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Abstract

We evaluate the effects of outsourcing and labour taxation on wage formation and equilibrium unemployment and labour demand in dual labour markets, when there is both unionized and competitive determination of wages in the home country. Outsourcing promotes wage dispersion between the high-skilled and low-skilled workers. Higher domestic low-skilled wage tax, higher payroll tax and lower wage tax exemption increase optimal outsourcing. Higher amount of outsourced production will reduce equilibrium unemployment of low-skilled workers both in the presence and absence of labour taxation. Under outsourcing higher wage tax and higher tax exemption will have an ambiguous effect on equilibrium unemployment, while higher payroll tax will decrease equilibrium unemployment. In the presence of outsourcing raising wage tax and tax exemption to keep the relative tax burden per worker constant, this higher tax progression will decrease the wage rate and increase the labour demand of low-skilled workers.

JEL Classification: E24, J21, J31, J51, J82, H22.

Keywords: Outsourcing, unionized and competitive domestic labour markets, labour taxation, equilibrium unemployment.

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I. Introduction

High wage differences across countries constitute an important explanation for the currently significant business practice of international outsourcing. These wage differentials could lead to outsourcing (see e.g. Sinn (2007) for details, and Stefanova (2006) concerning the East-West dichotomy of outsourcing). Glass and Saggi (2001) have studied the causes of outsourcing and its effects and they found that higher international outsourcing lowers the relative wage of domestic workers, while it increases the profits and thereby creates greater incentives for innovation. We are not aware of any existing study, which would have studied theoretically the employment consequences of international outsourcing when workers are heterogeneous and one skill-type is employed in a perfectly competitive market, and another in an imperfectly competitive market. Therefore we assume that the low-skilled workers are unionized, while the wage of the high-skilled workers adjusts to equilibrate the demand and supply of their labour services.

In this study we analyze the effects of international outsourcing to low-wage countries on equilibrium unemployment of low-skilled workers and labour demand in high-wage countries characterized by heterogenous in-house workers in the dual labour markets when there is both unionized and competitive determination of wages in the home country. In terms of results we will show that the own wage elasticity, the cross wage elasticity and the outsourcing elasticity for the low-skilled labour demand depend positively on the amount of outsourcing, and it also depends positively on the payroll tax, whereas the own wage elasticity, the cross wage elasticity and the outsourcing elasticity for the high-skilled labour demand are independent of the amount of outsourcing. In the presence of outsourcing the high-skilled wage formation by the monopoly labour union depends negatively on the low-skilled wage and the payroll tax, whereas the high-skilled wage is independent of the high-skilled wage tax parameters under Cobb-Douglas utility function. In terms of low-skilled wage determination in the presence of outsourcing a higher share of outsourced production and a higher productivity of outsourced production will decrease the wage for the low-skilled labour and increase the wage for the high-skilled labour, thereby inducing higher wage dispersion. A higher low-skilled wage tax will increase the wage for the low-skilled labour and a higher low-skilled wage tax exemption will decrease the wage for the low-skilled labour and increase the wage for the high-skilled labour, while a higher payroll tax for the firms will decrease the wage for the low-skilled labour and also under reasonable assumptions decrease the wage for the high-skilled labour.

In terms of optimal committed outsourcing policy parameters affect as follows: a higher domestic low-skilled wage tax and a higher unemployment benefit increase optimal outsourcing, while a higher tax exemption, ceteris paribus, decreases optimal outsourcing, and a higher payroll tax for the firms will have a positive effect on optimal outsourcing under reasonable assumptions.

Finally, in terms of the effects of outsourcing and some policy variables on equilibrium unemployment of low-skilled workers under alternative unemployment benefit specifications we have the following results: A higher amount of outsourced production will reduce equilibrium unemployment of low-skilled workers both in the absence and presence of progressive wage taxation and proportional payroll taxation.

In the presence of outsourcing concerning the assumption that the direct effect of tax parameters on wage formation dominates the indirect effect via outsourcing the higher wage tax and the higher tax exemption will have an ambiguous effect on equilibrium unemployment while the higher payroll tax will decrease equilibrium unemployment when the benefit-replacement ratio is fixed and less than one. In the absence of outsourcing the higher wage tax and the higher tax exemption will have a positive effect on equilibrium unemployment, while the higher payroll tax will have no effect. In the presence of outsourcing raising the wage tax and the tax exemption while keeping the relative tax burden per worker constant, this higher degree of tax progression will decrease the wage rate and labour demand of low-skilled workers. This result is qualitatively similar in the absence of outsourcing.

