House Price Fluctuations and Residential Sorting

Markus Haavio
Helsinki School of Economics, RUESG and HECER

and

Heikki Kauppi
University of Helsinki, RUESG and HECER

Discussion Paper No. 137
November 2006
ISSN 1795-0562
House Price Fluctuations and Residential Sorting*

Abstract

Empirical evidence indicates local jurisdictions are internally more heterogenous than standard sorting models predict. We develop a dynamic multi-region model, with fluctuating regional house prices, where an owner-occupying household's location choice depends on its current wealth and its current “match” and involves both consumption and investment considerations. The relative strength of the consumption motive and the investment motive in the location choice determines the equilibrium pattern of residential sorting, with a strong investment (consumption) motive implying sorting according to the match (wealth). The model predicts a negative relation between house price fluctuations and residential sorting in the match dimension. Also, movers should be more sorted than stayers. These predictions are consistent with evidence from US metropolitan areas when income, age and education are used as proxies for the “match”.

JEL Classification: D31, D52, R13, R21, R23

Keywords: Residential sorting, House prices, Incomplete markets, Owner-occupation, Household mobility

Markus Haavio
Department of Economics
Helsinki School of Economics
P.O. Box 1210 (Arkadiankatu 7)
FI-00100 Helsinki
FINLAND

e-mail: markus.haavio@hse.fi

Heikki Kauppi
Department of Economics
University of Helsinki
P.O. Box 17 (Arkadiankatu 7)
FI-00014 University of Helsinki
FINLAND

e-mail: heikki.kauppi@helsinki.fi

* We thank Seppo Honkapohja, Erkki Koskela, Matti Liski, Niku Määttänen, François Ortalo-Magné, Sven Rady and Juuso Välimäki for their useful comments and suggestions on earlier drafts. This paper is part of the research program of the Research Unit of Economic Structures and Growth (RUESG) at the University of Helsinki. Financial support from the Yrjö Jahnsson Foundation and the Research Foundation of the OKO Bank Group is gratefully acknowledged. The usual disclaimer applies.
\section{Introduction}

A central theme in regional and urban economics has been to examine how households sort themselves into neighborhoods and communities according to various socioeconomic characteristics, such as income, household size or education. Roughly speaking, the sorting approach predicts that local jurisdictions should be internally more homogenous than the larger geographical or economic unit of which they are a part. Also, the jurisdictions should differ from each other.

However, recent empirical evidence reveals that there is considerable heterogeneity within municipalities and local neighborhoods. Davidoff (2005), and Epple and Sieg (1999) find that ca. 90\% of the variation of household income in US metropolitan areas can be explained by within-community variance, and only ca. 10\% is accounted for by differences across jurisdictions. According to Ioannides (2004), in a typical American neighborhood, neighbors tend to differ significantly in terms of income, age and education, while local property values are more correlated. These findings are put into a historical perspective by Rhode and Strumpf (2003), who report that heterogeneity across US municipalities and counties, measured with respect to income and (so called) public good preference proxies (including education, age, race, nativity, party vote shares in presidential elections, owner-occupation rate, and religion) did not increase over the period 1850-1990 although migration costs fell, which should have made sorting easier.

In this paper we develop a dynamic model of residential sorting, which provides one possible explanation as to why local jurisdictions may be internally rather heterogenous, and not very distinct from each other. We also derive a number of empirical predictions about the degree of sorting under different circumstances and compare these predictions with observations from US metropolitan areas.

Our approach is based on the following main elements: (i) For owner-occupying households, housing is both a consumption good and an investment, and location choices involve both consumption and investment considerations. (ii) Regional house prices fluctuate, and the capital gains and losses made in the housing market play a major role in determining how a household’s wealth evolves over time. (iii) Borrowing constraints may prevent a
household from moving from a currently cheap location to a currently expensive location.

There are two locations in the economy, both having enough housing capacity for half of the households. In each period, the utility that a household derives from residing in different locations depends on the household’s current “match”. Empirically, the “match” can be interpreted as reflecting various demographic and socioeconomic characteristics of the household (other than wealth) such as education, the age of the household head(s), or the number of children, which affect housing and location decisions. In each period, one of the locations is considered being “desirable” while the other location is “less desirable”. The basic allocation problem arises from the fact that more than one half of the households (and possibly all of them) derive more utility from residing in the (currently) “desirable” location, and thus housing is in short supply there.

In equilibrium, a pattern of two-dimensional residential sorting emerges and location choices depend on both wealth and the “match”. It may be instructive to first have a brief look at simple polar cases. If residential sorting takes place mainly in the wealth dimension, the wealthiest households tend to live in the currently popular, and expensive, location, and those who reside in the unpopular location do so because they cannot afford a more expensive home; they are borrowing constrained. According to our theory, a household is currently wealthy (impecunious), because it has been fortunate (unlucky) in the timing of its housing market transactions; the current wealth position largely reflects past fortunes in the housing market, rather than some inherent characteristics of the household. Thus the theory predicts that under wealthwise sorting the locations should be internally rather heterogenous in the match dimension. Within the same neighborhood there may be households which have little in common, apart from the value of their home.

In the opposite case, where sorting takes place in the match dimension, those who have the best current match with the desirable location also live there. Given the empirical interpretation we attribute to the match, households living within the same jurisdiction should then resemble each other with respect to various socioeconomic characteristics (such as household size, the age of the household head(s), or education). Also different jurisdictions should differ from each other with respect to the distribution of these observable characteristics.
A major objective of the paper is to study under what circumstances sorting happens primarily in the wealth dimension, and when location choices are made principally according to the match. Residential sorting in the match dimension requires that the basic allocation problem in the economy is not mainly solved through the borrowing constraint, but rather by self-selection. Here self-selection essentially means that some households, which receive a higher current utility stream from the desirable location, voluntarily choose the less desirable area. The incentives to make such a choice arise from the basic elements (i), (ii) and (iii), underlying our approach, and they reflect a trade-off between the consumption motive and the investment motive of housing.

To be more specific, we assume that the relative ranking of the locations is not set once and for all, but with a certain probability there may be a regional shock, so that the ranking is reversed, and regional house prices change. The regional shock may reflect e.g. altering labor market conditions, changes in the supply of public goods and services, or the evolution in the tastes and the needs of the population. Alternatively, the house price dynamics may be interpreted as reflecting (in a reduced form) the interaction between housing demand and supply. According to this interpretation, an area is currently expensive, because housing supply has not yet increased to absorb a positive demand shock.\footnote{When the housing supply adjusts, and the demand shock is absorbed, the prices will fall. However, since construction takes time, and may also require changes in local public policies, such as rezoning, this may only happen after a long, and random, delay. See Capozza et al. (2004), Evenson (2003) or Mayer and Somerville (2000).}

This pattern of house price fluctuations implies that choosing the currently popular and expensive location today involves the risk of incurring capital losses, if regional house prices fall, and then facing the borrowing constraint in the future, when the match with the (then) expensive location may be better than today. By contrast, buying a property in the currently less popular area provides the opportunity to realize capital gains. Thus while the currently “desirable” and expensive location is, for most households, more attractive from the consumption point of view, the currently cheap location offers better investment opportunities.

The pattern of residential sorting that emerges in equilibrium essentially reflects the
relative strength of the consumption motive and the investment motive of housing. If regional shocks are large and/or persistent, the consumption motive dominates. The households make their location choices mainly by comparing current benefit streams. Since only a small group (always less than one-half of the households, perhaps no household) voluntarily chooses the less desirable location based on the consumption motive only, the regional allocation of households basically boils down to differences in wealth. Also, regional house price differences, as well as capital gains and losses realized in the housing market, are large, compared with typical household wealth.

When regional shocks are small and/or transient, the investment motive is stronger (in relative terms, compared with the consumption motive). Caring about their future prospects, many households, which would receive a larger immediate welfare stream from the desirable location, voluntarily choose the less desirable location, in the hope of making capital gains. Typically, a household resides in the desirable location, only if its current match with that location is truly good. The regional allocation of households then happens mainly through self-selection, according to the match, rather than based on wealth differences and borrowing constraints. The fact that many households voluntarily choose the less desirable location is also reflected in house prices. In equilibrium, regional house price differences, and capital gains and losses, are small, in comparison with typical household wealth.

The model produces two main empirical predictions. First, the size of house price fluctuations should be negatively correlated with the degree of residential sorting in the match dimension. Second, movers should be more sorted than stayers. We confront these predictions of the model with data from US metropolitan areas. Consistent with the predictions, we find that those metropolitan areas, where house price fluctuations have been large, also tend to have a more diverse mix of different educational, age and income groups, whereas metropolitan areas where prices have been less volatile tend to have a less diverse population, with certain age, educational and income categories under- or overrepresented, compared with the national average.

We also have data concerning the distribution of age, education and income, at the municipality level, and thus we can examine residential sorting within metropolitan areas.
We find that metropolitan areas (such as Seattle) where house prices have been volatile tend to display lower degrees of residential sorting (so that there is heterogeneity within, rather than between, municipalities) than metropolitan areas (such as Atlanta), where price swings have been small.

In the second part of our empirical analysis, we establish that, among owner-occupying households, movers are more sorted (with respect to age, education and income) than stayers. Based on our theory, we interpret these differences as resulting from housing market related wealth shocks. For example, some stayer households whose characteristics are ill-suited for their present location, may be unable to move, since, due to a depreciation in local property values, they have negative housing equity. To further check if this wealth shock based mechanism might be (a part of) the explanation, we also look at renters (who are not subject to wealth shocks in the housing market) in our data. Interestingly, we find that among renters, stayers are more sorted than movers.

As a final result, the model predicts a non-linear relation between wealth and mobility: households with intermediate wealth levels should be more mobile than poor households and wealthy households. While we do not address this issue in the empirical part of the study, Henley (1998) has documented that this humpshaped relationship holds for British households.

Generally speaking, our theoretical framework combines themes, which are typically addressed in two separate branches of literature. (i) Most papers in the literature on residential sorting use static general equilibrium models. Earlier sorting models routinely assumed that households differ with respect to one characteristic only (typically income), and predicted perfect stratification along that dimension, a prediction clearly in contradiction to the empirical evidence, cited above. The more recent two-dimensional sorting models by Epple and Platt (1998), Epple and Sieg (1999) and Epple, Romer and Sieg (2001) are more successful in explaining the data. In these models, households differ both with respect to income and with respect to tastes, and there is imperfect sorting along both dimensions. An alternative approach to account for the observed diversity of

---

households within jurisdictions is based on the heterogeneity of the housing stock (e.g., Nechyba (2000)). In contrast to the present paper, the atemporal nature of these models means that housing and location choices do not involve investment considerations, and there is no feedback from house price fluctuations to household wealth. On the other hand, in the sorting literature, the attractiveness of different jurisdictions typically arises endogenously as a part of the equilibrium (e.g., the supply of local public goods and services is determined in a political economy equilibrium), whereas we take the process that determines the desirability of different locations as given.

(ii) The second branch of literature analyzes housing wealth as an important component of a household’s asset portfolio. While the double nature of housing, as a consumption good and as an investment, and house price fluctuations play an important role here, this literature essentially focuses on the optimization problem of an individual household, and the implications for residential sorting are not examined.

A few recent papers take up a roughly similar mix of issues as we do here. Ortalo-Magné and Rady (2006a) examine tenure choice and income heterogeneity in booming cities, where house prices rise, and home-owners, who make capital gains, may choose to stay put, even when newcomers typically earn higher incomes. The model has two periods and, with a certain probability, the newcomers enter the city in the second period. The households differ in one dimension, income/wealth; whereas the first period distribution is taken as given, the second period distribution is scrambled by the wealth shocks realized in the housing market. By contrast, we study sorting in two dimensions, wealth and the match, more in line with Epple and Platt (1998), Epple and Sieg (1999) and Epple, Romer and Sieg (2001).

---

3 A few papers (e.g., Bénabou (1996), Fernandez and Rogerson (1996)) analyze sorting in a dynamic context. Even in these models, however, the households are typically assumed to be renters, and they are also assumed to choose their location once and for all (in the first period), so that realized capital gains and losses do not shape the equilibrium pattern of residential sorting.


5 Gyourko et al. (2006) study the influx of high income households into (so called) “superstar cities”, i.e., prime locations, where land is scarce. Over time, both housing prices and household incomes rise in these locations, and the “superstar cities” become increasingly distinct from the rest of the economy. In the model, the driving force behind these processes is population growth, and the evolution of the
Glaeser and Gyourko (2005) study the joint process of falling house prices and neighborhood change in declining cities. Due to the durability of housing, a negative shock leads to a sharp fall in housing prices, but only a slow and gradual decline in city size. Low housing costs in a city attract low-income households. However, in contrast to our framework, the model assumes away the possibility that capital losses realized in the housing market may affect residential location choices.

The plan of the paper is as follows. Section 2 presents some empirical observations, which underlie and motivate our modelling approach. The model is developed in Section 3. Section 4, which contains our main theoretical results, shows how the equilibrium pattern of residential sorting reflects the relative strength of the consumption motive and the investment motive of housing. The section also establishes a relationship between house price fluctuations and residential sorting, and compares the degree of sorting among movers and stayers. To communicate the main ideas in the simplest possible framework, Sections 3 and 4 assume that after a regional shock, all matches are broken, and new match realizations are independently drawn; as a consequence, wealth and the match are independently distributed. In Section 5 we show that the main results still hold when the match follows a general Markov process, and wealth and the match can be correlated. Finally Section 6 concludes.

2 Empirical background

2.1 Housing investments, house prices and incomplete markets

The modelling approach we adopt in this paper is consistent with the following observations:

(1) In most developed countries, owner-occupied housing is the single most important investment for a typical household. For example, in the late 1990’s, single family owner-
occupied housing composed 2/3 of household wealth in the UK, 1/3 of household wealth in the US, and 2/3 of the assets of a US household with median wealth.\textsuperscript{6}

(2) House prices are often highly volatile, and in different regions property values tend to rise and fall asynchronously, so that also relative regional prices may vary considerably over time. Figures 1 and 2 illustrate this finding with price data from four British regions and US metropolitan areas; in the figures, the country average is normalized to 1.\textsuperscript{7} Other OECD countries with large regional price fluctuations include Denmark, Finland, Ireland, and Spain. Relative prices can fluctuate significantly even at a more local level, e.g. between London boroughs. For example, in 1995 average house prices were 3\% lower in Hackney than in Greenwich, but in 2001 Hackney was 20\% more expensive than Greenwich\textsuperscript{8}; see also Iacoviello and Ortalo-Magné (2003). For similar findings on the Boston metropolitan area, see Case and Mayer (1996).

(3) While (relative) house prices may vary quite a bit in the short and medium run, in many cases there appears to be a long-run equilibrium relationship between house prices in different areas\textsuperscript{9}, or between local house prices and local economic fundamentals, such as average income or construction costs\textsuperscript{10}. If prices are currently below (above) the long-run equilibrium level, they are likely to rise (fall) some time in the future.\textsuperscript{11}

\textsuperscript{6}Banks et al. (2002), Federal Reserve’s 2001 Survey of Consumer Finances.

\textsuperscript{7}According to Shiller (1993, Ch 5 p. 79) real estate booms and busts in US cities have been regionally asynchronous and prize movements often dramatic. Del Negro and Otrok (2005) find that, with the exception of the current housing boom, US house price dynamics have been mainly driven by local or regional, rather than national, shocks. For further evidence on US prices, see also Case and Shiller (1989), Malpezzi (1999), Case, Quigley and Shiller (2005), or Himmelberg, Mayer and Sinai (2005). For British evidence, see Muellbauer and Murphy (1997), or Cook (2003).

\textsuperscript{8}Source: Land Registry, http://www.landreg.gov.uk.

