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# Dynamic Moral Hazard and Project Completion

## Abstract

We analyse a simple model of dynamic moral hazard in which there is a clear and tractable trade-off between static and dynamic incentives. In our model, a principal wants an agent to complete a project. The agent undertakes unobservable effort, which affects in each period the probability that the project is completed. The principal pays only on completion of the project. We characterise the wage that the principal sets, with and without commitment. We show that the commitment wage declines over time, in order to give the agent incentives to exert effort.

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# 1 Introduction

It has been estimated that approximately 90% of American companies use, to some extent at least, agency firms to find workers; see Fernandez-Mateo (2003). According to Finlay and Coverdill (2000), between 13% and 20% of firms use private employment agencies “frequently” to find a wide variety of workers. The proportion is higher when searching for senior executives. Recent surveys in the pharmaceutical sector estimate almost two-thirds of senior executive hires involve a ‘headhunter’. Typically, it takes a number of months to find and recruit a candidate; for example, the same pharmaceutical source states that the average length of time taken to fill a post is between four and six months from the start of the search. See Pharmafocus (2007). Payment to headhunters takes two forms. Contingency headhunters are paid only when they place a candidate successfully (a typical fee is 20–35% of the candidate’s first year’s salary). Retained headhunters receive an initial payment, some payment during search, and a bonus payment on success. See Finlay and Coverdill (2000). There is risk attached to search: between 15% and 20% of searches in the pharmaceutical sector fail to fill a post; Pharmafocus (2007).

Residential real estate accounts for a large share of wealth—33% in U.S. in 2005, according to Merlo, Ortalo-Magné, and Rust (2006), and 26% in the UK in 1999, according to the Office of National Statistics. The value of residential house sales in England and Wales between July 2006 and July 2007 was of the order of 10% of the UK’s GDP.<sup>1</sup> According to Office of Fair Trading (2004), over nine out of ten people buying and selling a home in England and Wales use a real estate agent. In the UK, the majority of residential properties are marketed under sole agency agreement: a single agent is appointed by a seller to market their property. (The details of the UK market are given in Merlo and Ortalo-Magné (2004) and Merlo, Ortalo-Magné, and Rust (2006).) The median time in the UK to find a buyer who eventually buys the property is 137 days. (Once an offer is accepted, it still takes on average 80–90 days until the transfer is completed.) The real estate agent can affect buyer arrival rates, through exerting marketing effort. But there

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<sup>1</sup>The UK’s GDP in 2006 was £1.93 trillion, according to the ONS. The UK Land Registry reports that roughly 105,000 houses were sold each month over the period July 2006—July 2007, at an average price of around £180,000 each. See Land Registry (2007).

is also exogenous risk (such as general market conditions) that affect the time to sale. In most agreements, the real estate agent is paid a proportion of the final sale price on completion.

In both of these examples, a principal hires an agent to complete a project. The principal gains no benefit until the project is completed. The agent can affect the probability of project completion by exerting effort. The task for the principal is to provide the agent with dynamic incentives to provide effort. The task of this paper is to analyse the dynamic incentives that arise in these settings and the contracts that are written as a result.

Continuation values matter for both sides. For the agent, its myopic incentives are to equate the marginal cost of effort with the marginal return. But if it fails to complete the project today, it has a further chance tomorrow. This dynamic factor, all other things equal, tends to reduce the agent's effort towards project completion. Similarly, the principal's myopic incentives trade off the marginal costs of inducing greater agent effort (through higher payments) with the marginal benefits. But the principal also knows that the project can be completed tomorrow; all other things equal, this tends to lower the payment that the principal pays today for project completion. On the other hand, the principal also realises that the agent faces dynamic incentives; this factor, on its own, tends to increase the payment to the agent.

Our modelling approach allows us to resolve these different incentives to arrive at analytical conclusions. We can do this with some degree of generality; for example, we allow for the principal and agent to have different discount rates. This then allows us isolate the different channels that are at work in the model. Three features lie behind our approach. First, we look at pure project completion: the principal cares only about final success and receives no interim benefit from the agent's efforts. Secondly, we deal with the continuous-time limit of the problem. As noted by Sannikov (forthcoming), this can lead to derivations that are much more tractable than those in discrete time models. Finally, in our model, the agent's participation constraint is not binding; consequently, its continuation value is strictly positive while employed by the principal. (This arises

because of a limited liability constraint.) The principal uses the level and dynamics of the agent's continuation value to generate incentives for effort.

We start by comparing two stationary problems, in which the principal pays the sequentially rational wage (i.e., has no ability to commit to a contract to pay the agent), and in which the principal can commit to pay a constant wage to the agent. Given our set-up, the principal pays only on completion of the project: the agent receives no interim payments. In the sequentially rational solution, a change in the wage offer currently offered by the principal has no effect on future wage offers. As a result, there is no dynamic incentive effect to consider. In contrast, when the principal commits to a constant wage over time, by paying more today, the principal must also pay more tomorrow. As a result, the principal then also increases the continuation value of the agent. This dynamic incentive decreases the agent's current effort. Consequently, the sequentially rational wage offer is higher than the wage offered by a principal who commits to a constant wage.

This result gives an immediate intuition for what the wage profile with full commitment looks like: we show that it must be decreasing over time. This is the only way in which the principal can resolve in its favour the trade-off between static incentives (which call for a high current wage) and dynamic incentives (which call for lower future wages). We establish this result for all discount rates, regardless of whether the principal or the agent has the higher discount rate. Further, we show that there are two cases. When the principal is less patient than the agent, the wage, and hence agent's effort, must converge to zero—this is the only way in the which the principal can reduce the agent's continuation value and so induce high effort. On the other hand, when the principal is more patient than the agent, the wage and agent's effort converge to strictly positive levels. In this case, the principal can rely on the agent's impatience to provide incentives for current effort.

We can also say something about the principal's use of deadlines to provide incentives. We can show that when the principal commits to a constant wage and can employ only one agent, a deadline is not used. But it is clear that the best outcome for the principal is

to use a sequence of agents, each for a single period. So suppose that the principal can fire an agent and then find a replacement with some probability: replacement agents arrive according to a Poisson process. We show that the principal's optimal deadline decreases with the arrival rate of replacement agents.

We conclude the analysis by considering how the main results might change when project quality matters. (For most of the paper, we assume that the completed project yields a fixed and verifiable benefit to the principal.) The issue is complicated; but we provide at least one setting in which our results hold even with this complication.

At first glance, our results look similar to those in papers that look at unemployment insurance: see e.g., Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997). In these papers, a government must make payments to an unemployed worker to provide a minimum level of expected discounted utility to the worker. The worker can exert effort to find a job; the government wants to minimise the total cost of providing unemployment insurance. Shavell and Weiss (1979) show that the optimal benefit payments to the unemployed worker should decrease over time. Hopenhayn and Nicolini (1997) establish that the government can improve things by imposing a tax on the individual when it finds work.

We find that the principal's optimal payment under full commitment decreases over time. The unemployment insurance literature finds decreasing unemployment benefits over time. Despite this similarity, our results are quite different. Perhaps the easiest way to see this is to note that both Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997) require that the agent (worker) is risk averse. Without this assumption, neither paper can establish a decreasing profile of payments. In contrast, we allow for a risk neutral agent. We ensure that the principal does not simply sell the project to the agent by imposing limited liability. The limited liability constraint creates a positive continuation value for the agent. The principal controls this continuation value in order to give the agent incentives for effort. If instead we have a risk averse agent and no limited liability, then, in our model, the principal would employ a constant penalty while the project is not completed, and a constant payment on completion. Hence our work

identifies the agent's continuation value, and not its risk aversion, as a key factor driving decreasing payments.

