

Overinvestment vs. Underinvestment in Educational Signals: A Search-Theoretic Approach

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Abstract

We consider Spence's (1973) idea of job market signalling in a matching model, where workers invest in education to accumulate human capital and to signal their privately known 'types'. As search frictions become infinitesimal, our model reproduces the 'Riley outcome'; i.e. each type, except the least able worker, overinvests in education. Otherwise, the bargaining wage induces underinvestment in education at least within a subset of types. Most notably, the 'minimum signalling profile' is not necessarily Pareto-dominant. If workers' innate types and education are sufficiently strong complements in terms of productivity, the model may exhibit multiple steady state equilibria.

JEL Classification: J24, J31, J41

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1 Introduction

In his path-breaking work, Spence (1973, 1974) suggested that difficulties in observing those attributes of jobseekers which correlated with labor productivity would result in an equilibrium where the wage offers were based on the observable educational levels; i.e. employers would use educational signals as a sorting device. However, the viability of such a 'signalling' (=separating) equilibrium remained unclear in Spence's analysis. The necessary preconditions for the existence of a separating regime would be that workers with greater productivity also face smaller marginal cost in acquiring education and that the equilibrium wage function (contingent upon the signals) would induce the workers of each 'type' to choose different educational levels.

As a natural starting point, it was first examined whether such a wage function could arise as an extension of the Walrasian price vector. It was shown by Rothschild and Stiglitz (1976) (in the context of a two class model) and Riley (1975) (with continuum of types) that separating regime¹ generally fails to reach stability in Walrasian competitive equilibrium context². In the current model, the problems related to Walrasian equilibrium are avoided by utilizing a search-theoretic approach with decentralized trading. Assuming pairwise matching, each market participant have access to at most one trading opportunity at a time and wages are determined under a strategic bargaining process between an unemployed worker and a firm. Our focus is on wage functions that support separating equilibrium so that the educational signals fully reveals workers' types, and there is no need for additional 'signalling' for example through unexpected wage demands³.

Our analysis is not explicitly game-theoretic⁴ but rather close to Riley's (1979) competitive equilibrium model - the key difference being decentralized trading process postulated in the current setting. Indeed, our model reproduces the so called 'Riley outcome' as a limiting case. According to 'Riley outcome', the Pareto-dominant signalling profile is such that the least able type chooses his first-best educational level while the more able workers overinvest in schooling in order to separate themselves from the less able colleagues. However,

¹Or *informational consistency* in Riley's (1979) terminology.

²As a refinement of the conventional Walrasian equilibrium, Riley (1979) introduced restrictions to the competitive behavior of agents. Riley's 'reactive equilibrium' obtained stability.

³Generally, strategic bargaining models with asymmetric information can easily lead to a great multiplicity of perfect Bayesian equilibria. This is because, the less informed party may have reasons to *believe* that informed agents try to transmit information about their types, not only with the observable signal, but also by making unexpected offers. See for example Muthoo (1999, ch. 9.8) and Fudenberg and Tirole (1991, ch. 10.4).

⁴Perfect Bayesian equilibrium and its refinements - 'perfect equilibrium' á la Selten (1975), 'sequential equilibrium' by Kreps and Wilson (1982) as well as further refinements of sequential equilibrium introduced by Cho and Kreps (1987) and by Banks and Sobel (1987) - offer powerful tools for analyzing Spence's idea in game theoretic settings.

given that education enhances productivity *per se*⁵, we find that the overinvestment result ceases to hold generally if both parties gain a positive fraction of the surplus generated by the match. We show that the bargaining wage induces underinvestment in education at least within a subset of types. Search frictions on workers' side of the market tend to amplify underinvestment. Moreover, the 'minimum signalling profile' is not necessarily Pareto-dominant. In our case, Pareto-dominant signalling profile entails underinvestment in education among the most (least) able types if workers' bargaining power is relatively low (high).

The possibility of underinvestment in education under asymmetric information has been previously studied at least by Stiglitz (1975), who pointed out that 'social return' on schooling may in some cases exceed 'private return', if the signalling somehow improves the 'match' between workers and jobs⁶. Stiglitz' argument implicitly assumes heterogeneity in both sides of the labor market - a feature that is not incorporated in the current model, where the bilateral trading and search frictions suffice to give rise to inefficiently low levels of schooling.

A steady state general equilibrium will be derived according to the matching framework postulated by Laing, Palivos and Wang (1995), whose model, in turn, rests on the pioneering works of Diamond (1982), Mortensen (1982) and Pissarides (1984, 2000). Our key finding is that, if worker's innate 'type' and education are sufficiently strong complements in terms of productivity, and/or if workers' bargaining power is sufficiently weak, the underinvestment problem may hinder job creation when the labor market is 'slack'. Therefore, improvements in workers' position - either through less severe search frictions or increased bargaining power - reduce underinvestment in schooling and may actually stimulate market entry by firms under certain parameter values. Due to this feature, multiple steady state equilibria may arise. Reminiscent of the result derived by Laing et al. (1995) in a pure 'human capital' context, our model suggests that small improvements in either market infrastructures or common technological knowledge may lead to a considerable leap from a sub-optimal equilibrium among multiple steady states to a unique and more efficient steady state.