We proceed as follows. Section II presents the time sequence of the decisions regarding some policy issues associated with labour taxes, outsourcing, wage setting for low-skilled workers and labour demand for high-skilled and low-skilled workers and the wage setting for high-skilled workers. We study the segmented labour demand for heterogenous labour force and wage formation of high-skilled workers due to market equilibrium under proportional payroll tax in section III. The focus on wage formation by the monopoly labour union for low-skilled workers under linearly progressive wage tax, levied on workers, and proportional payroll tax, levied on firms, is analyzed in section IV. Section V explores how the optimal committed production mode from the firms' point of view in the presence of partly imperfectly competitive and segmented labour market depends on various important policy variables. In section VI we explore some policy issues concerning equilibrium unemployment and labour demand of low-skilled domestic workers. Finally, we present comments in section VII.

II. Basic Framework

We analyze a model with heterogeneous workers and international outsourcing. The production combines labour services by high-skilled workers and low-skilled workers. Low-skilled labour services can be provided either by the firm's own workers, or obtained from abroad through international outsourcing.

Establishing international outsourcing is time-consuming, and reversing such decisions is often costly. Therefore, we assume that the firms have to commit to outsourcing before they hire domestic labour. Whether the firms or the government moves first, is an open question, a priori. We assume that the government decides on taxation and unemployment benefits before the firms decide on their international outsourcing. There are two motivations for this. First of all, major overhauls of tax systems are rather rare, and thus tax systems appear more stable than outsourcing decisions by individual firms. Second, the tax parameters to which we assume the government to commit could be viewed as an equilibrium outcome of a repeated game. Without commitment on the government's part, the tax parameters that the firms expect the government to choose ex ante would simply correspond to what is optimal for the government to choose ex post. The timing of event is depicted as Figure 1. The government sets its policy at stage 1. At stage 2, the firms make irreversible investment in outsourcing. At stage 3, conditional of policy choices by the government and the outsourcing decisions by the firms, the labour union determines the wage for the low-skilled workers. When deciding on its wage demand, the monopoly union of each industry takes into account how this affects the demand for labour by the firms. We assume that there are many industries, so that each labour union represents only a small fraction of the total labor force. At stage 4, firms decide on domestic employment. The wages of the high-skilled labour adjust to equalize labour demand and labour supply.

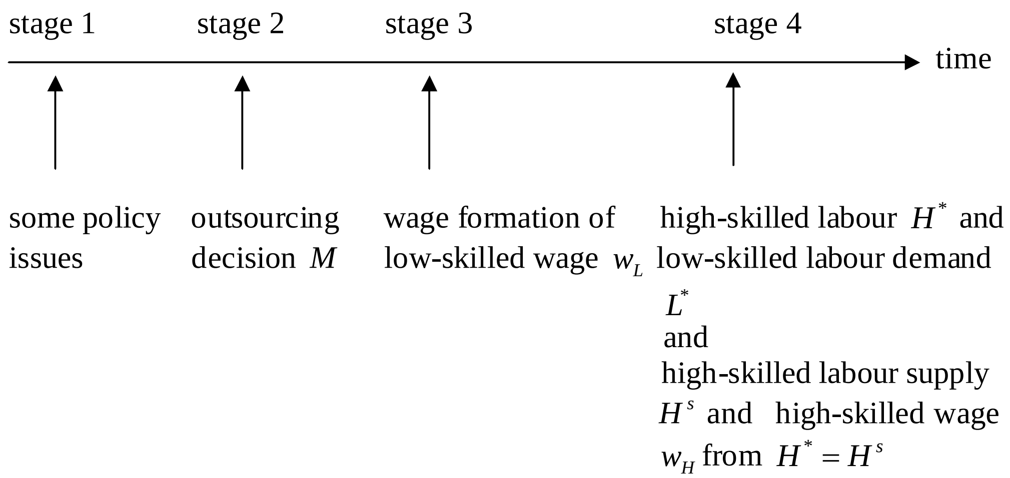

Figure 1: Time sequence of decisions

The decisions in terms of these things at each stage are analyzed by using backward induction. There are high-skilled and low-skilled workers and we assume that high-skilled wage formation is determined by the equality of the high-skilled labour demand and labour supply and the low-skilled formation is determined by the labour union subject to labour demand.[1] This timing structure seems plausible as a starting point when the implementation of a production mode with outsourcing compared with domestic labour demand and wage formation requires irreversible investment concerning the establishment of a network of foreign supplies. Of course, the relative timing of wage formation and outsourcing might in some few cases to be different in certain circumstances, which would be relevant if the firms flexibly adjust their