We also argue that our paper identifies much more clearly the intertemporal incentives in this type of dynamic moral hazard problem. We show explicitly how continuation values affect current choices. By allowing for different discount rates between the principal and the agent, we can close off different channels in the model in order to highlight their effects. The simplicity of our set-up allows us to consider issues—such as deadlines and project quality—that are not dealt with in the unemployment insurance papers.

Our work is, of course, related to the broader literature on dynamic moral hazard problems: particularly the more recent work on continuous-time models. This literature has demonstrated in considerable generality the benefits to the principal of being able to condition contracts on the intertemporal performance of the agent. By doing so, the principal can relax the agent's incentive compatibility constraints. See e.g., Malcomson and Spinnewyn (1988) and Laffont and Martimort (2002). More recently, Sannikov (2007), Sannikov (forthcoming) and Williams (2006) have analysed principal-agent problems in continuous time. For example, in Sannikov (forthcoming), an agent controls the drift of a diffusion process, the realisation of which in each period affects the principal's payoff. When the agent's action is unobserved, Sannikov characterises the optimal contract quite generally, in terms of the drift and volatility of the agent's continuation value in the contract. For example, he shows that the drift of the agent's value always points in the direction where it is cheaper to provide the agent with incentives.

An immediate difference between this paper and e.g., Sannikov (forthcoming) is that we concentrate on project completion. We think this case is of independent interest for a number of different economic applications. But we also think that our setting, while less general in some respect than Sannikov's, serves to make very clear the intertemporal incentives at work.

The rest of the paper is structured as follows. Section 2 lays out the basic model. Section 3 looks at the sequentially rational solution in which the principal has no commitment ability. Section 4 looks at the situation when the principal commits to a wage

that is constant over time. The contrast between this and the sequentially rational solution gives a strong intuition for the properties of the wage that the principal sets when it has full commitment power (and so can commit to a non-constant wage). The latter is analysed in section 5. Section 6 looks at the issue of deadlines; section 7 considers the issues that arise when the agent can affect the quality of the completed project. Our overall conclusions are stated in section 8.

## 2 The Model

Consider a continuous-time model where an agent must exert effort in any period in order to have a positive probability of success in a project. Assume that the effort choices of the agent are unobservable but the success of the project is verifiable; hence payments can be contingent only on the event of success or no success.

The principal and the agent are risk neutral; but the agent is credit constrained so that payments from principal to agent must be non-negative in all periods. (Otherwise the solution to the contracting problem would be trivial: sell the project to the agent.) In fact, the agent could be allowed to be risk averse: the key assumption for our analysis is limited liability.

The instantaneous probability of success when the agent exerts the effort level  $a$  within a time interval of length  $\Delta t$  is  $a\Delta t$  and the cost of such effort is  $c(a)\Delta t$ . We make the following assumption about the cost function.

**Assumption 1**     •  $c'(a) \geq 0, c''(a) \geq 0, c'''(a) \geq 0$  for all  $a \geq 0$ .

- $c(0) = 0$  and  $\lim_{a \rightarrow \infty} c'(a) = \infty$ .
- $c'(a) + ac''(a)$  and  $(ac'(a) - c(a))/a$  are strictly increasing in  $a$ , and equal to zero at  $a = 0$ .
- $ac'(a) - c(a) - a^2c''(a) \leq 0$  for all  $a \geq 0$ .

This assumption is satisfied e.g., for quadratic costs:  $c(a) = \gamma a^2$ , where  $\gamma > 0$ .

Consider contracts of the form where the principal pays  $w(t) \geq 0$  to the agent if a success takes place in time period  $t$  and nothing if there is no success. This is the only form of contract that the principal will use: it is clearly not optimal to make any payment to the agent before project completion. Success is worth  $v \geq 0$  to the principal. Both the principal and the agent discount, with discount rates of  $r_P$  and  $r_A$  respectively.

We consider several models of contracting between the principal and the agent. We solve first for the sequentially rational wage level. We then consider the case where the principal must choose a stationary wage at the beginning of the game. We show that the sequentially rational wage exceeds the stationary commitment level. We then show that under full commitment, any stationary wage profile is dominated by a non-stationary, non-increasing one. Finally, we consider the use of deadlines for providing incentives.

### 3 Sequentially rational wage level

We start by supposing that the principal offers a spot wage contract for each period to the agent (or has the power to offer a temporary bonus for immediate success). We consider wage proposals of the form

$$w(s) = \begin{cases} w & \text{for } s \in [t, t + \Delta t), \\ \tilde{w} & \text{for } s \geq t + \Delta t. \end{cases}$$

There is no loss of generality in this, since, as we shall see, the sequentially rational solution involves a constant wage. The crucial feature of this wage proposal is that the current wage  $w$  can be different from the future wage  $\tilde{w}$ .

The agent's dynamic optimization problem can be characterized by a Bellman equation. Let the agent's value function from time  $t + \Delta t$  onwards be  $\tilde{W}$ ; let its value function at time  $t$  be  $W$ . Then

$$W = \max_a \{a\Delta t w - c(a)\Delta t + e^{-r_A \Delta t} (1 - a\Delta t) \tilde{W}\}.$$

This Bellman equation can be rewritten:

$$W - \tilde{W} = \max_a \{ (a(w - \tilde{W}) - c(a) - r_A \tilde{W}) \Delta t \}. \quad (1)$$

The first-order condition (which is also sufficient by convexity of  $c$ ) is:

$$c'(a) = w - \tilde{W}.$$

Denote the solution to this by  $a(w; \tilde{w})$ . Note that if  $w > \tilde{w}$ , then  $a(w; \tilde{w}) > a(w; w) > a(\tilde{w}; \tilde{w})$ . The implicit function theorem implies that

$$\frac{\partial a(w; \tilde{w})}{\partial w} = \frac{1}{c''(a(w; \tilde{w}))}. \quad (2)$$

The stationarity of the problem means that the sequentially rational wage will, in fact, be constant:  $w = \tilde{w}$ . Hence  $W = \tilde{W}$ , so that the agent's first-order condition can be written as

$$w - c'(a(w; w)) - \left( \frac{a(w; w)c'(a(w; w)) - c(a(w; w))}{r_A} \right) = 0. \quad (3)$$

The principal's Bellman's equation is:

$$V = \max_w \{ a(w; \tilde{w}) \Delta t (v - w) + e^{-r_P \Delta t} (1 - a(w; \tilde{w}) \Delta t) \tilde{V} \}.$$

$\tilde{V}$  is independent of  $w$  and hence the first-order condition is

$$w = v - \frac{a(w; w)(a(w; w) + r_P)}{r_P} \frac{1}{\frac{\partial a(w; \tilde{w})}{\partial w}}$$

Substitution gives

$$w = v - \frac{a(w; w)(a(w; w) + r_P)}{r_P} c''(a(w; w)) \quad (4)$$

which along with equation (3) gives the sequentially rational wage  $w^S$  and the agent's

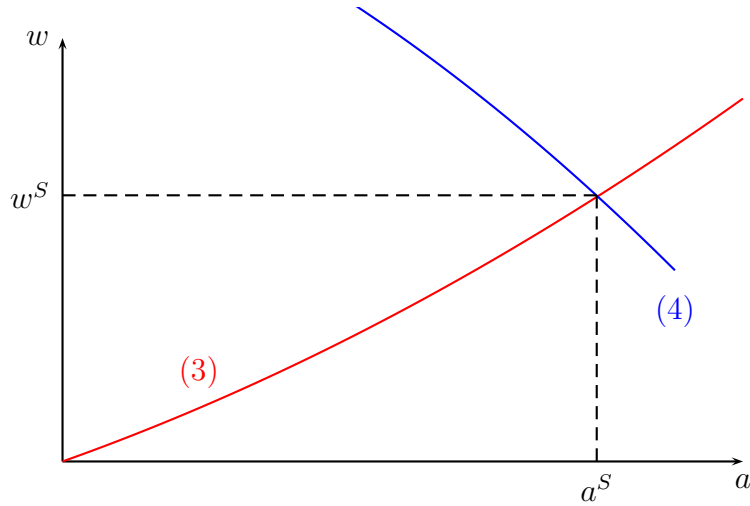


Figure 1: The sequentially rational solution with quadratic costs

effort level  $a^S$ .

