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# Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment

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# Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment\*

## Abstract

We analyze the following questions associated with outsourcing and profit sharing under imperfectly competitive labour markets. How does strategic outsourcing influence wage formation, profit sharing and employee effort when firms commit to optimal profit sharing before wage formation or decide for profit sharing after wage formation. What is the relationship between outsourcing, profit sharing and equilibrium unemployment under various cases depending on whether in other industries profit share is or is not a part of the compensation scheme. We also characterize the optimal production mode in terms of strategic outsourcing. We find that if the firm will decide on profit sharing before the wage negotiation, higher outsourcing decreases wage whereas profit sharing has an ambiguous effect. Under flexible profit sharing the wage is higher if optimal profit share is small enough. For equilibrium unemployment we find in the case when there is no profit sharing in other industries, that outsourcing will decrease equilibrium unemployment. If profit sharing is a part of the outside option then the effect is ambiguous.

**JEL Classification:** E23, E24, J23, J33, J74, J 82.

**Keywords:** Outsourcing, profit sharing, labour market imperfections, employee effort, equilibrium unemployment

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## 1. Introduction

High wage differences across countries constitute an important explanation for the currently significant business practice of international outsourcing. For example, 1,10 € per hour in China is very low to 27 € per hour e.g. in Denmark, Germany or Norway. These wage differentials could lead to outsourcing (see e.g. Sinn (2007) for details, see also Stefanova (2004) concerning the East-West dichotomy of outsourcing). Glass and Saggi (2001) have studied the causes of outsourcing and its effects and they found that higher international outsourcing lowers the relative wage of domestic workers, while increases the profits and thereby creates greater incentives for innovation.

It is known that higher wages affect worker's productivity which is influenced by their effort. Of course, according empirical evidence another way to stimulate the effort is profit sharing. Profit sharing is an empirically important phenomenon in many OECD countries. Pendleton et al. (2001) have presented detailed data on profit sharing schemes in 14 EU-countries. For example, among western EU-countries in 1999/2000 a double-digit percentage of the workplaces uses profit sharing in Austria, Finland, France, Germany, Ireland, Netherlands, Portugal, Spain, Sweden and United Kingdom. The lowest incidences are found in Denmark, Italy and Greece. For further evidence regarding the incidence of profit sharing, we refer to the DICE data base, collected by CESifo, <http://www.CESifo.de> as well as to Conyon and Freeman (2001).

In terms of profit sharing Koskela and Stenbacka (2004a) have offered a framework to analyze employment, effort, wages and profit sharing when firms face stochastic revenue shocks. Moreover, they have investigated the interaction between labour and credit market imperfections in the presence of profit sharing (see Koskela and Stenbacka (2004b)). In these papers they have analyzed profit sharing which is committed, i.e. decided before wage negotiation. Koskela and Stenbacka (2006) have also studied the differences between committed profit sharing and flexible profit sharing, which is decided after wage formation. They have shown that the optimal profit share under commitment is higher than under flexibility because through a profit share commitment the firms can induce wage moderation. In these papers they

have also studied the relationship between profit sharing and equilibrium unemployment.

As profit sharing is now commonly incorporated in the compensation schemes, and international outsourcing has recently increased among western EU-countries and in the United States, then it is important to study their relationship and implications for workers' effort, wage formation and unemployment when profit sharing is also a part of a compensation scheme in other industries or not. This is the topic, which is our focus in this paper. We assume that firms commit to outsourcing before profit sharing, wage negotiation, labour demand and effort determination. Moreover, and importantly, we also analyze the implications of two alternative time sequences in terms of profit sharing decision: (i) firms might commit profit sharing before base wage negotiation or (ii) it might also sometimes be relevant the case according to which firms decide about profit sharing only after knowing the result of base wage negotiation.

In our framework we analyze the following questions associated with outsourcing and profit sharing under imperfectly competitive labour markets by using the scenario without outsourcing: How does strategic outsourcing influence wage formation, profit sharing and employee effort, when firms commit to optimal profit sharing before wage formation or decide profit sharing after wage formation. We also analyze the relationship between outsourcing, profit sharing and equilibrium unemployment under various cases depending on whether in other industries profit share is or is not a part of the compensation scheme. Finally, we briefly look at the long-run perspective for the optimal production mode in terms of strategic outsourcing.

First we show that in the presence of outsourcing the wage elasticity of labour demand depends positively both on the amount of outsourcing and on the base wage, but negatively on the size of profit sharing. As a result we also show that in the case of committed profit sharing strategic outsourcing has a negative effect on wage formation. This lies in conformity with empirics and results from our assumption of perfect substitutability between outsourcing and effective domestic labour. Under flexible profit sharing the wage is higher if optimal flexible profit share is small enough. We also find that the profit share under commitment in the presence of

outsourcing is not necessary larger than profit share under flexibility. Only if there is a wage moderation effect in the committed case, we are in line with the literature, which argues that the optimal profit share under commitment is higher than the profit share under flexibility. If the wage rate increases by contrast the opposite result occurs. In the flexible case we show that an increasing share of outsourcing or a higher wage rate will lower the profit share so that there is a negative relationship between outsourcing or base wage and optimal flexible profit share.

If there is no profit sharing as a part of outside option in other industries higher outsourcing will decrease equilibrium unemployment while profit sharing will have an ambiguous effect on equilibrium unemployment, but in the absence of outsourcing higher profit sharing will decrease equilibrium unemployment. If there is profit sharing a part of outside option in other industries outsourcing and profit sharing will have ambiguous effects on equilibrium unemployment. Also in the absence of outsourcing profit sharing will have an ambiguous effect on equilibrium unemployment. In terms of optimal long-run strategic outsourcing wage moderation will have the positive indirect marginal profit in the presence of committed profit sharing due to wage moderation, but in the presence of flexible profit sharing this effect is a priori ambiguous.

We proceed as follows: Section 2 presents the basic structure of theoretical framework and two different time sequences in terms of profit sharing decision in the presence of outsourcing activity. The determination of labour demand by firms and effort by workers are presented in section 3. Section 4 investigates the wage formation by monopoly labour union in the presence of strategic outsourcing and committed profit sharing, and section 5 studies the wage formation by monopoly labour union with strategic outsourcing and flexible profit sharing, which is decided after wage formation. Section 6 explores the implications of strategic outsourcing and different time decisions of profit sharing on equilibrium unemployment under various cases. Section 7 studies briefly the optimal long-run outsourcing given the wage formation, the profit sharing, the labour demand and the employee effort. Finally, we present conclusions in section 8.

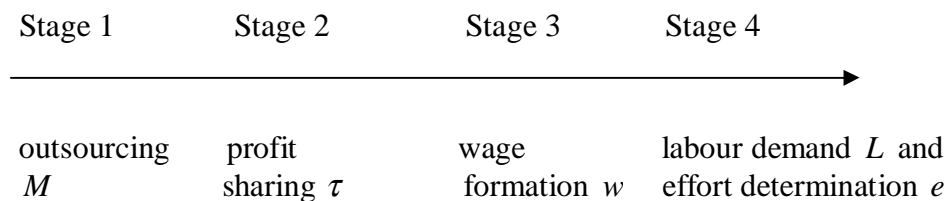
## 2. Basic Framework

We consider a representative firm and assume that output depends not only on the units of labour but also on the effort supplied by workers, i.e. the workers' productivity. This lies in conformity with the efficiency wage hypothesis (for a survey and several important seminal articles, see e.g. the book edited by Akerlof and Yellen (1986)). We analyze two alternative timing decisions made representatives by the firm, the labour union and the worker.

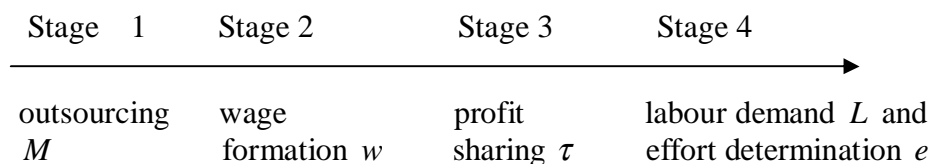
The timing structure (I) captures the idea that the representative firm commits both to outsourcing and profit sharing in anticipation of wage determination. After wage formation the representative firm determines employment and the representative worker decides on effort provision. The partly alternative timing structure (II) will change the timing of determination of profit sharing and wage determination by keeping other timing aspects similar as in (I). In this case the representative firm is flexible in the decision of profit sharing by deciding it after wage formation. We summarize these alternative timing decisions in Figure 1.<sup>1</sup>

**Figure 1:** *Alternative time sequences of decisions in terms of employment, effort, wage formation, profit sharing and outsourcing*

(I) *Strategic outsourcing and committed profit sharing:*



(II) *Strategic outsourcing and flexible profit sharing:*



<sup>1</sup> Whether profit sharing is committed or flexible in terms of base wage formation is an important new topic for empirical research.