\textsuperscript{9}That is, regional house prices are cointegrated. For evidence from British regions, see MacDonald and Taylor (1993), Alexander and Barrow (1994) or Cook (2003). For evidence from US census regions, as well as for a comparison between the US and the UK, see Meen (2002). Also, Pollakowski and Ray (1997) find that in the Greater New York metropolitan area, the evolution of local house prices in a municipality can be predicted using lagged price changes in neighboring jurisdictions. Tirtiroglu (1992) reports similar findings from Hartford CT.


\textsuperscript{11}The adjustment may take a long time. Factors that slow down the process include high population density, the scarcity of land, and strict public regulations. See Capozza et al. (2004) or Evenson (2003).
(4) Capital gains and losses made in the housing market can be remarkably large in comparison with typical household incomes and savings. To illustrate this point, Table 1 shows maximum and minimum house-price-to-income ratios in four major US cities over the period 1979-1996. In the UK, the average annual capital gain in the London market between 1983 and 1988 corresponded to 72% of the mean annual disposable household income in the UK over that period, and exceeded by the factor of 7.8 average yearly household savings. Between 1989 and 1992, the annual capital loss of a typical London homeowner was equivalent to 77% of average disposable household income, and 8.4 times average household savings.

(5) Empirical studies of household mobility reveal that capital losses made in the housing market may seriously limit a household’s ability to move from one location to another. This is the case especially if a household has negative equity, i.e. the value of the mortgage exceeds the value of the house.\textsuperscript{12}

(6) As a general rule, housing market risks are uninsurable. For example Shiller (1993, 2003) lists home equity insurance as one of the key financial markets currently missing. Shiller (1993), and Shiller and Weiss (1999) discuss the potential problems, both economic and psychological, involved in providing hedging against house price swings, as well as ways to overcome these problems.\textsuperscript{13}


\textsuperscript{13}In the UK, real estate futures were traded in the London Futures and Options Exchange (London Fox) in 1991, from May through October. Trading volume was low, and the market was closed when it was reported that the exchange had allegedly attempted to create a false impression of high trading value by false trades (Shiller (1993, Ch 1)). In the early 2000’s house price index derivatives were (re)introduced by two spread betting firms IG Index and City Index (Iacovello and Ortalo-Magné (2003)). The interest shown by the British public has been minimal and IG Index has already withdrawn the products from the market.

In the US, there are a few local experiments with home equity insurance. The Oak Park Experiment has been running since 1977 and the South-West Home Equity Assurance Program was initiated in 1988. Both of these programs are in Chicago and insure homeowners against price declines caused by neighborhood change. More recently, the Yale/Neighborhood Reinvestment Corporation Home Equity Guarantee Project has developed home equity insurance products, to be initially used in Syracuse, New York. See Shiller (2003, Ch 8).
2.2 House price fluctuations, household mobility and residential sorting

To provide a benchmark against which the predictions of the model can be judged, we study some simple relations between house price fluctuations and residential sorting, and analyze the degree of sorting among movers and stayers. There is much discussion and data on large fluctuations in relative house prices in the US metropolitan areas (MSAs). Also, recent studies (Ortalo-Magné and Rady (2006a), Glaeser and Gyourko (2005)) suggest that rising or falling house prices affect the income distribution in a MSA. In a similar vein, we first ask whether the size of house price fluctuations is related to how much each MSA’s population structure deviates from the US as a whole. We then move to examine how house price variations in different MSAs are related to measures of residential sorting between local municipalities (so called Minor Civil Divisions or MCDs) within each MSA. MCDs, in particular, have been found to exhibit much less residential sorting than conventional sorting models predict (see Rhode and Strumpf (2003)). Finally, we compare residential sorting of movers to that of stayers across (so called) Public Use Microdata Areas (PUMAs) in the whole US.\footnote{Each PUMA has a population of approximately 100 000. For further information, see the appendix.} This part is related to the work by Ortalo-Magné and Rady (2006a), who study income distributions among movers and stayers. For a detailed description of the data and their sources, see the empirical appendix.

The predictions of our model (see Section 4) are consistent with the following empirical observations. The fourth observation is based on empirical work by Henley (1998) on the UK.

(P1) MSAs, where house prices have been volatile, tend to have a rather diverse population, with the size of different age, education and income groups largely corresponding to the national population structure, while MSAs with less volatile prices typically deviate more from the national population structure, with certain age, education and income categories over- or underrepresented, compared with the national average. The underlying OLS regression results are reported on the first three columns of Table 2. The data on population structure come from the 1990 census. As an indicator of how much a MSA
deviates from the US average we use the measure

\[ DV = \sum_{m=1}^{M} \frac{(S_m - S_{mUS})^2}{S_{mUS}}, \]  

where \( S_m \) is the share of age, education or income group \( m \) in the MSA and \( S_{mUS} \) is the corresponding share at the national level. As a measure of house price fluctuations we use the standard deviation of the house price \( p_{it} \) of the MSA \( i \) over the sample period (1985-2000), where \( p_{it} = \log(I_{it}/I_t) \), \( I_{it} \) is the house price index in MSA \( i \) in period \( t \), and \( I_t \) is the US house price index in period \( t \).\(^{15}\)

(P2) There is a negative correlation between the size of house price fluctuations and the degree of residential sorting within an MSA. That is, MSAs with volatile house prices tend to exhibit low degrees of residential sorting, with heterogeneity within, rather than between, municipalities. MSAs with small house price fluctuations tend to display higher degrees of sorting, so that municipalities differ more clearly from each other with respect to the distribution of age, education and income. The OLS regression results are reported on the last three columns of Table 2. The measure of residential sorting we use is the Gini coefficient\(^{16}\)

\[ GC = \frac{1}{2} \sum_{m} \sum_{k} \sum_{j} \frac{N_k N_j}{N^2 S_m (1 - S_m)} \left| S_{mk} - S_{mj} \right| \]  

where \( N \) is the population of the MSA, \( N_i \) is the population of municipality \( i \), \( S_m \) is the share of group \( m \) in the MSA and \( S_{mi} \) is the share of group \( m \) in municipality \( i \). The Gini coefficient takes values between 0 and 1, with small (large) values associated with low (high) degrees of sorting. The measure of house price fluctuations is the same as above.\(^{17}\)

(P3) Among owner-occupying households, movers are more sorted than stayers. If two mobile households choose the same jurisdiction, these newcomers typically share some

\(^{15}\)The price index data are from the Office of Federal Housing Enterprise Oversight. As an alternative indicator of the size of house price swings we used the measure \( \max_t(p_{it}) - \min_t(p_{it}) \). The OLS regression results were qualitatively similar.

\(^{16}\)See Rhode and Strumpf (2003).

\(^{17}\)As we do not have data on municipality level house prices, we cannot study changes in relative prices within a MSA.
common characteristics; they also tend to differ from other mobile households, which choose a different location. By contrast, stayers living within the same jurisdiction tend to have less in common with each other. The first two columns of Table 3 report the Gini coefficients for owner-occupying movers and stayers; we classify an individual as a mover, if (s)he has resided in his/her current home for less than five years. For each characteristic (age, education, income) and each group (movers and stayers), we have computed a single Gini coefficient, which describes the degree of residential sorting across PUMAs in the whole US. Thus, \( N \) in (2) now stands for the US population, \( N_i \) is the population of PUMA \( i \), \( S_m \) is the share of group \( m \) in the US, and \( S_{mi} \) is the share of group \( m \) in PUMA \( i \).

Based on our theory, we interpret the low degree of sorting among owner-occupying stayers as reflecting housing market related wealth shocks. Some households, which would like to move out of an area where property prices fall, may be unable to do so because they have negative equity (see observation 5 in Section 2.1). Alternatively, some stayers, who bought their home when prices were lower, may be unwilling to leave when the area becomes more expensive. Interestingly, among renters (who do not face these wealth shocks), the pattern of sorting is reversed: stayers are more sorted than movers; see the last two columns of Table 3. Some further evidence on sorting among movers and stayers, together with statistical tests, is provided in the appendix.

(P4) Henley (1998) has documented that in the UK there is a humpshaped relation between wealth and household mobility. According to Henley (p. 425), “levels of housing wealth are an important factor in explaining mobility and the relationship between the two is not linear.” British households with large negative equity are virtually immobile. Also wealthy households tend to move rather little. Households with intermediate levels of wealth are the most mobile. See especially Figure 2 in Henley (1998).
3 The model

3.1 The basics of the economy

We develop a dynamic two-region model of residential sorting, based on the following main elements. First, the basic allocation problem arises, since in each period one of the locations is more desirable than the other one, and housing is in short supply in the popular area. Second, as in the models by Epple and Platt (1998), Epple and Sieg (1999), and Epple, Romer and Sieg (2001), there is heterogeneity, and sorting, in two dimensions. In particular, we want to analyze, whether the basic allocation problem can be (mainly) solved through self-selection, so that there is sorting according to the match (i.e. socioeconomic characteristics such as age, education or household size), or if borrowing constraints play a major role, so that there is sorting according to wealth. Third, wealth dynamics are driven by capital gains and losses made in the housing market. As in Ortalo-Magné and Rady (2006a), changes in house prices are caused by regional shocks. Fourth, for a household to care about the house price fluctuations, and to view its home as an investment, there must be a chance that the household will want to sell its home, and buy another house, sometime in the future. Here we follow the matching/search literature\(^\text{18}\) (e.g. Wheaton (1990), Williams (1995), Krainer and LeRoy (2002)), in assuming that the match between a household and a housing location may be broken, and the household finds it optimal to move.

Our basic assumptions are the following.

The economy has two locations. Each location has an equal, fixed, stock of identical houses. Each house is occupied by a single household and no one household is ever homeless. All households are owner-occupyers and there is no rental housing. For convenience, assume that the stock of houses and the mass of households each comprises a continuum of size unity.

There are infinite discrete time periods indexed by \(t = 0, 1, \ldots\). In each period, one of the locations is deemed to be “desirable” while the other one is “less desirable”. When a

\(^{18}\)Unlike these models, we do not introduce any search frictions.
period changes, the relative ranking of the locations is reversed with probability \( \pi \in (0, 1) \).

We also consider a small region interpretation of the model, with a continuum of locations. Then in each period, one half of the locations are “desirable” while the remaining locations are “less desirable”, and when a period changes, a measure \( \pi \) of the locations is hit by a regional shock. The long-run equilibrium of the model is essentially identical under both interpretations.

The households differ in the utility premium they derive from residing in the desirable location. The household specific component of the premium is captured by the match, \( \theta \): a high realization of \( \theta \) implies a good match with the currently desirable location, while a low (negative) realization implies a good match with the less desirable location.\(^{19}\) The aggregate heterogeneity of households is unchanged over time, and \( \theta \) has a stationary distribution, with a cumulative distribution function \( G(\theta) \), on some support \([\theta_L, \theta_H]\). Without loss of generality, we assume that the median match \( \theta_m = 0 \), i.e. \( G(0) = \frac{1}{2} \).

A household with current match \( \theta \) receives per period utility \( \frac{1}{2} \varepsilon + \theta \), when living in the currently desirable location. The per period utility of anyone household living in the less desirable location is \( -\frac{1}{2} \varepsilon \). Here the parameter \( \varepsilon > 0 \) measures regional welfare differences. \( \varepsilon \) also gauges the size of regional shocks: if a location is hit by a shock, the utility stream it offers to the (median) household changes from \( \frac{1}{2} \varepsilon \) to \( -\frac{1}{2} \varepsilon \), or vice versa.

Given these assumptions, all households with a current match \( \theta > -\varepsilon \) derive a positive utility premium from residing in the desirable location. The measure of these households is \( 1 - G(-\varepsilon) > \frac{1}{2} \). In particular, if \( \theta_L > -\varepsilon \) and \( G(-\varepsilon) = 0 \), all households would rather live in the popular area. Since the measure of houses in the desirable location is one half, housing is in short supply in the popular region.

A household’s match may change over time. First, if the neighborhood or jurisdiction where the household resides is hit by a regional shock, the match between the household and the location is broken, and a new match is independently drawn from the distribution function \( G(\theta) \).\(^{20}\) Second, even if the overall popularity of the jurisdiction remains

\(^{19}\)As will become clear below, even households with low realizations of \( \theta \) may derive a positive premium from the desirable location. However, even if this is the case, households with low \( \theta \) loose less if they reside in the undesirable location than households with higher realizations of \( \theta \).

\(^{20}\)An underlying premise is that a location which was popular (unpopular) in period \( t \) and another
unaltered, between periods the match may change for some idiosyncratic, or household specific, reason\textsuperscript{21}, with probability $\lambda \in [0, 1]$, and the new match is independently drawn from the distribution $G(\theta)$. In particular, the assumption that the match is always independently drawn, when there is a regional shock and households’ wealth positions change, means that wealth and the match are independently distributed. This property simplifies the analysis. However, for our main results to go through, it is sufficient that the match sometimes, and somehow, changes, following either a regional or an idiosyncratic shock. In Section 5, we drop the assumption of independent draws, and the match is allowed to follow a general Markov process, with possibly different transition dynamics after a regional and an idiosyncratic shock.

Finally, the households live forever and discount future utilities by a common factor $\beta \in (0, 1)$.

In any period, the aggregate welfare is maximized, if all households with $\theta > \theta_m = 0$ are allocated to the (currently) desirable location, those with $\theta < 0$ live in the less desirable location, and the group (always of measure zero, if $G$ is continuous) with $\theta = 0$ is divided between the locations so that capacity constraints on housing are not violated. In other words, there is perfect residential sorting according to the match. If this allocation rule is followed, the aggregate utility in any period is $w^* = \frac{1}{2}E[\theta \mid \theta \geq 0]$.

### 3.2 Wealth dynamics

In the market outcome, the location choice depends on not only the match, but also on wealth. In this section, we study how a household’s wealth evolves over time.

The only choice available to the households is whether to own a house in the currently popular area or in the unpopular area: a household cannot sell a house without buying another one, and vice versa.\textsuperscript{22} We choose the minimum feasible level of housing wealth location which is popular (unpopular) in period $t + 1$ are likely to be “desirable” (“undesirable”) in different ways; thus it is plausible to assume that the match that the household had with the period-$t$ desirable (undesirable) location does not carry over to the period-$(t + 1)$ desirable (undesirable) area.

\textsuperscript{21}The match changes for similar reasons as in the search models by Wheaton (1990) and Williams (1995). Examples include change of household size or educational status and evolution in tastes when members of the household age.

\textsuperscript{22}These properties follow from our basic assumptions: (i) no household can be homeless (being homeless

15
as the origin and fix the value of a cheap home to 0. We also normalize the house price in a popular location to 1. This normalization means that house price swings, as well as capital gains and losses made in the housing market, are always of size unity. However, we shall below show how their magnitude under different circumstances can be measured in a meaningful way, by comparing them with the value of financial assets, and with average household wealth.

Consistent with empirical evidence, we assume that capital gains and losses made in the housing market (as well as idiosyncratic shocks affecting the match $\theta$) are uninsurable. The incomplete markets setting we consider here is the simplest possible one. In addition to owning a home, the households can carry wealth to the future by holding a single non-interest bearing financial asset, which can be interpreted as outside money. The real supply of money is $M/p$, where $M$ is the fixed nominal money supply, and $1/p$ is the price of money in terms of housing (in good locations); $p$ is endogenously determined in equilibrium.

We could also easily introduce pure credit, or inside money and allow the households to borrow up to a certain limit, without changing any of the results. Since the households have no income sources outside the housing market, the steady state interest rate cannot be positive; with a positive interest rate, a household with negative initial financial asset holdings exceeds any finite debt limit with a positive probability. Then in the steady state the interest rate is zero, so that inside and outside money are perfect substitutes. These simplifying assumptions (no income sources outside the housing market, and, by implication, zero interest rate) are adopted so as to focus on the role of capital gains and losses in wealth dynamics (see observation 4 in Section 2.1).