Equations (3) and (4) are reaction functions for the dynamic game; their intersection point determines the sequentially rational equilibrium. The equations give relationships between wage  $w$  and effort  $a$ , which can also be interpreted in terms of the demand and supply of effort. The agent's supply of effort, given by equation (3), is an upward-sloping curve in  $(a, w)$  space: the agent requires a higher wage in order to put in more effort. The principal's (inverse) demand for effort, given by equation (4), is downward-sloping: the higher the effort put in, the more likely it is that the principal has to pay the wage, and so the lower the wage that the principal wants to set. Equilibrium is determined by the unique intersection point of the reaction functions: an effort level  $a^S$  and wage level  $w^S$ .

The solution is illustrated in figure 1, which plots equations (3) and (4) for the case of quadratic costs:  $c(a) = \gamma a^2$ , where  $\gamma > 0$ . In this example, equation (3) gives

$$w(a) = \frac{\gamma a^2 + 2\gamma r_A a}{r_A}$$

and equation (4) gives

$$w(a) = v - \frac{2a\gamma(a + r_P)}{r_P}.$$

## 4 Commitment to a single wage offer

We now contrast the sequentially rational solution to the alternative case in which the principal commits to a wage  $w$  for the duration of the game and the agent maximizes utility by choosing effort optimally in each period.

The agent's dynamic optimization problem is characterized by the Bellman equation:

$$W = \max_a \{a\Delta tw - c(a)\Delta t + e^{-r_A\Delta t}(1 - a\Delta t)W\}.$$

Letting  $\Delta \rightarrow 0$  and rearranging, we obtain

$$r_A W = \max_a \{aw - c(a) - aW\}.$$

The agent's first-order condition (which is also sufficient by convexity of  $c$ ) is:

$$c'(a) = w - W.$$

Substituting  $W$  from the first-order condition into Bellman's equation gives:

$$W = \frac{ac'(a) - c(a)}{r_A}; \tag{5}$$

finally, this gives

$$w - c'(a) - \left( \frac{ac'(a) - c(a)}{r_A} \right) = 0 \tag{6}$$

which determines the optimal effort level  $a(w)$ , as a function of  $w$ , in this case. The first-order condition has two components. The first,  $w - c'(a)$ , relates to the myopic incentives that the agent faces, equating the wage to its marginal cost of effort. The second,  $-(ac'(a) - c(a))/r_A$ , describes the dynamic incentives. Since  $c(\cdot)$  is convex, this term is non-positive. When  $r_A$  is very large (the agent discounts the future entirely), only the myopic incentives matter. When  $r_A$  is very small (no discounting), only dynamic incentives matter; the agent then exerts very low effort.

The implicit function theorem implies that

$$\frac{da}{dw} \equiv a'(w) = \frac{r_A}{(a + r_A)c''(a)} > 0. \quad (7)$$

Notice that the agent's current effort is *less* elastic in this case than in the sequentially rational solution. This is because a change in the constant commitment wage increases both the current and future wages. An agent faced with a higher future wage has a higher continuation value, and is therefore less willing to supply effort now.

Consider next the principal's optimization problem. For a fixed level of  $a$ , the value to the principal is:

$$V(a, w) = \frac{a(w)(v - w)}{a(w) + r_P}.$$

The optimality condition is

$$V'(w) = \frac{d}{dw}V(a(w), w) = \frac{\partial V(a, w)}{\partial a} \frac{da}{dw} + \frac{\partial V}{\partial w} = 0.$$

Hence (simplifying) we have:

$$w = v - \frac{a(w)(a(w) + r_P)}{r_P} \frac{1}{a'(w)}. \quad (8)$$

Equation (8) shows the principal's balance of myopic and dynamic incentives. When  $r_P$  is very large (so that the principal discounts the future entirely), the first-order condition reduces to the myopic marginal equality:

$$(v - w)a'(w) - a = 0.$$

When  $r_P$  is very small (no discounting), the first-order condition is dominated by dynamic incentives and the principal sets a zero wage.

Substituting for  $a'(w)$  gives

$$w = v - \frac{a(w)(a(w) + r_A)(a(w) + r_P)}{r_A r_P} c''(a(w)), \quad (9)$$

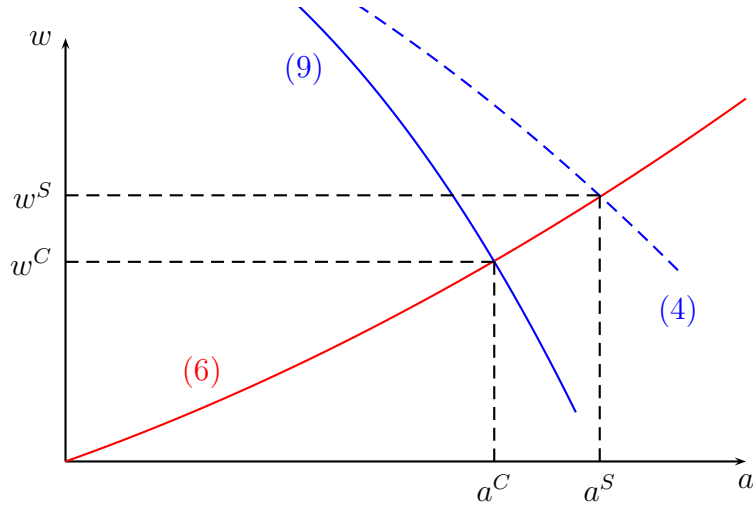


Figure 2: The constant commitment and sequentially rational solutions with quadratic costs

which, along with equation (6), can be solved for the equilibrium effort level  $a^C$  and the optimal wage  $w^C$ .

Equations (6) and (9) are the reaction functions for the dynamic game with commitment to a constant wage. As in the sequentially rational solution, the agent's reaction function is an upward-sloping curve in  $(a, w)$  space; the principal's reaction is downward-sloping. Equilibrium is determined by the unique intersection point of the reaction functions: an effort level  $a^C$  and wage level  $w^C$ .

The solution is illustrated in figure 2, which plots equations (6) and (9) for the case of quadratic costs:  $c(a) = \gamma a^2$ , where  $\gamma > 0$ . In this example, equation (6) gives

$$w(a) = \frac{\gamma a^2 + 2\gamma r_A a}{r_A}$$

and equation (9) gives

$$w(a) = v - \frac{2a\gamma(a + r_A)(a + r_P)}{r_A r_P}.$$

Comparison of equations (2) and (7) shows that the agent's current effort is *more* elastic in the sequentially rational case, because a marginal change in the current wage does not (necessarily) raise all future wages as well. In terms of reaction functions, the agent's reaction function is the same in the sequentially rational and constant commitment

cases. The principal's reaction is higher in the sequentially rational case: the principal is willing to pay a higher wage, for any given effort level. This is illustrated in figure 2 (for the quadratic cost case), which shows the upward shift in the principal's reaction function.

Consequently, the following proposition follows immediately.