This timing structure seems plausible when the implementation of a production mode with outsourcing requires irreversible investments concerning the establishment of a network of foreign suppliers. Of course, the relative timing of wage formation in the presence of outsourcing might be different in certain circumstances. Such a reversed timing structure would be relevant if the firms would flexible adjust their production mode, and decide whether to initiate foreign outsourcing once the domestic wage is determined.<sup>2</sup>

In the following sections we turn to an analysis of these two alternative decisions taking place at the different stages of the interaction between the representative firm, the monopoly labour union and the representative worker by using the backward induction and solving the game in reverse order.

### 3. Labour Demand and Employee Effort

Here we characterize the optimal labour demand by the representative firm and the effort by the representative worker in stage 4 by taking both profit sharing  $\tau$ , wage formation  $w$ , and outsourcing  $M$  as given. The technology is assumed to satisfy the following revenue function<sup>3</sup>

$$R(e, L, M) = \frac{1}{\alpha} (eL + M)^\alpha \quad (1)$$

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<sup>2</sup> Skaksen (2004) has analyzed this case using a Cobb-Douglas production function also in the case of homogenous domestic labour, but both in the absence of effort determination of workers and profit sharing of firms. Also Braun and Scheffel (2007) have developed a simple two-stage game between a monopoly union and a firm by assuming that the union sets wages before the firm decides on the degree of outsourcing and the level of production. They also abstract from effort determination of workers and profit sharing of firms. They argue that under flexible outsourcing the costs of outsourcing has an ambiguous effect on the wage set by the union.

<sup>3</sup> Specifying the inverse product demand function according to a monopolistic product market competition (for details, see the seminal paper by Dixit and Stiglitz (1977)) in a simple way as

$$p = \frac{1}{\alpha} D^{-(1-\alpha)}, \quad \alpha \in (0,1),$$

gives the following inverse elasticity of demand  $-p_D D / p = (1-\alpha)$

so that  $-D_p p / D = \frac{1}{1-\alpha} > 1$ . By assuming  $F(e, L, M) = eL + M$  and  $F = D$  gives another suggestion for (1). In what follows we do not elaborate the potential role of product market competition for our issues.

where  $L$  is unit of labour,  $e$  describes the effort determination,  $M$  indicates the amount of outsourcing and  $0 < \alpha < 1$ . We assume that outsourcing and effective labour are perfect substitutes. Outsourcing cost is defined by the convex function  $c(M)$  with  $c'(M), c''(M) > 0$ .

The disutility of effort is assumed to satisfy the following convex function  $g(e) = \gamma e^{1/\gamma}$  with  $0 < \gamma < 1$ , i.e.  $g'(e), g''(e) > 0$ . The individual utility function for the employed worker is (2a) and for the unemployed worker (2b)

$$u = w + \frac{\tau}{L} \pi - g(e) \quad (2a)$$

$$\bar{u} = b \quad (2b)$$

where  $w$  is the base wage,  $\tau$  is the profit share,  $\pi$  captures the firm's profit and  $b$  stands for the unemployed worker's exogenous outside option.

The profit function can now be expressed as

$$\pi = \frac{1}{\alpha} (eL + M)^\alpha - wL - c(M). \quad (3)$$

Given  $M$ ,  $w$  and  $e$  the first-order condition for the firm's optimal labour demand can be expressed as

$$\pi_L = (eL + M)^{-(1-\alpha)} e - w = 0 \quad (4)$$

and the second-order condition  $\pi_{LL} = -(1-\alpha)(eL + M)^{-(2-\alpha)} e^2 < 0$ . The first-order condition can be re-expressed as

$$L = w^{-\eta} e^{\eta-1} - \frac{M}{e} \quad (5)$$

where the *direct* wage elasticity of labour demand is  $\eta = -L_w w / L = 1/(1-\alpha) > 1$ .

According to (5)  $L = L\left(\underset{-}{w}, \underset{-}{M}, \underset{+}{e}\right)$  so that higher wage rate and higher outsourcing, which is a perfect substitute for domestic labour, will decrease labour demand. Moreover, higher employee's effort will increase labour demand. Labour demand (5) does not directly depend on profit sharing, which lies in conformity with empirical evidence (see e.g. Wadhvani and Wall (1990), Cahuc and Dormont (1997)).

The first-order condition in terms of effort determination for equation (2a) is

$$u_e = \frac{\tau}{L} \pi_e - g'(e) = 0. \quad (6)$$

Using  $g'(e) = e^{(1/\gamma)-1}$  and  $\pi_e = (eL + M)^{-(1-\alpha)} L$  equation (6) implies

$$e = (\tau w)^\gamma \quad (7)$$

where  $\gamma = \frac{e_\tau \tau}{e}$  is the elasticity of effort with respect to profit sharing (see about this, Koskela and Stenbacka (2006)). Therefore the optimal effort by worker is a positive function of both base wage,  $w$ , and profit sharing,  $\tau$ , i.e.  $e_w = \frac{\gamma e}{w}$  and  $e_\tau = \frac{\gamma e}{\tau}$ , so that profit sharing and base wage enhance productivity by increasing effort provision and thereby affects labour demand indirectly. But outsourcing will have no effect in the case of perfect substitutability between outsourcing  $M$  and employee effort  $e$ .<sup>4</sup>

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<sup>4</sup> This finding lies in conformity with empirics (see e.g. Booth and Frank (1999), Cable and Wilson (1990), Cahuc and Dormont (1997), Kruse (1992) and Wadhvani and Wall (1990)). Of course, we have to mention that these issues have not been studied to our knowledge empirically in the presence of outsourcing.

## 4. Wage Formation by Monopoly Labour Union with Strategic Outsourcing and Committed Profit Sharing

Now we continue the timing structure (I) to analyze the case, where the representative firm commits to the profit share prior to the base wage formation and by taking outsourcing as given and allowing for their effects on labour demand and effort determination.

### 4.1. Wage Formation

By analyzing the base wage formation by monopoly labour union under committed outsourcing and committed profit sharing in stage 3 the objective function of monopoly labour union is

$$V = (w - b)L + \tau \pi - g(e)L + bN \quad (8)$$

where  $b$  captures the exogenous minimum income for all labour union members  $N$ . Maximizing (8) in terms of base wage subject to labour demand (5), effort determination (7) and given outsourcing and profit sharing gives the following first-order condition

$$V_w = L + L_w(w - b) + \tau \pi_w - g(e)L_w - Lg'(e)e_w = 0. \quad (9)$$

This first-order condition for the monopoly labour union's base wage can be now expressed as follows<sup>5</sup>

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<sup>5</sup> By calculating the following parts  $g'(e)e_w = e^{(1/\gamma)-1}e_w = \gamma\tau^{1-\gamma+\gamma}w^{1-\gamma+\gamma-1} = \gamma\tau > 0$ , and  $\pi_w = -L + (eL + M)^{-\frac{1}{\eta}}Le_w = -L\left[1 - (eL + M)^{-\frac{1}{\eta}}e_w\right] = -(1-\gamma)L < 0$ , we can rewrite the first-order condition as follows  $V_w = 0 \Leftrightarrow \frac{L}{w}\left[w + \frac{L_w w}{L}(w - b) - \tau(1-\gamma)w - \frac{L_w w}{L}\tau\gamma w - \tau\gamma w\right] = 0$  which gives (10).

$$w = \left[ \frac{\hat{\eta}}{\hat{\eta}(1-\gamma\tau) - (1-\tau)} \right] b \quad (10)$$

where

$$\hat{\eta} \equiv -\frac{L_w w}{L} = \eta(1-\gamma) \left( 1 + \frac{M}{eL} \right) + \gamma = \eta \left( 1 + \frac{M}{eL} \right) - \left( \eta \left( 1 + \frac{M}{eL} \right) - 1 \right) \gamma \quad (11)$$

is the total wage elasticity of labour demand (see Appendix 1 for details). Therefore, the total wage elasticity  $\hat{\eta}$  depends on the base wage rate, the amount of outsourcing and the effort determination. This means that it should be emphasized that the wage determination (10) is expressed in the implicit (not explicit) form, because the wage elasticity of labour demand associated with the mark-up

$A^c = \left[ \frac{\hat{\eta}}{\hat{\eta}(1-\gamma\tau) - (1-\tau)} \right]$  depends also on the base wage.

The base wage elasticity of labour demand depends positively on the amount of outsourcing, i.e.

$$\hat{\eta}_M = \eta(1-\gamma) \frac{(eL+M)}{(eL)^2} = \frac{\eta(1-\gamma)}{eL} \left( 1 + \frac{M}{eL} \right) > 0. \quad (12a)$$

This positive relationship results from the fact that higher outsourcing will increase the ratio between outsourcing and effective labour, i.e.  $M/(eL)$ . This lies in conformity with empirics (see e.g. Hasan et al (2007), Slaughter (2001) and Senses (2006)).