---

23 This possibility is briefly addressed in Section 3.4.
24 The (non-degenerate) asset market equilibrium of a pure credit economy (see Hugget 1993) necessarily involves some households having negative positions.
25 See Ljungqvist and Sargent (2004, Ch 17.10).
26 In models which analyze both the evolution of household wealth and recurrent housing choices, it is rather normal to adopt simplifying assumptions about wealth dynamics. For example Ortalo-Magné and Rady (2006a, 2006b) abstract from households’ consumption/saving decisions, by assuming that there is non-housing consumption only in the last period of life, when all accumulated wealth is consumed.
Denote financial asset holdings by $a$ and let $h$ be housing. $h$ is equal to 1, if the household owns a house in a desirable location, and equal to 0, if the house is in an undesirable location. We also define a household’s total wealth ($n$), which consists of both financial wealth (money) and housing wealth

\[ n_t = a_t + h_t. \]  

(3)

In any given period $t$, the household’s budget constraint is

\[ h_t + a_t = a_{t-1} + (1 - s_t)h_{t-1} + s_t(1 - h_{t-1}), \]  

(4)

where $s_t$ is an indicator function which is equal to 1 if there is a regional shock between periods $t - 1$ and $t$, and 0 otherwise. Combining (3) and (4) yields

\[ n_{t+1} = n_t + s_{t+1} (1 - 2h_t). \]  

(5)

Given our simple wealth dynamics, the household’s wealth position ($n$) changes if and only if the household makes a capital gain or suffers a capital loss in the housing market. If, prior to the shock, the household owned a property in a then unpopular location, ($h_t = 0$) the household makes a capital gain and climbs one rung in the wealth ladder; if the house was in an expensive area ($h_t = 1$) before the change of fortunes, the household suffers a loss and falls one rung down.

There is a lower limit for financial asset holdings $a_{\text{min}}$, that a household is not allowed to exceed. A simple and fairly natural normalization is adopted here by fixing the minimum balance to be zero, $a_{\text{min}} = 0$, but allowing a negative minimum balance, say $-b$, would just involve a change of origin, without altering the analysis or any of the results.\(^\text{27}\) Since the minimum wealth level is $n = 0$ (the minimum level of housing wealth is 0, and the minimum level of financial asset holdings is 0) and since households make capital gains and losses of size unity, we can now assume, without loss of generality, that wealth only

\(^{27}\)See Aiyagari (1994) or Ljungqvist and Sargent (2004, Ch. 17.10).
takes non-negative integer values \( n = 0, 1, 2, \ldots \). At wealth levels \( n \geq 1 \), a household may freely choose its housing location, and its wealth portfolio may consist of \( n \) units of real money balances and a cheap house \((h = 0)\), or \( n - 1 \) units of financial assets and an expensive home \((h = 1)\). If \( n = 0 \), the household owns a house in an undesirable location, \( h = 0 \), and since it has no money, \( a = a_{\min} = 0 \), it cannot afford a house in a desirable location: choosing \( h = 1 \) would imply \( a = -1 < a_{\min} \), and this is not allowed. The liquidity (or borrowing) constraint that limits a household’s location choices can be expressed as follows:

\[
h_t = 0 \text{ if } n_t = 0. \tag{6}
\]

### 3.3 The household’s problem

Consider the optimization problem of any one household. At each time \( t \) it chooses its location \( h_t \in \{0, 1\} \) so as to maximize the expected discounted utility stream

\[
E_\theta \sum_{t=0}^{\infty} \beta^t \left[ h_t \left( \frac{1}{2} \epsilon + \theta_t \right) - (1 - h_t) \frac{1}{2} \epsilon + \beta \left\{ (1 - \pi) \left[ (1 - \lambda) V(\theta, n) + \lambda V(n) \right] + \pi \left[ (1 - h_t) V(n + 1) + h V(n - 1) \right] \right\} \right],
\]

subject to (5) and (6). The problem can be conveniently presented in a recursive form. Let \( V(\theta, n) \) be the (ex post) value function of a household with current type \( \theta \) and current wealth \( n \). Also define the household’s ex ante value function \( V(n) = E_\theta [V(\theta, n)] \), which describes the household’s expected prospects when the household faces a shock (idiosyncratic or regional) and does not yet know its new match. The value function \( V(\theta, n) \) satisfies the Bellman equation

\[
V(\theta, n) = \max_{h \in \{0, 1\}} h \left( \frac{1}{2} \epsilon + \theta \right) - (1 - h) \frac{1}{2} \epsilon + \beta \left\{ (1 - \pi) \left[ (1 - \lambda) V(\theta, n) + \lambda V(n) \right] + \pi \left[ (1 - h) V(n + 1) + h V(n - 1) \right] \right\}, \tag{7}
\]

subject to (6). In the current period, the household’s utility is \(-\frac{1}{2} \epsilon\) or \(\frac{1}{2} \epsilon + \theta\), depending on its location choice. Its prospects for the next period are discounted by \(\beta\) and are given inside the curly brackets. With probability \((1 - \pi)(1 - \lambda)\) the household is not exposed to any shocks, and it will face the same value function \( V(\theta, n) \) as today. With the
complementary probability \([1 - (1 - \pi) (1 - \lambda)]\) the match is broken and the household’s prospects are captured by the ex ante value function. If the match changes for household specific reasons, the wealth of the household remains unaltered and future welfare is given by \(V(n)\). If there is a regional shock, not only the match changes, but also house prices rise or fall, and depending on housing location, the household makes a capital gain or suffers a capital loss, resulting in expected future welfare \(V(n + 1)\) or \(V(n - 1)\).

At each unconstrained wealth level \(n \geq 1\), the household chooses the desirable location if and only if

\[
\theta + \varepsilon > \pi \beta [V(n + 1) - V(n - 1)] .
\]  (8)

The condition (8) involves a useful decomposition of the decision problem into the consumption motive, figuring on the left-hand side, and the investment motive, visible on the right-hand side. The strength of the consumption motive depends only on the current match \(\theta\) and the measure of regional disparities \(\varepsilon\). If there were no need to care about the future, all households with \(\theta > -\varepsilon\) would choose the currently desirable region, while only those with \(\theta < -\varepsilon\) would (voluntarily) live in the less popular area. The downside of choosing a currently popular and expensive location is that a household may suffer capital losses, if regional house prices fall, and may then be borrowing constrained in the future, when the match \(\theta\) with an expensive location is better than today. By contrast, opting for a currently less popular and less expensive area entails the chance of making capital gains. These considerations are captured by the investment motive. Due to the investment motive, even some households with \(\theta > -\varepsilon\), i.e. households whose immediate benefits are higher in the desirable location, may voluntarily choose the unpopular area. As the right-hand side of (8) indicates, the strength of the investment motive depends on wealth and it does not depend on the current match.

At each wealth level \(n\), there is then a critical value of the match

\[
\theta^*_n = \begin{cases} 
\theta_H & \text{if } n = 0 \\
-\varepsilon + \pi \beta [V(n + 1) - V(n - 1)] & \text{if } n \geq 1 
\end{cases}
\]  (9)
and the household’s location choice rule assumes a simple threshold form:

\[
h(\theta, n) = \begin{cases} 
1 & \text{if } \theta > \theta^*_n \\
0 & \text{if } \theta \leq \theta^*_n 
\end{cases}
\]  

(10)

Figure 3 shows the critical match \(\theta^*_n\) with different values of \(n\) when \(\theta\) is uniformly distributed on \([-\frac{1}{2}, \frac{1}{2}]\), \(\varepsilon = 1\), \(\beta = .95\), and \(\pi = .3\). Clearly, \(\theta^*_n\) decreases with \(n\), and wealthier households are ready to choose the desirable location even with a more modest match. This is a general property of \(\theta^*_n\), and it stems from the fact that the ex ante value function is concave. (Concavity is proved in the appendix.) Also, this finding has a natural interpretation. Assets are valued since they provide the option to make unconstrained choices in the future. However, if a household is wealthy, additional assets are of less value: the more assets the household has, the more distant is the prospect of being borrowing constrained at some point in the future. To put it differently, the investment motive is more important for poor households than for wealthy households.

The appendix shows that for very wealthy households, the investment motive all but vanishes:

\[
\lim_{n \to \infty} \pi \beta \left[ V(n + 1) - V(n - 1) \right] = 0,
\]

(11)

and as a consequence \(\lim_{n \to \infty} \theta^*_n = -\varepsilon\). In particular, if \(\theta_L > -\varepsilon\) (and all households prefer the desirable location from the consumption point of view), there is a wealth level \(\pi < \infty\) such that \(\theta^*_n < \theta_L\) for \(n \geq \pi\) and all households with \(n \geq \pi\) choose the desirable location. In Figure 3, \(\theta_L = -\frac{1}{2} > -1 = -\varepsilon\), and \(\pi = 3\).

Essentially, the preceding discussion reveals that there is a pattern of two-dimensional residential sorting:

**Proposition 1** Residential sorting takes place both according to the match and according to wealth. Wealthy households and households with a high match realization tend to choose the popular location, while poor households and households with a low match realization live in the less desirable location.

Next we show that the relative strength of the consumption motive and the investment
motive, as well as location choices, depend on the size and frequency (persistence) of regional shocks.\footnote{The household’s location choice rule does not depend on the parameter $\lambda$. To understand this finding, notice that the consumption motive depends only on the current match, while the investment motive is unaffected, because every time a capital gain or loss is realized, the destruction of the match nullifies the effects of previous idiosyncratic shocks. The parameter $\lambda$ does, however, affect the household’s ex post prospects: differentiating (7) reveals that the more permanent the types are, the more the ex post value function $V(\theta, n)$ depends on the current match. Also, as shown in Section 4.2, it has an effect on the volume of residential mobility.}

The parameter $\varepsilon$ measures interregional welfare differences, and the size of regional shocks. An increase in $\varepsilon$ strengthens the households’ consumption motive to choose the desirable location in the current period. On the other hand, it also reinforces the incentives to accumulate assets (investment motive), since a household stands to lose more if it faces the borrowing constraint at some point in the future. However, since future losses are discounted and only occur by chance, while the higher welfare stream is available right away, the effect on the consumption motive dominates. Hence, the larger the regional differences or shocks, the more likely an unconstrained household chooses the currently desirable location:

\textbf{Lemma 1} For all $n \geq 1$, $\frac{d\theta^*}{d\varepsilon} < 0$.

\textbf{Proof} See the appendix. \hfill $\blacksquare$

The parameter $\pi$ measures the frequency of regional shocks. As $\pi$ does not affect the utility streams available in different locations, its changes have no impact on the consumption motive. However, a change in $\pi$ affects the investment motive. The higher $\pi$, the more likely a household living in the popular area suffers a capital loss, while the more likely a household living in the unpopular area makes a capital gain. Then, at any unconstrained wealth level, a household needs a better match before it chooses to live in the currently popular location:

\textbf{Lemma 2} For all $n \geq 1$, $\frac{d\theta^*}{d\pi} > 0$.

\textbf{Proof} See the appendix. \hfill $\blacksquare$
3.4 Equilibrium

The previous section showed how a household chooses its location based on its current wealth and its current match. On the other hand, a household’s current wealth depends on its past fortunes in the housing market and its past location choices. Then the long-run wealth distribution arises as a result of households’ moving policies. Location choices and the stationary wealth distribution together determine the long run equilibrium of the model.

Denote by \( f(n) \) the size of wealth class \( n \) and let \( f^h_a(n) \) be the frequency of households at wealth level \( n \), with house value \( h \in \{0,1\} \) and \( a = n - h \) units of financial assets. If there is a regional shock, all \( f(n) \) households which were previously in wealth class \( n \) either go up to \( n+1 \) or fall to \( n-1 \), depending on their house location. They are replaced by \( f^{0}_{n-1}(n-1) \) class \( n-1 \) households which have made a capital gain and \( f^{1}_{n+1}(n+1) \) class \( n+1 \) households which have suffered a capital loss. The wealth distribution is stationary if and only if

\[
f(n) \equiv f^{0}_{n}(n) + f^{1}_{n-1}(n) = f^{0}_{n-1}(n-1) + f^{1}_{n}(n+1)
\]

for all \( n \). We also consider the model version, with a continuum of atomistic regions. Between any periods, a measure \( \pi \) of the locations is hit by a regional shock, and the wealth distribution is stationary if and only if

\[
f(n) = (1 - \pi) f(n) + \pi (f^{0}_{n-1}(n-1) + f^{1}_{n}(n+1)).
\]

It is easy to conclude that (13) reduces to (12): as a consequence, both model variants have the same long-run wealth distribution and the same long-run equilibrium.\(^{29}\)

There are no wealth classes below 0 (i.e., \( f(n) = 0 \) for \( n < 0 \)) and at wealth level 0 the households can only choose an unpopular location (i.e., \( f^{1}_{-1}(0) = 0 \)). These restrictions

\(^{29}\) The environment that an individual household faces is identical in both model variants: there is a regional shock with probability \( \pi \). Then the households’ location choices, analyzed in Section 3.3, are identical in both cases.
and (12) then imply the sequence

\[ f^0_n(n) = f^1_n(n+1), \text{ for all } n \geq 0. \] (14)

In words, each group with \( n \) units of financial assets and a cheap home is of equal size as the group with the same amount of financial assets and an expensive house: the distribution of financial assets is identical in both locations. This property means that the economy (as characterized by the distribution of housing wealth and financial assets) looks exactly the same at the end of any given period and at the beginning of the subsequent period even if the popularity ranking of the locations is reversed.

To characterize the wealth distribution, note that the households’ location choice rule (10) implies

\[ f^0_n(n) = G(\theta^*_n) f(n) \text{ and } f^1_{n-1}(n) = (1 - G(\theta^*_n)) f(n). \] (15)

Using (15) in (14) we obtain a recursive equation

\[ f(n+1) = \gamma(n+1) f(n), \text{ for } n = 0, 1, ... \] (16)

where

\[ \gamma(n) \equiv \frac{G(\theta^*_{n-1})}{1 - G(\theta^*_n)}, \text{ for } n = 1, 2, ... \] (17)

Equation (16) and the sequence \( \gamma(n) \) determine the wealth distribution.\(^{30}\) By (9) we have \( G(\theta^*_0) = G(\theta_H) = 1 \) and thus \( \gamma(1) \geq 1 \). On the other hand, \( \gamma(n) \) is decreasing in \( n \) for all \( n \geq 1 \) and \( \lim_{n \to \infty} \gamma(n) = \frac{G\left(\theta_H\right)}{1 - G\left(\theta_H\right)} < 1 \). Thus, equation (16) indicates that the wealth distribution is single-peaked, with wealth classes in the middle having more mass than those on the tails, a property which is consistent with observed empirical wealth distributions. In particular, \( \lim_{n \to \infty} f(n) = 0. \(^{31}\)

Next, we proceed to studying the housing market equilibrium. The demand for housing in location \( h \) is given by \( \sum_{n=0}^{\infty} f^h_{n-h}(n), \ h \in \{0, 1\} \). Using (15), (16) and (17) it can be

\[^{30}\text{If } \theta_L > -\varepsilon, f(n) = 0 \text{ for all } n > \frac{1}{\varepsilon}.\]

\[^{31}\text{If } \theta_L > -\varepsilon, f(n) = 0 \text{ for all } n > \frac{1}{\varepsilon}.\]
seen that these sums converge.\textsuperscript{32} Then, by (14) and the restriction \( f_{1-1}^0 (0) = 0 \), we have
\[
\sum_{n=0}^{\infty} f_n^0 (n) = \sum_{n=0}^{\infty} f_{n-1}^1 (n).
\]
Given that the aggregate mass of households is unity
\[
\sum_{n=0}^{\infty} f (n) = \sum_{n=0}^{\infty} \left( f_n^0 (n) + f_{n-1}^1 (n) \right) = 1,
\]
it follows that
\[
\sum_{n} f_{n-h}^h (n) = \frac{1}{2}, \quad h \in \{0, 1\}.
\] (18)

But these equations indicate that the demand for housing, on the left-hand side, is equal to the supply of housing \( \left( \frac{1}{2} \right) \), in both locations. Households’ location choices combined with the endogenously arising long-run wealth distribution guarantee that housing markets clear: If few households willingly choose the less desirable location (in wealth classes \( n \geq 1 \) the threshold value \( \theta_n^* \) is low, and consequently \( \sum_{n=1}^{\infty} f_n^0 (n) \) is small), in the long-run equilibrium many households end up living there because they are borrowing constrained \( (f_0^0 (0) \) is large).

