</td>
<td>64.5</td>
</tr>
<tr>
<td>Mexico</td>
<td>15.4</td>
<td>23.4</td>
<td>8.0</td>
<td>78.1</td>
</tr>
<tr>
<td>Netherlands</td>
<td>43.6</td>
<td>50.7</td>
<td>7.1</td>
<td>56.6</td>
</tr>
<tr>
<td>New Zealand</td>
<td>20.7</td>
<td>33.0</td>
<td>12.3</td>
<td>37.3</td>
</tr>
<tr>
<td>Norway</td>
<td>36.9</td>
<td>43.2</td>
<td>6.3</td>
<td>25.4</td>
</tr>
<tr>
<td>Poland</td>
<td>43.1</td>
<td>45.7</td>
<td>2.6</td>
<td>33.7</td>
</tr>
<tr>
<td>Portugal</td>
<td>32.6</td>
<td>39.4</td>
<td>6.8</td>
<td>60.0</td>
</tr>
<tr>
<td>Slovak Republic</td>
<td>42.0</td>
<td>48.3</td>
<td>6.3</td>
<td>52.1</td>
</tr>
<tr>
<td>Spain</td>
<td>38.0</td>
<td>45.5</td>
<td>7.5</td>
<td>43.3</td>
</tr>
<tr>
<td>Sweden</td>
<td>48.0</td>
<td>51.7</td>
<td>3.7</td>
<td>17.0</td>
</tr>
<tr>
<td>Switzerland</td>
<td>28.8</td>
<td>36.5</td>
<td>7.7</td>
<td>46.7</td>
</tr>
<tr>
<td>Turkey</td>
<td>42.7</td>
<td>44.5</td>
<td>1.8</td>
<td>12.5</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>31.2</td>
<td>40.6</td>
<td>9.4</td>
<td>35.1</td>
</tr>
<tr>
<td>United States</td>
<td>29.6</td>
<td>34.1</td>
<td>4.5</td>
<td>22.5</td>
</tr>
</tbody>
</table>


Legend: Tax rates are for the year 2004 for a single person with 100% of average wage, relative to the gross wage including the social security contributions paid by employees. Column (3) shows the difference between marginal and average rate of income tax. As an approximation it is assumed that for each country the tax schedule consists of a tax exemption and a constant marginal tax rate. The exchange rate between US-Dollar and Euro was assumed to be unity. Social assistance level does not include housing costs. Numbers of social assistance are from 2002 taken from OECD (2004), Benefits and Wages, OECD Indicators.

In the second main part of the paper we then analyze revenue-neutral tax reforms that change the degree of tax progression and derive the qualitative effects such tax reforms have on the negotiated wage, individual effort and aggregate employment. Table 1 highlights the importance of such an analysis. All OECD labor tax systems in all OECD countries are progressive and show significant differences in the degree of tax progression. We measure tax progression by the difference of marginal and average tax...
rates that are shown in the first and second column. This difference, reported in the third column, is known as the average wage tax progression ARP (see Lambert 2001 and our section 5). The higher this difference, the more progressive wage taxation is. The highest difference is for France, with 19.2 percentage points, and the lowest one for Canada, with only 1.4 percentage points.

Our first main result shows that an increase in wage tax progression always leads to wage moderation. In this respect our model shows that the wage moderation effect of higher tax progression that is present in both the efficiency wage model and the bargaining model carries over to the more general case when both wage setting motives are at work. The effect on effort and, consequently, on labor demand, however, is ambiguous. Although it remains true that introducing tax progression raises employment, it turns out that the claim “tax progression is good for employment” (Koskela and Vilmunen 1996) only applies for moderate degrees of tax progression.

In the following section 2 we present the basic structure of the model and describe the time sequence of decisions with respect to wage bargaining, labor demand and individual effort determination. The workers’ individual effort determination and the firms’ labor demand are elaborated in section 3. Section 4 uses the Nash bargaining approach to analyze wage negotiations subject to firms’ labor demand and workers’ effort determination and presents the essential comparative static results. Section 5 applies the analysis to revenue-neutral changes in the labor tax structure and explores the effects of tax progression on the negotiated wage, individual effort and employment. The main findings are summarized in section 6.

---

1 To make these figures comparable with our stylized model framework below, we refer all tax rates to the gross wage including payroll taxes paid by the employer.
2. Basic framework

Concerning the time sequence of decisions we assume that the government behaves as a Stackelberg leader who fixes the tax parameters in the first stage. To raise revenues, the government can employ a wage tax \( t \), which is levied on the gross wage minus a tax exemption \( a \). Thus the tax base for the wage tax \( t \) equals \((w-a)L\), where \( L \) denotes total employment. In the presence of a positive tax exemption \( a \), the marginal tax rate \( t \) exceeds the average tax rate \( t^a \equiv t(1-a/w) \) so that we have a linearly progressive tax system. The net-of-tax wage workers receive is given by \( w^n = (1-t)w + ta \). We abstract from payroll taxes.