Next we characterize the relationship between the base wage elasticity of labour demand and the base wage rate which comes via  $eL$ , i.e. we have

$$\hat{\eta}_w = \left[ \frac{\eta(1-\gamma)}{eL} \right]^2 \frac{M}{w} (eL+M) > 0. \quad (12b)$$

Therefore, the wage elasticity depends positively on the base wage rate in the

presence of outsourcing. In the absence of outsourcing, this effect is, however, zero, i.e.  $\hat{\eta}_w|_{M=0} = 0$ .

The relationship between the base wage elasticity and profit sharing can be written in the following way

$$\hat{\eta}_\tau = \eta(1-\gamma) \left[ \frac{-M(e_\tau L + L_\tau e)}{(eL)^2} \right] < 0 \quad (12c)$$

where  $e_\tau = \frac{\gamma e}{\tau} > 0$  and  $L_\tau = w^{-\eta}(\eta-1)e^{\eta-2}e_\tau + \frac{Me_\tau}{e^2} > 0$  so that

$e_\tau L + L_\tau e = \frac{\gamma}{\tau} \eta(eL + M)$ . Equation (12c) can be written as

$$\hat{\eta}_\tau = -\frac{\eta^2(1-\gamma)\gamma}{\tau} \frac{M}{eL} \left( 1 + \frac{M}{eL} \right) < 0.$$

The base wage elasticity depends negatively on profit sharing, because higher profit sharing will decrease the ratio between outsourcing and effective labour, i.e.  $M/eL$ . It is important to emphasize that there will be no effect in the absence of outsourcing, i.e.  $\hat{\eta}_\tau|_{M=0} = 0$ .

We can now summarize our findings as follows.

**Proposition 1:** *In the presence of outsourcing the wage elasticity of labour demand depends positively on the amount of outsourcing and on the base wage and negatively on the size of profit sharing.*

In the absence of outsourcing the total wage elasticity is slightly different. In this case the total wage elasticity is smaller, i.e.  $\hat{\eta}|_{M=0} = \eta - (\eta-1)\gamma$  (see about this, Koskela and Stenbacka (2006)). This implies the following monopoly labour union's base wage formation

$$w|_{M=0} = \frac{\eta - (\eta-1)\gamma}{[\eta - (\eta-1)\gamma](1-\gamma\tau) - (1-\tau)} b. \quad (13)$$

Next we characterize the comparative statics in a different way than in the explicit formulations. After characterizing the base wage elasticity of labour demand in terms of various parameters we now analyze the effects of these parameters on the wage formation by the monopoly labour union both under committed outsourcing and committed profit sharing.

Differentiating equation (10) with respect to the base wage and outsourcing gives (see Appendix 2)

$$\frac{dw}{dM} = - \frac{\frac{\hat{\eta}_M w(1-\tau)}{\hat{\eta}}}{\left[ \hat{\eta}(1-\gamma\tau) - (1-\tau) + \frac{\hat{\eta}_w w(1-\tau)}{\hat{\eta}} \right]} = - \frac{\frac{\hat{\eta}_M w}{\hat{\eta}}}{\left[ \hat{\eta} \frac{(1-\gamma\tau)}{(1-\tau)} - 1 + \frac{\hat{\eta}_w w}{\hat{\eta}} \right]} < 0. \quad (14)$$

Therefore, higher outsourcing will decrease the base wage formation because higher outsourcing will increase wage elasticity of labour demand. This lies in conformity with empirics under our assumption according to which there is substitutability between outsourcing and domestic labour (this also lies in conformity with empirics, see e.g. Munch and Skaksen (2005)).

Differentiating equation (10) with the base wage and profit sharing gives

$$\frac{dw}{d\tau} = - \frac{\frac{[\hat{\eta}_\tau(1-\tau) + \hat{\eta}(1-\hat{\eta}\gamma)]w}{\hat{\eta}}}{\left[ \hat{\eta}(1-\gamma\tau) - (1-\tau) + \frac{\hat{\eta}_w w(1-\tau)}{\hat{\eta}} \right]} = - \frac{\left[ \frac{\hat{\eta}_\tau}{\hat{\eta}} + \frac{(1-\hat{\eta}\gamma)}{(1-\tau)} \right]w}{\left[ \hat{\eta} \frac{(1-\gamma\tau)}{(1-\tau)} - 1 + \frac{\hat{\eta}_w w}{\hat{\eta}} \right]} = ? \quad (15)$$

(see Appendix 2). Therefore under this framework with outsourcing the effect of committed profit sharing on the base wage formation by monopoly labour union is a priori ambiguous because under outsourcing the profit sharing will have a negative effect to wage formation via the mark-up, but also a positive effect due to a negative effect on wage elasticity.

In the absence of outsourcing equation (15) can be re-expressed as follows

$$\left. \frac{dw}{d\tau} \right|_{M=0} = - \frac{(\eta - (\eta - 1)\gamma)[1 - (\eta - (\eta - 1)\gamma)]}{[(\eta - (\eta - 1)\gamma)(1 - \gamma\tau) - (1 - \tau)]^2} < 0 \quad (16)$$

so that profit sharing will decrease the mark-up of wage formation.<sup>6</sup>

We can now summarize our findings as follows

**Proposition 2:** *In the presence of outsourcing and given committed profit sharing a higher share of outsourcing will decrease the wage, whereas profit sharing has an ambiguous effect on the base wage. But in the absence of outsourcing higher profit sharing will decrease the base wage.*

## 4.2. Committed Profit Sharing

In the timing structure of decisions (I) in stage 2 the representative firm commits to profit sharing so that profit is maximized subject to labour demand (5), effort determination (7) and wage formation by the monopoly labour union (10) and by taking outsourcing as given, i.e.

$$\text{Max}_{\tau} \bar{\pi} = (1 - \tau) \left[ \frac{1}{\alpha} (eL + M)^{\alpha} - wL - c(M) \right] \quad (17a)$$

s.t.

$$L = w^{-\eta} e^{\eta-1} - \frac{M}{e} \quad (17b)$$

$$e = (\tau w)^{\gamma} \quad (17c)$$

$$w = \left[ \frac{\hat{\eta}}{\hat{\eta}(1 - \tau\gamma) - (1 - \tau)} \right] b \quad (17d)$$

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<sup>6</sup> Using  $\eta = \frac{1}{1 - \alpha}$  we have  $\eta - (\eta - 1)\gamma = \frac{1 - \alpha\gamma}{1 - \alpha} > 0$  and  $1 - (\eta - (\eta - 1)\gamma)\gamma = 1 - \frac{(1 - \alpha\gamma)\gamma}{1 - \alpha} > 0$  so that  $\left. \frac{dw}{d\tau} \right|_{M=0} < 0$ .

The first-order condition is  $-\pi + (1-\tau)\pi_\tau = 0$ , where the indirect profit can be expressed as  $\pi = \frac{1}{\eta-1} [w^{1-\eta} e^{\eta-1}] + \frac{wM}{e} - c(M)$ . The derivative of profit with respect to profit sharing by allowing for the wage and effort effects of profit sharing is

$$\pi_\tau = \frac{wL}{\tau} \left( \gamma - \frac{w_\tau \tau}{w} \right) \quad (18)$$

because  $L_\tau = 0$  due to the envelope theorem ( $\pi_L = 0$ ).<sup>7</sup> Next we have to solve the optimal committed profit sharing by using equations (18) and the indirect profit in  $-\pi + (1-\tau)\pi_\tau = 0$  so that given outsourcing  $M$  the optimal committed profit sharing can be presented as (see Appendix 3)

$$\tau^c = \frac{(\eta-1) \left( \gamma - \frac{w_\tau \tau}{w} \right)}{\left[ 1 + (\eta-1) \left( \gamma - \frac{w_\tau \tau}{w} \right) + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]}. \quad (19a)$$

This is an implicit form because concerning the RHS of (19a) employee effort, labour demand and base wage formation depend on profit sharing.