Given the results of Lemmas 1 and 2, we can analyze what happens to the wealth distribution when the size or the frequency of regional shocks changes.

**Lemma 3** When regional shocks become smaller (\( \varepsilon \) decreases) or more frequent (\( \pi \) increases), the wealth distribution shifts to the right, in the sense of first-order stochastic dominance.

**Proof** Define the cumulative distribution function \( F (n; \varepsilon, \pi) = \sum_{i=0}^{n} f (i) \). By Lemmas 1 and 2, the \( \theta_n^* \)-schedule shifts up when \( \varepsilon \) decreases or \( \pi \) increases. This then increases \( \gamma (n) \) in (17) so that by (16) the ratio \( f (n+1) / f (n) \) goes up for all \( n \geq 0 \). It follows that \( dF (n; \varepsilon, \pi) / d\varepsilon \geq 0 \) and \( dF (n; \varepsilon, \pi) / d\pi \leq 0 \), for each \( n \). ■

Taken together, Lemmas 1-3 allow us to assess whether the housing markets are cleared mainly through self-selection or through rationing via borrowing constraints.

**Remark 1** When regional shocks become smaller (\( \varepsilon \) decreases) or more frequent (\( \pi \) increases), more households willingly choose the less desirable location, and fewer households

\[
\frac{f_n^0 (n) / f_{n-1}^0 (n-1)}{f_{n-1}^1 (n) / f_n^1 (n)} = \frac{f_n^1 (n+1) / f_n^1 (n)}{f_{n-1}^1 (n) / f_{n-1}^1 (n)} = \frac{G(\theta_n^*)}{1 - G(\theta_n^*)} \equiv \hat{\gamma} (n).
\]
Convergence follows, since
\[
\lim_{n \to \infty} \hat{\gamma} (n) = \frac{G(\varepsilon)}{1 - G(\varepsilon)} < 1.
\]
live there because of the borrowing constraint.

In addition to the households’ location choice rule and the invariant wealth distribution, the third constituent of the equilibrium is the relative price of housing and financial assets, \( p \). To solve for \( p \), consider the asset market clearing condition

\[
E[a] = \frac{M}{p},
\]

(19)

where the left-hand is the aggregate demand for financial assets and the right-hand side is the net supply, equal to real outside money.\(^\text{33}\) Using (3) and the housing market equilibrium condition \( E[h] = \frac{1}{2} \), (19) can be rewritten as \( E[n] = \frac{1}{2} + \frac{M}{p} \), and the relative price of housing and financial assets is

\[
p = \frac{M}{E[n] - \frac{1}{2}}.
\]

(20)

Notice that \( p \) also measures the monetary size of capital gains and losses made in the housing market.\(^\text{34}\)

In particular, when \( \pi \) increases, or \( \varepsilon \) decreases, the investment motive becomes stronger (in relative terms), driving up the demand for financial assets, and their relative price \( 1/p \). As a result, the share of financial assets in total wealth, \( E[a]/E[n] = (E[n] - \frac{1}{2})/E[n] \), increases, while the share of housing (in popular locations), \( \frac{1}{2}/E[n] \), decreases. Also house price fluctuations become smaller, compared with household wealth, and a typical household is better equipped to withstand capital losses.\(^\text{35}\)

Remark 2 Assume that regional shocks become smaller (\( \varepsilon \) decreases) or more frequent (\( \pi \) increases). Then (i) the monetary size of house price fluctuations, \( p \), decreases, and

\(^{33}\)The equilibrium we establish here essentially resembles the equilibrium of the simple Bewley-type model considered by Ljungqvist and Sargent (2004, Ch 17.10.4), where outside money and inside money (credit) are perfect substitutes, and the interest rate is zero.

\(^{34}\)If the households are allowed to borrow in terms of financial assets, and the borrowing limit, denoted in monetary terms, is \(-B\), the asset market equilibrium condition reads \( E[a] = \frac{M+B}{p} \), and

\[
p = \frac{(M+B)}{(E[n] - \frac{1}{2})}.
\]

\(^{35}\)Notice that \( \lim_{n \to \infty} \frac{n+1}{n} f(n+1) = \lim_{n \to \infty} \gamma_n = \frac{G(-\varepsilon)}{1+\alpha(-\varepsilon)} < 1 \). Thus the sum \( E[n] \equiv \sum_{n=0}^{\infty} n f(n) \) converges, and \( E[n] \) is always finite.
(ii) the price fluctuations become smaller compared with household wealth, measured by average wealth, median wealth, or any other quantile of the wealth distribution.

**Proof** (i) The result follows from eq. (19) and Lemma 3. (ii) The size of house price fluctuations is normalized to 1. Lemma 3 implies that average wealth $E[n]$ increases (decreases) together with $\pi$ ($\varepsilon$); also any quantile $n_q$ of the wealth distribution, where $n_q = \max\{n\}$ such that $F(n) \leq q$, $q \in [0, 1]$, increases (or remains constant) when $\pi$ increases or $\varepsilon$ decreases. ■

4 Residential sorting

4.1 Main patterns

This section studies the aggregate behavior of the economy. We begin by analyzing social welfare. Addressing this normative issue will then allow us to characterize the form of residential sorting emerging in equilibrium, since in the present model high social welfare is associated with location choices based on the match, rather than wealth. The ultimate objective of the section is to establish a relation between the size of house price fluctuations and the pattern of residential sorting, so that the predictions of the model can be compared with the empirical observations (P1) and (P2) presented in Section 2.2.

As a first step we show that the average match in the desirable location, $E[\theta | h = 1]$, can be used as a measure of social welfare. Consider any given period. Since the households choose their location according to the threshold rule (10), the average utility at wealth level $n$ is

$$u(n) = -G(\theta_n^*) \frac{1}{2} \varepsilon + (1 - G(\theta_n^*)) \left( \frac{1}{2} \varepsilon + E[\theta | \theta \geq \theta_n^*] \right).$$

Summing over all wealth classes, and taking into account that $\sum_n f(n) G(\theta_n^*) = \sum_n f(n) = \frac{1}{2}$ (by the housing market equilibrium (18)), yields the overall welfare in any given period:

$$w = \sum_{n=0}^{\infty} f(n) u(n) = \sum_{n=1}^{\infty} f_{n-1} (n) E[\theta | \theta \geq \theta_n^*] = \frac{1}{2} E[\theta | h = 1]. \quad (21)$$
An alternative way to approach social welfare is to imagine that a new household enters the economy. The entrant is assigned to wealth class \( n \) with probability \( f(n) \), and its expected intertemporal prospects are then given by the (ex ante) value function \( V(n) \). The household’s prospects ex ante, i.e. before it knows its wealth and its match, are

\[
W = \sum_{n=0}^{\infty} f(n) V(n).
\]

The appendix shows that these two measures of social welfare are equivalent, up to a constant multiplier: \( W \) is the present value of a program with a (constant) per-period payoff \( w \),

\[
W = w/(1 - \beta).
\]

This equality is also needed in the proof of the following proposition.

**Proposition 2** Social welfare increases, when (i) the size of regional shocks \( \varepsilon \) decreases, or (ii) regional shocks become more frequent \( \pi \) increases.

**Proof** See the appendix.

Intuitively, social welfare is high, when housing markets are cleared through self-selection rather than borrowing constraints; see Remark 1. Combing Proposition 2 and Remark 2 also reveals that small (large) house price fluctuations tend to be associated with high (low) levels of social welfare.

The normative Proposition 2 is next used as a building block, as we characterize residential sorting in the match dimension, and the relation between sorting and house price fluctuations.

**Proposition 3** When (i) the size of regional shocks \( \varepsilon \) decreases or (ii) the regional shocks become more frequent \( \pi \) increases, the degree of residential sorting in the match dimension increases in the following sense. (a) In each location \( h \in \{0, 1\} \), the average match \( E[\theta | h] \) becomes more distinct from the economywide average \( E[\theta] \). (b) The locations become more distinct from each other and the between-locations variance of the match increases. (c) The locations become internally more homogenous in the sense that
the within-location variance of the match decreases.

**Proof** When conditions (i) and/or (ii) hold, it follows from Proposition 2 that $E[\theta | h = 1]$ increases. (a) Then, since $\frac{1}{2}E[\theta | h = 1] + \frac{1}{2}E[\theta | h = 0] = E[\theta]$, and $E[\theta]$ is a constant, it follows that $E[\theta | h = 0]$ decreases. Thus the difference $|E[\theta | h] - E[\theta]|$ increases for $h \in \{0, 1\}$. (b) Item (a) implies that the between-locations variance $\text{Var}(E[\theta | h]) = \frac{1}{2}(E[\theta | h = 0] - E[\theta])^2 + \frac{1}{2}(E[\theta | h = 1] - E[\theta])^2$ increases. (c) The economywide variance of the match $\text{Var}(\theta)$ can be decomposed $\text{Var}(\theta) = \text{Var}(E[\theta | h]) + E[\text{Var}(\theta | h)]$. Since $\text{Var}(\theta)$ is a constant, it follows from item (b) that the within-locations component $E[\text{Var}(\theta | h)]$ must decrease. ■

**Corollary 1** The smaller or the more frequent the regional shocks are, (i) the smaller are house price fluctuations and (ii) the more residential sorting there is in the match dimension.

**Proof** The result follows from Proposition 3 and Remark 2. ■

Next we proceed to analyzing sorting in the wealth dimension. Above we noted that the distribution of financial assets is identical in both location types. Then given that $E[a | h = 1] = E[a | h = 0]$ interregional wealth differences derive entirely from different house values

$$E[n | h = 1] - E[n | h = 0] = E[h | h = 1] - E[h | h = 0] = 1. \quad (24)$$

To assess the magnitude of these interregional wealth differences in a meaningful way, we compare them with typical household wealth in the economy:

**Proposition 4** When regional shocks become larger ($\varepsilon$ increases) or less frequent ($\pi$ decreases), interregional wealth differences become larger compared with typical household wealth, as measured by (economywide) average wealth, median wealth or any other quantile of the (economywide) wealth distribution.

**Proof** The result follows from equation (24) and Lemma 3. ■

28
The following proposition is about polar cases.

**Proposition 5** (a) When $\varepsilon \to 0$ or $\delta \equiv \frac{\pi \beta}{1-\beta(1-\pi)} \to 1$, there is perfect sorting in the match dimension (and no sorting in the wealth dimension). In any given period, a household chooses the desirable location if and only if $\theta > \theta_m$. (b) If $\theta_L + \varepsilon > \pi \beta \frac{E[\theta]-\theta_L}{1-\beta}$, there is perfect sorting in the wealth dimension (and no sorting in the match dimension). A household resides in the less desirable location if and only if it is borrowing constrained.

**Proof** See the appendix.

The equilibrium pattern of residential sorting, with different values of $\varepsilon$, is illustrated in Figure 4. In each panel, the cumulative wealth distribution is measured on the horizontal axis, and the cumulative match distribution on the vertical axis. Then area has a simple frequency mass interpretation (with one quarter of the area of the unit square corresponding to one quarter of the households etc.). The figure shows a clear pattern, with the degree of residential sorting in the match dimension decreasing, and the degree of wealthwise sorting increasing, as the size of the regional shocks grows. Also the magnitude of house price fluctuations, measured by $P \equiv \frac{1}{2} \frac{1}{E[\theta]}$, $P \in [0,1]$, grows together with the size of the shocks (see Remark 2). Panels a (no shocks) and d (large shocks) correspond to polar cases, with perfect sorting in the match dimension and in the wealth dimension, respectively (and no sorting in the complementary dimension). Panels b and c are intermediate cases, with shocks of intermediate size, and imperfect sorting along both dimensions. A similar set of figures could be also presented with respect to $\pi$.

The pattern of residential sorting that emerges in equilibrium essentially reflects the relative strength of the consumption motive and the investment motive of housing. If the consumption motive dominates (large $\varepsilon$ and/or small $\pi$), the households choose their location mainly by comparing current benefit streams, and few of them willingly pick the less desirable region. Then the regional allocation of households basically boils down to differences in wealth: typically a household resides in a cheap location if and only if it cannot afford a more expensive home. When the investment motive is stronger, compared with the consumption motive (small $\varepsilon$ and/or large $\pi$), many households, which would receive a larger immediate welfare stream from the desirable location, voluntarily choose
the less desirable location, so as to make capital gains, and to avoid capital losses and future credit constraints. Typically, a household lives in the desirable location only if its current match with that location is truly good. The regional allocation of households then happens mainly through self-selection, according to the match, rather than based on wealth differences and borrowing constraints.

To connect the model to empirics, we invoke the interpretation that the match $\theta$ can be thought of as reflecting various socioeconomic characteristics of the household (other than wealth) such as age, education, income or household size. Here we address two implications of the model.\textsuperscript{36}

First, if house price fluctuations are large, the match distribution in an individual location should roughly replicate the economywide distribution. If price fluctuations are small, the local match distribution should differ more clearly from the aggregate distribution. (See Proposition 3, especially item (a), and Corollary 1; see also Figure 4.) These predictions appear to be roughly consistent with observation (P1): If house prices have been volatile in a metropolitan area (an individual location), the shares of different age, education and income groups in the MSA tend to largely correspond to the average shares in the US (the aggregate economy). If prices have been relatively stable, the MSA tends to deviate more from the US average.

Second, if house prices are volatile, different locations should have largely similar match distributions. If house price fluctuations are small, there should be more sorting in the match dimension, and the locations should differ from each other. (See Proposition 3, especially items (b) and (c) and Corollary 1; see also Figure 4.) These predictions are largely congruent with observation (P2): The size of house price fluctuations is negatively correlated with the degree of residential sorting within a metropolitan area. The larger the house price fluctuations, the less the municipalities (individual locations) within the MSA (the aggregate economy) tend to differ from each other with respect to age, education and income.

\textsuperscript{36}As the reader will notice, when linking the locations of the model to real-world economic and geographical units, two different interpretations of the model are used.
4.2 Movers and stayers

In our analysis of US data, we observed that, among owner-occupying households, movers are geographically more sorted than stayers in terms of age, education and income (observation (P3) in Section 2.2). Also, the empirical mobility literature has found an interesting non-linear (humpshaped) relationship between wealth and mobility (observation (P4) in Section 2.2). In this section, we show that the empirical predictions of our model are consistent with these observations.

We begin by demonstrating a simple humpshaped relation between wealth and mobility. Take any given wealth class \( n \). At the beginning of any period, the portion \( 1 - G(\theta^*_n) \) of households own a house in the desirable location; since equations (14) hold in the steady state, this is true even after a regional shock. Between any two periods, \( (1 - s) \lambda + s \) households are hit by a shock, which breaks their match. Then the share \( ((1 - s) \lambda + s) G(\theta^*_n) \) of the households, which are in the popular area at the beginning of the period, get a realization \( \theta < \theta^*_n \) and move to the unpopular area. Therefore, mobility from the desirable to the undesirable location in wealth class \( n \) is equal to \( ((1 - s) \lambda + s) G(\theta^*_n)[1 - G(\theta^*_n)] \).