[1] This has also been analyzed a little bit in Lingens and Waelde (2006), but they have abstracted from outsourcing issues.

production mode, and decide whether to initiate foreign outsourcing after the domestic low-skilled wage is determined.[2]

III. High-Skilled and Low-Skilled Labour Demand and Wage Formation of High-Skilled Workers under Progressive Wage Tax and Proportional Payroll Tax

III.1. High-Skilled and Low-Skilled Labour Demand

At the last stage, the firm decides on the high-skilled labour demand H and the low-skilled labour demand L in order to maximize the profit function, taking the acquired amount of outsourcing, M, as given.

$$\underbrace{Max}_{(H,L)}\ \pi = F(H,L,M) - \tilde{w}_H H - \tilde{w}_L L - g(M) \tag{1}$$

When deciding on its labour demand, each firm takes the gross wage for high-skilled labour, $\tilde{w}_H = w_H(1+s)$, the gross wage for low-skilled labour, $\tilde{w}_L = w_L(1+s)$, and the outsourced low-skilled labour input M as given, where s is the proportional payroll tax levied on the firm. Under outsourced production firms acquire the low-skilled labour input at the factor price c, which is lower than the wage of domestic low-skilled workers. In order to obtain M units of outsourced low-skilled labour input firms have to make irreversible investment $g(M) = 0.5cM^2$ with $g'(M) = cM > 0$ and $g''(M) = c > 0$ into the establishment of networks of suppliers in the relevant low-wage country.

2 Skaksen (2004) has analyzed this case using a Cobb-Douglas production function applied only to a homogenous domestic labour force. Also Braun and Scheffel (2007) have developed a simple two-stage game between a monopoly union and a firm by assuming that the labour union sets wages before the firms decide on the degree of outsourcing. They have argued that under such flexible outsourcing the cost of outsourcing has an ambiguous effect on the wage set by the labour union.

We follow Koskela and Stenbacka (2007) and assume a general and reasonable Cobb-Douglas-type production function with the decreasing returns to scale according to $F(H,L,M)=\left[H^{a}(L+\gamma\, M)^{1-a}\right]^{\rho}$, where the parameters ρ and a are assumed to satisfy: $0<\rho<1$ and $\frac{1}{2}<a<1$. This latter specification means that the marginal productivity of the high-skilled labour is higher than that of the low-skilled labour. The parameter $\gamma>0$ captures the productivity of the outsourced low-skilled labour input relative to the domestic low-skilled labour input. The marginal products of high-skilled and low-skilled labour are $F_{H}=\rho X^{\rho-1}aH^{a-1}(L+\gamma\, M)^{1-a}$ and $F_{L}=\rho X^{\rho-1}H^{a}(1-a)(L+\gamma\, M)^{-a}$, respectively, where $X=H^{a}(L+\gamma\, M)^{1-a}$. The outsourced low-skilled labour input affects the marginal products of the domestic high-skilled and low-skilled labour inputs after calculations as follows:

$$F_{HM}=\rho^{2}X^{\rho-1}aH^{a-1}(1-a)\gamma(L+\gamma M)^{-a}>0 \qquad (2a)$$

$$F_{LM}=-\rho X^{\rho-1}H^{a}(1-a)\gamma(L+\gamma M)^{-a-1}\left[1-\rho(1-a)\right]<0 \qquad (2b)$$

Thus, for this production function the domestic high-skilled labour input and the outsourced low-skilled labour input are complements, whereas the low-skilled domestic labour input and the outsourced low-skilled labour input are substitutes in terms of the marginal product effects of outsourcing. Also one can calculate in the similar way that the domestic high-skilled and low-skilled labour from $F(H,L,M)=\left[H^{a}(L+\gamma\, M)^{1-a}\right]^{\rho}$ are complements, i.e. $F_{HL}>0$. Given both the outsourcing decision and the wages the first-order conditions characterizing the domestic high-skilled and low-skilled labour demands are

$$\pi_{H}=\rho\left[H^{a}(L+\gamma\, M)^{1-a}\right]^{\rho-1}aH^{a-1}(L+\gamma\, M)^{1-a}-\tilde{w}_{H}=0 \qquad (3a)$$

$$\pi_{L}=\rho\left[H^{a}(L+\lambda\, M)^{1-a}\right]^{\rho-1}(1-a)H^{a}(L+\gamma\, M)^{-a}-\tilde{w}_{L}=0\ . \qquad (3b)$$