In the absence of outsourcing the optimal committed profit sharing can be re-expressed from (19a) as follows

$$\tau^c \Big|_{M=0} = \frac{(\eta-1) \left( \gamma - \frac{w_\tau \tau}{w} \right)}{\left[ 1 + (\eta-1) \left( \gamma - \frac{w_\tau \tau}{w} \right) \right]} \quad (19b)$$

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<sup>7</sup> The derivative  $\pi_\tau = \frac{1}{\eta-1} \left[ (1-\eta)w^{-\eta} e^{\eta-1} w_\tau + (\eta-1)w^{1-\eta} e^{\eta-2} e_\tau + M \frac{w_\tau e - e_\tau w}{e^2} \right]$  gives

$$\pi_\tau = \frac{w}{e} \frac{1}{\tau} \left( \gamma - \frac{w_\tau \tau}{w} \right) (w^{-\eta} e^\eta - M) \text{ so that we have (18).}$$

(see about this, Koskela and Stenbacka (2006)). Comparison between (19a) and (19b) shows that in the presence of outsourcing the optimal committed profit share is smaller than in the absence of outsourcing, i.e.  $\tau^c < \tau^c|_{M=0}$  because  $\frac{w_\tau \tau}{w} < 0$  in the absence of outsourcing by increasing profit sharing, while it is ambiguous and thereby smaller in the presence of outsourcing. Moreover, in the denominator of (19a)  $\eta \frac{M}{eL} - (\eta - 1) \frac{c(M)}{wL} > 0$  and it is zero in (19b). In both cases (19a) and (19b) higher wage elasticity with respect to profit sharing,  $w_\tau \tau / w$ , will have a negative effect on the optimal committed profit sharing.

We can now summarize our findings as follows

**Proposition 3:** *In the presence of outsourcing the optimal committed profit share is smaller than in the absence of outsourcing because in the absence of outsourcing profit share elasticity of wage formation is negative, but in the presence of outsourcing it is a priori ambiguous.*

## 5. Wage Formation by Monopoly Labour Union with Strategic Outsourcing and Flexible Profit Sharing

We now use the timing structure (II) to analyze the wage formation before the flexible profit sharing by the representative firm. After that and by taking outsourcing as given and committed before wage and profit sharing determinations we allowing for their effects on labour demand and employee effort.

### 5.1. Flexible Profit Sharing

First we study the optimal profit sharing in stage 3 decided after outsourcing and wage formation subject to labour demand and employee effort determinations. Now the profit sharing is decided to maximize profit by taking both the base wage and the outsourcing as given, i.e.

$$\text{Max}_\tau \bar{\pi} = (1-\tau) \left[ \frac{1}{\alpha} (eL + M)^\alpha - wL - c(M) \right] \quad (20a)$$

s.t.

$$L = w^{-\eta} e^{\eta-1} - \frac{M}{e} \quad (20b)$$

$$e = (\tau w)^\gamma \quad (20c)$$

The first-order condition is similar as in the case of committed profit sharing in terms of the first-order condition, i.e.  $-\pi + (1-\tau)\pi_\tau = 0$ , where the indirect profit is  $\pi = \frac{1}{\eta-1} [w^{1-\eta} e^{\eta-1}] + \frac{wM}{e} - c(M)$ . But as we show the optimal profit sharing is slightly different in the case of flexible profit sharing decision.

To allow for the envelope theorem due to  $\pi_L = 0$  so that  $L_\tau$  is not taken into account, the partial derivative of the profit in terms of profit sharing is  $\pi_\tau = w^{1-\eta} e^{\eta-2} e_\tau - \frac{wM}{e^2} e_\tau = \left( \frac{\gamma}{\tau} \right) \left[ w^{1-\eta} e^{\eta-1} - \frac{wM}{e} \right] = \left( \frac{\gamma}{\tau} \right) wL$  so that the first-order condition can be written as  $\frac{1}{\eta-1} wL + \frac{\eta}{\eta-1} \frac{wM}{e} - c(M) = \left( \frac{\gamma}{\tau} \right) (1-\tau) wL$ . This can be solved for the following optimal flexible profit sharing in the presence of outsourcing

$$\tau^f = \frac{(\eta-1)\gamma}{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]}. \quad (21a)$$

This is also like in the case of committed optimal profit sharing (see equation (19a)) an implicit form because both employee effort and labour demand also depend on profit sharing (see equations (5) and (7)) concerning the RHS of (21a).

In the absence of outsourcing profit sharing can be expressed as follows

$$\tau^f \Big|_{M=0} = \frac{(\eta-1)\gamma}{1 + (\eta-1)\gamma}. \quad (21b)$$

Comparison between (21a) and (21b) shows that in the presence of outsourcing the

optimal flexible profit share is smaller than in the absence of outsourcing, i.e.

$$\tau^f < \tau^f|_{M=0} \text{ because } \eta \frac{M}{eL} - (\eta - 1) \frac{c(M)}{wL} > 0.$$

Comparing the relationship between the optimal profit share under commitment (equation (19a)) and under flexibility (equation (21a)) it depends on what is the relationship between the wage rate and profit sharing. If  $w_\tau$  is negative (positive) then optimal profit share under commitment is larger (smaller) than that associated with flexibility,  $\tau^c > \tau^f$  ( $\tau^c < \tau^f$ ). Of course in the absence of outsourcing we have higher optimal committed profit share than optimal flexible profit share, i.e.

$$\tau^c|_{M=0} > \tau^f|_{M=0} \text{ by comparing equations (19b) and (21b).}$$

Now we analyze the effects of the parameters outsourcing and base wage on flexible profit sharing under strategic outsourcing (see Appendix 4 for details). The effect of  $M$  can be obtained by differentiating (21a) to get<sup>8</sup>

$$\frac{d\tau^f}{dM} = \frac{-\frac{1}{eL} \left[ \frac{(\eta - 1)\gamma Z}{\left[1 + (\eta - 1)\gamma + \eta \frac{M}{eL} - (\eta - 1) \frac{c(M)}{wL}\right]^2} \right]}{\left[ 1 + \frac{\frac{1}{eL} (\eta - 1)\gamma X}{\left[1 + (\eta - 1)\gamma + \eta \frac{M}{eL} - (\eta - 1) \frac{c(M)}{wL}\right]^2} \right]^2} \quad (22a)$$

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<sup>8</sup> In (22a)  $X = -\eta^2 \frac{M}{eL} \frac{\gamma}{\tau} (eL + M) + (\eta - 1) \frac{c(M)}{wL} \frac{\gamma}{\tau} ((\eta - 1)eL + \eta M)$  which we re-write so that holds  $X = -\frac{\gamma}{\tau} \left[ \eta \left( \eta \frac{M}{eL} - (\eta - 1) \frac{c(M)}{wL} \right) (eL + M) - (\eta - 1) \frac{c(M)}{wL} (eL) \right] < 0$ . We specify the parameter  $Z$  as  $Z = \eta \left( 1 + \frac{M}{eL} \right) - (\eta - 1) \frac{e}{w} \left( c'(M) + \frac{c(M)}{eL} \right) > 0$  which is equivalent to  $\eta \frac{M}{eL} - (\eta - 1) \frac{c(M)}{wL} + \eta - (\eta - 1) \frac{c'(M)e}{w} > 0$ . Also we assume although  $X < 0$  that  $(\eta - 1)\gamma(eL) + X\tau^2 > 0$ .

and using equation (21a), i.e.  $1 + (\eta - 1)\gamma + \eta \frac{M}{eL} - (\eta - 1) \frac{c(M)}{wL} = \frac{(\eta - 1)\gamma}{\tau}$  we can re-express (22a) as follows

$$\frac{d\tau^f}{dM} = \frac{-\frac{1}{eL} \frac{Z\tau^2}{(\eta-1)\gamma}}{\left[1 + \frac{\frac{1}{eL} X\tau^2}{[(\eta-1)\gamma]}\right]} = -\frac{Z\tau^2}{(\eta-1)\gamma(eL) + X\tau^2} < 0. \quad (22b)$$

Our assumptions,  $Z > 0$  and  $X < 0$ , but  $(\eta - 1)\gamma(eL) + X\tau^2 > 0$ , sound to be reasonable if optimal flexible profit sharing is small enough so that in this case optimal flexible profit sharing depends negatively on outsourcing.

Differentiating (21a) with respect to base wage gives<sup>9</sup>

$$\frac{d\tau^f}{dw} = \frac{-\frac{1}{eL} \left[ \frac{(\eta-1)\gamma Y}{\left[1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL}\right]^2} \right]}{\left[1 + \frac{\frac{1}{eL} (\eta-1)\gamma X}{\left[1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL}\right]^2}\right]^2}. \quad (23a)$$

Using equation (21a), we get

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<sup>9</sup> In (23a)  $Y = (1 - \gamma)\eta^2 \frac{M}{eL} \frac{\gamma}{\tau w} (eL + M) + (\eta - 1) \frac{c(M)}{w} \frac{e}{w} (1 - \hat{\eta})$ . For  $Y$  holds  $Y > 0$  which means that  $(1 - \gamma)\eta^2 \frac{M}{eL} \frac{\gamma}{\tau} (eL + M) > \frac{c(M)e}{w} (\eta - 1)(\hat{\eta} - 1)$  where  $\hat{\eta} - 1 = (1 - \gamma) \left( \eta - 1 + \eta \frac{M}{eL} \right)$  so that  $\eta^2 \frac{M}{eL} \frac{\gamma}{\tau} (eL + M) > \frac{c(M)e}{w} (\eta - 1) \left( \eta - 1 + \eta \frac{M}{eL} \right)$ .

$$\frac{d\tau^f}{dw} = \frac{-\frac{1}{eL} \frac{Y\tau^2}{(\eta-1)\gamma}}{\left[1 + \frac{\frac{1}{eL} X\tau^2}{[(\eta-1)\gamma]}\right]} = -\frac{Y\tau^2}{(\eta-1)\gamma(eL) + X\tau^2} < 0. \quad (23b)$$

If optimal flexible profit sharing is small enough, then under this assumption  $(\eta-1)\gamma(eL) + X\tau^2 > 0$  so that the base wage rate will have the negative effect on flexible profit sharing, while the base wage rate will have no effect in the absence of outsourcing, i.e.  $\left.\frac{\partial\tau^f}{\partial w}\right|_{M=0} = 0$ .