At stage 2 firms and trade unions bargain with respect to the gross wage.\(^2\) They take the tax parameters as given and anticipate the consequences the negotiated gross wage has for labor demand by firms and the resulting net labor income has for individual effort determination by workers. After the wage negotiations are settled the firms decide at stage 3 about their labor demand. Since firms cannot perfectly observe effort, the firms have to anticipate the workers’ individual effort decisions. At the final stage 4 workers make their individual effort choice.

The time sequence of decisions is summarized in Figure 1. In the subsequent sections we derive the decisions taking place at different stages by using backward induction.

**Figure 1: Time sequence of decisions**

\[1^{\text{st}} \text{ stage} \quad 2^{\text{nd}} \text{ stage} \quad 3^{\text{rd}} \text{ stage} \quad 4^{\text{th}} \text{ stage} \]

- Tax policy \((t, a)\)
- Wage bargaining \((w)\)
- Labour demand \((L)\)
- Effort determination \((e)\)

\(^2\) Since tax parameters are given from the viewpoint of firms and trade unions, it does not matter whether they bargain over gross or net-of-tax wages (see Koskela and Schöb 2002).
3. Individual effort determination and labor demand

We start analyzing the 4th stage where workers decide about their working effort, taking the tax policy, the negotiated wage and aggregate employment as given. Then we analyze stage 3 where firms determine employment.

3.1. Individual effort determination

We focus on the choice that a single worker faces when employed by a representative firm in a static framework. Effort cannot be fully controlled by firms. They can set a standard effort that we normalize to one. If workers meet this standard their jobs are secure. If they shirk by providing less effort, however, firms can fire them. The probability of detection depends positively on monitoring effort. Following Bental and Demougin (2006), we consider an isoelastic probability function of employment \( e^d \) where \( d \in [0; 1] \) denotes the (constant) probability elasticity of effort. The probability of being laid off is thus \( 1 - e^d \). Assuming a representative risk-neutral worker and applying a specific utility function \( V \) that is additively separable and quasi-linear, we obtain

\[
V^w = e^d [w^n - g(e)] + (1 - e^d)b,
\]

where \( b \) denotes the workers’ outside option that equals some exogenous unemployment income, and \( g(e) \) denotes the disutility of effort \( e \) as a convex function, i.e. \( g'(e), g''(e) > 0 \). Working time per worker is fixed and normalized to unity.

For the following it is convenient to define the workers’ surplus as the difference \( s \equiv w^n - g(e) - b \). This allows us to rewrite the utility function as \( V^w = e^d s + b \), which split the utility into the expected surplus when working with effort \( e \) and the basic income \( b \) the household receives in any case. The optimal individual effort level can be derived from the first-order condition \( V^w_e = de^{d-1}s - e^d g'(e) = 0 \). The worker chooses an effort level where the expected utility loss of working harder, which occurs with probability \( e^d \) equals the expected utility gain from an increased probability of staying in
employment and receiving the surplus $s$. Using the parameterization $g(e) = e^\theta / \theta$, $\theta > 1$, the effort function becomes:

\[
ed = \left( \frac{d\theta}{d + \theta} \right)^\frac{1}{\theta} \left( w^n - b \right)^\frac{1}{\theta} \equiv \left( w^n - b \right)^\frac{1}{\theta}.
\]

It is straightforward to show that individual effort is increasing in the net-of-tax wage rate, and decreasing in the outside option. This implies that we have $e_t < 0$, because this lowers the net-of-tax wage and thus reduces the penalty when caught shirking. Accordingly we observe $e_w > 0$ and $e_a > 0$. In fact, we have $e_t = -\frac{(w-a)}{t}e_a$, a property we will employ later on. The wage elasticity of effort is

\[
\varepsilon = \frac{e_w}{e} = \frac{w(1-t)}{\theta(w^n - b)} > 0.
\]

The respective partial derivatives with respect to the outside option $b$, the tax exemption $a$ and the tax rate $t$ are

\[
\varepsilon_a = -\frac{w(1-t)t}{\theta(w^n - b)^2} < 0,
\]

\[
\varepsilon_t = \frac{w(b-a)}{\theta(w^n - b)^2}.
\]

The partial derivatives (4) and (5) depend on effects the respective parameter have on the net-of-tax wage relative to the income surplus of working. With respect to an increase in the tax rate, this effect is ambiguous since a rise in the wage tax lowers $w(1-t)$ but at the same time raises the effective tax credit $ta$. A higher tax rate always increases the difference between the net-of-tax rate in absolute terms, but it may lower the relative difference, which is decisive for the elasticity, if the tax exemption $a$ is very generous. If $b = a$, the wage elasticity of effort is unaffected by $t$ since in this case we have $(w^n - b) = (1-t)(w - b)$. A higher tax exemption $a$ implies that a wage rate increase has a lower relative impact on the net-of-tax wage and thus implies a lower wage elasticity of
effort. Only if $b > a$, a tax rate increase raises the impact a wage rate increase has on effort: the higher is $t$, the stronger is the relative increase of $w^a - b$ due to a wage increase and thus the relative effect on individual effort! 

The direct effect of a change in the tax exemption is unambiguous. An increase in the tax exemption implies that a marginal wage increase has now a lower relative impact.