These first-order conditions imply the following relationship between the high-skilled labour (H) and the low-skilled labour inclusive of outsourcing ($L+\gamma\ M$)

$$H=\frac{w_L}{w_H}\frac{a}{1-a}(L+\gamma\ M) \quad . \tag{4}$$

Substituting (4) into (3b) gives (see Appendix A) the low-skilled labour demand, which can be expressed as follows

$$L^*=mw_L^{-\varepsilon_L^L}w_H^{-\varepsilon_H^L}(1+s)^{-\varepsilon}-\gamma\ M\ , \tag{5}$$

where $m=\left[\rho a^{a\rho}(1-a)^{1-a\rho}\right]^{\frac{1}{1-\rho}}>0$, $\varepsilon_L^L=-\frac{L_{w_L}w_L}{L}=\frac{1-\rho a}{1-\rho}>1$ denotes the own wage elasticity of the low-skilled labour and $\varepsilon_H^L=-\frac{L_{w_H}w_H}{L}=\frac{\rho a}{1-\rho}>0$ denotes the cross wage elasticity of the low-skilled labour with respect to the high-skilled wage in the absence of outsourcing, when $M=0$.[3] These elasticities are higher with weaker decreasing returns to scale. Higher own wage and cross wage will affect negatively the low-skilled labour demand. In the absence of outsourcing the payroll tax elasticity of the low-skilled labour is $\varepsilon_s^L=\varepsilon=-\frac{L_s(1+s)}{L}=\frac{1}{1-\rho}>1$ because of the decreasing returns to scale. According to (5), a more extensive outsourcing activity will decrease the low-skilled labour demand. This feature is consistent with empirical evidence.[4]

In the presence of outsourcing M the wage elasticities of the low-skilled labour, $-\frac{L^*_{w_L}w_L}{L^*}\Big|_{M>0}$ and $-\frac{L^*_{w_H}w_H}{L^*}\Big|_{M>0}$, can be written as follows

$$\hat{\varepsilon}_L^L=\varepsilon_L^L\left(1+\gamma\frac{M}{L^*}\right) \tag{6a}$$

and

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3 In the presence of perfect substitutability between two types of labour inputs, i.e. between L and M , we would have $\gamma=1$, but it is important to mention that qualitative results are similar.

4 For instance Diehl (1999) has presented empirical evidence from German manufacturing industries in support of this hypothesis. Moreover, Görg and Hanley (2005) have used plant-level data of the Irish electronic sector to empirically conclude that international outsourcing reduces plan-level labour demand.

$$\hat{\varepsilon}_H^L = \varepsilon_H^L\left(1+\gamma\frac{M}{L^*}\right) \qquad (6b)$$

so that $\frac{\partial \hat{\varepsilon}_L^L}{\partial M} = \varepsilon_L^L \gamma\left[\frac{L^* - ML_M^*}{L^{*2}}\right] = \varepsilon_L^L \frac{\gamma}{L^*}(1+\gamma\frac{M}{L^*}) = \frac{\gamma}{L^*}\hat{\varepsilon}_L^L > 0$ and $\frac{\partial \hat{\varepsilon}_H^L}{\partial M} = \varepsilon_H^L \gamma\left[\frac{L^* - ML_M^*}{L^{*2}}\right] = \varepsilon_H^L \frac{\gamma}{L^*}(1+\gamma\frac{M}{L^*}) = \frac{\gamma}{L^*}\hat{\varepsilon}_H^L > 0$. These are in conformity with empirical evidence according to which higher outsourcing increases the wage elasticity of low-skilled labour demand.[5]

Moreover, the elasticity of low-skilled labour with respect to outsourcing is positive, i.e. $\varepsilon_M^L = -\frac{L_M^* M}{L^*} = \frac{\gamma M}{\left[m w_L^{-\varepsilon_L^L} w_H^{-\varepsilon_H^L} - \gamma M\right]} > 0$. Differentiating this with respect to M gives

$$\frac{\partial \varepsilon_M^L}{\partial M} = \frac{m w_L^{-\varepsilon_L^L} w_H^{-\varepsilon_H^L}\gamma}{\left[m w_L^{-\varepsilon_L^L} w_H^{-\varepsilon_H^L} - \gamma M\right]^2} = \frac{(L^* + \lambda M)\gamma}{L^{*2}} = \frac{\gamma}{L^*}(1+\gamma\frac{M}{L^*}) > 0\,. \qquad (7)$$

which means that higher outsourcing will increase the outsourcing elasticity of the low-skilled labour. Differentiating (6a) with respect to the payroll tax gives