We can now summarize our findings as follows

**Proposition 4:** *In the presence of outsourcing and flexible profit sharing under reasonable assumptions higher base wage and higher outsourcing will decrease profit sharing but in the absence of outsourcing the base wage will have no effect on flexible profit sharing.*

## 5.2. Wage Formation under Flexible Profit Sharing

We now analyze the base wage formation in stage 2 by monopoly labour union under committed outsourcing and flexible profit sharing. The objective function can be written as

$$V = (w-b)L + \tau\pi - g(e)L + bN \quad (24)$$

where  $b$  captures the exogenous minimum income for all labour union members. Maximizing (24) in terms of the base wage subject to labour demand (5), effort determination (7), and profit sharing determination (23b), gives

$$V_w = L + L_w(w-b) + \tau\pi_w + \pi\tau_w - g(e)L_w - Lg'(e)e_w = 0 \quad (25)$$

where there is the new term  $\pi \tau_w$  compared with the case of committed profit sharing formulation (9). Using the assumption (23b) with respect to base wage gives  $\pi \tau_w < 0$ . Therefore higher wage rate will have negative effect on flexible profit sharing so that the base wage by monopoly labour union under committed outsourcing and flexible profit sharing is smaller than in the case of committed outsourcing and committed profit sharing. By using the earlier calculations according to which  $\pi_w = -(1-\gamma)L$  and  $g'(e)e_w = \gamma\tau$  we can solve the first order condition (25) as follows

$$w \left( \hat{\eta}(1-\gamma\tau) - (1-\tau) - \tau_w \frac{\pi}{L} \right) = \hat{\eta} b \quad (26)$$

where there is the new term  $-\tau_w \frac{\pi}{L}$  compared with the case of committed profit sharing. Rewriting of (26) gives the following implicit wage formation equation (see Appendix 5)

$$w = \left[ \frac{\hat{\eta}}{\hat{\eta}(1-\gamma\tau) - (1-\tau) - \tau_w \frac{\pi}{L}} \right] b = \left[ \frac{\hat{\eta}}{\hat{\eta}(1-\gamma\tau) - (1-\tau)(1+\gamma\tau\Gamma)} \right] b \quad (27)$$

with  $\Gamma = \frac{wY}{(\eta-1)\gamma(eL) + X\tau^2}$ . If optimal flexible profit sharing is small enough, then

$\Gamma > 0$  so that in this case in the presence of outsourcing the denominator in (27) is smaller than the one in (10) so that under  $\Gamma > 0$  the mark-up in terms of wage formation is higher under flexible profit share.

We can summarize this as follows

**Proposition 5:** *In the presence of outsourcing and flexible profit sharing the base wage formation is bigger than in the case of committed profit sharing if optimal flexible profit sharing is small enough.*

## 6. Strategic Outsourcing, Profit Sharing and Equilibrium Unemployment

We now move on to explore the implications of profit sharing and outsourcing on equilibrium unemployment. Our goal is to characterize the equilibrium unemployment as a function of institutional features of labor market, defined by the benefit replacement ratio, the structure of the compensation system and the given outsourcing.

In the case of committed profit sharing the base wage formation by the monopoly labour union has the form

$$w_i = A_i^c b \quad (28)$$

in industry  $i$ , where the wage mark-up is defined by  $A_i^c = \frac{\hat{\eta}}{\hat{\eta}(1-\gamma\tau) - (1-\tau)} > 1$ . For simplicity, we focus on the situation with identical industries in terms of the wage mark-up so that  $A_i^c = A^c$ . In a general equilibrium the outside option  $b$  will be re-interpreted to be the relevant outside option. We specify two alternative outside options. If in other industries there is no profit sharing, then the outside option can be specified as

$$b = (1 - u^c)w + u^c B \quad (29a)$$

where  $u^c$  denotes the unemployment rate in the case of committed profit sharing,  $B$  the unemployment benefit and  $w$  is the base wage formation and an unemployed worker faces the probability  $(1 - u^c)$  of being employed in another industry (for a standard justification we refer e.g. to Nickell and Layard (1999) and Layard et al. (2005), pp. 100-101). If the compensation scheme is similar of being employment in other industries, the outside option can be specified as

$$b = (1 - u^c) \left( w + \tau \frac{\pi}{L} \right) + u^c B. \quad (29b)$$

Equation (29b) captures the idea that all identical industries adopt profit sharing so that an unemployed worker faces the probability  $(1 - u^c)$  of being employed in another industry, which makes use of a similar compensation scheme. We further restrict in these outside options to the case of a constant benefit-replacement ratio  $q = B/w$  in the presence of unemployment so that  $0 < q < 1$ .

Combining (28) and (29a) and the assumption of a constant benefit-replacement ratio,  $q$ , we can rewrite the wage equation (28) as follows  $w = A^c(1 - u^c)w + A^c u^c q w$ . The aggregate unemployment rate can now be expressed according to

$$u^c = \frac{1 - \frac{1}{A^c}}{1 - q} \quad (30)$$

where the assumption is  $q < \frac{1}{A^c}$ . In the presence of outsourcing

$A^c = \frac{\hat{\eta}}{\hat{\eta}(1 - \gamma\tau) - (1 - \tau)}$  and in the absence of outsourcing

$A^c|_{M=0} = \frac{\eta - (\eta - 1)\gamma}{[\eta - (\eta - 1)\gamma](1 - \gamma\tau) - (1 - \tau)} > 1$ . Combining (28) and (29b) and the

assumption of a constant benefit-replacement ratio,  $q$ , we can rewrite the wage

equation (28) as follows  $w = A^c(1 - u^c)w + A^c(1 - u^c)\tau \frac{\pi}{L} + A^c u^c q w$ . The aggregate

unemployment rate can now be expressed according to  $u^c = \frac{1 - \frac{1}{A^c} + \tau \frac{\pi}{wL}}{1 - q + \tau \frac{\pi}{wL}}$ . This can

be presented in the presence of outsourcing as follows

$$u^c = \frac{1 - \frac{1}{A^c} + \frac{\tau}{\eta - 1} K}{1 - q + \frac{\tau}{\eta - 1} K} \quad (31a)$$

where  $K = 1 + \eta \frac{M}{eL} + (\eta - 1) \frac{c(M)}{wL}$  and the assumption is  $q < \frac{1}{A^c}$ . In the absence of outsourcing under the monopoly labour union's wage formation but committed profit sharing in all industries we have the following equilibrium unemployment

$$u^c|_{M=0} = \frac{1 - \frac{1}{A^c|_{M=0}} + \frac{\tau}{\eta - 1}}{1 - q + \frac{\tau}{\eta - 1}}. \quad (31b)$$

First we look at the implications of outside option (29a) on equilibrium unemployment according to which there is no profit sharing as a part of outside option in other industries. In the presence of outsourcing differentiating (30) with respect to outsourcing and profit sharing gives

$$\frac{du^c}{dM} = \frac{A_M^c}{[(1-q)A]^2} < 0 \quad \text{and} \quad \frac{du^c}{d\tau} = \frac{A_\tau^c}{[(\eta-1)A]^2} = ? \quad (32)$$

where  $A_M^c = -\frac{(1-\tau)\hat{\eta}_M}{[\hat{\eta}(1-\gamma\tau) - (1-\tau)]^2} < 0$  and  $A_\tau^c = -\frac{[(1-\tau)\hat{\eta}_\tau + \hat{\eta}(1-\hat{\eta}\gamma)]}{[\hat{\eta}(1-\gamma\tau) - (1-\tau)]^2} = ?$

Therefore, if there is no profit sharing as a part of outside option in other industries higher outsourcing will decrease equilibrium unemployment while profit sharing will have an ambiguous effect on equilibrium unemployment. In the absence of outsourcing higher profit sharing will decrease equilibrium unemployment because in this case  $A_\tau^c|_{M=0} < 0$ .