The paper is organized as follows: In Section 2, we first characterize the pairwise

⁵Naturally, 'underinvestment' in education may occur only if one assumes that education *per se* has a positive effect on labor productivity. A considerable empirical literature provides rather conflicting views on this issue (traditional human capital theory initiated by Becker, 1964 in one hand and evidence from so called 'sorting models' (e.g., Kang and Bishop, 1986 and Altonji, 1995) on the other). However, Chatterji, Seaman and Singell (2003) find quite strong evidence that workers invest in education *both* because it improves their productivity *and* because it distinguishes them from workers with less schooling.

⁶Stiglitz's argument rests on the assumption that the abilities that correlate with schooling improve productivity on some 'specialized' vacancies but not on all jobs.

trading process and derive the possible signalling profiles. After that, the results will be compared with the competitive equilibrium benchmark, the 'Riley outcome'. Section 3 provides the steady state analysis and Section 4 concludes the paper.

2 The Model

The economy is populated by a continuum of infinitely lived and risk neutral workers and by a continuum of impersonal firms. Firms open vacancies that require a labor input of exactly one worker.

2.1 Heterogeneous labor

Workers are heterogeneous and the 'type' of a worker is denoted by $\theta \in \Theta$. This parameter serves as a measure for labor productivity so that a worker with higher θ can produce more output than a colleague with lower θ .

* *Assumption 1:* The type of a worker θ is distributed on $\Theta = [\theta_l, \theta_h]$ according to a strictly increasing function $F(\theta)$. $F(\theta)$ is common knowledge, while the actual realization of θ is each worker's private information.

2.2 Schooling

Before entering labor market, workers may devote effort in acquiring some level of schooling, $s \in S = [0, \infty)^7$, in educational sector. For simplicity, it is assumed that schooling is a one shot investment, the cost of which, $C(\theta; s)$ depends on the chosen educational level, s , and on the type of the agent, θ .

* *Assumption 2:*

$$C_s > 0, C_\theta < 0, \text{ and } C_{s\theta} < 0, C_{ss} > 0.$$

where the lower indices denote partial derivatives. $C(\theta; s)$ is common knowledge.

Assumption 2 is the first necessary precondition for the existence of a separating equilibrium, where educational level fully reveals the innate productivity of each worker.

After obtaining diploma from the educational sector, workers are ready to enter the labor market and start searching for a job as an unemployed worker. Each worker type

⁷Hence, there is no upper bound for how much a single worker can choose to take education. Having an upper bound for schooling would only limit the set of feasible separating equilibria; for a detailed discussion, see Riley (1979).

decides upon educational level by maximizing the net of the expected return from education and the schooling cost:

$$s \in \arg \max_{s \in \mathcal{S}} U_i(\theta_i; s, w(s)) = J_i^U(\theta_i; w(s)) - C_i(\theta_i; s),$$

where J_i^U represents the family of utility functions describing the discounted value of being unemployed with innate characteristics θ_i and with educational level s . Note that separating equilibrium requires that the available wage rate in the labor market must be conditional upon the observable level of schooling; i.e. $w = w(s)$.

2.3 Search

We utilize a simple continuous-time search model where an unemployed worker locates a firm with an open vacancy at a Poisson arrival rate α and a firm locates an unemployed worker at rate β . Search effort is assumed to be costless but time consuming. Each economic agent discounts future income with the common discount rate, $r > 0$. Individual agents treat the flow probabilities as exogenously given, even though α and β , as well as the equilibrium measures for unemployed workers and open vacancies, will be endogenously determined in a steady state general equilibrium in Section 3.

2.4 Utilities from trading

We follow here Laing, Palivos and Wang (1995) by assuming that once an unemployed worker and an unfilled job have been matched, the worker earns a perpetual⁸ flow of income, the discounted value of which reads as

$$\int_t^\infty e^{-(\tau-t)r} w(s) d\tau = W(s).$$

Henceforth, $W(s)$ is called as the wage function prevailing in the labor market, even though it literally denotes the present value of the life-time earnings received by a worker labelled with an educational level s .

Assuming that working effort does not cause any disutility, risk-neutrality implies that the discounted value of being employed is given by

$$J^E(\theta; s) = W(s). \tag{1}$$

Since an unemployed worker locates a potential employer at rate α , the value of being unemployed obtains

$$rJ^U(\theta; s) = \alpha (J^E(\theta; s) - J^U(\theta; s)), \tag{2}$$

⁸Thus, there is no exogenous 'job destruction' and thereby no risk of falling back into the pool of unemployed workers.

Hence, each successful match in the labor market is followed by a perpetual stream of output which is determined by worker's type, θ , and his educational level, s . The present value of this infinite flow of output is denoted by $V(\theta; s)$.