$$\frac{\partial \hat{\varepsilon}_L^L}{\partial s} = \varepsilon_L^L\left[-\frac{\gamma M L_s^*}{L^{*2}}\right] = \frac{\varepsilon_L^L}{(1+s)}\frac{\gamma M}{L^*}\varepsilon(1+\gamma\frac{M}{L^*}) = \frac{\varepsilon}{(1+s)}\frac{\gamma M}{L^*}\hat{\varepsilon}_L^L > 0 \qquad (8)$$

according to which the payroll tax in the presence of outsourcing will have a positive effect of the wage elasticity of the low-skilled labour demand. Of course there is no wage elasticity effect of payroll tax in the absence of outsourcing, i.e. $\left.\frac{\partial \hat{\varepsilon}_L^L}{\partial s}\right|_{M=0} = 0$. In

[5] Senses (2006) has provided empirical evidence according to which a production mode with more outsourcing seems to increase the wage elasticity of labour demand. Also Slaughter (2001) and Hasan, Mitra and Ramaswamy (2007) have shown in terms of empirics that international trade has increased the wage elasticity of labour demand.

the presence of outsourcing the payroll tax elasticity of the low-skilled labour, $-\frac{L_s^*(1+s)}{L^*}\bigg|_{M>0}$, is

$$\hat{\varepsilon} = \varepsilon\left(1+\gamma\frac{M}{L^*}\right) \tag{9}$$

where $\varepsilon_s^L = \varepsilon = \frac{1}{1-\rho} > 1$ so that higher outsourcing raises this elasticity as well, i.e. $\frac{\partial\hat{\varepsilon}}{\partial M} = \varepsilon\frac{\gamma}{L^*}(1+\gamma\frac{M}{L^*}) = \frac{\gamma}{L^*}\hat{\varepsilon} > 0$.

Finally, substituting the RHS of equation (5) into the relationship between the high-skilled and low-skilled labour presented in equation (4) gives the following optimal demand for the high-skilled labour

$$H^* = \frac{ma}{1-a} w_H^{-\varepsilon_H^H} w_L^{-\varepsilon_L^H} (1+s)^{-\varepsilon}, \tag{10}$$

where $\varepsilon_H^H = -\frac{H_{w_H}^* w_H}{H^*} = \frac{1-\rho(1-a)}{1-\rho} > 1$, $\varepsilon_L^H = -\frac{H_{w_L}^* w_L}{H^*} = \frac{\rho(1-a)}{1-\rho} > 0$ and $\varepsilon_s^H = \varepsilon = -\frac{H_s^*(1+s)}{H^*} = \frac{1}{1-\rho} > 1$. These elasticities are also higher with weaker decreasing returns to scale, but unlike the case with the low-skilled labour, both the own wage and cross wage labor demand elasticities, and the payroll tax elasticity for high-skilled labour are independent of outsourcing. Higher own wage, cross wage and payroll tax will affect negatively the high-skilled labour demand.

We can now summarize our findings regarding the properties of the domestic demand for labour in the presence of outsourcing as follows.

Proposition 1 *In the presence of outsourcing*

(a) the own wage elasticity, the cross wage elasticity and the outsourcing elasticity for the low-skilled labour demand depend positively on the amount of outsourcing, and they also depend positively on the payroll tax, whereas

(b) the own wage elasticity, the cross wage elasticity and the outsourcing elasticity for the high-skilled labour demand are independent of the amount of outsourcing and the payroll tax.

Proposition 1 reveals an asymmetry in how the demand for high-skilled and low-skilled labor react to the amount of outsourcing and the level of payroll taxes. An increase in outsourcing or payroll taxes would increase the own wage elasticity, the cross wage elasticity and the outsourcing elasticity for the low-skilled labour demand, while having no effect on the elasticities for the high-skilled labour demand.

III.2. Wage Formation for High-Skilled Workers in the Presence of Progressive Wage Tax and Proportional Payroll Tax

III.2.1 Optimal Labour Supply of High-Skilled Workers

We assume that the market equilibrium for the high-skilled wage w_H follows from the equality of labour demand and the labour supply in the case of Cobb-Douglas utility function, where the elasticity of substitution between consumption and leisure is one. First we derive labour supply and after that the wage formation from market equilibrium by taking the low-skilled wage w_L as given.

We assume that government can employ the proportional wage tax t_H for high-skilled worker, which is levied on the wage rate w_H minus tax exemption e. Thus the total tax base in this case is $(w_H - e)H$, where H is labour supply. In the presence of positive tax exemption the marginal wage tax exceeds the average wage tax rate $t_H(1-e/w_H)$ so that the system is linearly progressive.[6] The net-of-tax wage, the high-skilled worker receives, is $\hat{w}_H = (1-t_H)w_H + t_H e$.