**Proposition 1** *In the sequentially rational solution, both the wage and the effort level are higher than in the constant commitment case:  $w^S \geq w^C$  and  $a^S \geq a^C$ .*

## 4.1 Comparative statics of equilibrium

The comparative statics of the equilibrium action and wage can also be investigated. Of particular interest is how the action and wage depend on the separate discount rates  $r_P$  and  $r_A$ . Two cases are of particular interest:

1.  $r_P = r_A = +\infty$ : both the principal and the agent are myopic, ignoring all continuation values and playing the game as if it were one-shot.
2.  $r_P < r_A = +\infty$ : the agent is myopic, but the principal is not.

These two cases allow us to identify the dynamic incentives in the model, by shutting down various channels in turn. The second case, with  $r_A = +\infty$ , also has a useful interpretation, as a case where the principal operates for an infinite number of periods, employing a sequence of different agents each for one period. This case will be useful when analysing wages with deadlines in section 6.

In the myopic case, with  $r_P = r_A = +\infty$ , the agent's and principal's continuation values are equal to zero. The agent's first-order condition is then

$$w = c'(a). \tag{10}$$

The principal's first-order condition is

$$w = v - ac''(a). \tag{11}$$

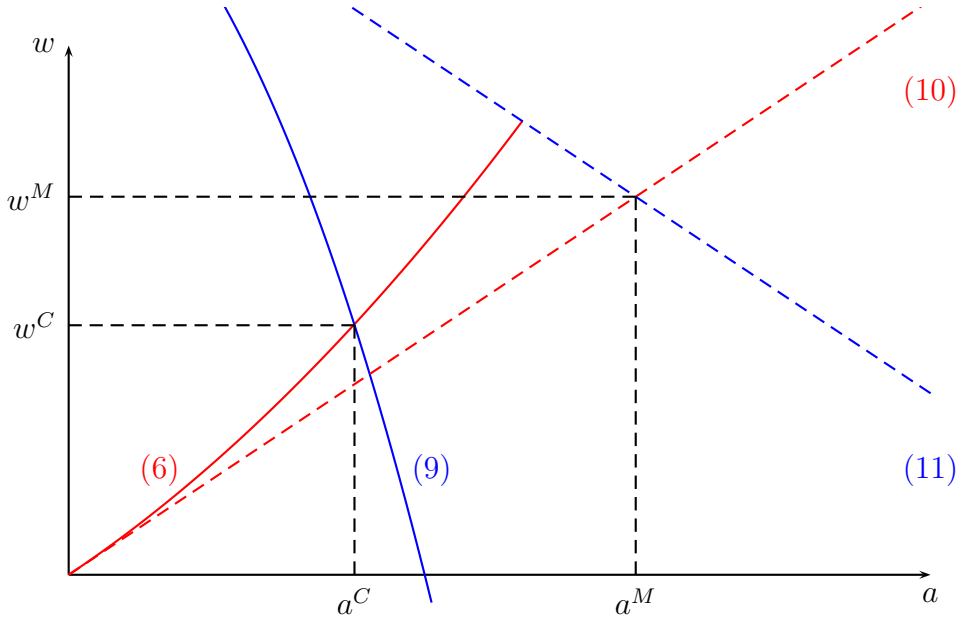


Figure 3: The constant commitment and myopic solutions

Equations (10) and (11) define the myopic wage  $w^M$  and effort  $a^M$ .

It is straightforward to show that the myopic effort  $a^M$  is greater than the effort in the constant commitment case  $a^C$ . The comparison with the constant commitment wage  $w^C$  is more difficult. Figure 3 (using quadratic costs) illustrates why. Equation (10) defines a curve in  $(a, w)$  space that lies below the curve defined by equation (6). That is, in the dynamic problem, the agent requires a higher wage to exert the same effort level as in the static situation. Clearly, this is due to the continuation value that is present in the dynamic problem. Equation (11) defines a curve in  $(a, w)$  space that lies above the curve defined by equation (9): the dynamic principal offers a lower wage than the static principal, for any given effort level. The reason is the same: the prospect of continuation value in the dynamic problem leads the principal to lower the wage. Both shifts lead to a lower effort level in the dynamic problem; but have an ambiguous effect on the wage.

Now consider the case  $r_P < r_A = +\infty$ . The agent is myopic, and so its first-order condition is

$$c'(a(w)) = w.$$

The principal's first-order condition is

$$w = v - \frac{a(w)(a(w) + r_P)}{r_P} c''(a(w)). \quad (12)$$

Let the effort level in this case be  $a^{R,\infty}$  and the wage level  $w^{R,\infty}$ . (The notation will become clearer in section 6). As in the previous case, the effect of increasing  $r_A$  to infinity on the effort level is easy to establish, but the effect on wage is ambiguous. An increase in the agent's discount rate always increases the equilibrium effort level. This occurs because the agent's reaction function shifts downwards, while the principal's reaction function shifts upwards. The shifts are illustrated in figure 4 for the quadratic cost case. The figure also summarises the different cases that we have considered. The principal's reaction functions are labelled 'P', subscripted with the values of the discount rates. The agent's reaction functions are labelled 'A'. The sequentially rational solution is labelled 'S'; the constant commitment solution with  $r_P$  and  $r_A$  finite is labelled 'C'; the myopic case (with  $r_P = r_A = \infty$ ) is labelled 'M'; and the agent replacement case (with  $r_P < r_A = \infty$ ) is labelled 'R'.

The figure shows that we can make the following general statements.

**Proposition 2** • *Effort levels:  $a^C \leq a^S \leq a^{R,\infty} \leq a^M$ .*

• *Wage levels:  $w^C \leq w^S$ ;  $w^{R,\infty} \leq w^S$ ; and  $w^{R,\infty} \leq w^M$ .*

Only a little more can be said about equilibrium wages in particular cases. For example, suppose that costs are quadratic:  $c(a) = \gamma a^2$ . We can then establish the following.

**Proposition 3** *Suppose that the cost function is quadratic:  $c(a) = \gamma a^2$ , with  $\gamma > 0$ . In this case: (i)  $w^C < w^M$ ; (ii)*

**Proof.** With quadratic costs,  $a^M = v/4\gamma$  and  $w^M = v/2$ . The proof works by determining the action levels in equations (6) and (9) that result by setting  $w^C = w^M = v/2$ . We establish that the action level from equation (9) is less than the action level from equation

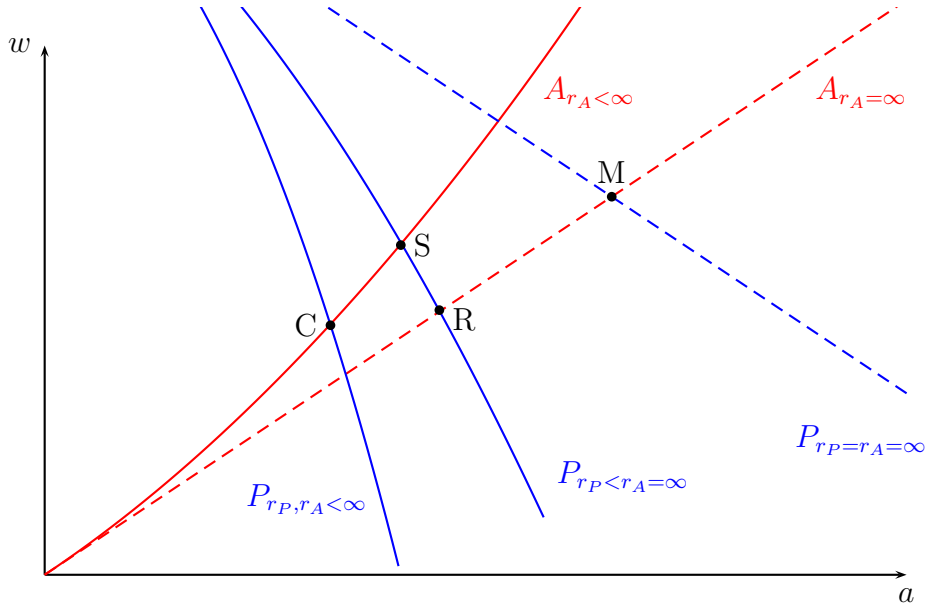


Figure 4: Summary of the cases

(6). This then necessarily means that  $w^C < w^M$ . From equation (6),

$$\frac{v}{2} = 2\gamma a + \frac{\gamma a^2}{r_A}; \quad (13)$$

from equation (9),

$$\frac{v}{2} = 2\gamma a + \frac{2\gamma a^2(r_A + r_P + a)}{r_A r_P}. \quad (14)$$