Likewise, it is easy to conclude that mobility from the undesirable to the desirable location equals the same measure. Then overall mobility in wealth class \( n \) is

\[
\mu(n) = ((1 - s) \lambda + s) 2G(\theta^*_n)[1 - G(\theta^*_n)].
\]

Clearly, there is more mobility in those periods when the economy is hit by a regional shock and \( s = 1 \). Under the atomistic locations interpretation, in any given period, mobility at wealth level \( n \) is \( \overline{\mu}(n) = ((1 - \pi) \lambda + \pi) 2G(\theta^*_n)[1 - G(\theta^*_n)] \). Notice also that in the two-region case, \( \overline{\mu}(n) \) is the long-run average mobility at wealth level \( n \).

Essentially, \( \mu(n) \) or \( \overline{\mu}(n) \), defines a humpshaped relation between wealth and mobility:\(^{37}\)

\(^{37}\)Notice that the measure \( \mu(n) \) (or \( \overline{\mu}(n) \)) answers the following question: Assume that a household has wealth \( n \) in a given period \( t \). What is the probability that the household moves during the period? An alternative question might be: What is the probability that the household lives in different locations in period \( t \) and in period \( t + 1 \)? The answer to this question is an alternative mobility measure \( \tilde{\mu}(n) = (1 - st_{t+1}) \lambda 2G(\theta^*_n)[1 - G(\theta^*_n)] + st_{t+1} [G(\theta^*_n)G(\theta^*_{n+1}) + (1 - G(\theta^*_n)) (1 - G(\theta^*_{n-1}))] \). If there is no regional shock between periods \( t \) and \( t + 1 \) (that is, \( st_{t+1} = 0 \), there is a humpshaped relation between wealth
Proposition 6 Assume that $\pi \geq 2$. Then (i) mobility is increasing in wealth at low wealth levels, and decreasing in wealth at high wealth levels, and (ii) households with intermediate levels of wealth are more mobile than rich households and poor households. In particular, borrowing constrained households are completely immobile. If $\theta_L > -\varepsilon$, so that $\pi < \infty$, all households with $n \geq \pi$ are immobile.

Proof Equation (25) implies that the measure of mobility $\mu(n)$ (or $\pi(n)$) is a downward opening parabola, with its peak at $G(\theta_n) = \frac{1}{2}$. Also $\mu(n) = 0$ at the extreme points $G = 0$ and $G = 1$. According to Proposition 1, $\theta_n^*$, and thus $G(\theta_n^*)$, is decreasing in $n$. Also, $G(\theta_n^*) > \frac{1}{2}$ at low values of $n$, with $G(\theta_0^*) = 1$. On the other hand $G(\theta_n^*) < \frac{1}{2}$ at high levels of $n$, since $\lim_{n \to \infty} \theta_n^* = -\varepsilon$ and $G(-\varepsilon) < \frac{1}{2}$. In particular, if $\theta_L > -\varepsilon$ we have $G(\theta_n^*) = 0$ for all $n \geq \pi$, where $\pi < \infty$. ■

This pattern of mobility essentially reflects the varying strength of the investment motive at different wealth levels. Rich households, with a weak investment motive, want to live in the popular location with most match realizations, and only rarely find it optimal to move. Poor households typically stay in the unpopular location; for the borrowing constrained this is obviously the only alternative. At intermediate levels of wealth, the investment motive is neither extremely strong nor very weak; these households choose the expensive (cheap) location with high (low) realizations of $\theta$, and when the match is broken, they often find it optimal to change location.

Next we proceed to comparing the degree of residential sorting among movers and stayers. In any given period, we classify as a mover a household which has moved during that period. The following results are proved in the appendix.

Proposition 7 (a) In both location types, movers have a better match with their (new) home region than stayers, in the sense of first order stochastic dominance. (b) Movers are more sorted than stayers in the match dimension.

When interpreting item (a) of the proposition, remember that a good match with a cheap location means that a household has a low realization of $\theta$.

and mobility, as measured by $\tilde{\mu}(n)$. If there is a regional shock ($s_{t+1} = 1$), the relation may take many possible forms, including humpshaped and monotonously increasing.
Item (a) reflects the fact that those who move from one location to another tend to have rather strong match-related reasons to make that choice, while those who stay put may do so largely because they have been lucky or unlucky in the housing market. For example, households which move from the desirable location to the less desirable location, choose a cheap area, although they could afford a more expensive house (their former home). As a result, in a cheap location newcomers tend to have low realizations of $\theta$. By contrast, at least a part of the old residents live in a cheap area because they have suffered capital losses in the housing market, and are now borrowing constrained; among this group there are households with a high realization of $\theta$, and a poor match with the (currently) unpopular location. Likewise, households which move from a cheap location to an expensive location tend to have high current realizations of $\theta$, while among old residents there are households which have a more modest match with the (now) popular area. These stayers with lower realizations of $\theta$ have been lucky in the timing of housing market transactions: many of them have bought their home before the rise of local house prices.\footnote{More generally, and more formally, the appendix shows that in cheap locations, the wealth distribution of movers first order stochastically dominates the wealth distribution of stayers, while in the expensive locations, the opposite is true.} Item (b) is a rather straightforward corollary of item (a). Since movers are better matched with their home region than stayers in both location types, movers are obviously more sorted than stayers.

5 More general match dynamics

In this section, we drop the assumption that, after a shock, the match is independently drawn, and allow the match to follow a general Markov process. Compared with the basic model, this extension brings about two major changes. First, the strength of the investment motive now varies with the match, and we can address issues such as attachment to the home region and household specific plans to move. Second, in the long-run equilibrium, wealth and the match are (typically) correlated, rather than independently distributed. Empirically, the interpretation that the match may reflect various socioeco-
nomic characteristics of the household becomes more plausible, when the match can be serially correlated in a general way. While the equilibrium pattern of residential sorting is more complex than in the basic setting, the main results (Propositions 2, 3 and 4, Corollary 1) carry over; in particular there is a negative relation between house price fluctuations and sorting in the match dimension.

There are \( J \geq 2 \) different match realizations. If the match changes for idiosyncratic, or household specific, reasons \((s = 0)\), the transition probabilities from one match to another are given by a transition matrix \( \Lambda_0 \). If there is a regional shock \((s = 1)\), the transition probabilities are (possibly) given by a different matrix \( \Lambda_1 \).

To guarantee the existence of a stationary joint distribution for wealth and the match, we adopt the small region interpretation of the model, and assume that there is a continuum of atomistic locations. In each period, a measure \( \pi \) of the matches is broken due to regional shocks, and a measure \( \lambda \) for household specific reasons. Let \( \pi = \xi \sigma \) and \( \lambda = (1 - \xi) \sigma \), where \( \sigma \in (0, 1) \) is the overall probability that the match is broken, and \( \xi \in (0, 1] \) measures the relative frequency of regional and idiosyncratic shocks. The parameter \( \sigma \) can be interpreted as reflecting the overall degree of turbulence in the economy. The stationary marginal distribution of the match is defined as the eigenvector associated with a unit eigenvalue of \( \Lambda' \), where \( \Lambda \equiv (1 - \xi) \Lambda_0 + \xi \Lambda_1 \).\(^{39}\) Notice that if the frequency of shocks \((\sigma)\) changes, but the relative probabilities of regional and idiosyncratic shocks \((\xi \text{ and } 1 - \xi)\) remain constant, the stationary match distribution is unaltered.

Next we proceed to households’ location choices. The value function \( V(\theta, n) \) satisfies the Bellman equation

\[
V(\theta, n) = \max_{h \in \{0, 1\}} \left\{ h \left( \frac{1}{2} \varepsilon + \theta \right) - (1 - h) \frac{1}{2} \varepsilon + \beta \left\{ (1 - \sigma) V(\theta, n) \right\} + \lambda E_{\tilde{\theta}} \left[ V(\tilde{\theta}, n) \mid \theta, s = 0 \right] + \pi E_{\tilde{\theta}} \left[ (1 - h) V(\tilde{\theta}, n + 1) + h V(\tilde{\theta}, n - 1) \mid \theta, s = 1 \right] \right\},
\]

subject to (6). At any unconstrained wealth level \( n \geq 1 \), the household chooses a currently

\(^{39}\)We assume that the matrix \( \Lambda \) is indecomposable, so that it induces a unique long-run match distribution, but otherwise we do not impose any restrictions on the structure of the stochastic matrices \( \Lambda_0 \) and \( \Lambda_1 \).
desirable location if and only if

\[ \theta + \varepsilon > \pi \beta E_0 \left[ V\left( \bar{\theta}, n + 1 \right) - V\left( \bar{\theta}, n - 1 \right) \mid \theta, s = 1 \right] . \]  

(27)

Importantly, the investment motive, figuring on the right-hand side of (27) now depends on the current match \( \theta \) (and on the distribution of future matches \( \tilde{\theta} \), conditional on the current match). Thus, unlike in the basic model presented in Section 3, two households with the same level of wealth \( n \), but a different match, may differ with respect to the strength of the investment motive.

The following proposition characterizes households’ location choices.

**Proposition 1’**  
(i) For all \( \theta > -\varepsilon \), there exists \( n^*(\theta) \geq 1 \), such that \( h(\theta, n) = 0 \) if \( n < n^*(\theta) \) and \( h(\theta, n) = 1 \) if \( n \geq n^*(\theta) \). (ii) \( n^*(\theta_H) = 1 \). (iii) For all \( \theta < -\varepsilon \), and all \( n \), \( h(\theta, n) = 0 \).

Part (i) of the proposition indicates, that conditional on the match, there is perfect sorting in the wealth dimension. As in the basic model, the investment motive gets weaker when the household accumulates more assets. Then, given that \( \theta > -\varepsilon \), wealthy households reside in expensive locations while poor households live in cheap locations.\(^{40}\) Unlike in the basic model, there is not necessarily perfect sorting in the match dimension, conditional on wealth. At the ends of the match distribution, location choices are similar as in the basic model: A household with the highest possible match realization (\( \theta_H \)) resides in a popular location, whenever it can do so (item (ii)).\(^{41}\) Also, households with \( \theta < -\varepsilon \), which prefer unpopular locations for consumption reasons, always live in a cheap area.

\(^{40}\)Proving this (rather obvious) property is straightforward. Here is a sketch of the proof. Consider a household facing a finite horizon problem. It can be shown that in any period \( t \), (a) location choices are characterized by match-specific threshold rules \( n^*_t(\theta) \) and (b) the difference \( V_t(\theta, n + 1) - V_t(\theta, n - 1) \) is non-increasing in \( n \), for all \( \theta \). These results are first proved for the final period \( t = T \). Moving backward in time, it is straightforward to prove by induction that (a) and (b) hold in any period \( t \). Finally, when the time horizon is very long (\( T \rightarrow \infty \)), location choices in the early periods approach the optimal choices in the infinite horizon economy.

\(^{41}\)A household with \( \theta > -\varepsilon \) may choose an unpopular location for investment reasons, as it does not want to be borrowing constrained in the future, when the match may be better than today. Obviously there is no reason to defer gratification, if the current match is the best possible one. This result is easy to prove by induction; see footnote 41.
In the intermediate range $\theta \in (-\varepsilon, \theta_H)$, it is possible, however, that $n^*(\theta_i) > n^*(\theta_j)$ for some $\theta_i > \theta_j$. Then for $n \in [n^*(\theta_j), n^*(\theta_i))$, $h(\theta_i, n) = 0$ and $h(\theta_j, n) = 1$; that is, the household with the lower match realization ($\theta_j$) resides in an expensive area, while the household with the higher realization ($\theta_i$) lives in a cheap area. This possibility arises, since $\theta_i$ may have a stronger investment motive than $\theta_j$.

As an example, consider a situation with four match realizations $\theta_1 < -\varepsilon < \theta_2 < \theta_3 < \theta_4$ and a transition matrix

$$
\Lambda_1 = \begin{bmatrix}
\zeta & 1 - 3\zeta & \zeta & \zeta \\
1 - 3\zeta & \zeta & \zeta & \zeta \\
\zeta & \zeta & \zeta & 1 - 3\zeta \\
\zeta & \zeta & 1 - 3\zeta & \zeta
\end{bmatrix}
$$

where $\zeta$ is very small (close to zero). (For simplicity we assume that $\lambda = 0$, so that there is no need to specify $\Lambda_0$.) Given the match dynamics $\Lambda_1$, in numerical simulations it is easy to pick parameter values $\{\theta_i\}_{i=1}^4, \varepsilon, \pi$ and $\beta$, such that $n^*(\theta_3) > n^*(\theta_2)$, although $\theta_3 > \theta_2$. (Consistent with Proposition 1’, $n^*(\theta_4) = 1$ and $n^*(\theta_1) = \infty$.) Here type $\theta_2$ has a weak investment motive, since after a regional shock its new match realization is likely to be $\theta_1 < -\varepsilon$: these households are attached to the home area, and it is unlikely that they want to move even if the home region becomes “undesirable”. Then it does not matter if local house prices fall, since the households do not intend to sell.42 Type $\theta_3$ has a strong investment motive, as it wants to live in an expensive area in the future, when the match is likely to be $\theta_4 = \theta_H$. For these households, a major function of the current house is to serve as a stepping stone to the future home. As the example illustrates, in many cases the connection between the match and the strength of the investment motive may be interpreted as reflecting household specific needs to move. The investment motive tends to be strong, if the household plans to move in the near future. The investment motive tends to be weak if the household intends to stay put for a long time.

Next we consider the long-run joint distribution of wealth and the match. As the new

42In a similar vein, Sinai and Souleles (2005) argue that owner-occupation is not risky, if a household plans to stay put for a long time.
match is not independently drawn after a shock, in general wealth and the match are not independently distributed.\footnote{Correlation arises, since (i) the current wealth position depends on past location choices (and luck), (ii) past location choices were influenced by past match realizations, and (iii) the current match is correlated with past match realizations. The vector difference equation, which implicitly defines the long-run joint distribution is presented in the appendix. The appendix also establishes the equilibrium of the model.}

The fact that wealth and the match can be correlated is an appealing feature in the following sense: In this paper we argue that under wealthwise sorting, regions and local jurisdictions tend to be internally rather heterogenous in the match dimension (i.e. with respect to socioeconomic characteristics such as age, education, income or household size). Preferably, this property should arise endogenously, rather than by assumption. In the structure of the extended model, there is nothing that rules out the possibility that, say, high (low) match realizations are overrepresented among wealthy (poor) households. That is, in principle a high degree of sorting in the wealth dimension could be associated with a high degree of sorting in the match dimension.

Under perfect sorting in the wealth dimension, the independence of wealth and the match emerges endogenously. In such a situation, all households at a given wealth level make the same location choice. However, if location choices do not depend on the match, also wealth dynamics are independent of $\theta$, and the (long-run) distributions must be independent. In other words, a household’s current wealth position reflects the household’s past fortunes in the housing market and bears no relation to the household’s inherent characteristics (captured by the match). In equilibrium households which reside in expensive locations are wealthier than households which live in cheap locations, but otherwise these geographically separated groups do not differ: perfect sorting in the wealth dimension is associated with no sorting in the match dimension. (It is worth reemphasizing that if there is not perfect sorting in the wealth dimension, wealth and the match are typically correlated in equilibrium.)

Overall, since the strength of the investment motive can depend on the household’s match, and since wealth and the match can be correlated, the pattern of residential sorting that emerges in equilibrium can be quite complex. Nevertheless, the pattern of sorting still
reflects the relative importance of the consumption motive and the investment motive in the location choice. The appendix proves that the main results of the paper, Propositions 2, 3 and 4, and Corollary 1, still hold, with the exception that $\pi$ is substituted by $\sigma$. If $\lambda = 0$, so that there are no idiosyncratic shocks, these results hold verbatim.