* *Assumption 3*:

$$V_\theta > 0, V_s > 0 \text{ and } V_{ss} < 0.$$

Assumption 3 simply states that both the worker's innate type and education contribute to the productivity of labor. However, schooling affects V at a diminishing rate.

Regarding employers' payoffs, the value available from a filled vacancy is

$$\pi^F(\theta; s) = V(\theta; s) - J^E(\theta; s). \quad (3)$$

Moreover, given that firms locate workers at rate β , the expected present value of having an unfilled vacancy is given by

$$r\mathbb{E}[\pi^U(\theta; s)] = \beta(\mathbb{E}[\pi^F(\theta; s)] - \mathbb{E}[\pi^U(\theta; s)]), \quad (4)$$

where

$$\mathbb{E}[\pi^j(\theta; s)] = \int_{\Theta} \pi^j(\theta; s) dF(\theta), \quad j = U, F.$$

2.5 Bargaining⁹

Upon meeting, worker and firm negotiate over the division of the surplus generated by the match. The bargaining process is modelled as a strategic bargaining game where the unemployed worker is allowed to propose a wage demand in a 'take-it-or-leave-it' fashion with probability γ while the employer is in a similar position with the complementary probability $1 - \gamma$.¹⁰ Generally, the dominant strategy in the bargaining game is to demand or offer a wage that makes the trading partner indifferent between accepting the offer or continuing search. Hence, the party who *receives* the wage offer is driven to his reservation utility level. When the unemployed worker is to propose the offer, the resulting wage demand captures the net of the social gain from the match, $V(\theta; s)$, and employers' expected reservation utility, $\mathbb{E}[\pi^U(\theta; s)]$. On the other hand, when the employer makes the offer, he offers a

⁹As already noted above, focusing on separating regimes guarantees that the bargaining with educational signals can be treated as if there were no asymmetric information concerning workers' innate abilities. Under separating regime, the observable educational level of a worker fully reveals the unobservable productivity of the particular worker.

¹⁰This type of strategic bargaining produces a sharing rule that coincides with the outcome from generalized Nash bargaining.

future income stream equivalent to the value of being unemployed, $J^U(\theta; s)$. Therefore, the bargaining wage is the weighted average of these two offers:

$$W(s) = \gamma (V(\theta; s) - \mathbb{E}[\pi^U(\theta; s)]) + (1 - \gamma) J^U(\theta; s). \quad (5)$$

2.6 Separating equilibrium

Definition 1 *Separating equilibrium is a wage-function $W(s)$ s.t. (i) workers of type θ choose*

$$s = g^{-1}(\theta),$$

where

$$g^{-1}(\theta) \in \arg \max_{s \in S} U(\theta, s, W(s)) = \{J^U(\theta, W(s)) - C(\theta, s)\} \quad \forall \theta,$$

and (ii) $\exists [s, \infty] \subset S$ s.t. $g : [s, \infty] \rightarrow \mathbb{R}_+$ is strictly and monotonically increasing with

$$\theta_l \geq g(s),$$

and (iii) $W(s)$ results from the sharing rule expressed in (5).

The condition (i) in Definition 1 requires that the educational signals result from workers' optimization problem at the *ex ante* stage, given the prevailing wage function in the labor market. the condition (ii), in turn, requires that for each educational level there is exactly one worker type who have chosen that particular signal. Moreover, the condition (ii) presumes that employers can infer higher schooling effort to indicate higher ability. Finally, the condition (iii) states that, given workers' optimal signalling strategies, the equilibrium wage schedule results from the sharing rule expressed in equation (5).

The equilibrium wage function supporting separating regime will be derived in the spirit of backward induction: We start with the condition (iii) and determine wages as a function of the educational signals. Then, given the wage function, we proceed with the condition (i) and derive worker's optimal schooling effort at the *ex ante* stage. As a final step, we need to make sure that the possible signalling profiles are consistent with the condition (ii).

Given that workers have chosen the educational levels according to the separating regime, plugging (2) and (4) into the sharing-rule in (5), and utilizing (1) and (3), we have

$$W(s) = \frac{\gamma(\alpha + r)}{\gamma\alpha + r} (V(h(s), s) - \mathbb{E}[\pi^U(h(s); s)]), \quad (6)$$

where $h(s) = \theta$.

Now, consider a schooling level $s + \Delta$. Then the corresponding wage satisfies¹¹

$$W(s + \Delta) = \frac{\gamma(\alpha + r)}{\gamma\alpha + r} (V(h(s + \Delta), s + \Delta) - \mathbb{E}[\pi^U(h(s); s)]). \quad (7)$$

Subtracting (6) from (7), rearranging terms and dividing both sides by Δ gives the difference quotient:

$$W(s + \Delta) - W(s) = \frac{\gamma(\alpha + r)}{\gamma\alpha + r} [V(h(s + \Delta), s + \Delta) - V(h(s), s)].$$

Now, letting $\Delta \rightarrow 0$, we have the first derivative of the wage function w.r.t. schooling:

$$W'(s) = \frac{\gamma(\alpha + r)}{\gamma\alpha + r} [V_\theta h'(s) + V_s], \quad (8)$$

where the lower indices again denote partial derivatives.