Next we look at the implications of outside option (29b) on equilibrium unemployment according to which there is profit sharing as a part of outside option in

all identical industries. In the presence of outsourcing differentiating (31a) with respect to outsourcing gives (see Appendix 6)<sup>10</sup>

$$\frac{du^c}{dM} = \left[ 1 - q + \frac{\tau}{\eta - 1} K \right]^{-2} \left( \frac{\tau}{\eta - 1} K_M \left( \frac{1}{\eta - 1} - q \right) + \frac{A_M^c}{(A^c)^2} \left( 1 - q + \frac{\tau}{\eta - 1} K \right) \right) = ? \quad (33)$$

The impact of outsourcing on equilibrium unemployment in this case is a priori ambiguous for the following reasons. Higher outsourcing will decrease the mark-up and therefore will have a negative effect on equilibrium unemployment due to lower wage elasticity of labour demand, but higher outsourcing will also increase profit relative to wage costs so that outside option will increase and therefore will have a positive effect on equilibrium unemployment.

In the presence of outsourcing differentiating (31a) with respect to profit sharing gives<sup>11</sup> (see Appendix 6)

$$\frac{du^c}{d\tau} = \left[ 1 - q + \frac{\tau}{\eta - 1} K \right]^{-2} \left( \frac{\tau}{\eta - 1} K_\tau \left( \frac{1}{\eta - 1} - q \right) + \frac{A_\tau^c}{(A^c)^2} \left( 1 - q + \frac{\tau}{\eta - 1} K \right) + \frac{1}{\eta - 1} \left[ K \left( 1 - \frac{\tau}{\eta - 1} \right) - 1 + \frac{1}{\eta - 1} \right] \right) = ? \quad (34a)$$

According to (34a) the impact of profit sharing on equilibrium unemployment in this case is a priori ambiguous for the following reasons. Higher profit sharing will have an ambiguous effect on the mark-up but higher profit sharing will also increase profit relative to wage costs so that outside option will increase and therefore will have a positive effect on equilibrium unemployment.

<sup>10</sup> Because our former assumption we can show that holds  $K_M = \frac{1}{eL} \left[ \eta \left( 1 + \frac{M}{eL} \right) - (\eta - 1) \frac{e}{w} \left( c'(M) + \frac{c(M)}{eL} \right) \right] > 0$ .

<sup>11</sup> This result holds because  $A_\tau^c = ?$  (see equation (15)) and

$$K_\tau = \frac{1}{eL} \left[ \eta^2 \frac{M}{eL} \frac{\gamma}{\tau w} \left( 1 + \frac{M}{eL} \right) + (\eta - 1)(\hat{\eta} - 1) \frac{ec(M)}{w^2} \right] > 0.$$

In the absence of outsourcing equation (34a) by using  $K_\tau = 0$  and  $K = 1$  can be re-expressed as

$$\left. \frac{du^c}{d\tau} \right|_{M=0} = \left[ 1 - q + \frac{\tau}{\eta - 1} \right]^{-2} \left( \frac{A_\tau^c}{(A_\tau^c)^2} \left( 1 - q + \frac{\tau}{\eta - 1} \right) + \frac{1}{\eta - 1} \left[ -\frac{\tau}{\eta - 1} + \frac{1}{A_\tau^c} \right] \right) = ? \quad (34b)$$

which is also ambiguous in terms of equilibrium unemployment.

In the case of flexible profit sharing the base wage formation by the monopoly labour union has the form  $w_i = A_i^f b$  in industry  $i$ , where the mark-up is defined by

$$A_i^f = \frac{\hat{\eta}}{\hat{\eta}(1 - \gamma\tau) - (1 - \tau) - (1 - \tau)\gamma\Gamma},$$

which is smaller than in the case of committed profit sharing. By one cannot fixed the effects of outsourcing and profit sharing on the mark-up due to the new part in the denominator of the mark-up, i.e.  $-(1 - \tau)\gamma\Gamma$ . The equilibrium unemployment in the flexible case when there is no profit sharing in other industries is expressed in (35a) and when also profit sharing is in the compensation scheme in other industries in (35b)

$$u^f = \frac{1 - \frac{1}{A^f}}{1 - q} \quad (35a)$$

$$u^f = \frac{1 - \frac{1}{A^f} + \frac{\tau}{\eta - 1} K}{1 - q + \frac{\tau}{\eta - 1} K} \quad (35b)$$

In the absence of outsourcing under the monopoly labour union's wage formation we have the following equilibrium unemployment in the presence of flexible profit sharing in all industries

$$u^f \Big|_{M=0} = \frac{1 - \frac{1}{A^f \Big|_{M=0}} + \frac{\tau}{\eta - 1}}{1 - q + \frac{\tau}{\eta - 1}} \quad (36)$$

where  $A^f \Big|_{M=0} = \frac{\eta - (\eta - 1)\gamma}{[\eta - (\eta - 1)\gamma](1 - \tau\gamma) - (1 - \tau)} > 1$  and  $\Gamma = 0$ . Implications of outside

option (29a) gives  $\frac{du^f}{d\tau} \Big|_{M=0} < 0$  and outside option (29b) gives  $\frac{du^f}{d\tau} \Big|_{M=0} = ?$  like in

the case of committed profit sharing.

We can now summarize equilibrium unemployment aspects in the presence of outsourcing and profit sharing when labour markets are imperfectly competitive as follows.

**Proposition 6:**

- (1) *If there is no profit sharing as a part of outside option in other industries higher outsourcing will decrease equilibrium unemployment while profit sharing will have an ambiguous effect on equilibrium unemployment, but in the absence of outsourcing higher profit sharing will decrease equilibrium unemployment.*
- (2) *If there is profit sharing as a part of outside option in other industries outsourcing and profit sharing will have ambiguous effects on equilibrium unemployment both under committed and flexible profit sharing. Also in the absence of outsourcing profit sharing will have an ambiguous effect on equilibrium unemployment.*

## 7. Optimal Strategic Outsourcing

So far we have restricted to a medium or short-run perspective where the firm has committed to the magnitude of outsourcing activity prior to wage determination,

profit sharing, labour demand and employee effort. Now we turn to explore the initial stage 1, where the firm commits to the outsourcing activity. It is assumed that the long-run production mode decision may internalize the effect of the share of outsourced production on wage formation depending on the time sequence decision of profit sharing.

In the long-run the firm is assumed to have rational expectations regarding subsequent outcomes and determines the magnitude optimal committed outsourcing

so as to maximize profit  $\pi = (1 - \tau) \left( \frac{\eta}{\eta - 1} [eL + M]^{\frac{\eta-1}{\eta}} - wL - c(M) \right)$  subject to labour

demand (5) (allowing for the envelope theorem according to  $\pi_L = 0$ ) and effort determination (7). Moreover, in the presence of committed profit sharing profit maximization is also subject to wage formation (10) and profit sharing (19a) (allowing for the envelope theorem according to  $\pi_\tau = 0$ ), while in the presence of flexible profit sharing also subject to profit sharing (21a) (allowing for the envelope theorem according to  $\pi_\tau = 0$ ) and wage formation (27).

Allowing the envelope theorem both in terms of the optimal profit sharing ( $\pi_\tau = 0$ ) and the optimal labour demand ( $\pi_L = 0$ ) we differentiate

$\pi = \frac{\eta}{\eta - 1} [eL + M]^{\frac{\eta-1}{\eta}} - wL - c(M)$  with respect to  $M$ . Using  $eL = w^{-\eta} e^\eta - M$  and

$wL = w^{1-\eta} e^{\eta-1} - w \frac{M}{e}$  we can express the profit function as

$$\pi = \frac{1}{\eta - 1} w^{1-\eta} e^{\eta-1} + w \frac{M}{e} - c(M). \quad (37)$$

Differentiating (37) with respect to  $M$  and allowing both its direct effects and the indirect effects via the base wage and the effort determination gives

$\pi_M = -w^{-\eta} e^{\eta-1} w_M + w^{1-\eta} e^{\eta-2} e_w w_M + \frac{w}{e} + \frac{M}{e} w_M - w \frac{M}{e^2} e_w w_M - c'(M) = 0$ . This can

be written as follows  $\pi_M = -w_M \left[ w^{-\eta} e^{\eta-1} - \frac{M}{e} - w^{-\eta} e^{\eta-1} \gamma + \frac{M}{e} \gamma \right] + \frac{w}{e} - c'(M) = 0$

so that we have the first-order condition

$$\pi_M = \left( \frac{w}{e} - w_M L(1-\gamma) \right) - c'(M) = 0. \quad (38)$$

The second-order condition is  $\pi_{MM} = \frac{w_M}{e} (2-\gamma) - w_{MM} L(1-\gamma) - c''(M) < 0$ .