6 Conclusions

Recent empirical evidence indicates that local jurisdictions are internally more heterogeneous, and less distinct from each other, than standard economic models of residential sorting predict. Motivated by these findings, this paper developed a dynamic model of two-dimensional sorting, with the following main properties. (i) A household’s location choice depends both on its current wealth, and on its current “match”, where the “match” may reflect various socioeconomic characteristics of the household. (ii) For an owner-occupying household, a house is both a consumption good and an investment, and location choice involves both aspects. (iii) Regional house prices fluctuate, and the resulting capital gains and losses affect household wealth. (iv) After suffering capital losses, a household may face a borrowing constraint, which prevents mobility from a cheap housing location to an expensive location.

The pattern of residential sorting that emerges in equilibrium depends on the relative strength of the consumption motive and the investment motive of housing. The potency of these two motives is shown to depend on the size of and frequency (or persistence) of regional shocks. (When regional shocks, which affect the benefit streams available in the different locations, are large, or when these shocks are persistent, the consumption motive is strong in comparison with the investment motive.) When the consumption motive is strong, in comparison with the investment motive, the households essentially care about today, rather than worry about tomorrow. Then residential sorting takes place primarily in the wealth dimension. In each period, the wealthiest households live in the currently desirable, and expensive locations, while those who reside in the less popular areas do so because they cannot afford a more expensive home. With this pattern of wealthwise sorting, neighborhoods or local jurisdictions are internally heterogenous with respect to
socioeconomic characteristics other than wealth: neighbors may have little in common apart from the value of their home.

When the investment motive is strong, compared with the consumption motive, there is sorting according to the match. Households, which care about their future prospects, voluntarily choose a location which is currently unpopular and cheap, but where property values may rise in the future, and only live in the currently popular area, when their match with that location is truly good. Then, given the empirical interpretation of the match, neighbors should be alike.

The model produces two main empirical predictions. First, the size of house price fluctuations should be negatively correlated with the degree residential sorting in the match dimension. Second, movers should be more sorted than stayers. These predictions are consistent with evidence from US metropolitan areas when income, age and education are used as proxies for the “match”.

Mathematical Appendix

Location choice

The household’s decision problem boils down to the choice of the sequence of optimal thresholds $\theta^*_n$. Since $x_n = G(\theta^*_n)$ is a monotonous function of $\theta^*_n$, also $x_n$ can be treated as a choice variable, and the household’s decision problem (equations (7)-(10)) can be summarized by the following Bellman equation

$$V(n) = \max_{x_n} u(x_n) + \beta \{(1 - \pi)V(n) + \pi[x_nV(n + 1) + (1 - x_n)V(n - 1)]\}, \quad (28)$$

subject to $x_0 = 1$, where

$$u(x_n) \equiv \left(\frac{1}{2} - x_n\right)\varepsilon + \int_{x_n}^1 G^{-1}(x)dx$$

is the expected utility stream at wealth level $n$. Notice that $\frac{d^2u(x_n)}{dx_n^2} = -\frac{1}{G'(\theta^*_n)} < 0$. Thus (28) defines a maximization problem with a concave objective function and linear
constraints. As a consequence the value function \( V(n) \) is concave.

We also show that the condition (11) holds, or equivalently \( \lim_{n \to \infty} \theta^*_n = -\varepsilon \). If not, then \( \lim_{n \to \infty} \theta^*_n = \hat{\theta}^* > -\varepsilon \). Since \( \theta^*_n \) is a non-increasing sequence, and, by assumption, the feasible values of \( \theta^*_n \) lie on a finite interval, \( \theta^*_n \in \left[ \hat{\theta}^*, \theta_H \right] \), we have \( \lim_{n \to \infty} (\theta^*_{n+k} - \theta^*_n) = 0 \) for all finite, positive integers \( k \geq 1 \). But then \( \lim_{n \to \infty} (u_{n+k} - u_n) = 0 \) for all \( k \geq 1 \). As a consequence, \( \lim_{n \to \infty} [V(n+1) - V(n-1)] = 0 \), and \( \lim_{n \to \infty} \theta^*_n = -\varepsilon \). A contradiction.

Let \( v(n) \equiv V(n+1) - V(n-1) \) and \( \Delta x_n \equiv x_{n+1} - x_{n-1} \); since \( \theta^*_n \) is a non-increasing sequence, \( \Delta x_n \in [-1, 0] \). Also define the operator \( L \)

\[
L [z(n)] = (1 - \pi) z(n) + \pi [x_{n+1} z(n+1) + (1 - x_{n-1}) z(n-1)],
\]

where \( z(n) \) is a generic function of \( n \). Since \( V(n) \) satisfies the recursive equation (28), \( v(n) \) satisfies the recursive equation

\[
v(n) = \int_{x_{n+1}}^{x_{n-1}} G^{-1}(x) dx - \Delta x_n \varepsilon + \beta L [v(n)].
\]  

(29)

Finally, the expression for \( \theta^*_n \), eq. (9) can be rewritten as \( \theta^*_n = Q(n; \varepsilon, \pi) \equiv -\varepsilon + \pi \beta v(n) \) for \( n \geq 1 \).

**Proof of Lemma 1** Define \( q^\varepsilon(n) \equiv \frac{dv(n)}{d\varepsilon} \). Differentiating (29) yields \( q^\varepsilon(n) = -\Delta x_n + \beta L [q^\varepsilon(n)] \). (Notice that indirect effects can be ignored due to the envelope theorem.) Let \( q_\text{max}^\varepsilon = \max q^\varepsilon(n) \) and \( n^\varepsilon = \arg \max q^\varepsilon(n) \). Now \( q_\text{max}^\varepsilon \leq -\Delta x_n + \beta q^\varepsilon \max (1 + \pi \Delta x_n) \), and \( q_\text{max}^\varepsilon \leq \frac{-\Delta x_n + \varepsilon}{1 - \beta (1 + \pi \Delta x_n)} \leq \frac{1}{1 - \beta (1 - \pi)} \). Finally \( \frac{d\theta^*_n}{d\varepsilon} = \frac{dQ(n; \varepsilon, \pi)}{d\varepsilon} = -1 + \pi \beta q^\varepsilon(n) \leq -1 + \pi \beta q_\text{max}^\varepsilon \leq -\frac{1 - \beta}{1 - \beta (1 - \pi)} < 0 \). □

**Proof of Lemma 2** Define \( q^\pi(n) = \frac{d\pi v(n)}{d\pi} \). Then multiplying both sides of (29) by \( \pi \), differentiating the resulting equation by \( \pi \), and simplifying, yields \( q^\pi(n) = (1 - \beta) v(n) + \beta L [q^\pi(n)] \). Let \( q^\pi_\text{min} \equiv \min q(n) \) and \( n^\pi = \arg \min q(n) \). Now \( q^\pi_\text{min} \geq (1 - \beta) v(n^\pi) + \beta q^\pi_\text{min} (1 + \pi \Delta x_n^\pi) \), and \( q^\pi_\text{min} \geq \frac{(1 - \beta) v(n^\pi)}{1 - \beta (1 + \pi \Delta x_n^\pi)} \geq \frac{(1 - \beta) v(n^\pi)}{1 - \beta (1 - \pi)} > 0 \). Finally \( \frac{d\theta^*_n}{d\pi} = \frac{dQ(n; \varepsilon, \pi)}{d\pi} = \beta q^\pi(n) \geq \beta q^\pi_\text{min} > 0 \). □
Proof of Proposition 2

(i) We begin by deriving equation (23), which is needed in the proof of the proposition. Using vector notation, equation (28) can be rewritten as follows

\[ V = \max_{\{x_n\}} u + \beta [(1 - \pi) I + \pi A] V \]  

(30)

for \( n \geq 1 \) (and \( x_0 = 1 \)) where \( V \) is the (ex ante) value function, stacked as a column vector, \( u \) is a column vector with elements \( u_n = u(x_n) \), and \( A \) is a transition matrix, with elements \( A_{i,j} = 1 - x_i \) if \( j = i - 1 \), \( A_{i,j} = x_i \) if \( j = i + 1 \) and \( A_{i,j} = 0 \) otherwise. Premultiplying both sides of (30) by the stationary wealth distribution \( f' \) yields \( f' V = f'u + f'\beta [(1 - \pi) I + \pi A] V \). The distribution \( f \) is induced by the transition matrix \( A \), and it satisfies the equation \( f'A = f' \). But then \( w = f'u = (1 - \beta) f' V = (1 - \beta) W \).

(ii) As proving the proposition with respect to \( \pi \) and \( \varepsilon \) involves the same steps, we introduce a generic parameter \( \rho \), where \( \rho \in \{\pi, \varepsilon\} \). Also, let \( x \) be the vector with the \( n \)th element \( x_n \). Now

\[ \frac{dw}{d\rho} = \frac{\partial w}{\partial \rho} + \frac{\partial w}{\partial x} \frac{dx}{d\rho} = \frac{\partial w}{\partial x} \frac{dx}{d\rho} = (1 - \beta) \frac{\partial W}{\partial x} \frac{dx}{d\rho} = (1 - \beta) V' \frac{df}{dx} \frac{dx}{d\rho} \]

Equality \( (a) \) involves a decomposition into the direct effect and the indirect effect. \( (b) \) follows from the fact that \( w \) does not depend directly on \( \pi \) and \( \varepsilon \) (see (21)), and thus \( \frac{\partial w}{\partial \rho} = 0 \). \( (c) \) follows from equality (23). \( (d) \) uses the definition of \( W \), (22), and the envelope theorem: since the threshold \( \theta^*_n \), and thus also \( x_n \), is optimally chosen in all wealth classes \( n \geq 1 \), a small policy change does not affect the value function \( V(n) \).

By Lemma 3 we know that the wealth distribution shifts to the right, in the sense of first-order stochastic dominance, when \( \pi \) increases or \( \varepsilon \) decreases. As the value function \( V(n) \) is increasing in \( n \), this shift in the stationary distribution translates into higher social welfare:

\[ \frac{dw}{d\pi} = (1 - \beta) V' \frac{df}{dx} \frac{dx}{d\pi} \geq 0, \quad \frac{dw}{d\varepsilon} = (1 - \beta) V' \frac{df}{dx} \frac{dx}{d\varepsilon} \leq 0. \]
Proof of Proposition 5

(a) Match dimension. The household chooses \( \{ x_n \} \), so as to maximize the value function \( V \), where \( V \) satisfies the recursive equation \( V = \delta A V + \left( 1 - \delta \right) \frac{u}{1 - \beta} \). (This equation follows directly from (30).) Iterating forward, we get \( V = \left( 1 - \delta \right) \sum_{t=0}^{\infty} \left( \delta A \right)^t \frac{u}{1 - \beta} \). Next notice that \( \lim_{t \to \infty} A^t = 1 \otimes f' \) (where \( \otimes \) is Kronecker product). Thus when \( \pi \to 1 \) and \( \beta \to 1 \), so that \( \delta \to 1 \), maximizing \( V \) becomes essentially equivalent to maximizing \( f' u = w = \frac{1}{2} E [ \theta | h = 1 ] \). The objective function \( w = \frac{1}{2} E [ \theta | h = 1 ] \) is maximized iff there is perfect sorting in the match dimension.

(b) Sorting in the wealth dimension. The putative equilibrium strategy is of the following form: \( h(0, \theta) = 0 \) for all \( \theta \) (due to the borrowing constraint), \( h(n, \theta) = 1 \) for all \( \theta \) and \( n \geq 1 \). Then in equilibrium \( f_0 = f_1 = \frac{1}{2} \) and \( f_n = 0 \) for all \( n \geq 2 \).

Given this strategy, it is easy to calculate the ex ante values of the program \( V(n) \) at different wealth levels \( n \geq 0 \). In particular, one can show that \( V(2) - V(0) = (1 - \delta) \frac{\varepsilon + E[\theta]}{1 - \beta} \). Given the optimal location choice rule (8), the putative strategy is optimal for the household iff it always prefers the desirable location at wealth level \( n = 1 \), i.e., iff \( \theta + \varepsilon > \pi \beta (V(2) - V(0)) = \pi \beta \left( 1 - \delta \right) \frac{\varepsilon + E[\theta]}{1 - \beta} \) for all \( \theta \).

\[ \theta + \varepsilon > \pi \beta \left( V(2) - V(0) \right) = \pi \beta \left( 1 - \delta \right) \frac{\varepsilon + E[\theta]}{1 - \beta} \] for all \( \theta \).

In particular, the condition (31) must hold for the lowest possible realization of the match \( \theta_L \). Inserting \( \theta = \theta_L \), and slightly manipulating (31), yields the condition for residential sorting in the wealth dimension: \( \theta_L + \varepsilon > \pi \beta \frac{E[\theta | h = 1] - \theta_L}{1 - \beta} \). ■

Proof of Proposition 7

(a) We define cumulative distribution functions \( G(\theta | h, m) \) separately for four groups, conditioning on the household's present location \( (h \in \{0, 1\}) \), and on whether the household has moved in the present period \( (m = 1, \text{if the household has moved, and } m = 0, \text{if the household has not moved}) \). So, for example, \( G(\theta | h = 0, m = 1) \) is the distribution function for those households, which moved at the beginning of the period (from an
expensive location) and currently live in a cheap location. We also define the functions

\[ DG(\theta \mid h) \equiv G(\theta \mid h, m = 1) - G(\theta \mid h, m = 0), \quad h \in \{0, 1\} \]  

(32)

which allow us to compare (in the sense of first order stochastic dominance) the distributions of newcomers and old residents, who live in the same location (0 or 1).

To prove the proposition, we need to construct \( G(\theta \mid h, m), h, m \in \{0, 1\} \).

(i) As a first step, we characterize the match distributions of households living in the desirable and in the undesirable location, conditional on wealth class \( n \). Given the threshold location choice rule (10), the distribution in the desirable location \( G(\theta \mid h = 1, n) = \begin{cases} G(\theta \mid \theta \geq \theta_n^*) & \text{for } \theta \geq \theta_n^* \text{ (and 0 for } \theta < \theta_n^*) \end{cases} \) is left-truncated, with truncation point \( \theta_n^* \), while the distribution in the undesirable location \( G(\theta \mid h = 0, n) = \begin{cases} G(\theta \mid \theta < \theta_n^*) & \text{for } \theta \leq \theta_n^* \text{ (and 1 for } \theta > \theta_n^*) \end{cases} \) is right-truncated with the same truncation point \( \theta_n^* \). It is easy to see that \( \frac{\partial G(\theta \mid \theta \geq \theta_n^*)}{\partial \theta} \leq 0 \) and \( \frac{\partial G(\theta \mid \theta < \theta_n^*)}{\partial \theta} \leq 0 \) for all \( \theta \).

This property means that if we compare two wealth levels \( n_1 \) and \( n_2 \), such that \( n_1 < n_2 \), and consequently \( \theta_n^* < \theta_{n_2}^* \), the higher threshold \( \theta_{n_1}^* \) in group \( n_1 \) implies that the distribution \( G(\theta \mid h, n_1) \) first-order stochastically dominates the distribution \( G(\theta \mid h, n_2) \) for \( h \in \{0, 1\} \). More formally

\[ G(\theta \mid h, n_1) \leq G(\theta \mid h, n_2) \text{ for all } \theta, \text{ when } n_1 < n_2 \text{ and } h \in \{0, 1\}. \]  

(33)

(ii) As a second step, we need to study the conditional wealth distributions, contingent on housing location and mobility. The main objective is to establish a first-order stochastic dominance relation between movers and stayers in each location.