According to the condition (ii) in Definition 1, we must have $h'(s) > 0$, which means that the innate type of a worker, θ , and his educational choice, s , are positively related¹². Moreover, it should be clear by Assumption 2 that in a separating equilibrium the wages must be increasing in schooling; i.e. $W'(s) > 0$.

Workers' optimal signalling profile is determined by the following program for each type $\theta \in \Theta$:

$$\max_s J^U(\theta, W(s)) - C(\theta, s),$$

and the first-order necessary condition gives

$$\frac{\alpha}{\alpha + r} W'(s) - C_s = 0.$$

Utilizing (8), the first order condition becomes

$$\frac{\gamma\alpha}{\gamma\alpha + r} [V_\theta h'(s) + V_s] - C_s = 0, \quad (9)$$

which can be solved for $h'(s)$ to yield

$$h'(s) = \frac{\frac{\gamma\alpha + r}{\gamma\alpha} C_s - V_s}{V_\theta}, \quad (10)$$

Equation (10) is an ordinary differential equation. As noted by Riley (1979), this kind of ODE has family of solutions of type

$$\theta = h(s) = g(s; \alpha, \gamma, k), \quad (11)$$

¹¹Note that Δ does not appear at $\mathbb{E}[\pi^U(\cdot, \cdot)]$, because the employers' 'outside option' is the same upon every match.

¹²If $h'(s) < 0$, it would mean that more able workers would choose lower educational levels than the less able colleagues. But in that case, due Assumption 1, it would always pay for the 'low-type' workers to imitate the 'high-types'. Therefore, a signalling profile that is decreasing along with worker's type cannot establish a stable separating equilibrium.

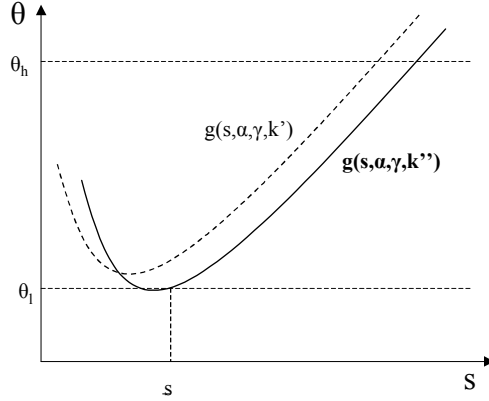


Figure 1: One possible separating equilibrium

where k is an integrating constant. From (11), one can solve the equilibrium signalling as a function of worker's type:

$$s = g^{-1}(\theta; \alpha, k). \quad (12)$$

Clearly, the necessary precondition for a separating equilibrium $h'(s) > 0$ is not automatically satisfied in (10). However, by convexity of C and concavity of V w.r.t. s , we know that the numerator of (10) inevitably turns positive after some threshold \underline{s} . It is then a matter of choosing k such that $\theta_l \geq g(\underline{s}; \cdot)$ (required by cond. (ii)). Reminiscent of the discussion by Riley (1979), $g(s; \alpha, \gamma, k'')$ in Figure 1 establishes a separating equilibrium while $g(s; \alpha, \gamma, k')$ does not.

Finally, it is instructive to derive the equilibrium wage function, given the equilibrium signalling. Plugging (10) back into (8), and using (11) and (12), we get

$$W_s(g^{-1}(\theta)) = \frac{\alpha + r}{\alpha} C_s(\theta, g^{-1}(\theta)),$$

so that the equilibrium wage schedule yields

$$W(g^{-1}(\theta)) = \frac{\alpha + r}{\alpha} C(\theta, g^{-1}(\theta)) + K, \quad (13)$$

where constant K can be interpreted as a base salary to be paid for each worker, regardless of his type. According to (13), the equilibrium wage function is such that, at the time of investment in schooling, the present value of the marginal increase in wages in response to a marginal increase in educational level just off-sets the marginal costs from additional schooling effort. Interestingly, the parameter representing the relative strength of the worker at the wage negotiation, γ , is absent in (13); i.e. 'no-excess-returns-from-schooling' holds regardless of γ .

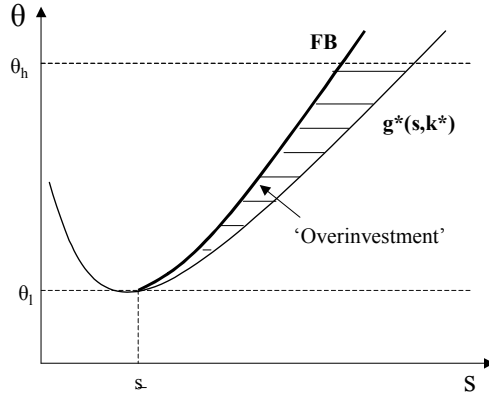


Figure 2: The 'Riley outcome'

2.7 'Riley outcome' as a limiting case

Riley (1979) considered signalling in a competitive equilibrium model¹³, which can be seen as a limiting case of the current setting. Assume that matching in the labor market is highly efficient so that workers locate employers very easily; i.e. α approaches infinity. Then, equation (10) obtains

$$h'(s) = \frac{C_s - V_s}{V_\theta},$$

which is equivalent to the formula derived by Riley (1979). In Riley's case, social optimum would require that each type θ chose an educational level where the marginal cost and marginal benefit from schooling balance; i.e. $C_s = V_s$. However, in a second-best solution, separating equilibrium requires $C_s > V_s$. Thus, signalling profiles that establish separating equilibrium must entail too much investment in education compared to the socially efficient level.