3.2. Labor demand

In the 3rd stage of the game, each firm takes the tax parameters and the negotiated wage as given and decides about the labor demand $L$ by taking into account how the representative worker will adjust effort. To derive an explicit solution, we postulate a decreasing returns-to-scale Cobb-Douglas production function in terms of labor and effort:

$$f(eL) = \frac{\delta}{\delta-1} (eL)^{\delta-\frac{\delta}{\delta-1}}, \quad \delta > 1.$$ 

Profit is given by $\pi = f(eL) - wL$. Since firms anticipate the effort level, workers will provide $(V_e = 0)$, the first order profit maximization condition is $\pi_L = 0 = f'(eL)e - w$. Using the specification (7) gives the following labor demand function

$$L = w^{-\delta} e^{\delta-1}.$$ 

The partial derivative of labor demand with respect to the tax parameters and the negotiated wage rate and are

$$L_t = L \frac{(\delta-1)}{e} e^{-\delta} > 0, \quad L_w = L \frac{(\delta-1)}{e} w^{-\delta} > 0,$$

$$L_w = -\delta w^{(\delta-1)} e^{\delta-1} + w^{-\delta} e^{\delta-2} (\delta-1)e_w = -\frac{L}{w} (\delta(1-\delta) + \delta) < 0.$$ 

Since the wage tax and the tax exemption are levied on workers, they only affect labor demand via the workers’ individual effort, which depends on the net-of-tax wage rate.
The wage rate $w$ affects labor demand in two different ways. Note that the standard assumption that profit decreases in the wage rate implies that the wage elasticity of effort is smaller than one, i.e. $\varepsilon < 1$. For the concave production function (6) the wage elasticity of labor demand depends on both the technological parameter $\delta$ and the wage elasticity of individual effort $\varepsilon$ as defined in (3):

(8) \[ \frac{-L_w w}{L} \equiv \delta^* = \varepsilon(1-\delta) + \delta. \]

The wage elasticity of labor demand is lower compared to the case where wages do not affect effort. It now depends negatively on the wage elasticity of effort. For $0 \leq \varepsilon < 1$ we have $1 < \delta^* \leq \delta$. Hence, in the presence of unobservable individual effort determination the wage elasticity of labor demand depends on the tax structure and thus tax policy. If, for instance, a tax reform increases the wage elasticity of effort, labor demand would become less elastic. A wage rise would then be less costly for a trade union since the firm will then lay off less workers.

The firm’s indirect profit function, which we will use in the next section, can be obtained by substituting labor demand (7) into the profit function:

(9) \[ \pi^*(w,e) = f(w^{-\delta} e^\delta) - w^{1-\delta} e^{\delta-1} = \frac{w^{1-\delta} e^{\delta-1}}{(\delta - 1)}. \]

Having analyzed workers’ and firms’ behavior with respect to effort and labor demand we can now turn to the collective wage bargaining of stage 2.

4. Collective wage bargaining

To derive the negotiated wage we apply the Nash bargaining solution within a ‘right-to-manage’ model according to which employment is unilaterally determined by the firms.
The wage bargaining takes place in anticipation of the optimal employment decision by the firms (8) and the optimal individual effort decision by workers (2).

The trade union maximizes the sum of the workers utility $V^w$, and the utility of the unemployed. Since those being caught shirking and fired are replaced by unemployed workers, the expected utility of an unemployed is

$$V^u = \left(1 - (1 - e^d) \frac{L}{N - L}\right) b + (1 - e^d) \frac{L}{N - L} (w^u - g(e^*)) .$$

While we assume that an single worker who is caught shirking will become and remain unemployed and receive $b$, from the viewpoint of the trade union, an unemployed member will replace a laid off worker with the lay-off probability, which is $1 - e^d$ times the employment share. We can rewrite the linear utilitarian objective function of the trade union as

$$\hat{U} = V^w L + V^u (N - L) = s(e^*) L^* + b N ,$$

where the first term captures the workers surplus from employment and the second term captures the exogenously given minimum income for all $N$ members. $L^*$ denotes optimal employment and $e^*$ optimal effort in the $s$ term. We denote the relative bargaining power of the union by $\beta$, and that of the firm by $(1 - \beta)$, and assume that the threat points of the trade union and the firm are described by $U^0 = Nb$ and $\pi^0 = 0$, respectively. Applying the Nash bargaining solution the negotiating parties decide on the wage $w$ in order to solve

$$\max_{(w)} \Omega(w) = U^{\beta} \pi_+^{1 - \beta} , \text{ s.t. } V_c = \pi_L = 0 ,$$

where $U = \hat{U} - U^0 = s(e^*) L^*$ is the bargaining surplus to the trade union by including the disutility of effort and $\pi_+^*$ is the indirect profit, presented in equation (9). The Nash bargaining solution satisfies the following first-order condition.
As shown in appendix A, we can solve the first-order condition (13) to find the following implicit Nash bargaining solution for the wage rate in the presence of individual effort determination

\[ w = \left( \frac{\beta + (\delta - 1)(1 - \varepsilon)}{\delta - 1(1 - \varepsilon) + \beta} \frac{d}{(d + \theta)} \right) \left[ \frac{g(e^*) + b - ta}{1 - t} \right] = M \left[ \frac{g(e^*) + b - ta}{1 - t} \right], \]

where \( d/(d + \theta) < 1 \) and thus \( M > 1 \) for \( \varepsilon \leq 1 \). The negotiated gross wage rate depends on the exogenous income \( b \) when unemployed, the wage tax \( t \) and the tax exemption \( a \). Furthermore it also depends on the disutility from providing effort \( g(e^*) \) and the term \( M \), which we can interpret as the mark-up. Apart from exogenous parameters this mark-up also depends on the wage elasticity of effort.