Denote the mass of households with wealth \( n \), and group \((h, m)\), by \( \varphi_m^h(n) \). Now

\[
\begin{align*}
\varphi_0^0(n) & = f_0^0(n) \psi(\theta_n^*) \equiv f_0^0(n) \{(1 - s)[(1 - \lambda) + \lambda G(\theta_n^*)] + s G(\theta_n^*)\} \\
\varphi_0^1(n) & = f_{n-1}^1(n) \widehat{\psi}(\theta_n^*) \equiv f_{n-1}^1(n) \{(1 - s)[(1 - \lambda) + \lambda (1 - G(\theta_n^*))] + s (1 - G(\theta_n^*))\} \\
\varphi_1^0(n) & = \varphi_1^0(n) = f_n^0(n) (1 - G(\theta_n^*)) [(1 - s) \lambda + s] = f_{n-1}^1(n) G(\theta_n^*) [(1 - s) \lambda + s]
\end{align*}
\]

43
Also let \( \hat{\varphi}_{rh}^h (n) \equiv \varphi_{rm}^h (n) / \sum_i \varphi_{rm}^h (i) \) be the relative share of wealth class \( n \) in group \((h,m)\). 

Next, to compare the wealth distributions, we need the size ratios of adjacent wealth classes in different groups. Denote \( \hat{\gamma}_{rh}^h (n) \equiv \hat{\varphi}_{rh}^h (n + 1) / \hat{\varphi}_{rh}^h (n) = \varphi_{rm}^h (n + 1) / \varphi_{rm}^h (n) \). 

Now using the equations (34) we get
\[
\frac{\hat{\gamma}_{0}^0 (n)}{\hat{\gamma}_{1}^0 (n)} = \frac{\psi (\theta_{n+1}^*)}{\psi (\theta_{n}^*)} \frac{1 - G (\theta_{n+1}^*)}{1 - G (\theta_{n}^*)} \leq 1 \tag{35}
\]
\[
\frac{\hat{\gamma}_{0}^1 (n)}{\hat{\gamma}_{1}^1 (n)} = \frac{\hat{\psi} (\theta_{n+1}^*)}{\hat{\psi} (\theta_{n}^*)} \frac{G (\theta_{n+1}^*)}{G (\theta_{n+1}^*)} \geq 1 \tag{36}
\]

These inequalities hold, since clearly \( \psi (\theta_{n+1}^*) / \psi (\theta_{n}^*) \leq 1 \), \( (1 - G (\theta_{n+1}^*)) / (1 - G (\theta_{n}^*)) \leq 1 \), \( \hat{\psi} (\theta_{n+1}^*) / \hat{\psi} (\theta_{n}^*) \geq 1 \) and \( G (\theta_{n+1}^*) / G (\theta_{n+1}^*) \geq 1 \). The inequality (35) allows us to compare the wealth distributions of mover and stayer households, which currently reside in the cheap location. The inequality tells that, for any adjacent wealth classes \( (n + 1) \) and \( n \), the ratio \( \hat{\varphi}_{rm}^h (n + 1) / \hat{\varphi}_{rm}^h (n) \) is larger for movers than for stayers. But this means that in the cheap location newcomers are wealthier than the old residents, in the sense of first-order stochastic dominance. The inequality (36) then implies that in the expensive location the opposite is true, and old residents are wealthier than newcomers, in the sense of first-order stochastic dominance.

(iii) As a final step, we combine the results of steps (i) and (ii), and construct the conditional match distribution functions
\[
G (\theta \mid h, m) = \sum_n \hat{\varphi}_{rh}^h (n) G (\theta \mid h, n), \text{ for } h, m \in \{0, 1\}. \tag{37}
\]

That is, the conditional match distributions \( G (\theta \mid h, m) \) are convex combinations of the location-contingent distributions \( G (\theta \mid h, n) \) at different wealth levels \( n \). In each group \((h,m)\), the weight assigned to the distribution function \( G (\theta \mid h, n) \) corresponds to the relative size of wealth class \( n \) in the group, \( \hat{\varphi}_{rm}^h (n) \).

Using (32) and (37), we get
\[
DG (\theta \mid h = 0) = \sum_n \left[ \hat{\gamma}_{0}^1 (n) - \hat{\gamma}_{0}^0 (n) \right] G (\theta \mid h = 0, n) \geq 0,
\]
\[
DG (\theta \mid h = 1) = \sum_n \left[ \hat{\gamma}_{1}^1 (n) - \hat{\gamma}_{1}^0 (n) \right] G (\theta \mid h = 1, n) \leq 0 \tag{38}
\]
for all \( \theta \). The inequalities follow from stochastic dominance, results (33), (35) and (36). The expressions (38) mean that in a currently cheap location, the match distribution of old residents stochastically dominates the match distribution of newcomers, while the in a currently expensive location the opposite is true. Thus we have proved that in both areas movers (with \( m = 1 \)) tend to have a better match with the location than stayers (\( m = 0 \)). (Remember, that a low (negative) realization of \( \theta \) implies a good match with a currently unpopular location).

\( (b) \) To address the degree of residential sorting among movers and stayers, we further define \( DG(\theta | m) \equiv G(\theta | h = 1, m) - G(\theta | h = 0, m), \ m \in \{0, 1\} \). Then \( DG(\theta | m = 1) \) tells how the distribution of households which have moved from a cheap location to an expensive location differs from the distribution of those households which have moved the other way round; also \( DG(\theta | m = 0) \) allows us to compare the distributions of immobile households living in different locations. Finally, to compare the degree of residential sorting between movers and stayer, we define the function \( RS_{m/s}(\theta) \equiv DG(\theta | m = 1) - DG(\theta | m = 0) \). It is clear than both among movers and among stayers, those who live in the desirable location typically have a higher value of \( \theta \) than those who reside in the less desirable location, that is \( DG(\theta | m) \leq 0 \) for all \( \theta \) and for \( m \in \{0, 1\} \). Now we use the function \( RS_{m/s}(\theta) \) to address the question: among which group (movers or stayers) are the households residing in different locations more distinct from each other. In particular, if \( RS_{m/s}(\theta) \leq 0 \) for all \( \theta \), movers are more sorted in this sense. But

\[
RS_{m/s}(\theta) = DG(\theta | m = 1) - DG(\theta | m = 0) \\
= G(\theta | h = 1, m = 1) - G(\theta | h = 0, m = 1) \\
- [G(\theta | h = 1, m = 0) - G(\theta | h = 0, m = 0)] \\
= DG(\theta | h = 1) - DG(\theta | h = 0) \leq 0,
\]

where the inequality follows from (38).
More general match dynamics

Let \( v(\theta, n) \equiv V(\theta, n + 1) - V(\theta, n - 1) \) and \( \Delta h(\theta, n) \equiv h(\theta, n + 1) - h(\theta, n - 1) \). Also define the operator \( \hat{L} \),

\[
\hat{L} [z(\theta, n)] \equiv (1 - \sigma) z(\theta, n) + \lambda E_\theta [z(\tilde{\theta}, n) \mid \theta, s = 0] \\
+ \pi E_\theta [h(\theta, n - 1) z(\tilde{\theta}, n - 1) + (1 - h(\theta, n + 1)) z(\tilde{\theta}, n + 1) \mid \theta, s = 1],
\]

where \( z(\theta, n) \) is a generic function of \( \theta \) and \( n \). Since \( V(\theta, n) \) satisfies the Bellman equation (26), the function \( v(\theta, n) \) satisfies the recursive equation

\[
v(\theta, n) = \Delta h(\theta, n)(\varepsilon + \theta) + \beta \hat{L}[v(\theta, n)].
\]  

(39)

For all \( \theta \) and all \( n \geq 1 \), the household’s location choice rule assumes the form \( h(\theta, n) = 1 \) iff \( \theta \geq \tilde{Q}(\theta, n; \varepsilon, \sigma) \) (and 0 otherwise), where \( \tilde{Q}(\theta, n; \varepsilon, \sigma) \equiv -\varepsilon + \pi \beta E_\theta [v(\tilde{\theta}, n) \mid \theta, s = 1]. \)

**Lemma 1’** For all \( \theta \) and \( n \geq 1 \), \( \frac{d\tilde{Q}(\theta, n; \varepsilon, \sigma)}{d\varepsilon} < 0. \)

**Proof** Define \( \tilde{q}^\varepsilon(\theta, n) \equiv \frac{d\tilde{Q}(\theta, n; \varepsilon, \sigma)}{d\varepsilon} \). Differentiating (39) with respect to \( \varepsilon \) shows that \( \tilde{q}^\varepsilon(\theta, n) \) satisfies the equation \( \tilde{q}^\varepsilon(\theta, n) = \Delta h(\theta, n) + \beta \hat{L}[\tilde{q}^\varepsilon(\theta, n)]. \) Next define \( \tilde{q}_{\max} \equiv \max \tilde{q}^\varepsilon(\theta, n) \) and \( \{\theta^*, \tilde{n}^*\} \equiv \arg \max \tilde{q}^\varepsilon(\theta, n) \). Then \( \tilde{q}_{\max} \leq \Delta h(\theta^*, \tilde{n}^*) + \beta \tilde{q}_{\max} \left(1 - \pi \Delta h(\theta^*, \tilde{n}^*)\right) \), and \( \tilde{q}_{\min} \leq -1 + \pi \beta \tilde{q}_{\max} \leq -\frac{1}{\beta(1 - \pi)}. \) Finally, \( \frac{d\tilde{Q}(\theta, n; \varepsilon, \sigma)}{d\varepsilon} = -1 + \pi \beta E_\theta [\tilde{q}^\varepsilon(\theta, n) \mid \theta, s = 1] \leq -1 + \pi \beta \tilde{q}_{\max} \leq -\frac{1}{\beta(1 - \pi)} < 0. \)

**Lemma 2’** For all \( \theta \) and \( n \geq 1 \), \( \frac{d\tilde{Q}(\theta, n; \varepsilon, \sigma)}{d\sigma} > 0. \)

**Proof** Define \( \tilde{q}^\sigma(\theta, n) \equiv \frac{d\tilde{Q}(\theta, n; \varepsilon, \sigma)}{d\sigma} \). Multiplying both sides of (39) by \( \sigma \), differentiating with respect to \( \sigma \), and simplifying, shows that \( \tilde{q}^\sigma(\theta, n) \) satisfies the equation \( \tilde{q}^\sigma(\theta, n) = (1 - \beta) v(\theta, n) + \beta \hat{L}[\tilde{q}^\sigma(\theta, n)]. \) Next define \( \tilde{q}_{\min} \equiv \min \tilde{q}^\sigma(\theta, n) \) and \( \{\theta^*, \tilde{n}^*\} \equiv \arg \min \tilde{q}^\sigma(\theta, n) \). Then \( \tilde{q}_{\min} \geq (1 - \beta) v(\theta^*, \tilde{n}^*) + \beta \tilde{q}_{\min} \left(1 - \pi \Delta h(\theta^*, \tilde{n}^*)\right) \), and \( \tilde{q}_{\min} \geq \frac{(1 - \beta)v(\theta^*, \tilde{n}^*)}{1 - \beta(1 - \pi)} > 0. \) Finally, \( \frac{d\tilde{Q}(\theta, n; \varepsilon, \sigma)}{d\sigma} = \xi \beta E_\theta [\tilde{q}^\sigma(\theta, n) \mid \theta, s = 1] \geq \xi \beta \tilde{q}_{\min} > 0. \)

**Stationary distribution.** Let \( \hat{f}_n(\theta_j) \) denote the long-run frequency mass of households with match \( \theta_j \) and wealth \( n \), and let \( \hat{f}_n \) be a \( J \times 1 \) vector, with the \( j \)th element \( \hat{f}_n(\theta_j) \). Also let \( H_n, n \geq 1 \), be a \( J \times J \) diagonal matrix, with the \( j \)th diagonal element
The stationary distribution satisfies the following set of recursive equations

$$\hat{f}_n = (1 - \sigma) \hat{f}_n + \lambda \hat{f}_n A_0 + \pi \left( \hat{f}_{n-1} B_{n-1} + \hat{f}_{n+1} H_{n+1} \right) A_1$$

for all $n = 0, 1, ...$ Simplifying yields

$$\hat{f}_n = (1 - \xi) \hat{f}_n A_0 + \xi \left( \hat{f}_{n-1} B_{n-1} + \hat{f}_{n+1} H_{n+1} \right) A_1$$

(40)

Notice in particular that the parameters $\varepsilon$ and $\sigma$ do not appear in (40), and thus the joint distribution of wealth and the match depends on these parameters only indirectly, through changes in policies.

**Equilibrium.** Postmultiplying both sides of equation (40) by the unit vector $1$, and taking into account the fact that $A_0 1 = A_1 1 = 1$, yields a set of recursive equations for the marginal distribution of wealth

$$f(n) = f_{n-1}^0 (n - 1) + f_{n+1}^1 (n + 1)$$

(41)

where $f(n) = \hat{f}_n 1$ is the frequency mass of households at wealth level $n$, $f_{n}^0 (n) = \hat{f}_n B_{n} 1$ is the mass of households at wealth level $n$ residing in an unpopular location, and $f_{n-1}^1 (n) = \hat{f}_n H_{n} 1$ is the mass of households at wealth level $n$ residing in a popular location. But equation (41) is identical to equation (12) so that equilibrium follows in the same way as in Section 3.3.

**Lemma 3’** Define the cumulative distribution function $\hat{F}(\theta_j, n; \varepsilon, \sigma) = \sum_{i=0}^{n} \hat{f}_i (\theta_j)$. Then $\frac{d\hat{F}(\theta_j, n; \varepsilon, \sigma)}{d\varepsilon} \geq 0$ and $\frac{d\hat{F}(\theta_j, n; \varepsilon, \sigma)}{d\sigma} \leq 0$ for all $n$ and $\theta_j$.

**Proof** Define a history as a collection of match realizations and regional shock realizations $H_t = \{(\theta, s_t)\}_{t=0}^{t}$. Notice that histories are exogenous in the sense that they do not depend on the households’ location choices. Denote a state by $y = (\theta, n)$. Consider two location choice rules $h^0$ and $h^1$ such that for some state $\hat{y}$, $h^0(\hat{y}) = 0$ and $h^1(\hat{y}) = 1$, and for all other states $y \neq \hat{y}$, $h^0(y) = h^1(y) = h(y)$ (where $h(y)$ is the common policy).

Next notice that there is a mapping from histories $H_t$ to states $y_t$, conditional on
policy \( h^i, i \in \{0, 1\} \). That is, at any date \( t \), the household’s wealth \( n^i_t = n^i(H_t) \) and the state \( y^i_t = y^i(H_t) \), where \( i \in \{0, 1\} \) refers to the policy that the household follows.

Consider two households. Household 0 follows policy \( h^0 \), while household 1 follows policy \( h^1 \). Assume the households have the same history \( H_t \). Define \( \nu_t \equiv n^0_t - n^1_t \) and notice that by equation (5) it obeys the law of motion \( \nu_{t+1} = \nu_t + 2s_{t+1}(h^1(y^1_t) - h^0(y^0_t)) \).

Obviously,
\[
\Delta \nu_t \equiv \nu_{t+1} - \nu_t = 2s_{t+1}(h^1(y^1_t) - h^0(y^0_t)) \in \{-2, 0, 2\}. \tag{42}
\]
Assume that for some period \( t, \nu_t = 0 \) so that also \( y^0_t = y^1_t \). Given the properties of \( h^0 \) and \( h^1 \) it is evident that
\[
\Delta \nu_t \in \{0, 2\}, \text{ if } \nu_t = 0. \tag{43}
\]
(\( \Delta \nu_t = 2 \text{ iff } y^0_t = y^1_t = \hat{y} \text{ and } s_{t+1} = 1 \)). Next, assume the households have the same initial wealth, \( \nu_0 = 0 \). From (42) and (43) it follows that \( \nu_t = 2k, k \in \{0, 1, 2, ...\} \) for all \( t = 0, 1, 2, ... \). The essential finding is that, given identical histories and equal initial wealth, household 1 cannot be wealthier than household 0.