We assume that labour supply of the high-skilled worker is determined by utility maximization. In the case of the Cobb-Douglas utility function the elasticity of

6 For a seminal paper about tax progression, see Musgrave and Thin (1948), and for another elaboration, see e.g. Lambert (2001, chapters 7-8).

substitution is equal to one in terms of consumption C and leisure $1-H$ in the utility function, i.e. $U(C,H)=C^{\mu}(1-H)^{1-\mu}$, $0<\mu<1$. Maximizing $U(C,H)=C^{\mu}(1-H)^{1-\mu}$ s.t. $\hat{w}_H H=C$ with respect to labour supply H gives $U_H=\mu(\hat{w}_H H)^{\mu-1}(1-H)^{1-\mu}-(1-\mu)(\hat{w}_H H)^{\mu}(1-H)^{-\mu}=0$ so that

$$H^s=\mu \tag{11}$$

Therefore under this assumption the net-of-tax wage $\hat{w}_H=(1-t_H)w_H+t_H e$ will have no effect on labour supply when the substitution and income effects of wage rate cancel each other. It is important to emphasize that a central findings in the empirical labour market literature is that labour supply tends to be quite unresponsive along the intensive margin (see for empirical evidence, e.g. Immervoll, Kleven, Kreiner and Saez (2007) and Blundell and MaCurdy (1999)). Therefore, we would like to focus on this finding concerning the market equilibrium of high-skilled workers.

III.2.2 Market Equilibrium for High-Skilled Wage Formation

Unlike in the case of low-skilled workers we assume that the high-skilled wage w_H is determined by the market equilibrium concerning the equality of the labour demand function and the labour supply function. In the case of C-D utility function the equality $H^*=H^s$ gives $\frac{ma}{1-a}w_H^{-\varepsilon_H^H}w_L^{-\varepsilon_L^H}(1+s)^{-\frac{\varepsilon}{\varepsilon_H^H}}=\mu$, which can be expressed as

$$w_H=\left[\frac{\mu(1-a)}{ma}\right]^{-\frac{1}{\varepsilon_H^H}}w_L^{-\frac{\varepsilon_L^H}{\varepsilon_H^H}}(1+s)^{-\frac{\varepsilon}{\varepsilon_H^H}} \tag{12}$$

where $\varepsilon_L^H/\varepsilon_H^H=\frac{\rho(1-a)}{1-\rho(1-a)}>0$ and $\varepsilon/\varepsilon_H^H=\frac{1}{1-\rho(1-a)}>1$. The comparative statics in terms of w_H and w_L is

$$\frac{\partial w_H}{\partial w_L}=-\frac{\varepsilon_L^H}{\varepsilon_H^H}\left[\frac{\mu(1-a)}{ma}\right]^{-\frac{1}{\varepsilon_H^H}}w_L^{-\frac{\varepsilon_L^H}{\varepsilon_H^H}-1}(1+s)^{-\frac{\varepsilon}{\varepsilon_H^H}}=-\frac{\varepsilon_L^H}{\varepsilon_H^H}\frac{w_H}{w_L}<0. \tag{13}$$

Equation (13) lies in conformity with empirics concerning the negative relationship between high-skilled and low-skilled wages. It has been empirically shown that higher outsourcing will decrease wage formation of low-skilled workers and increase wage formation of high-skilled workers, i.e. that wage dispersion will increase[7]

The effect of payroll tax on wage formation of high-skilled workers is under our utility assumption

$$\frac{\partial w_H}{\partial s} = -\frac{\varepsilon}{\varepsilon_H^H}\left[\frac{\mu(1-a)}{ma}\right]^{-\frac{1}{\varepsilon_H^H}} w_L^{-\frac{\varepsilon_L^H}{\varepsilon_H^H}}(1+s)^{-\frac{\varepsilon}{\varepsilon_H^H}-1} = -\frac{\varepsilon}{\varepsilon_H^H}\frac{w_H}{1+s} < 0 \tag{14}$$

so that higher payroll tax will decrease the wage rate of high-skilled workers because it decreases labour demand in the case of fixed labour supply (concerning empirical evidence, see. e.g. Daveri and Tabellini (2000), and Bingley and Lanot (2002)).

We can now summarize our findings regarding the properties of the high-skilled wage determination in the presence of outsourcing as follows.

Proposition 2 *In the presence of outsourcing*

(a) the high-skilled wage depends negatively on the low-skilled wage and the payroll tax, whereas

(b) the high-skilled wage is independent of the high-skilled wage tax parameters in the case of high-skilled workers' Cobb-Douglas utility function.