It is clear, therefore, that the action level in equation (13) is greater than the action level in equation (14). This concludes the proof of the first part.  $\square$

Even with quadratic costs, no clear comparison can be made between e.g.,  $w^C$  and  $w^{R,\infty}$ . Numerical solution with particular values for  $v, r_A$  and  $r_P$  shows that  $w^C$  can be greater or less than  $w^{R,\infty}$ , depending on the size of  $\gamma$ , the cost parameter. Outside of the quadratic cost case, the ordering wage levels in the different cases can be changed by altering the degree of convexity of the cost function and the discount rates. The effect of these changes is to alter the extent to which the principal's and agent's reaction functions shift as the comparative statics are done.

## 5 Commitment to non-stationary wages

The analysis so far shows that the optimal commitment wage is not stationary in general. If the optimal commitment wage is stationary, then it must be at the level given in section 4. In section 3, we considered deviations for  $\Delta t$  units of time from a stationary wage. Since the analysis shows that it is optimal for the principal to offer a different wage if continuation wages are at  $w^C$ , we conclude immediately that the optimal commitment path of wages cannot be constant.

In order to derive the optimal commitment wage schedule, we use the method developed by Spear and Srivastava (1987) and Phelan and Townsend (1991) and write the optimal contract in terms of the agent's continuation value as the state variable. Consider an arbitrary reward function  $w(t)$ . The agent's Bellman's equation is given by:

$$r_A W(t) = \max_a \{a(w(t) - W(t)) - c(a) + \dot{W}(t)\}. \quad (15)$$

The agent's first-order condition is

$$w(t) = W(t) + c'(a(t)). \quad (16)$$

Substituting into the Bellman equation gives

$$\dot{W}(t) = r_A W(t) - (a(t)c'(a(t)) - c(a(t))). \quad (17)$$

Equations (16) and (17) are constraints on the principal's problem.

The principal's Bellman equation is

$$r_P V(W) = \max_a \left\{ a(t)(v - V(W) - W - c'(a(t))) + (r_A W - (a(t)c'(a(t)) - c(a(t))))V'(W) \right\} \quad (18)$$

where equation (16) has to been used to substitute in for the wage and equation (17) for

$\dot{W}$ . The principal's first-order condition is

$$v - V(W) - W - (c'(a(t)) + a(t)c''(a(t))) - a(t)c''(a(t))V'(W) = 0; \quad (19)$$

from the properties of the cost function (see assumption 1), this is necessary and sufficient for an optimum. Differentiating this first-order condition with respect to time gives

$$\begin{aligned} -V'(W)\dot{W} - \dot{W} - (2c''(a(t)) + a(t)c'''(a(t)))\dot{a}(t) \\ - (c''(a(t)) + a(t)c'''(a(t)))V'(W)\dot{a}(t) - a(t)c''(a(t))V''(W)\dot{W} = 0. \end{aligned} \quad (20)$$

The envelope theorem on equation (18) gives

$$-a(t)(V'(W) + 1) - (r_P - r_A)V'(W) + V''(W)\dot{W} = 0. \quad (21)$$

Combining equations (17), (20) and (21) gives

$$\begin{aligned} (c''(a(t)) + (V'(W) + 1)(c''(a(t)) + a(t)c'''(a(t))))\dot{a}(t) \\ = -(V'(W) + 1)(r_A W - (a(t)c'(a(t)) - c(a(t)))) \\ + a^2(t)c''(a(t))(r_P - r_A)V'(W). \end{aligned} \quad (22)$$

Equations (17) and (22) determine the dynamics of the system.

There are two further optimality conditions. The principal is free to choose the level of the agent's value at the start of the program. The initial value  $W_0$  is determined by the (necessary) condition  $V'(W_0) = 0$ . The second optimality condition is the transversality condition that the system converges to a steady state in which  $\dot{a} = \dot{W} = 0$  and  $a$  and  $W$  are bounded.

We are now able to characterise the dynamics of the optimal commitment contract.

**Proposition 4** *In the full commitment solution, the agent's continuation value  $W(t)$ , the optimal wage profile  $w(t)$ , and the agent's effort level  $a(t)$  are all decreasing over time. If  $r_P \geq r_A$ , then the continuation value, wage and effort levels converge to zero:*

$\lim_{t \rightarrow \infty} W(t) = \lim_{t \rightarrow \infty} w(t) = \lim_{t \rightarrow \infty} a(t) = 0$ . If  $r_P < r_A$ , then the wage and effort levels converge to strictly positive levels. The initial effort level, and hence all levels, are below the myopic effort:  $a(t) \leq a^M$ .

**Proof.** The proof uses a phase diagram in  $(a, W)$  space. Three aspects need to be analysed: the dynamics of  $a$  and  $W$ ; and the sign of  $V'(\cdot)$  in equilibrium.

The sign of  $\dot{W}$  is, from equation (17), determined by the sign of  $r_A W - (ac'(a) - c(a))$ . This defines an upward-sloping function in  $(a, W)$  space, given by

$$W_{\dot{W}=0}(a) \equiv \frac{ac'(a) - c(a)}{r_A}$$

so that for  $W > (<)W_{\dot{W}=0}(a)$ ,  $\dot{W} > (<)0$ . Note that  $W_{\dot{W}=0} = 0$ .

To determine the sign of  $V'(\cdot)$  in equilibrium, manipulate the principal's Bellman equation and first-order condition to give

$$(r_A W - ac'(a) + c(a) + (a + r_P)ac''(a))V'(W) = r_P(v - W - c'(a)) - (a + r_P)ac''(a).$$

Consider the left-hand side of this equation. From assumption 1,  $r_A W - ac'(a) + c(a) + (a + r_P)ac''(a) \geq 0$  for all non-negative values of  $a$  and  $W$ . Hence the sign of  $V'(W)$  is determined by the sign of  $r_P(v - W - c'(a)) - (a + r_P)ac''(a)$ . This defines a function in  $(a, W)$  space, given by

$$W_{V'=0}(a) \equiv v - c'(a) - \left( \frac{a + r_P}{r_P} \right) ac''(a).$$

This is a downward-sloping function, with an intercept  $W_{V'=0}(0) = v$ ; it hits the horizontal axis at an effort level  $\hat{a}$  strictly less than the myopic level  $a^M$  (defined by  $v - c'(a^M) - a^M c''(a^M) = 0$ ). For values of  $(a, W)$  lying below (above) this function,  $V'(W)$  is positive (negative); along the function,  $V'(W) = 0$ . The function  $W_{\dot{W}=0}(a)$  is, therefore, split into two portions by the function  $W_{V'=0}(a)$ ; call the intersection point of the two functions  $(a^*, W^*)$ .

Now consider the dynamics of  $a$ , determined by equation (22). When  $V'(\cdot) = 0$  (in

particular, at the optimal initial choice of  $W$ ), the term on the left-hand side,  $c''(a(t)) + (V'(W)+1)(c''(a(t))+a(t)c'''(a(t)))$ , is non-negative (using assumption 1). The right-hand side is equal to

$$-(r_A W - (a(t)c'(a(t)) - c(a(t))) + a^2 c''(a)),$$

which by assumption 1 is negative for all non-negative values of  $a$  and  $W$ . Hence  $\dot{a} \leq 0$  at the optimal initial choice of  $W$ .