According to so called 'Riley outcome' (Figure 2), Pareto-dominant signalling profile is the one where the least able worker (θ_l) chooses his first-best education level while the more able workers acquire education in a manner which is just enough to separate them from the less able colleagues.

2.8 Signalling under search frictions

Under non-trivial search frictions, socially optimal educational level (first-best) satisfies

$$s^{fb} = \arg \max_s \left\{ \frac{\alpha}{\alpha + r} V(\theta, s) - C(\theta, s) \right\},$$

¹³Though, he needed to introduce a refinement to conventional Walrasian equilibrium concept in order to obtain stability of the separating equilibrium. Riley's refinement was 'reactive equilibrium'.

and the first-order condition of the first-best can be written as

$$\frac{\alpha + r}{\alpha} C_s = V_s. \quad (14)$$

However, under asymmetric information, separating equilibrium requires that

$$\frac{\gamma\alpha + r}{\gamma\alpha} C_s > V_s.$$

Obviously, if $\gamma = 1$, our model still coincides with the 'Riley outcome'. However, when $\gamma < 1$, the numerator of (10) turns positive already 'earlier' than at the point where the least able worker invests his first-best level. In other words, it is possible to have signalling profiles that entail underinvestment and still support separating equilibrium.

More specifically,

Lemma 1

$$\begin{aligned} \frac{\partial g^{-1}(\theta; \alpha, \gamma)}{\partial \alpha} &> 0, \frac{\partial^2 g^{-1}(\theta; \alpha, \gamma)}{\partial \alpha^2} < 0 \text{ and} \\ \frac{\partial g^{-1}(\theta; \alpha, \gamma)}{\partial \gamma} &> 0, \frac{\partial^2 g^{-1}(\theta; \alpha, \gamma)}{\partial \gamma^2} < 0. \end{aligned}$$

Proof. See Appendix A1. ■

According to Lemma 1, for any type θ , the level of education obtained is an increasing but concave function of both the ease with which unemployed workers will locate vacant jobs (α) and workers' relative bargaining power (γ). Note that when either γ or α is large enough, the slope of the signalling profile is flatter than the slope of the first-best education profile. Thus, the overinvestment result is still likely to hold for the most able types. On the other hand, if either γ or α approaches zero, the slope of the signalling profile approaches vertical line leading to infinitesimal separation in educational levels. In that case, the signalling profile capturing the minimum investment in schooling would lead to underinvestment throughout the set Θ . Obviously,

Proposition 1 *If either workers' bargaining power is sufficiently weak or the search frictions on workers' side of the market are severe, the 'minimum signaling profile' is not Pareto-dominant.*

Instead, Pareto-dominant signalling profile is the one where the inefficiencies due under- and overinvestment are minimized. Figure 3 illustrates the trade-off. If either γ or α is relatively large, the 'minimum signaling profile' is still likely to be Pareto-dominant - underinvestment among the 'low-types' reduces the inefficiencies caused by overinvestment among the 'high-types'. However, if either γ or α is sufficiently low, the 'minimum signaling

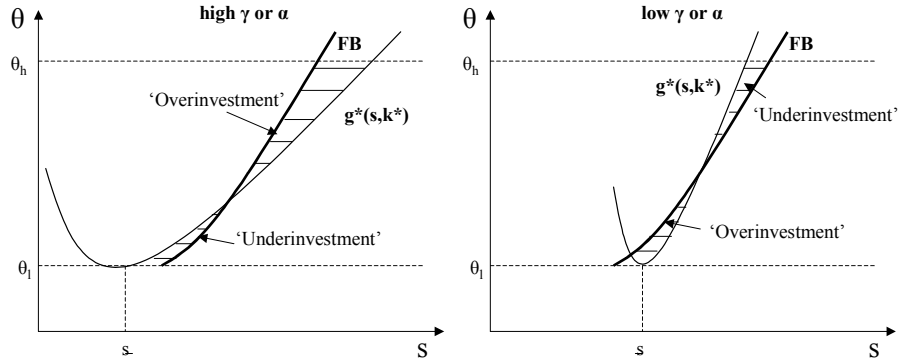


Figure 3: Pareto-dominant signalling profiles

profile' may be suboptimal; the Pareto-dominant signaling profile entails some overinvestment among the least able workers in order to reduce underinvestment among the most able workers.