In the first sight, it may appear surprising that the high-skilled wage reacts negatively to the low-skilled wage tax, but is independent of their own wage tax. The intuition for this relies on our assumption that the high-skilled workers have a C-D utility function.

7 See evidence from various countries which lies in conformity with this, e.g. Braun and Scheffel (2007), Feenstra and Hanson (1999, 2001), Hijzen, Görg and Hine (2005), Hijzen (2007), Egger and Egger (2006), Munch and Skaksen (2005), Yan (2006), Riley and Young (2007) and Geishecker and Görg (2008)

With it, income and substitution effects of a tax increase on the labor supply cancel each other out.

IV. Wage Formation by Monopoly Labour Union for Low-Skilled Workers in the Presence of Progressive Wage Taxation and Proportional Payroll Taxation

Now we analyze the wage formation of low-skilled workers and continue to consider the acquired amount of outsourcing, M as given. We analyze the wage formation by the labour monopoly union, which determines the wage for low-skilled workers in anticipation of optimal in-house low-skilled labour demand and of market equilibrium for the high-skilled wage w_H. In western European countries, which we like to focus labour market institutions are closed to this than in other countries (see e.g. Freeman (2008), see also Cahuc and Zylberberg (2004), p. 401-403 concerning the monopoly trade union specification).

IV.1. Wage Formation by the Monopoly Labour Union

We investigate the wage formation by monopoly labour union when there is proportional payroll tax, and the linearly progressive wage tax for low-skilled workers. The market equilibrium for the high-skilled wage w_H follows from the equality of labour demand and the labour supply by focusing the case of C-D utility function. The monopoly labour union determines the wage for low-skilled workers in anticipation of optimal in-house employment decisions by the firm. We assume that government can employ a proportional tax rate t_L, which is levied on the wage rate w_L minus a tax exemption e. Thus the total tax base in this case is $(w_L - e)L^*$. In the presence of a positive tax exemption the marginal wage tax exceeds the average wage tax rate $t_L(1-e/w_L)$ so that the system is linearly progressive. The net-of-tax wage is $\hat{w}_L = (1-t_L)w_L + t_L e$.

The objective function of the labour union is assumed to be $V = ((1-t_L)w_L + t_L e - b_L)L^* + b_L N = (\hat{w}_L - b_L)L^* + b_L N$, where b_L is the (exogenous) outside option available to the low-skilled workers and N is the number of labour union members. Given the amount of outsourcing, the monopoly labour union sets wage both for the low-skilled workers so as to maximize the surplus according to

$$\underbrace{\max}_{(w_L)} \; V = (\hat{w}_L - b_L)L^* + b_L N \;\text{ s.t. }\; \pi_L = 0 \;\text{ and }\; H^* = H^s \tag{15}$$

where in the presence of payroll tax $H^* = \frac{ma}{1-a} w_H^{-\varepsilon_H^H} w_L^{-\varepsilon_L^H} (1+s)^{-\varepsilon}$ and $H^s = \mu$, which implies $w_H = \left[\frac{\mu(1-a)}{ma}\right]^{-\frac{1}{\varepsilon_H^H}} w_L^{-\frac{\varepsilon_L^H}{\varepsilon_H^H}} (1+s)^{-\frac{\varepsilon}{\varepsilon_H^H}}$ (see equations (10), (11) and (12)).

The first-order condition associated with (15) is

$$V_{w_L} = 0 \Leftrightarrow \frac{L^*}{w_L}\left[(1-t_L)w_L + ((1-t_L)w_L + t_L e - b_L)\left(\frac{L^*_{w_L} w_L}{L^*} + \frac{L^*_{w_H} w_H}{L^*}\frac{\partial w_H}{\partial w_L}\frac{w_L}{w_H}\right)\right]. \tag{16}$$

This can be written as follows

$$V_{w_L} = (1-t_L)w_L(1-(\hat{\varepsilon}_L^L + \hat{\varepsilon}_H^L \frac{\partial w_H}{\partial w_L}\frac{w_L}{w_H})) + (b_L - t_L e)(\hat{\varepsilon}_L^L + \hat{\varepsilon}_H^L \frac{\partial w_H}{\partial w_L}\frac{w_L}{w_H}) = 0 \tag{17}$$