The function defined by  $\dot{a} = 0$  is

$$W_{\dot{a}=0}(a) \equiv \frac{ac'(a) - c(a)}{r_A} - \frac{ac''(a)}{r_A} \left( a + (r_P - r_A) \frac{V'(W)}{V'(W) + 1} \right).$$

Note that when  $V' = 0$ ,

$$W_{\dot{a}=0}(a) \equiv \frac{ac'(a) - c(a) - a^2 c''(a)}{r_A} \leq 0$$

from assumption 1. Hence the function  $W_{\dot{a}=0}(a)$  crosses the function  $W_{V'=0}(a)$  at a point below the horizontal axis. If  $r_P \geq r_A$ , then  $W_{\dot{a}=0}(a) \leq 0$  for all  $a \in [0, \hat{a}]$ . If  $r_P < r_A$ , then  $W_{\dot{a}=0}(a) > W_{\dot{W}=0}(a)$  for sufficiently small  $a$ . Since  $W_{\dot{a}=0}(a)$  is continuous in  $a$ , the  $W_{\dot{a}=0}(a)$  curve must therefore cross the  $W_{\dot{W}=0}(a)$  at a value of  $a$  that is strictly less than  $a^*$ .

We can now determine the dynamics of  $a$  and  $W$ . The region of particular interest for the analysis is defined as follows. Let

$$\mathcal{W}(a) \equiv \{W \in \mathbb{R}_+ | W \leq W_{\dot{W}=0}(a) \text{ and } W \leq W_{V'=0}(a) \text{ and } W \geq W_{\dot{a}=0}(a)\}$$

for  $a \in [0, \hat{a}]$ . Let

$$\mathcal{E} \equiv [0, \hat{a}] \mathcal{W}(a).$$

For  $(a, W) \in \mathcal{E}$ , both  $\dot{a}$  and  $\dot{W}$  are non-positive. The region  $\mathcal{E}$  is illustrated as the shaded regions in figures 5 and 6. If  $r_P \geq r_A$ , then  $\mathcal{E}$  is defined as the (lower) area between the curves  $W_{\dot{W}=0}(a)$  and  $W_{V'=0}(a)$ . If  $r_P < r_A$ , then  $\mathcal{E}$  is further defined by the curve

$W_{\dot{a}=0}(a)$ .

An initial choice of  $W$  on the portion of the  $W_{V'=0}(a)$  curve above the  $W_{\dot{W}=0}(a)$  cannot be optimal. The reason is that the dynamics from this point involve  $\dot{W} \geq 0$ . Hence any path from such a point cannot converge to a steady state, and by transversality cannot be optimal.

Hence the optimal initial choice of  $W$  must lie on the portion of the  $W_{V'=0}(a)$  curve below the  $W_{\dot{W}=0}(a)$ . (Note that this must involve an initial effort level less than  $\hat{a}$ , and hence less than the myopic level  $a^M$ .) The initial dynamics from such a point are  $\dot{W} \leq 0$  and  $\dot{a} \leq 0$ . The resulting path lies in the region  $\mathcal{E}$ , and hence  $\dot{W} \leq 0$  and  $\dot{a} \leq 0$  along all parts of an optimal path. The dynamics of  $w(t)$  then follows from (16).

If  $r_P \geq r_A$ , then assumption 1 ensures that the function  $W_{\dot{a}=0}(a)$  is negative for all values of  $a \in [0, \hat{a}]$ . Hence the only feasible steady state is  $a = W = 0$ . If  $r_P < r_A$ , then there is a second steady state with strictly positive  $a \in (0, a^*)$  and  $W \in (0, W^*)$ ; the optimal path converges to this steady state.  $\square$

Figures 5 and 6 illustrate the proof of the proposition. In the shaded area in the figures,  $\dot{a}$  and  $\dot{W}$  are non-positive. The optimal initial point must lie on the portion of the  $W_{V'=0}(a)$  curve that is in bold. Any optimal path from that portion of the curve must move into the shaded area; hence  $\dot{a}$  and  $\dot{W}$  must be non-positive along the entire path. Possible steady states are marked with a dot; clearly, they must lie on the curve along which  $\dot{W} = 0$ . If  $r_P \geq r_A$ , then only one steady state exists: the origin, with  $a = W = 0$ . Otherwise, two steady states exist (as shown in the figure). The optimal path converges to the steady state with  $a = W = 0$ , if  $r_P \geq r_A$ . Otherwise, it converges to a steady state with strictly positive levels of  $a$  and  $W$ . The latter is consistent with the previous result that in the limit, as  $r_A \rightarrow \infty$ , the equilibrium effort level converges to the (positive) level  $a^{R,\infty}$ .

These results are intuitive. Limited liability means that the agent earns positive rents from the contract. The principal uses the dynamics of these rents to provide the agent with incentives. In particular, the full commitment contract ensures that the agent's continuation value is decreasing in equilibrium. By ensuring this, the principal gives the

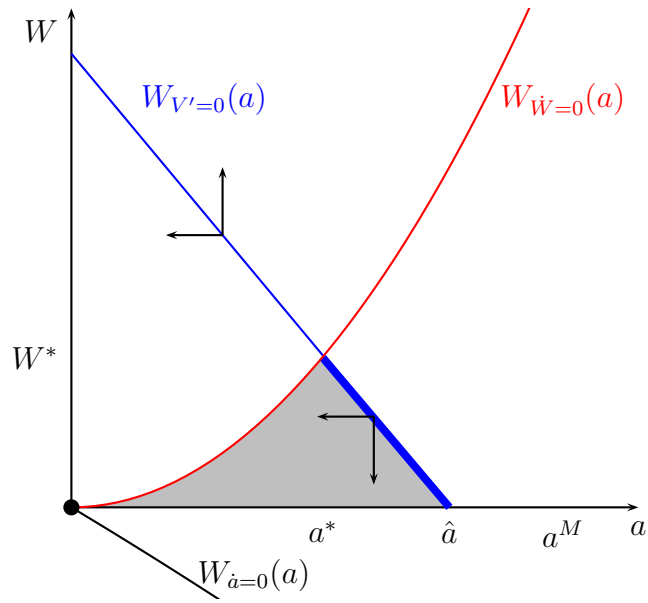


Figure 5: Phase diagram for the problem with full commitment with  $r_P \geq r_A$

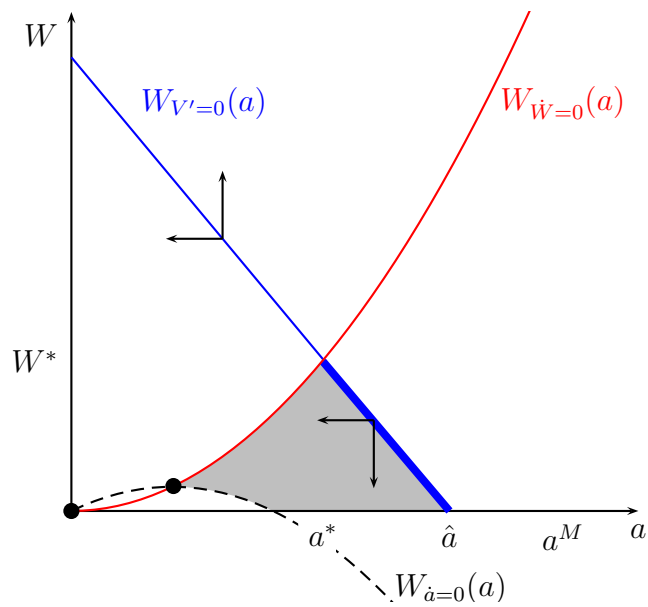


Figure 6: Phase diagram for the problem with full commitment with  $r_P < r_A$

agent incentives to exert effort today. The continuation value is driven downwards by a decreasing wage; but the agent's effort also decreases over time. When the agent is more patient than the principal, the principal has to drive the agent's wage (and hence effort) down to zero in the long-run in order to generate incentives for effort. But then the agent is less patient, the principal can rely on the agent's impatience to generate incentives. In this case, the long-run wage and effort are both strictly positive. In all cases, the principal induces less effort from forward-looking agent: equilibrium effort is less than the myopic level.