It remains undecided, however, according to which signalling profile the workers will school themselves. Even though the minimum signalling profile is not necessarily Pareto-dominant, it still seems the most plausible candidate. Since there are no extra returns available from schooling, it seems unlikely that workers would choose any higher education than the minimum level that is just enough to separate them from the less able workers. Therefore, the 'minimum signalling profile' is implicitly assumed throughout the next section.

3 General equilibrium analysis

The analysis presented in this section follows rather closely Laing et al. (1995), who investigate the effects of human capital accumulation on market entry and growth. Our main results are seemingly similar to theirs, even though our model draws from the 'signalling hypothesis'. The economic interpretations are somewhat different, however.

3.1 Entry and exit

It is assumed that new workers are born at an exogenously given and constant rate, η . From employers' side, however, we assume unrestricted entry. The cost of opening a vacancy is denoted by ϕ , which can be thought to capture all the other factors of production except the worker. Firms will open new vacancies until the expected profit from a filled vacancy equals the fixed cost, ϕ . After a successful match, both the hired worker and the filled vacancy

exit the market forever.

3.2 Pairwise matching

The number of unemployed workers is denoted by u while v is our measure for the number of open vacancies. The matching function, $m_0 M(u, v)$, gives the total number of matches at each point of time as a function of two inputs, u and v . Parameter m_0 describes the exogenous matching technology.

* *Assumption 4:* Matching function $M : R_+^2 \rightarrow R_+$ is strictly increasing and strictly concave, satisfies the Inada-conditions, and exhibits constant returns to scale (CRS).

In a steady state, pairwise matching requires:

$$\alpha u = \beta v. \quad (15)$$

3.3 Steady state equilibrium

Definition 2 A steady-state equilibrium is characterized by a wage schedule $W(h(s), s, \alpha^*, \gamma)$ and a corresponding signalling profile $g(s; \alpha^*, \gamma, k)$, and a quadruple $(\alpha^*, \beta^*, u^*, v^*)$ s.t.

- (i) $\mathbb{E}[\pi^U] = \phi$ (free entry)
- (ii) $\alpha^* u^* = \beta^* v^* = m_0 M(u^*, v^*)$, (matching condition)
- (iii) $\alpha^* u^* = \eta$ (steady-state condition).

The pairwise matching condition, together with the CRS-property of the matching function, implies that

$$\beta = m_0 M\left(\frac{\beta}{\alpha}, 1\right), \quad (16)$$

which implicitly defines the locus of the 'Beveridge-curve' (the steady state relationship between unfilled jobs and unemployment) in $\alpha\beta$ -plane. Utilizing the equilibrium wage schedule given in (13), the free-entry condition can be written as

$$\mathbb{E}[\pi^U] = \frac{\beta}{\beta + r} \int_{\Theta} [V(\theta, g^{-1}(\theta, \alpha)) - W(g^{-1}(\theta, \alpha))] dF(\theta) = \phi. \quad (17)$$

Now, the equations (16) and (17) determine the steady state meeting rates, α^* and β^* . Since $u^* = \eta/\alpha^*$ and $v^* = \eta/\beta^*$, the two equations solve the system completely, and a steady state equilibrium can be found at the intersection of the 'Beveridge-curve' (BC), and the free-entry curve (FE).

Following immediately from Assumption 4, the locus of the 'Beveridge-curve' is downward-sloping and convex in $\alpha\beta$ -plane. In turn, the slope of the free-entry curve obtains

$$\frac{d\beta}{d\alpha} = -\frac{\int_{\Theta} [V_s - W_s] \frac{\partial g^{-1}}{\partial \alpha} dF(\theta)}{r/(\beta + r)^2}. \quad (18)$$

Lemma 2 *If $\gamma \rightarrow 1$, the signalling profile approaches the 'Riley outcome' and the free-entry locus is monotonously upward-sloping in $\alpha\beta$ -plane.*

Proof. See Appendix A2. ■

Lemma 3 *If $\gamma \ll 1$, the FE-locus may exhibit decreasing segments in $\alpha\beta$ -plane.*

Proof. See Appendix A3. ■

By the analysis carried out in the previous section, we know that if $\gamma \ll 1$, underinvestment in education arises, especially if α is relatively low. Therefore, it may not pay to open new vacancies unless workers have sufficient incentives to invest in schooling, which - by Lemma 1 - happens when the flow probability α rises. Hence, in some cases higher α may stimulate market entry by firms leading to tougher competition on labor force and higher search frictions on employers' side (lower β).¹⁴

As $\alpha \rightarrow \infty$, the signalling profile approaches the 'Riley outcome' and the free-entry locus becomes monotonously increasing. Therefore, we may conclude that the free-entry locus necessarily turns increasing after some threshold level for α , say α_h . Moreover, if $\alpha \rightarrow 0$ so that $W_s \rightarrow \infty$, we also know that the free-entry locus must be upward-sloping for sufficiently low levels of α , say $\alpha < \alpha_l$. As depicted in Figure 4, if $\alpha_l < \alpha_h$, the free-entry locus has a decreasing segment within the interval $[\alpha_l, \alpha_h]$.