Before we discuss the general case, we will first briefly discuss several special cases, which can be analyzed within the framework developed here.

**A. Observable effort**

When effort is observable and verifiable, it can become part of the wage contract. If the contract specifies some fixed effort level \( \bar{e} \), we obtain the standard right-to-manage model of union bargaining where the wage depends on the bargaining power of the trade union and the (constant) wage elasticity of labor demand in the case of a Cobb-Douglas production function. Since a constant individual effort \( \bar{e} \) implies \( \varepsilon = 0 \) and a zero probability of being caught shirking, \( d = 0 \), we have

\[ w = \left( \frac{\beta}{\delta - 1} \right) \left[ \frac{g(\bar{e}) + b - ta}{1 - t} \right] = M \left[ \frac{g(\bar{e}) + b - ta}{1 - t} \right], \]

which implies a surplus of \( s = \left( \frac{\beta}{\delta - 1} \right) (g(\bar{e}) + b - ta) \). From (16) we can easily derive the special cases of a monopoly union.
and the competitive labor market outcome where unions have no bargaining power and
the gross wage only compensates for the disutility of working

\[
\left| w_{|c=d=0, \beta=1} = \left( \frac{\delta}{\delta - 1} \right) \left[ \frac{g(\bar{v}) + b - ta}{1 - t} \right] \right|
\]
in which case the firm exploit the complete workers’ surplus, i.e. \( s = 0 \).

**B. Unobservable effort without bargaining**

When \( \beta = 0 \), the firm unilaterally sets the wage. From the first-order condition \( \pi_w^* = 0 \) it
follows immediately that the firm act according to the well-known Solow-condition
(Solow 1979), i.e. we have \( \varepsilon = 1 \) and thus

\[
(16) \quad w = \left( \frac{\theta}{\theta - 1} \right) \left[ \frac{b - ta}{1 - t} \right] \equiv M|_{\varepsilon=0} \left[ \frac{b - ta}{1 - t} \right].
\]

The model therefore also captures the essence of the efficiency models with an mark-up
over the total outside option.

**C. Unobservable effort with bargaining: comparative statics**

For the general case we have \( (1-\varepsilon) > 0 \) and the mark-up is larger than one when the
trade union has some bargaining power, \( \beta > 0 \). It increases with the relative bargaining
power of the trade union \( \beta \), and depends negatively on the direct wage elasticity of labor
demand \( \delta \). The wage rate now depends on several new terms that in addition to the
relative bargaining power, the wage elasticity of labor demand, the exogenous income
and the tax parameters enter the formula: (i) the exogenously given probability of
monitoring workers \( d \), (ii) the indirect effect \( g(e^*) \) via effort provision and (iii) the
elasticity of effort determination \( \varepsilon \). Furthermore, unlike in the case of observable effort,
the exogenous income \( b \) when unemployed, the wage tax rate \( t \), and the tax exemption
\( a \) will also affect the wage rate via the mark-up \( M \).
The impact of a better monitoring of workers on the negotiated wage is zero as the wage elasticity of effort is not affected by monitoring. We can thus focus in what follows on the comparative statics of the tax parameters and the outside option, respectively. Thereby we will call the term \((g(e^*) + b - ta)/(1-t)\) as the total outside option, which affects the negotiated wage rate.

The tax exemption affects the negotiated wage positively both via the mark-up and the total outside option as follows (see appendix B)

\[
(17) \quad w_a = \frac{1}{\Delta} \left( \frac{M}{1^4 2^4 4^3} \right) \left( \frac{g(e^*) + b - ta}{1-t} \right) + \frac{M}{\Delta} \left( \frac{g'(e^*)e_w - t}{1^4 2^4 4^3} \right) < 0.
\]

with \(\Delta = 1 - M_t e_w M^{-1} w - Mg'(e^*)e_w (1-t)^{-1} > 0\). In the Nash bargaining with observable effort (15), the mark-up is independent of \(a\). With unobservable effort, however, workers will increase effort when the tax exemption rises. This, cet. par. lowers the mark-up because a lower wage elasticity of effort implies a higher wage elasticity of labor demand (see equation (8)). A higher wage then induces less effort, which makes the worker less productive. As a consequence more layoffs result from a wage increase.

The effect of the wage tax rate can be expressed as follows

\[
(18) \quad w_t = \frac{1}{\Delta} \left( \frac{M}{1^4 2^4 4^3} \right) \left( \frac{g(e^*) + b - ta}{1-t} \right) + \frac{M}{\Delta} \left( \frac{g(e^*) + b - a + (1-t)g'(e^*)e_w}{1^4 2^4 4^3} \right) > 0,
\]

(see Appendix B). The total effect of a higher wage tax rate on the negotiated wage is a priori ambiguous. When we assume \(b \geq a\) both the effect on the mark-up is unambiguously positive and the effect on the total outside option with the given mark-up.