## 6 Wages with a deadline

Consider now the case where the principal can commit to a constant wage  $w$  until time  $T$ . In general, this wage policy is not optimal, although it captures in a stark way a key aspect of the full commitment policy: a declining wage. In this section, we analyse whether such a policy can ever be optimal.

The agent's Bellman's equation is as before:

$$r_A W = \max_a \{a(w - W) - c(a) + \dot{W}\};$$

but note now that the wage  $w$  is not a function of time. The agent's first-order condition is also unchanged:

$$w = W + c'(a(t)).$$

Differentiating this first-order condition with respect to time gives

$$c''(a)\dot{a} = -\dot{W}. \tag{23}$$

The agent's continuation value at  $T$  must be zero:  $W(T) = 0$ . Since the agent's value is non-negative, it must be that  $\dot{W} \leq 0$ . Hence equation (23) implies that  $\dot{a} \geq 0$ : the

agent's effort increases over time when the principal sets a constant wage and a deadline. The effort level at the terminal time must equal the myopic level, for the given wage. We use this fact in the proof of the following proposition (which, since it is lengthy, is given in the appendix).

**Proposition 5** *If  $r_P = r_A$ , then when committing to pay a constant wage to a single agent, the principal does not use a finite deadline.*

Proposition 5 shows that the principal will not use a deadline—with a constant wage and equal discount rates for the principal and the agent, at least. On the other hand, the optimal situation for the principal is to employ an infinite sequence of agents each for a single period—the benchmark analysed in section 2. To span these two cases, suppose that the principal can search for a replacement agent, but only after it has fired its current agent. Search takes time: a replacement agent is found according to a Poisson process with parameter  $\lambda$ . The model considered in proposition 5 sets  $\lambda = 0$ : there is no prospect of replacement. The model in section 2 has  $\lambda = +\infty$ : replacement can occur infinitely often. (This explains the notation in that section, where  $w^{R,\infty}$  is the wage when replacement occurs infinitely often.)

The principal's optimisation problem is

$$V(\lambda) \equiv \max_{w,T} \left\{ \int_0^T \exp\{-r_P t - A_{w,T}(t)\} a_{w,T}(t) (v - w) dt + \exp\{-r_P T\} \frac{\lambda}{r_P + \lambda} V(\lambda) \right\}. \quad (24)$$

Equation (24) makes explicit the dependence of the principal's value on the arrival rate  $\lambda$  of replacement agents. We are interested in characterising the dependence of the optimal choice of  $T(\lambda)$  on the arrival rate  $\lambda$ .

**Proposition 6**  *$T(\lambda)$  is non-increasing in  $\lambda$ .*

The proof of the proposition is fairly mechanical and is given in the appendix. The proposition shows the (expected) result that the principal uses a shorter deadline when it is easier to replace the agent. In the limit, of course, when agents can be replaced

infinitely often ( $\lambda \rightarrow \infty$ ), the principal is in the ideal position of using a sequence of agents, each for an interval  $dt \rightarrow 0$ .

## 7 Project quality

In the analysis so far, the benefit to the principal from a completed project is fixed and verifiable. More generally, we might suppose that the agent is able to affect the quality, and hence the benefit to the principal, of the completed project.

We shall not attempt a general analysis of this issue here. Instead, we outline a variant of our model where project quality has no effect the overall conclusions. Suppose that an agent can affect the probability of project completion by exerting effort, in the same way as in previous sections. But now, the quality of a completed project is drawn at random from a distribution  $F(v)$  where  $v \geq 0$ , which is common knowledge.

At this point, there are two possibilities, which (it will turn out) are equivalent. The first is that the realised project quality cannot be verified. (For example, the agent may be a headhunter and the completed project a candidate for an executive position in the principal's firm. The fit of the candidate for the principal's post may be very difficult to establish to a third-party.) Hence the principal cannot condition payment on the realised project quality.

The second possibility is that the project quality is verifiable. This then raises the possibility that the principal pays only when the realised project quality is at least as great as the current completion wage. But it is easy to see that this cannot be optimal. The agent's expected payment on completing at time  $t$  would be  $\mathbb{E}[w|v \geq w]$  where the expectation is taken with respect to the quality distribution  $F(\cdot)$ . The principal gains  $\mathbb{E}[v - w|v \geq w]$ . But the principal could offer this level of payment unconditionally (and hence present the agent with the same effort incentives); and then accept all completed projects. The principal would gain  $\mathbb{E}[v|v \leq w]$  from this.

So, the principal will continue to pay only on project completion. The problem is unaltered by this modification: the principal's benefit  $v$  is replaced by an expected benefit—

that is all. Hence our previous analysis continues to hold.

The crucial feature of this example is that the agent's effort affects only the arrival rate of project completion, but not the realised quality. If project quality is affected (stochastically) by effort levels, then the principal will adjust the wage in order to affect both the quality and rate of completion of the project. We leave this and other related issues to further work.

## 8 Conclusions

We have developed a model of dynamic moral hazard involving project completion which has allowed us to identify clearly the intertemporal incentives involved. The contrast between the sequentially rational solution and the contract with commitment to a constant wage points immediately to the form of the full commitment contract. It involves a completion payment that decreases over time. In this way, the principal decreases the agent's continuation value over time; and in this way, gives the agent incentives for effort. The framework that we have developed is very tractable and offers a base from which we intend to explore further dynamic incentives for this type of problem.

## Appendix

### Proof of Proposition 5

The proof has three steps.

1. For any given wage, the expected discounted costs of the agent's efforts are higher when the agent is employed in perpetuity than when a finite deadline is used. Let  $a_w$  be the agent's effort with no deadline (since the wage is constant, the effort is constant).  $a_{w,T}(t)$  is the agent's effort with a deadline  $T$ ; because of the deadline,

this effort varies with time. Then we shall show that

$$\begin{aligned} C_{w,\infty} &\equiv \int_0^\infty \exp\{-r_A t - a_w t\} c(a_w) dt = \frac{c(a_w)}{r_A + a_w} \\ &\geq C_{w,T} \equiv \int_0^T \exp\{-r_A t - A_{w,T}(t)\} c(a_{w,T}(t)) dt \end{aligned}$$

for finite  $T$ , where

$$A_{w,T}(t) \equiv \int_0^t a_{w,T}(s) ds.$$

The proof of this step is by contradiction: suppose not. The expected discounted cost  $C_{w,T}$  is continuous in  $T$  and  $\lim_{T \rightarrow \infty} C_{w,T} = C_{w,\infty}$ . Hence if  $C_{w,\infty}$  is not larger than  $C_{w,T}$  for all finite  $T$ , then there must be some  $T^* < \infty$  such that  $C_{w,T}$  is maximised at  $T = T^*$ . By continuity, for some cost level  $C \equiv C_{w,T^*} - \Delta$ , for a small, positive  $\Delta$ , there exist two times  $T_1 < T^* < T_2$  such that  $C_{w,T_1} = C_{w,T_2} = C$ . At  $T = T_1$ ,  $C_{w,T}$  must be increasing in  $T$ . In the limit, as  $\Delta \rightarrow 0$ , this means that  $rC_{w,T_1} > c(a_{w,T_1}(0))$ . At  $T = T_2$ ,  $C_{w,T}$  must be decreasing in  $T$ . In the limit, as  $\Delta \rightarrow 0$ , this means that  $rC_{w,T_2} < c(a_{w,T_2}(0))$ . Since, by construction,  $C_{w,T_1} = C_{w,T_2}$ , this means that  $c(a_{w,T_2}(0)) > c(a_{w,T_1}(0))$ , or  $a_{w,T_2}(0) > a_{w,T_1}(0)$ . But we have established that  $a_{w,T}(T) = a^M$  for any  $T$ . Equation (23) then implies that if  $T_2 > T_1$ , then  $a_{w,T_2}(0) < a_{w,T_1}(0)$ . Hence  $C_{w,\infty} \geq C_{w,T}$  for any finite  $T$ .