When $\alpha > \alpha_h$, the separating regime generally entails overinvestment in schooling. Thus, additional schooling effort along with increasing α does not increase productivity as much as it increases wage-costs, which discourages market entry by firms until the search frictions faced by employers are sufficiently low (i.e. β is high enough). On the other hand, when $\alpha < \alpha_l$, workers tend to underinvest in education. Even though this problem is reduced along with higher α , lowering search frictions may not alleviate underinvestment enough in order to boost market entry. That happens only when $\alpha \in [\alpha_l, \alpha_h]$.

3.4 Multiple equilibria

For multiple equilibria to arise, the downward-sloping property of the free-entry locus is a necessary¹⁵ precondition. Moreover, the downward-sloping part of the free-entry locus

¹⁴This is essentially the same intuition as in the model by Laing et al.'s (1995), where *homogenous* workers optimize their schooling effort at the *ex ante* stage. However, Laing et al. (1995) do not justify the shape of the free-entry locus analytically.

¹⁵But not sufficient, as clearly visible in Figure 5.

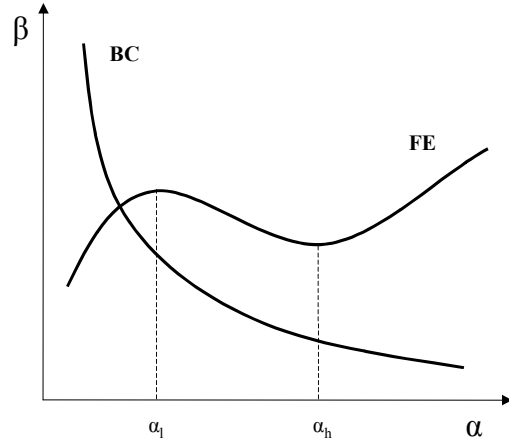


Figure 4: A unique steady-state equilibrium

needs to be steeper than the 'Beveridge-curve' when the two locuses intersect (point E_2 in Figure 5).

Proposition 2 *The labor market is the more likely to exhibit multiple equilibria the stronger is the complementarity between worker's type, θ , and education, s , and the weaker is workers' position at the wage-bargaining stage; i.e. the lower is γ .*

Proof. See Appendix A4. ■

The result presented in Proposition 2 should not be very surprising. The stronger is the complementarity between worker's type and educational level the more severe is the underinvestment problem. Therefore, a small reduction in search frictions (a slight increase in α) may drastically reduce underinvestment and have a large positive impact on productivity and market entry. Workers' relatively weak position in wage negotiations (low γ) amplifies this effect. Figure 5 depicts an example where the multiplicity of equilibria occurs.

Since the steady state unemployment is $u^* = \eta/\alpha^*$, E_1 in Figure 5 is characterized by high equilibrium unemployment and low market entry by firms. Under E_3 , in turn, the labor market is flooded by open vacancies, while the pool of unemployed workers is relatively small. E_2 is the intermediate equilibrium located at the intersection of the downward-sloping free-entry locus and the 'Beveridge-curve'.

The general pattern for the free-entry locus derived here is close to the pattern suggested in Laing et al. (1995). Therefore, also the possibilities for escaping multiple equilibria for a unique and a more desirable steady state equilibrium arise from similar economic fundamentals as in their model. Let us specify $V(\theta, s) = AQ(\theta, s)$, where A represents the