Hence, tax parameters in our model both with Nash wage bargaining and individual effort determination affect both via a change of the difference between net-of-tax wage income and outside option and via a change in the mark up.
We summarize our new characterization of the negotiated wage under individual effort determination in

**Proposition 1:** Unobservable individual effort determination strengthens the effects tax policy measures have on the negotiated wage, compared to the case where effort is observable. Decreasing the tax exemption lowers the negotiated wage. An increase in the wage tax rate increases the negotiated wage when \( b \geq a \).

We can easily verify that the effects indeed reinforce each other. If we take the partial derivative of (15), we obtain the comparative statics effect for the standard bargaining model with

\[
\begin{align*}
w_a &= M_{\varepsilon_a=0} \left( \frac{-t}{1-t} \right) < 0, \quad w_i = M_{\varepsilon_a=0} \left( \frac{g(\bar{r}) + b - a}{(1-t)^2} \right).
\end{align*}
\]

For \( b \geq a \) the effects tax parameter changes have on the negotiated wage when effort is observable are always reinforced when effort is not observable. The partial derivative of equation (16) with respect to \( a \) shows the same result for the efficiency wage model: the different wage setting motives thus reinforce the partial tax policy effects on gross wages. We shall note however that in the case where \( b \leq a \) and \( g(\bar{r}) + b - a > 0 \) we would obtain opposite partial effects for changes in the wage tax rate. An increase in the wage tax will then increase the gross wage when effort is observable but will lower the gross wage when effort is unobservable.

5. **Tax revenue-neutral change in tax progression in terms of wage formation, employment and individual effort**

We are now ready to analyze the impact a revenue-neutral restructuring of the labor tax structure, i.e. the degree of wage tax progression, has on wage formation, individual
effort determination and employment. The effect of wage tax progression, which keeps the tax revenue \( G = \left[t(w-a)L\right] \) constant, can be written in the following way:

\[
dG = 0 = (w-a)Ldt - tLda + \left[tL + t(w-a)L_a\right]dw.
\]

Recalling the definition of the average tax as \( t^a \equiv t(1-a/w) \), this can be expressed as

\[
(19) \quad \frac{da}{dg=0} = \frac{(w-a)}{t} dt + \frac{(t-t^a\delta^*)}{t} dw.
\]

An appropriate and intuitive way to define tax progression is to look at the average tax rate progression (ARP), which is given by the difference between the marginal tax rate \( t \) and the average tax rate \( t^a \), \( ARP = t - t^a \). The tax system is progressive if \( ARP \) is positive, and tax progression is increased if the difference increases (at a given income level, see Lambert 2001, chapters 7 and 8). The term \( t-t^a\delta^* \) indicates the marginal tax revenue per worker when the gross wage increases. It can be decomposed such that we have a tax progression effect and a tax level effect: \( ARP + t^a(1-\delta^*) \). The total effect is non-positive for a linear tax system with \( ARP = 0 \) since \((1-\delta^*)\leq 0\) but may eventually become positive if the tax system is sufficiently progressive since the employment effect is weighted by the average tax rate only. As we will see later on, the degree of tax progression is decisive for how a revenue-neutral change in tax progression affects both employment and individual effort.

5.1 Revenue neutral tax progression on the negotiated wage

The total effect of changes in the tax parameters \( t \) and \( a \) on the negotiated wage is

\[
(20) \quad dw = w_t dt + w_a da,
\]

with the partial derivatives derived in section 4. Substituting (19) into the RHS of (20) for \( da \) gives

\[
(21) \quad dw = w_t dt + \left(\frac{w-a}{t}\right) w_a dt + w_a \left[\frac{t-t^a\delta^*}{t}\right] dw.
\]
and thus, the total effect of a revenue-neutral increase in the wage tax rate is

\[
\frac{dw}{dt} \bigg|_{\delta G=0} = \frac{w_i}{1} + \frac{(w-a)}{t} \cdot w_a.
\]

In what follows, we assume Laffer-efficiency in the sense that a higher wage tax increases tax revenues while a higher tax exemption leads to lower tax revenues even when we take account of the indirect effects via changes in \(w\). With respect to the tax exemption we then have

\[
\hat{G}_u = -tL + G_w w_u = -tL \left(1 - w_a \frac{t - t^* \delta^*}{t}\right) < 0.
\]

Substituting the partial derivatives \(w_u\) from (17) and \(w_i\) from (18) into the numerator of (22) shows that the numerator is unambiguously positive (see appendix C). Hence, we have the following

**Proposition 2 (wage moderation):** A revenue-neutral increase in wage tax progression will moderate the negotiated wage in the presence of individual effort determination.