2. For any given wage, the agent's expected discounted payoff when it is employed in perpetuity is higher than when a finite deadline is used:

$$\begin{aligned} R_{w,\infty} &\equiv \int_0^\infty \exp\{-r_A t - a_w t\} a_w w dt = \frac{a_w w}{r_A + a_w} \\ &\geq R_{w,T} \equiv \int_0^T \exp\{-r_A t - A_{w,T}(t)\} a_{w,T}(t) w dt. \end{aligned}$$

This must hold because  $R_{w,\infty} - C_{w,\infty} \geq R_{w,T} - C_{w,T}$  and  $C_{w,\infty} \geq C_{w,T}$ . The first statement holds because, when faced with an infinite deadline, the agent can always choose its effort as if it were facing a finite deadline.

3. The principal's expected discounted payoff when it employs the agent in perpetuity

is higher than when a finite deadline is used. To see this, note first that the principal's expected discounted payoff is equal to the agent's expected discounted payoff, multiplied by a constant factor:

$$\int_0^\infty \exp\{-r_P t - a_w t\} a_w (v - w) dt = \frac{a_w (v - w)}{r_P + a_w} = \frac{v - w}{w} R_{w,\infty},$$

$$\int_0^T \exp\{-r_P t - A_{w,T}(t)\} a_{w,T}(t) (v - w) dt = \frac{v - w}{w} R_{w,T}.$$

In these equalities, we use the condition that  $r_A = r_P$ . Secondly, step 2 established  $R_{w,\infty} \geq R_{w,T}$ .

## Proof of Proposition 6

Write the principal's optimisation problem as

$$\max_{w,T} \{ \alpha(w, T)(v - w) + \exp\{-r_P T\} g(\lambda) \},$$

where

$$\alpha(w, T) \equiv \int_0^T \exp\{-r_P t - A_{w,T}(t)\} a_{w,T}(t) dt,$$

$$g(\lambda) \equiv \frac{\lambda}{r_P + \lambda} V(\lambda).$$

Since  $V(\lambda)$  must be non-decreasing in  $\lambda$ ,  $g(\lambda)$  is non-decreasing in  $\lambda$ . The first-order conditions for interior solutions are

$$\alpha_w (v - w) - \alpha = 0, \tag{25}$$

$$\alpha_T (v - w) - r_P \exp\{-r_P T\} g(\lambda) = 0 \tag{26}$$

where subscripts denote partial derivatives and the arguments of  $\alpha$  have been omitted for brevity. (Note the proposition 5 established that  $\alpha_T > 0$ ; it is straightforward to establish that  $\alpha_w > 0$  also.)

The comparative statics of the optimal controls can be established using Cramer's rule. The determinant of the Hessian is

$$H \equiv \begin{vmatrix} \alpha_{ww}(v-w) - 2\alpha_w & \alpha_{wT}(v-w) - \alpha_T \\ \alpha_{wT}(v-w) - \alpha_T & \alpha_{TT}(v-w) + r^2 \exp\{-r_P T\}g(\lambda) \end{vmatrix}.$$

From the second-order condition for interior solutions,  $H \geq 0$ . By Cramer's rule,

$$\begin{aligned} \frac{dT(\lambda)}{d\lambda} &= \frac{1}{H} \begin{vmatrix} \alpha_{ww}(v-w) - 2\alpha_w & 0 \\ \alpha_{wT}(v-w) - \alpha_T & r_P \exp\{-r_P T\}g'(\lambda) \end{vmatrix} \\ &= \frac{1}{H}(\alpha_{ww}(v-w) - 2\alpha_w)r_P \exp\{-r_P T\}g'(\lambda) \leq 0, \end{aligned}$$

where the inequality follows from the second-order conditions and  $g'(\lambda) \geq 0$ .

## References

- FERNANDEZ-MATEO, I. (2003): "How Free Are Free agents? Relationships and Wages in a Triadic Labor Market," Available at [http://www.london.edu/assets/documents/Isabel\\_paper.pdf](http://www.london.edu/assets/documents/Isabel_paper.pdf).
- FINLAY, W., AND J. COVERDILL (2000): "Risk, Opportunism, and Structural Holes: How Headhunters Manage Clients and Earn Fees," *Work and Occupations*, 27, 377–405.
- HOPENHAYN, H., AND J. P. NICOLINI (1997): "Optimal Unemployment Insurance," *Journal of Political Economy*, 105, 412–438.
- LAFFONT, J.-J., AND D. MARTIMORT (2002): *The Theory of Incentives: the Principal-Agent Model*. Princeton University Press.
- LAND REGISTRY (2007): "House Price Index," Discussion paper.
- MALCOMSON, J., AND F. SPINNEWYN (1988): "The Multi-Period Principal-Agent Problem," *Review of Economic Studies*, 55, 391–408.

- MERLO, A., AND F. ORTALO-MAGNÉ (2004): “Bargaining over Residential Properties: Evidence from England,” *Journal of Urban Economics*, 56, 192–216.
- MERLO, A., F. ORTALO-MAGNÉ, AND J. RUST (2006): “Bargaining and Price Determination in the Residential Real Estate Market,” Available at [http://gemini.econ.umd.edu/jrust/research/nsf\\_pro\\_rev.pdf](http://gemini.econ.umd.edu/jrust/research/nsf_pro_rev.pdf).
- OFFICE OF FAIR TRADING (2004): “Estate Agency Market in England and Wales,” Discussion Paper OFT693, Office of Fair Trading.
- PHARMAFOCUS (2007): “Finding top people for the top job,” Internet Publication, [http://www.pharmafocus.com/cda/focusH/1,2109,22-0-0-0-focus\\_feature\\_detail-0-491515,00.html](http://www.pharmafocus.com/cda/focusH/1,2109,22-0-0-0-focus_feature_detail-0-491515,00.html).
- PHELAN, C., AND R. TOWNSEND (1991): “Computing Multi-Period, Information-Constrained Optima,” *Review of Economic Studies*, 58, 853–881.
- SANNIKOV, Y. (2007): “Games with Imperfectly Observable Actions in Continuous Time,” *Econometrica*, 75(5), 1285–1329.
- (forthcoming): “A Continuous-Time Version of the Principal-Agent Problem,” *Review of Economic Studies*.
- SHAVELL, S., AND L. WEISS (1979): “The Optimal Payment of Unemployment Insurance Benefits over Time,” *Journal of Political Economy*, 87(6), 1437–1362.
- SPEAR, S., AND S. SRIVASTAVA (1987): “On Repeated Moral Hazard with Discounting,” *Review of Economic Studies*, 54, 599–617.
- WILLIAMS, N. (2006): “On Dynamic Principal-Agent Problems in Continuous Time,” Available at <http://www.princeton.edu/noahw/pa1.pdf>.