In addition to the direct marginal cost  $c'(M)$  there is the direct marginal profit  $\frac{w}{e}$  via outsourcing (see equation (38)) and the indirect marginal effects via the effect of outsourcing on wage, i.e.  $-w_M L(1-\gamma)$ . In the presence of committed profit sharing outsourcing moderates base wage so that the marginal profit will increase via  $-w_M L(1-\gamma) > 0$ . But in the presence of flexible profit sharing the indirect marginal profit  $-w_M L(1-\gamma)$  in terms of outsourcing is a priori ambiguous, because in this case the wage effect of outsourcing can be negative or positive.

We can summarize this as follows

**Proposition 7:** *In terms of optimal long-run strategic outsourcing wage moderation will have the positive indirect marginal profit in the presence of committed profit sharing due to wage moderation, but in the presence of flexible profit sharing this effect is a priori ambiguous.*

## 8. Conclusions

We have analyze the following questions associated with outsourcing and profit sharing under imperfectly competitive labour markets by using the scenario without outsourcing: How does strategic outsourcing, which we assume to be substitute for effective labour, influence wage formation, profit sharing and employee effort when firms commit to optimal profit sharing before wage formation or decide profit sharing

after wage formation. We also have studied the relationship between outsourcing, profit sharing and equilibrium unemployment as a function of various institutional features of the labour market. Finally, we have characterized the long-run perspective for the optimal production mode in terms of strategic outsourcing.

We have shown that in the presence of outsourcing the wage elasticity of labour demand depends positively on the amount of outsourcing and on the wage, but negatively on the size of profit sharing. As a result it has been presented that in the case of committed profit sharing strategic outsourcing has a negative effect on wage formation. This lies in conformity with empirics and results from our assumption of perfect substitutability between outsourcing and effective domestic labour. Under flexible profit sharing the wage is higher if optimal flexible profit share is small enough. But the impact of profit share on wage formation under commitment in the presence of outsourcing is not necessary larger than profit share under flexibility. Only if there is a wage moderation effect in the committed case we are in line with the literature, which argues that the optimal profit share under commitment is higher than the profit share under flexibility. If the wage rate increases by contrast the opposite result occurs. In the flexible case we show that a higher wage rate will lower the profit share so that there is negative relationship between base wage and optimal flexible profit share.

If there is no profit sharing as a part of outside option in other industries higher outsourcing will decrease equilibrium unemployment while profit sharing will have an ambiguous effect on equilibrium unemployment, but in the absence of outsourcing higher profit sharing will decrease equilibrium unemployment. If there is profit sharing as a part of outside option in other industries outsourcing and profit sharing will have ambiguous effects on equilibrium unemployment. Also in the absence of outsourcing profit sharing will have an ambiguous effect on equilibrium unemployment. Finally, in terms of optimal long-run strategic outsourcing wage moderation will have the positive indirect marginal profit in the presence of committed profit sharing due to wage moderation, but in the presence of flexible profit sharing this effect is a priori ambiguous.

There are several new research topics associated with these issues. One important issue is to study the implications of labour taxation and labor tax reforms on

effort, labour demand wage formation, profit sharing and equilibrium unemployment in the presence of outsourcing. Another topics are to extend the framework to allow for heterogeneity of workers in the domestic country in the presence of outsourcing and to allow for wage negotiations between labour unions and firms. Finally, it is also important to do empirical research associated with various results we have presented.

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## APPENDIX 1:

The derivative of labour demand (5) with respect to the base wage is

$$L_w = -\eta w^{-\eta-1} e^{\eta-1} + w^{-\eta} (\eta-1) e^{\eta-2} e_w + \frac{M e_w}{e^2} \quad \text{so that we have}$$

$$-L_w w = \eta w^{-\eta} e^{\eta-1} - w^{-\eta} e^{\eta-1} (\eta-1) \frac{e_w w}{e} - \frac{M}{e} \frac{e_w w}{e} = w^{-\eta} e^{\eta-1} (\eta(1-\gamma) + \gamma) - \frac{M}{e} \gamma. \quad (\text{A1})$$

This leads to (11). The effect of the base wage on the wage elasticity of labour demand is

$$\hat{\eta}_w = -\eta(1-\gamma) \frac{M}{(eL)^2} (e_w L + L_w e). \quad (\text{A2})$$

We can write  $e_w L = \frac{\gamma e}{w} \left( w^{-\eta} e^{\eta-1} - \frac{M}{e} \right) = \frac{\gamma}{w} (w^{-\eta} e^{\eta} - M)$  and

$L_w e = -\eta w^{-\eta-1} e^{\eta} + (\eta-1) w^{-\eta-1} e^{\eta} \gamma + \frac{M \gamma}{w}$ . Using  $w^{-\eta} e^{\eta} = eL + M$  we can show after

calculations that  $e_w L + L_w e = -\left( \frac{\eta(1-\gamma)}{w} \right) (eL + M)$ . Therefore, the total wage

elasticity of labour demand in terms of the base wage in the presence of outsourcing can be expressed in (12b). QED.

## APPENDIX 2:

Differentiating the implicit wage formation (10) with respect to the base wage and outsourcing gives

$$\left( 1 - \frac{[\hat{\eta}(1-\gamma\tau) - (1-\tau)] \hat{\eta}_w - \hat{\eta}(1-\gamma\tau) \hat{\eta}_w}{[\hat{\eta}(1-\gamma\tau) - (1-\tau)]^2} b \right) dw =$$

$$\frac{[\hat{\eta}(1-\gamma\tau) - (1-\tau)] \hat{\eta}_M - \hat{\eta}(1-\gamma\tau) \hat{\eta}_M}{[\hat{\eta}(1-\gamma\tau) - (1-\tau)]^2} b dM \quad (\text{A3})$$

which can be expressed as

$$\frac{dw}{dM} = - \frac{\frac{\hat{\eta}_M (1-\tau)}{[\hat{\eta}(1-\gamma\tau) - (1-\tau)]^2} b}{\left[ 1 + \frac{\hat{\eta}_w (1-\tau) b}{[\hat{\eta}(1-\gamma\tau) - (1-\tau)]^2} \right]} < 0. \quad (\text{A4})$$

Using equation (10), i.e.  $b = \frac{w[\hat{\eta}(1-\gamma\tau)-(1-\tau)]}{\hat{\eta}}$ , the relationship between the wage formation and outsourcing can be written as equation (14).

Differentiating the equation (10) with the base wage and profit sharing gives

$$\left[ 1 + \frac{\hat{\eta}_w(1-\tau)b}{[\hat{\eta}(1-\gamma\tau)-(1-\tau)]^2} \right] dw = \tag{A5}$$

$$\left( \frac{b}{[\hat{\eta}(1-\gamma\tau)-(1-\tau)]^2} [(\hat{\eta}(1-\gamma\tau)-(1-\tau))\hat{\eta}_\tau - \hat{\eta}(1-\gamma\tau)\hat{\eta}_\tau + \hat{\eta}^2\gamma - \hat{\eta}] \right) d\tau$$

which can be expressed as  $\frac{dw}{d\tau} = -\frac{[\hat{\eta}_\tau(1-\tau) + \hat{\eta}(1-\hat{\eta}\gamma)]b}{[\hat{\eta}(1-\gamma\tau)-(1-\tau)]^2} \left[ 1 + \frac{\hat{\eta}_w(1-\tau)b}{[\hat{\eta}(1-\gamma\tau)-(1-\tau)]^2} \right]$ , where  $\hat{\eta}_\tau < 0$  and

$\hat{\eta}(1-\hat{\eta}\gamma) = ?$  Using equation (10), i.e.  $b = \frac{w[\hat{\eta}(1-\gamma\tau)-(1-\tau)]}{\hat{\eta}}$ , the relationship between wage formation and outsourcing can be written as equation (15). QED.

### APPENDIX 3:

Using the first-order condition for profit share commitment given outsourcing, i.e.  $-\pi + (1-\tau)\pi_\tau = 0$  so that  $\pi_\tau > 0$  and we can rewrite it as follows

$$\frac{1}{\eta-1} [w^{1-\eta} e^{\eta-1}] + \frac{wM}{e} - c(M) = (1-\tau) \frac{wL}{\tau} \left( \gamma - \frac{w_\tau \gamma}{w} \right), \text{ which is equivalent to}$$

$$\frac{1}{\eta-1} \left[ wL + \frac{wM}{e} \right] - c(M) = (1-\tau) \frac{wL}{\gamma} \left( \gamma - \frac{w_\tau \gamma}{w} \right). \tag{A6}$$

This can be expressed as  $1 + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} = (1-\tau) \frac{1}{\tau} \left( \gamma - \frac{w_\tau \tau}{w} \right) (\eta-1)$  so that given outsourcing  $M$  the optimal committed profit sharing can be presented as

$$\tau^c = \frac{\left( \gamma - \frac{w_\tau \tau}{w} \right) (\eta-1)}{\left[ 1 + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} + \left( \gamma - \frac{w_\tau \tau}{w} \right) (\eta-1) \right]} \tag{A7}$$

QED.