The interpretation is straightforward as it turns out that the numerator in equation (24) denotes the compensated effect an increase of the tax rate has on the wage keeping the Nash maximand value constant (see appendix D). The revenue-neutral increase in the tax exemption fully offsets the income effect of the higher wage tax so that only the substitution effect of this progression-enhancing tax reform remains. This finding shows that the result from conventional ‘right-to-manage’ models in the absence of effort considerations (see e.g. Koskela and Vilmunen 1996) also applies when we allow for unobservable individual effort determination.
5.2 Revenue neutral tax progression on individual effort determination

The total effect of changes in the tax parameters \( t \) and \( a \) and the negotiated wage on effort determination is \( de = e_t dt + e_a da + e_w dw \). Substituting the RHS of the tax-revenue neutrality (19) for \( da \) gives

\[
\begin{align*}
\frac{de}{dt}_{da=0} &= e_t + \left( \frac{w-a}{t} - s \right) e_a + \left( t - t^* \delta^* \right) e_a \bigg|_{da=0} \\
&= e_t - s e_a + \left( t - t^* \delta^* \right) e_a.
\end{align*}
\]

(23)

It is only the induced wage-moderation effect that affects individual effort decisions. The term \( e_a/t \) measures the impact one additional Euro has on individual effort. A wage reduction of one Euro reduces the net-of-tax wage by \( (1-t) \) so that effort falls by \( e_a(1-t)/t \). The wage-moderation effect also affects the amount by which the tax exemption can be raised. It will be lower than the neutral effect of raising \( a \) by \( (w-a)/t \) if \( t - t^* \delta^* < 0 \). This always holds in a linear tax system but if the tax system becomes very progressive, i.e. \( 1 - t^* \delta^* > 0 \) individual effort eventually will fall. The case is the more likely, the smaller the wage elasticity of labor demand and the average tax burden are. If we assume a labor share of 2/3, we have \( \delta = 3 \), an average tax below 1/3 would suffice to let effort fall when progression rises. Formally, we have

\[
\left. \frac{de}{dt} \right|_{da=0} \begin{cases} > 0 \quad \iff \quad 1 - < t^* \delta^* \end{cases}
\]

(24)

A sufficient but not a necessary condition for individual effort to fall is \( t \delta < 1 \) since we have \( \delta^* < \delta \) and \( t_a < t \). These findings can be summarized in

Proposition 3 (individual effort determination): A revenue-neutral increase in wage tax progression will lower individual effort if (i) the wage elasticity

---

17
of labor demand and/or (ii) the marginal tax rate are sufficiently low. A sufficient condition is \( \delta < 1 \).

5.3 Revenue neutral tax progression and employment

Finally we consider the employment effect. The total effect of changes in the tax parameters \( t \) and \( a \) and the negotiated wage on employment is 
\[ dL = L_t dt + L_a da + L_x dw. \]
Substituting the RHS of (19) for \( da \) gives

\[
\left. \frac{dL}{dt} \right|_{dG=0} = L^* \left( \frac{\delta-1}{e^*} \left( e^*_t + \frac{(w-a)}{1} e^*_a \right) + \left( L^*_w + \frac{1}{t} (t^{-\delta^*}) L^*_a \right) \right) dw \bigg|_{dG=0}
\]

(25)

\[
= \left[ L^* \frac{(\delta-1)}{e^*} \left( \frac{1}{t} (t^{-\delta^*}) \right) e^*_a \right] - L^* \frac{\delta^*}{w} dw \bigg|_{dG=0}
\]

\[
= L^* \frac{(\delta-1)}{e^*} \left. \frac{de}{dt} \right|_{dG=0} - L^* \frac{\delta^*}{w} \left. \frac{dw}{dt} \right|_{dG=0}.
\]

The first two terms cancel out since they cover the change in \( t \) and \( a \) that cet. par. would leave the average tax burden and thus the net-of-tax wage constant. Hence, we are left with two effects. As we have seen in section 5.2, the tax reform affects individual effort. If – as is likely – effort decreases, labor productivity falls and cet. par. employment. On the other side, the wage-moderating effect increases labor demand for any given effort level. The total effect thus becomes ambiguous. From proposition 3 we can immediately infer

**Proposition 4 (rising employment):** A sufficient but not necessary condition that a revenue-neutral increase in wage tax progression will increase employment is \( t^{-\delta^*} \geq 1 \).

Substituting the RHS of (23) for \( \frac{de}{dt} \bigg|_{dG=0} \) in (25) we obtain
From equation (25) it follows immediately that starting from a linear tax system, employment will definitely rise. This leads to

**Proposition 5 (rising employment):** Introducing tax progression is good for employment when wages are negotiated and effort is determined individually.

Although we have seen that different wage setting motives reinforces tax policy effects on gross wages, this is not true anymore with respect to employment. With observable and verifiable effort, employment is always decreasing when tax progression rises. When effort is unobservable and not verifiable we find a countervailing effect via the adverse effect a rise in tax progression has on individual effort.

6. Conclusions

We provide an extended framework to study the implications of imperfectly observable individual effort of workers on the negotiated wage and the impact of a revenue-neutral change in the wage tax progression on wage negotiations, effort and employment. The first and most important result is that a higher degree of tax progression always leads to wage moderation. Our model confirms this result for the case of observable effort and wage bargaining as well as for the case where firms set efficiency wages unilaterally: the different wage setting motives reinforce partial tax policy effects present in either model. However, when effort is not observable and verifiable, the clear-cut effect well-known from the wage bargaining literature that tax progression is good for employment does not carry over to the case of imperfectly observable effort. In the general case, it remains true that *introducing* tax progression is good for employment, but if the adverse effect on effort becomes sufficiently large due to a too high degree of tax progression we cannot
rule out the case where employment falls as a consequence of a progressivity-enhancing tax reform.