Assume that there is a population of households following policy \( h^0 \), and another population following policy \( h^1 \). Also assume that all households, in either population, have the same initial wealth. As above, we refer to a household belonging to population 0 (1) as household 0 (1). Now, the proof of the lemma derives from the following observations.
(i) After any given (common) history \( H_t \), household 0 is at least as wealthy as household 1. (ii) After any given (common) history \( H_t \), household 0 and household 1 have the same match. (iii) The probability distribution over the histories does not depend on policy. (iv) In any period \( t \), and for any given current match, the wealth distribution under policy \( h^0 \) stochastically dominates the wealth distribution under policy \( h^1 \). (v) When \( t \to \infty \), the wealth distribution converges to the stationary distribution. Thus stochastic dominance applies to the stationary distribution. Finally, Lemmas 1’ and 2’ imply that when \( \varepsilon \) increases or \( \sigma \) decreases, the households may shift from policy \( h^0 \) to policy \( h^1 \), but the opposite shift (from policy \( h^1 \) to policy \( h^0 \)) never happens.

**Proposition 2’** When \( \varepsilon \) decreases or \( \sigma \) increases, social welfare grows.

**Proof** Let us define a KJ state Markov chain \( y \), where the \((nJ + j)\)th state is given by
the pair \((\theta_j, n)\). Notice that \(K\) (the number of wealth levels) is \(\pi + 1\), if \(\theta_L > -\varepsilon\), and otherwise \(K = \infty\). Let \(h\) be a \(KJ \times 1\) vector, with the \((nJ + j)\)th element \(h(\theta_j, n)\). Further define a \(KJ \times KJ\) diagonal matrix \(H\), with the vector \(h\) on the diagonal (and all off-diagonal elements equal to zero), and let the \(KJ \times KJ\) matrix \(\hat{A}\) be the transition matrix of the Markov chain \(y\).

The value function can be presented as a \(KJ \times 1\) vector \(\hat{V}\), where the \((nJ + j)\)th element is the value of the household’s program in state \((\theta_j, n)\). \(\hat{V}\) satisfies the Bellman equation

\[
\hat{V} = H (1_K \otimes \theta_j) + \left( h - \frac{1}{2} 1_{KJ} \right) \varepsilon + \beta \left[ (1 - \sigma) I + \sigma \hat{A} \right] \hat{V},
\]  

(44)

where \(\theta\) is the \(J \times 1\) vector of types \(\theta_j\). The stationary distribution of \(y\) is a \(KJ \times 1\) vector \(\hat{f}\). The distribution is induced by the transition matrix \(\hat{A}\) and it satisfies the equation

\[
\hat{f} = \hat{f} \hat{A}.
\]

Next we premultiply both sides of (44) by \(\hat{f}'\). Then using the fact that \(\hat{f}' = \hat{f}' \hat{A}\), and noting that \(\hat{f}' \left( h - \frac{1}{2} 1_{KJ} \right) = 0\), by the housing market equilibrium, yields

\[
\hat{W} = \hat{w} + \beta \hat{W} \iff \hat{W} = \hat{w} / (1 - \beta).
\]  

(45)

Given the equation (45), and Lemma 3’, Proposition 2’ can be proved following the same steps as in the proof of Proposition 2. See part (ii) of the proof. ■

**Proposition 3’** When \(\varepsilon\) increases or \(\sigma\) decreases, the degree of residential sorting in the match dimension decreases in the sense explained in Proposition 3.

**Proof** The result follows from Proposition 2’. See the proof of Proposition 3. ■

**Corollary 1’** There is a negative relation between the size of house price fluctuations and the degree of residential sorting in the match dimension.

**Proposition 4’** When \(\varepsilon\) increases or \(\sigma\) decreases, the degree of residential sorting in the
wealth dimension increases in the sense explained in Proposition 4.

**Proof** The results follows from Lemma 3'. See the proof of Proposition 4. ■

**Empirical Appendix**

**Further statistical evidence**

This appendix provides additional evidence as to how movers tend to be more sorted than stayers.

If movers are more sorted than stayers, then we expect that educational attainment, age and income are more dispersed across regions among movers than among stayers. Table 4 reports standard deviations over PUMA regions of the share of home-owners with a high school degree and the share of home-owners with at least a college degree, separately for movers and stayers. Clearly, both of the shares vary more across regions among movers than among stayers; and these differences are also statistically significant, as shown by the $p$-values of the Levene (1960) and the Brown-Forsythe (1974) tests for equal variance. Furthermore, Table 4 shows that owner-occupying movers’ age and income vary more across PUMA areas than those of stayers. As a robustness check, Table 4 also makes the same comparisons for people that live in rental housing. Because renters do not face similar housing market related wealth shocks as owners, moving renters need not be more sorted than staying renters. Consistent with this, the results of Table 4 indicate that moving renters are, in the most part, no more sorted than staying renters (and, in fact, the reverse can also be true).

As a final piece of evidence, we compare “short distance movers”, i.e. households which have moved within the same metropolitan area, and “long distance movers”, i.e. households, which have moved from another metropolitan area.\(^{44}\) Because “long distance movers” have more likely moved between two uncorrelated markets (so that the prices of the old and the new home may have evolved very differently), they should be more

\(^{44}\)We also use data on people that have moved from or to a non-MSA region. See the subsequent appendix for more details.
sorted than “short distance movers”. The Gini coefficients reported in Table 5 indicate that among owner-occupiers “long distance movers” are indeed more sorted than “short distance movers”, according to all three criteria.

Definitions of variables

Sorting measures computed from ICPSR data

The sorting measures applied in Table 2 are computed from extraction of data from the 1990 decennial Census, published in the ICPSR study 2889 (1990). Table 2 applies the data set 2 (DS2) where each variable is aggregated to the municipality (MCD) level. Because MCDs are geographically comprehensive, our MSA level observations are formed by summing up all relevant MCD level data.

The groups of types that we use in computing the $DV$ measures and the Gini coefficients in Table 2 are as follows. We use five categories for age: (1) “children” (those of 0-15 years old), (2) “youth” (16-24 years old), (3) “adults, early career” (25-44 years old), (4) “adults, late career” (45-64 years old), and (5) “seniors” (those at least 65 years old). For education, we have three groups: (1) less than a high school degree, (2) at least a high school degree but not a college degree, and (3) a college degree or more. The Census defines the education groups for only those who are at least 25 years old. This age category is used to normalize the education groups within each region. Finally, for income we apply all the 25 income groups available in the ICPSR study 2889. In each of the cases, the US level groups are obtained by a population weighted average of the MSA level groups. The education and income categories applied here are similar to those of the Gini coefficients considered by Rhode and Strumpf (2003, p. 1660) (see also their Data Appendix at www.unc.edu/~cigar/ or www.unc.edu/~prhode/).

The samples of observations applied in Table 2 derive from all those MSA level matches that we find between the house price volatility measure and the sorting measures in each case.

Sorting measures computed from IPUMS data

The sorting measures applied in Tables 3, 4 and 5 are computed from the Census data
provided at www.ipums.org. The web site provides detailed definitions for each variable. For each observation unit (i.e., person) in the 1% sample from the 1990 Census, we downloaded household id (SERIAL), age (AGE), educational attainment (EDUC99), household income (FTOTINC), tenure (OWNERSHP), migration information (MIGRATE5, MIGMET5, MIGPLAC5) and location indicators (PUMA, STATEFIP, METAREA). These data include observations on 2,479,568 persons from 1760 different PUMAs. The actual number of people in each PUMA is also obtained from www.ipums.org.

To compute the Gini coefficients in Table 3, we first classify each sample person into four categories depending on whether the person is an owner (OWNERSHP = 10) or a renter (OWNERSHP = 20) and whether the person is a mover (MIGRATE5 = 2) or a stayer (MIGRATE5 = 1). Persons with missing observations on OWNERSHP or MIGRATE5 are excluded from the calculations. To compute the Gini coefficients for age, education and income we apply similar categories as in Table 2. For computing the Gini coefficient for age, we estimate the shares of “children”, “youth”, etc. in each PUMA by computing the relative shares of the sample persons belonging to the relevant age category (for “children” the share of those 0-15 years old, etc.). For computing the education Gini coefficient, we restrict the sample to those at least 25 years old. The three education groups (consistent with those in Table 2) are formed by (1) EDUC99 ≤ 9, (2) 10 ≤ EDUC99 ≤ 11, and (3) 12 ≤ EDUC99. Finally, to compute the Gini coefficient for income, we first restrict the sample to household heads only (SERIAL = 1). Then we employ FTOTINC to classify each household into one of the 25 income ranges used in the ICPSR data, and compute the corresponding relative shares in each PUMA. In all cases (age, education and income), the US level shares are obtained as a population weighted average of the PUMA shares.

The PUMA observations of the variables considered in Table 4 are computed for household heads only, while the applied groupings (“Owners”, “Renters”, “Movers”, “Stayers”) are defined in the same way as in Table 3. “High school degree, %” is the relative share of household heads at least 25 years old that have 10 ≤ EDUC99 ≤ 11, “College degree, %” is the corresponding share of those that have 12 ≤ EDUC99 ≤ 17. Finally, “Age” and “Income”, respectively, refer to the average age (AGE) and income (FTOTINC) over the
relevant households in each case.

To compute the Gini coefficients in Table 5 we first restrict the sample to persons that own their house (OWNERSHP = 10) and that have moved recently (MIGRATE5 = 2). Within this subsample, we classify a person as a “short distance mover”, if his current MSA is the same as five years ago, i.e., if METAREA and MIGMET5 match; otherwise the person is classified as a “long distance mover”. In addition to data on persons that have moved from one MSA region to another, we also use data on persons that have moved from or to a non-MSA region. If a person has moved from an MSA region to a non-MSA region, or vice versa, he or she is recorded as a “long distance movers”, while a person that has moved between two non-MSA regions is recorded as a “long distance mover” only, if his or her current state of residence (STATEFIP) is different from that five years ago (MIGPLAC5). The Gini coefficients for age, education and income are formed by applying the same convention of groupings as in Table 3.

References


Figure 1: Relative house prices in the UK

Source: Nationwide Building Society
Figure 2: Relative house prices in the US
Figure 3: The $\theta_n^*$-curve when $\theta$ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\varepsilon = 1$, $\beta = .95$, and $\pi = .3$. 
Figure 4: Equilibrium pattern of residential sorting with different values of $\varepsilon$ when $\theta$ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\beta = .95$, and $\pi = .2$. The measure of house price fluctuations in the figures is $P = \frac{1}{2} \frac{1}{E[n]}$, $P \in [0,1]$. 
Table 1. Maximum and minimum house-price-to-income ratios, 1979-1996

<table>
<thead>
<tr>
<th>City</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boston</td>
<td>5.4</td>
<td>12.0</td>
</tr>
<tr>
<td>New York</td>
<td>5.3</td>
<td>12.0</td>
</tr>
<tr>
<td>Los Angeles</td>
<td>6.7</td>
<td>11.1</td>
</tr>
<tr>
<td>San Francisco</td>
<td>6.4</td>
<td>11.4</td>
</tr>
</tbody>
</table>

Source: Malpezzi, 1999
Table 2. OLS regression estimates of the effect of house price volatility on population structure and residential sorting

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>$DV_{Education}$</th>
<th>$DV_{Age}$</th>
<th>$DV_{Income}$</th>
<th>$GC_{Education}$</th>
<th>$GC_{Age}$</th>
<th>$GC_{Income}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>House price volatility</td>
<td>-0.20</td>
<td>-0.12</td>
<td>-0.23</td>
<td>-0.28</td>
<td>-0.13</td>
<td>-0.31</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.15)</td>
<td>(0.18)</td>
<td>(0.11)</td>
<td>(0.06)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.07</td>
<td>0.04</td>
<td>0.13</td>
<td>0.19</td>
<td>0.11</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(20.3)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Sample size</td>
<td>243</td>
<td>243</td>
<td>243</td>
<td>242</td>
<td>242</td>
<td>238</td>
</tr>
</tbody>
</table>

Notes: Dependent variable varies by column. Robust standard errors are given in parentheses. Sample size indicates the number of MSA level observations. $DV_{Education}$, $DV_{Age}$ and $DV_{Income}$ indicate the measures in (1) computed for education (with three groups), age (with five groups) and income (with 25 groups), respectively. $GC_{Education}$, $GC_{Age}$ and $GC_{Income}$ refer to the values of the Gini coefficients defined in (2) for education, age and income, respectively. Precise definitions of the groups in each case are given in the appendix.
Table 3. Gini coefficients for movers and stayers

<table>
<thead>
<tr>
<th></th>
<th>Owners</th>
<th></th>
<th>Renters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Movers</td>
<td>Stayers</td>
<td>Movers</td>
<td>Stayers</td>
</tr>
<tr>
<td>Education</td>
<td>0.28</td>
<td>0.24</td>
<td>0.29</td>
<td>0.31</td>
</tr>
<tr>
<td>Age</td>
<td>0.17</td>
<td>0.12</td>
<td>0.17</td>
<td>0.21</td>
</tr>
<tr>
<td>Income</td>
<td>0.37</td>
<td>0.28</td>
<td>0.31</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Notes: The entries of the table refer to Gini coefficients computed for the whole US using PUMA level data from the 1990 Census. Precise definitions of the groups in each of the cases (education, age, income) are given in the appendix.
Table 4. Comparing movers and stayers

<table>
<thead>
<tr>
<th></th>
<th>Movers</th>
<th>Stayers</th>
<th>Movers</th>
<th>Stayers</th>
<th>Levene</th>
<th>Brown-F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Owners</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High School degree, %</td>
<td>0.45</td>
<td>0.46</td>
<td>0.12</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>College degree, %</td>
<td>0.38</td>
<td>0.26</td>
<td>0.17</td>
<td>0.13</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Age</td>
<td>42.3</td>
<td>56.2</td>
<td>3.3</td>
<td>3.0</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Income</td>
<td>46039</td>
<td>42021</td>
<td>15503</td>
<td>13113</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Renters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High School degree, %</td>
<td>0.48</td>
<td>0.45</td>
<td>0.09</td>
<td>0.11</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>College degree, %</td>
<td>0.27</td>
<td>0.17</td>
<td>0.13</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Age</td>
<td>37.6</td>
<td>53.0</td>
<td>2.5</td>
<td>4.8</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Income</td>
<td>23405</td>
<td>21760</td>
<td>6924</td>
<td>6839</td>
<td>0.28</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Notes: The entries of the table are computed using PUMA level observations (total 1726). Each PUMA observation is obtained by averaging relevant observations (household heads) in the corresponding PUMA sample (from the 1990 Census). A household head is classified as a mover (a stayer), if he or she did not live (lived) in his or her current house five years ago. “High school degree, %” refers to the share of persons with a high school degree but not a college degree, “College degree, %” refers to the share of persons with at least a college degree, “Age” refers to the average age in years, while “Income” refers to the average annual income of household heads. (See the subsequent section for more detailed description of the variables.) “Levene” and “Brown-F.”, respectively, refer to the p-values of the Levene (1960) and Browne and Forsythe (1974) tests for the equality of variances.
Table 5. Gini coefficients for long and short distance movers

<table>
<thead>
<tr>
<th></th>
<th>Long distance</th>
<th>Short distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education</td>
<td>0.33</td>
<td>0.30</td>
</tr>
<tr>
<td>Age</td>
<td>0.26</td>
<td>0.17</td>
</tr>
<tr>
<td>Income</td>
<td>0.52</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Notes: The entries of the table refer to Gini coefficients computed for the whole US using PUMA level data from the 1990 Census. Precise definitions of the groups for each characteristic (education, age, income) are given in the appendix. “Short distance movers” refer to households which have moved within the same MSA, while “long distance movers” have migrated from another MSA. See the subsequent section for more detailed definitions.