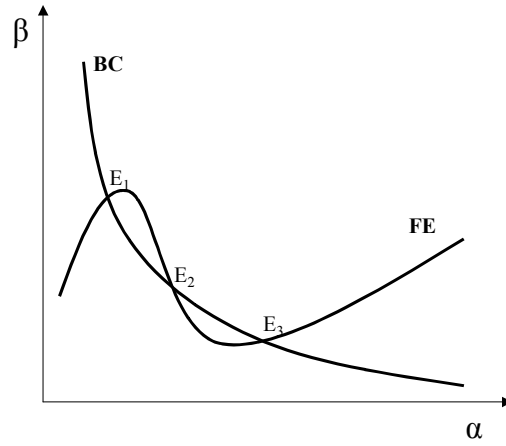


Figure 5: Multiple equilibria

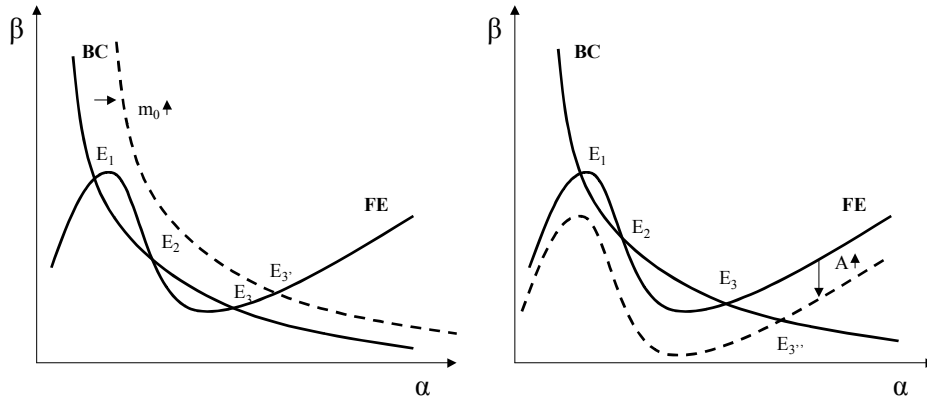


Figure 6: Effects of improved matching and a positive technological shock

general level of technological development in the economy, and $Q(\theta, s)$ characterizes the way in which worker's type, θ , and educational level, s , contribute to production. Figure 6 suggests that even small improvements in either market infrastructure, m_0 , or the level of technological development, captured by parameter A , can potentially lead to a considerable leap from initially inferior equilibrium to a unique and a more efficient equilibrium. Such a 'take-off' happens if the labor market equilibrium suddenly jumps from E_1 to $E_{3'}$ or $E_{3''}$. As noted by Laing et al., a possible 'take-off' can be interpreted as an example of Murphy's et. al (1991) 'big push' argument to economic development, in which improvements in economic infrastructure may 'push' the economy from 'cottage'-production to industrialization.

4 Concluding remarks

According to Riley (1979) and Spence (1981), signalling function of schooling tends to induce overinvestment in education. However, in a model with decentralized trading and search, we found that the conditions of the bilateral bargaining process crucially affect workers' incentives to invest in education. If workers' bargaining power is strictly less than one, search frictions tend to induce underinvestment in education at least within a subset of types. The 'Riley outcome' emerges only as a limiting case. Moreover, the Pareto-dominant signalling profile - the profile that minimizes inefficiencies caused by under- and overinvestment - is not necessarily the 'minimum signalling profile'.

On the market level, we found that underinvestment in schooling may hinder job creation when labor market is 'slack'. Even though workers' increasing likelihood to be matched generally weakens firms position in wage negotiations, it also alleviates underinvestment problem. Therefore, it is possible that higher contact rate on workers' side actually stimulates market entry by firms. This feature may also give rise to multiple steady state equilibria.

Reminiscent of the model by Laing et al. (1995), possibility of a 'Big Push' development arises: a (potentially) small improvement in either labor market's matching efficiency or exogenous technological level may help the economy to escape multiple equilibria for a unique and a more efficient steady state equilibrium.

5 Appendix

A1 Proof of Lemma 1

Proof. From (10) we know that $h_{s\alpha} < 0$. Since $g(s; \cdot) = \int h_s ds$,

$$\frac{\partial g(s; \cdot)}{\partial \alpha} = \int h_{s\alpha} ds < 0,$$

which in turn directly implies that $\partial g^{-1}(\theta; \cdot)/\partial \alpha > 0$.

Similarly, by (10), $h_{s\alpha\alpha} > 0$, which implies

$$\frac{\partial^2 g(s; \cdot)}{\partial \alpha^2} = \int h_{s\alpha\alpha} ds > 0,$$

and $\partial^2 g^{-1}(\theta; \cdot)/\partial \alpha^2 < 0$. Similar reasoning applies for elaborating the effect of γ . ■

A2 Proof of Lemma 2

Proof. For the free-entry -locus to be downward-sloping, it is obvious by (18) and Lemma 1 that the difference $V_s - W_s$ must be positive. In 'Riley outcome', separating equilibrium

requires

$$C_s > V_s.$$

But since in 'Riley outcome' $C_s = W_s$, we must conclude that $V_s - W_s < 0$. ■

A3 Proof of Lemma 3

Proof. Now, separating equilibrium requires that

$$\frac{\gamma\alpha + r}{\gamma\alpha} C_s > V_s.$$

On the other hand,

$$W_s = \frac{\alpha + r}{\alpha} C_s.$$

Thus, we may conclude that

$$W_s > \frac{\gamma(\alpha + r)}{\gamma\alpha + r} V_s,$$

which implies that

$$V_s - W_s < \frac{(1 - \gamma)r}{\gamma\alpha + r} V_s.$$

Thus, when

$$0 \leq V_s - W_s < \frac{(1 - \gamma)r}{\gamma\alpha + r} V_s,$$

$d\beta/d\alpha < 0$ in (18). ■

A4 Proof of Proposition 2

Proof. Let us restate here the formula for the slope of the FE-curve:

$$\frac{d\beta}{d\alpha} = - \frac{\int_{\Theta} [V_s - W_s] \frac{\partial g^{-1}}{\partial \alpha} dF(\theta)}{r / (\beta + r)^2}.$$

Clearly, the complementarity between θ and s makes V_s increasing in θ while W_s is unaffected by the complementarity. Hence, the expected value of the difference $V_s - W_s$ grows along with stronger complementarity.

Moreover, lower γ moderates W_s and increases $\partial g^{-1} / \partial \alpha$ (which is obvious by Lemma 1, since the effect of γ and α on g^{-1} are identical). Both of these effects work for steeper FE-locus. ■

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