References

Appendix A: the negotiated wage

This appendix develops the expressions for the terms $\pi^*_w/\pi^*$ and $U_w/U$ in the first-order condition (13) determining the Nash bargaining solution. We start by looking at the profit response by the firm to a change in the wage rate. The indirect profit function was presented in equation (9). By applying the envelope theorem, according to which the effect which take place through the labor demand vanish at the optimum, we find that

$$\pi^*_w = \frac{1}{(\delta-1)} \left[ (\tilde{\delta}-1)(w)^{1-\delta} e^{\delta-1} e_w - (\delta-1)(w)^{2-\delta} e^{\delta-1} \right]$$

(A1)

$$= (w)^{1-\delta} e^{\delta-1} w^{-1} \left[ e_w \frac{w}{e} - 1 \right] = -\frac{(w)^{1-\delta} e^{\delta-1}}{w} [1 - \varepsilon] < 0,$$

from which follows that

$$\frac{\pi^*_w}{\pi^*} = -\frac{(\delta-1)(1-\varepsilon)}{w} < 0.$$  

(A2)

as $\varepsilon < 1$. With respect to the trade union’s utility we find that

$$U_w = \frac{L}{w} \left[ w(1-t) - g' e_w w - \delta^* (w^o - g(e^*) - b) \right].$$

(A3)

where $\delta^* = \delta(1-\varepsilon) + \varepsilon = -\frac{L^* w}{L}$. Thus, it follows that

$$\frac{U_w}{U} = \frac{1}{w} \left[ \frac{w(1-t) - g' e_w w - \delta^* (w^o - g(e^*) - b)}{w^o - g(e^*) - b} \right].$$

(A4)

Substituting (A4) and (A2) into (14) yields

$$\beta \left[ \frac{w(1-t) - g' e_w w - \delta^* (w^o - g(e^*) - b)}{w^o - g(e^*) - b} \right] = (1-\beta)(\delta-1)(1-\varepsilon).$$

(A5)

This can be rewritten as:
\[ \beta \left[ w(1-t) - g' \epsilon\epsilon w - \delta^* (w^\rho - g(\epsilon^*) - b) \right] - w^\rho (1-\beta)(\delta-1)(1-\epsilon) \]

\[ = -(g(\epsilon^*) + b)(1-\beta)(\delta-1)(1-\epsilon) \iff \beta \left[ w(1-t) - g' \epsilon\epsilon w - \delta^* w^\rho \right] - w^\rho (1-\beta)(\delta-1)(1-\epsilon) \]

\[ = -(g(\epsilon^*) + b) \left[ (1-\beta)(\delta-1)(1-\epsilon) + \beta \delta^* \right] \iff \beta \left[ w(1-t) - g' \epsilon\epsilon w - \delta^* w(1-t) \right] - w(1-t)(1-\beta)(\delta-1)(1-\epsilon) \]

\[ = -g(\epsilon^*) + b - ta \left[ (1-\beta)(\delta-1)(1-\epsilon) + \beta \delta^* \right] \]

Using the definition of the total wage elasticity of labor demand \( \delta^* \) we obtain

\[ w(1-t) \left( (\delta-1)(1-\epsilon) + \beta \frac{g' \epsilon\epsilon w}{w(1-t)} \right) = \left[ g(\epsilon^*) + b - ta \right] \left[ \beta + (\delta-1)(1-\epsilon) \right]. \]

\[ w(1-t) \left( (\delta-1)(1-\epsilon) + \beta \frac{d}{(\theta + d)} \right) = \left[ g(\epsilon^*) + b - ta \right] \left[ \beta + (\delta-1)(1-\epsilon) \right]. \]

**Appendix B: comparative statics of the negotiated wage in terms of outside option, wage tax and tax exemption**

To see the effect, the parameters have on the mark-up, it is convenient to slightly change notation:

\[ M = \frac{\beta + (\delta-1)(1-\epsilon)}{(\delta-1)(1-\epsilon) + \beta \frac{d}{(\theta + d)}} = \frac{\beta + (\delta-1)(1-\epsilon)}{N}. \]

The mark-up with respect to \( \epsilon \) is:

\[ M_\epsilon = \frac{\beta(\delta-1)}{N^2} \frac{\theta}{(\theta + d)} > 0. \]

The mark-up with respect to effort \( e \) is \( M_e = 0 \). Condition (13) is an implicit function of \( w \). Thus the partial derivative with respect to e.g. \( a \) is:
\[ \begin{align*}
\frac{d\omega}{dt} & = \left[ 1 - M \varepsilon \right] \left( \frac{g(e^+ e^+) + b - ta}{1-t} \right) - M \left( \frac{g'(e^+) e^+}{1 - 1^2} \right) \\
& = \left[ M \varepsilon \right] \left( \frac{g(e^+ e^+) + b - ta}{1-t} \right) + M \left( \frac{1 + g'(e^+) e^+}{1 - 1^2} \right)
\end{align*} \]