#### APPENDIX 4:

By differentiating the implicit profit share function (21a) with respect to the profit sharing and the outsourcing gives the following total differential

$$\left( \frac{(\eta-1)\gamma \left[ -\frac{\eta M}{(eL)^2} (e_\tau L + L_\tau e) + \frac{(\eta-1)c(M)}{w} \frac{L_\tau}{L^2} \right]}{1 + \frac{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2}{}} \right) d\tau^f =$$

$$- \frac{(\eta-1)\gamma \left[ \frac{M}{e} \left( \frac{L - ML_M}{L^2} \right) - \frac{(\eta-1)}{w} \left( \frac{c'(M)L - L_M c(M)}{L^2} \right) \right]}{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2} dM \quad (\text{A8})$$

By using  $e_\tau = \frac{\gamma}{\tau} e$ ,  $L_\tau = (\eta-1)w^{-\eta}e^{\eta-1} \frac{\gamma}{\tau} + \frac{M}{e} \frac{\gamma}{\tau}$  and  $L_M = -1/e$  equation (A8) can

be re-expressed as

$$\left( \frac{(\eta-1)\gamma/eL \left[ -\eta^2 \frac{M}{eL} \frac{\gamma}{\tau} (eL + M) + (\eta-1) \frac{c(M)}{wL} \frac{\gamma}{\tau} ((\eta-1)eL + \eta M) \right]}{1 + \frac{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2}{}} \right) d\tau^f =$$

$$- \frac{(\eta-1)\gamma/eL \left[ \eta \left( 1 + \frac{M}{eL} \right) - (\eta-1) \frac{e}{w} \left( c'(M) + \frac{c(M)}{eL} \right) \right]}{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2} dM \quad (\text{A9})$$

which gives (22a).

By differentiating the implicit profit share function (21a) with respect to the profit sharing and the base wage gives the following total differential

$$\left( \frac{(\eta-1)\gamma \left[ -\frac{\eta M}{(eL)^2} (e_\tau L + L_\tau e) + \frac{(\eta-1)c(M)}{w L^2} L_\tau \right]}{1 + \frac{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2}{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2}} \right) d\tau^f = \quad (\text{A10})$$

$$- \frac{(\eta-1)\gamma \left[ \eta \frac{-M}{(eL)^2} (e_w L + L_w e) + (\eta-1) \frac{c(M)}{(wL)^2} (L + L_w w) \right]}{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2} dw$$

By using  $e_\tau = \frac{\gamma}{\tau} e$ ,  $L_\tau = (\eta-1)w^{-\eta}e^{\eta-1} \frac{\gamma}{\tau} + \frac{M}{e} \frac{\gamma}{\tau}$ ,  $e_w = \frac{\gamma}{w} e$  and

$L_w = w^{-\eta-1}e^{\eta-1}[(\eta-1)\gamma - \eta] + \frac{M}{e} \frac{\gamma}{w}$  equation (A10) can be re-expressed as

$$\left( \frac{(\eta-1)\gamma/eL \left[ -\eta^2 \frac{M}{eL} \frac{\gamma}{w} (eL + M) + (\eta-1) \frac{c(M)}{w} \frac{e}{w} (1 - \hat{\eta}) \right]}{1 + \frac{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2}{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2}} \right) d\tau^f = \quad (\text{A11})$$

$$- \frac{(\eta-1)\gamma/eL \left[ (1-\gamma)\eta^2 \left( 1 + \frac{M}{eL} \right) - (\eta-1) \frac{e}{w} \left( c'(M) + \frac{c(M)}{eL} \right) \right]}{\left[ 1 + (\eta-1)\gamma + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]^2} dw$$

which gives (23a). QED.

## APPENDIX 5:

According to equation (23b),  $\frac{d\tau^f}{dw} = \tau_w^f = -\frac{Y \tau^2}{(\eta-1)\gamma(eL) + X \tau^2} < 0$  so that

$-\tau_w \frac{\pi}{L} > 0$ , where  $\frac{\pi}{L} = \frac{w}{(\eta-1)} \left[ 1 + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right]$ . Now using equation (21a) we

have  $\frac{\pi}{L} = \frac{w\gamma(1-\tau)}{\tau}$  so that we can re-express

$$-\tau_w \frac{\pi}{L} = \frac{w(1-\tau) \gamma Y}{(\eta-1)\gamma(eL) + X \tau^2}, \quad (\text{A12})$$

where  $Y = (1-\gamma)\eta^2 \frac{M}{eL} \frac{\gamma}{\tau w} (eL+M) + (\eta-1) \frac{c(M)}{w} \frac{e}{w} (1-\hat{\eta}) > 0$  and

$$X = -\frac{\gamma}{\tau} \left[ \eta \left( \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \right) (eL+M) - (\eta-1) \frac{c(M)}{wL} (eL) \right] < 0. \text{ QED.}$$

## APPENDIX 6:

In the presence of outsourcing differentiating (31a) with respect to  $M$  gives -

$$\text{by using the notation } K = 1 + \eta \frac{M}{eL} - (\eta-1) \frac{c(M)}{wL} \text{ so that } u^c = \frac{1 - \frac{1}{A^c} + \frac{\tau}{\eta-1} K}{1 - q + \frac{\tau}{\eta-1} K}.$$

$$\begin{aligned} \frac{du^c}{dM} &= \left[ 1 - q + \frac{\tau}{\eta-1} K \right]^{-2} \left( \left( 1 - q + \frac{\tau}{\eta-1} K \right) \left( \frac{\tau}{\eta-1} K_M + \frac{A_M^c}{(A^c)^2} \right) - \left( 1 - \frac{1}{A^c} + \frac{\tau}{\eta-1} K \right) \frac{\tau}{\eta-1} K_M \right) = \\ & \left[ 1 - q + \frac{\tau}{\eta-1} K \right]^{-2} \left( \frac{\tau}{\eta-1} K_M \left( \frac{1}{A^c} - q \right) + \frac{A_M^c}{(A^c)^2} \left( 1 - q + \frac{\tau}{\eta-1} K \right) \right) = ? \end{aligned} \tag{A13}$$

where  $A_M^c < 0$  and

$$K_M = \eta \frac{eL+M}{(eL)^2} - (\eta-1) \frac{1}{eL} \left( \frac{Lec'(M) + c(M)}{wL} \right) = \frac{1}{eL} \left[ \eta \left( 1 + \frac{M}{eL} \right) - (\eta-1) \frac{e}{w} \left( c'(M) + \frac{c(M)}{eL} \right) \right] > 0.$$

In the presence of outsourcing differentiating (31a) with respect to profit sharing gives

$$\begin{aligned} \frac{du^c}{d\tau} &= \left[ 1 - q + \frac{\tau}{\eta-1} K \right]^{-2} \left( \left( 1 - q + \frac{\tau}{\eta-1} K \right) \left( \frac{\tau}{\eta-1} K_\tau + \frac{A_\tau^c}{(A^c)^2} \right) - \left( 1 - \frac{1}{A^c} + \frac{\tau}{\eta-1} K \right) \frac{\tau}{\eta-1} K_\tau + \right. \\ & \left. \frac{1}{\eta-1} \left[ K - \left( 1 - \frac{1}{A^c} + \frac{\tau}{\eta-1} K \right) \right] \tau K_\tau - \left( 1 - \frac{1}{A^c} + \frac{\tau}{\eta-1} K \right) \right) = \\ & \left[ 1 - q + \frac{\tau}{\eta-1} K \right]^{-2} \left( \frac{\tau}{\eta-1} K_\tau \left( \frac{1}{A^c} - q \right) + \frac{A_\tau^c}{(A^c)^2} \left( 1 - q + \frac{\tau}{\eta-1} K \right) + \frac{1}{\eta-1} \left[ K \left( 1 - \frac{\tau}{\eta-1} \right) - 1 + \frac{1}{A^c} \right] \right) = ? \end{aligned} \tag{A14}$$

where  $A_\tau^c = ?$  (see equation (15)) and

$$K_\tau = \eta \frac{-M(e_\tau L + L_\tau e)}{(eL)^2} + (\eta-1) \frac{c(M)L_\tau}{wL^2} = \frac{1}{eL} \left[ \eta^2 \frac{M}{eL} \frac{\gamma}{\tau w} \left( 1 + \frac{M}{eL} \right) + (\eta-1)(\hat{\eta}-1) \frac{ec(M)}{w^2} \right] > 0.$$

QED.