First we have to sign the term in square brackets:

\[(B2) \quad \Delta \equiv \left[ 1 - M \varepsilon \right] \left( \frac{g(e^+ e^+) + b - ta}{1-t} \right) - M \left( \frac{g'(e^+) e^+}{1 - 1^2} \right) \]

Adding the first and third term in the square brackets yields

\[
1 - M \left( \frac{g'(e^+) e^+}{1-t} \right) = \frac{w(1-t) - Me^8 \varepsilon}{w(1-t)} = M \frac{g(e^+ e^+) + b - ta - g(e^+) \theta e}{w(1-t)}
\]

\[
= M \left( \frac{1 - \frac{w(1-t)}{w^n - b}}{w(1-t)} \right) g(e^+) + b - ta = M \frac{(b - ta)(w^n - b - g(e^+))}{w(1-t)(w^n - b)} > 0.
\]

Thus we can sign \( \Delta \):

\[(B3) \quad \Delta = \left[ M \frac{(b - ta)(w^n - b - g(e^+))}{w^n - b - g(e^+)} \right] > 0 \]

With \( \Delta > 0 \) it is straightforward to sign the first term in equation (16) because \( M \varepsilon \theta c < 0 \).

The second term in equation (16) is also positive since

\[(B4) \quad g'(e^+) - t = -t \frac{w^n - b - g(e)}{w^n - b} < 0. \]

In equation (19) we have

\[(B5) \quad g(e^+) + b - a + (1-t)g'(e) = (b - a) \frac{w^n - b - g(e)}{w^n - b} , \]

which is positive if \( b - a > 0 \). QED.
Appendix C: the sign of the numerator of (22)

Substituting the partial derivatives (17) and (18) into the numerator of (22) yields (using (B4), (B5) and (4), (5), (13):

\[ \frac{w_t + \frac{(w-a)}{t}}{w_a} = M_{\epsilon} \left( \frac{\epsilon_t + \frac{(w-a)}{t} \epsilon_a}{1-t} \right) \left( g(e^+) + b - ta \right) + \frac{M}{\Delta} \left( \frac{g(e) + b - a + (1-t)g'(e)\epsilon_t}{(1-t)^2} + \left( \frac{w-a}{t} \right) \left( \frac{g'(e)\epsilon_a}{1-t} \right) \right) \]

\[ = -M_{\epsilon} \frac{w}{\Delta} \frac{w}{M} \frac{M}{\Delta} \frac{w^n - b - g(e)}{(w^n - b)} \left[ \frac{b-a}{1-t} - \frac{(w-a)}{1-t} \right] \]

\[ = -M_{\epsilon} \frac{w^2}{\Delta} \frac{w^n - b - g(e)}{\Delta} \frac{1-t}{(1-t)^2} < 0. \]

Appendix D: the Slutzky-decomposition for the total effect of the wage tax on the negotiated wage

Differentiating the indirect Nash maximand \( \Omega^* = U^0 \pi^{s+1} = \Omega^0 \), where \( U = sL' \) and \( \pi^* = f(e^+L') - w(1+s)L' \), with respect to \( t \) and \( a \) gives

\[(D1) \]

(i) \( \Omega^*_t = \beta U^{\beta-1} \pi^{s+1} U_t = -\beta U^{\beta-1} \pi^{s+1} L'(w-a) < 0, \]

(ii) \( \Omega^*_a = \beta U^{\beta-1} \pi^{s+1} U_a = \beta U^{\beta-1} \pi^{s+1} L't > 0. \)

The wage tax has a negative effect and tax exemption has a positive effect on the Nash maximand. Using the comparative statics the indirect Nash maximand can be inverted in terms of \( a \) for the following function \( a = h(t, \Omega^0) \). Substituting this for \( a \) in \( \Omega^* = U^0 \pi^{s+1} = V^0 \) gives the compensated indirect Nash maximand \( \Omega^* (t, h(t, \Omega^0)) = \Omega^0. \)

Differentiating this compensated indirect Nash maximand with respect to \( t \) gives \( \Omega^*_t + h_t \Omega^*_a = 0 \) so that \( h_t = -\Omega^*_a / \Omega^*_a = (w-a)/t. \) This describes the relationship of tax parameters to keep the Nash maximand constant.

---

3 See e.g. Diamond and Yaari (1972).
According to the duality theorem the Nash maximand wage function $w_t$, and the compensated wage function, $w^c_t$, at the same Nash maximand level are equal, so that we have $w(t, h(t, \Omega^0)) = w^c(t, \Omega^0)$. Differentiating this with respect to the wage tax gives $w_t + h_tw_u = w^c_t$ so that we obtain the Slutsky equation

$$w^c_t = w^c_t - \left(\frac{w-a}{t}\right) w_u,$$

where the total effect of the wage tax rate has been decomposed into the negative substitution effect ($w^c_t < 0$, see Appendix C) and the positive income effect $\left(\frac{w-a}{t}\right) w_u$. QED.