where the own wage elasticity of low-skilled labour demand is $\hat{\varepsilon}_L^L = \varepsilon_L^L\left(1+\frac{\gamma M}{L^*}\right)$ and the cross wage elasticity of low-skilled labour demand is $\hat{\varepsilon}_H^L = \varepsilon_H^L\left(1+\frac{\gamma M}{L^*}\right)$. These are not constant because the low-skilled labour demand, $L^* = m\, w_L^{-\varepsilon_L^L} w_H^{-\varepsilon_H^L} (1+s)^{-\varepsilon} - \gamma M$ depends negatively on the following variables: the high-skilled wage, the low-skilled wage, outsourcing, the productivity of the outsourced low-skilled labour input relative to the domestic low-skilled labour input, and the payroll tax. Equation (17) can be expressed as follows

$$w_L^*(M,w_H,\gamma)=\left(\frac{\hat{\varepsilon}_L^L-\hat{\varepsilon}_H^L\frac{\varepsilon_L^H}{\varepsilon_H^H}}{\hat{\varepsilon}_L^L-\hat{\varepsilon}_H^L\frac{\varepsilon_L^H}{\varepsilon_H^H}-1}\right)\hat{b}_L=\left(\frac{(\varepsilon_L^L-\varepsilon_H^L\frac{\varepsilon_L^H}{\varepsilon_H^H})(1+\gamma\frac{M}{L^*})}{(\varepsilon_L^L-\varepsilon_H^L\frac{\varepsilon_L^H}{\varepsilon_H^H})(1+\gamma\frac{M}{L^*})-1}\right)\hat{b}_L \tag{18}$$

where $\hat{b}_L=\dfrac{b_L-t_Le}{1-t_L}$. Therefore we have (see Appendix B)

$$w_L^*(M,w_H,\gamma,t_L,e,s)=\left(\frac{\bar{\varepsilon}_L^L}{\bar{\varepsilon}_L^L-1}\right)\hat{b}_L=\frac{\beta(L^*+\gamma M)}{(\beta-1)L^*+\beta\gamma M}\hat{b}_L \tag{19}$$

where the total wage elasticity allowing for the relationship between high-skilled and low-skilled wages is $\bar{\varepsilon}_L^L=\beta(1+\gamma\dfrac{M}{L^*})>1$, $\beta=\dfrac{1}{1-\rho(1-a)}$. It is important to emphasize that the optimal low-skilled wage (19) even for the monopoly labour union is an implicit form in the presence of outsourcing, because the numerator and denominator of the mark-up $A=\dfrac{\beta(L^*+\gamma M)}{(\beta-1)L^*+\beta\gamma M}$ depends on the low-skilled wage rate in a non-linear way. It cannot be solved explicitly for the optimal domestic low-skilled wage.

IV.2. Comparative Statics of Wage Formation

In order to characterize the effect of outsourcing on the low-skilled wage formation we therefore apply the implicit differentiation. Differentiating the wage formation (19) with respect to low-skilled wage and outsourcing gives

$$\left(1-\frac{\left[(\bar{\varepsilon}_L^L-1)\frac{\partial\bar{\varepsilon}_L^L}{\partial w_L}-\bar{\varepsilon}_L^L\frac{\partial\bar{\varepsilon}_L^L}{\partial w_L}\right]}{(\bar{\varepsilon}_L^L-1)^2}\hat{b}_L\right)dw_L^*=\frac{\left[(\bar{\varepsilon}_L^L-1)\frac{\partial\bar{\varepsilon}_L^L}{\partial M}-\bar{\varepsilon}_L^L\frac{\partial\bar{\varepsilon}_L^L}{\partial M}\right]}{(\bar{\varepsilon}_L^L-1)^2}\hat{b}_L dM \tag{20}$$

which can be expressed as $\frac{dw_L^*}{dM} = -\frac{\frac{\partial \bar{\varepsilon}_L^L}{\partial M}}{(\bar{\varepsilon}_L^L - 1)^2} \hat{b}_L \Big/ \left(1 + \frac{\frac{\partial \bar{\varepsilon}_L^L}{\partial w_L}}{(\bar{\varepsilon}_L^L - 1)^2} \hat{b}_L \right) < 0$. Using equation

(19) $\hat{b}_L = \frac{w_L(\bar{\varepsilon}_L^L - 1)}{\bar{\varepsilon}_L^L}$ and $\frac{\partial \bar{\varepsilon}_L^L}{\partial M} = \beta \frac{\gamma}{L^*}(1 + \gamma \frac{M}{L^*}) = \frac{\gamma}{L^*} \bar{\varepsilon}_L^L > 0$,

$\frac{\partial \bar{\varepsilon}_L^L}{\partial w_L} = \beta\gamma \frac{M}{w_L L^*} \varepsilon_L^L (1 + \gamma \frac{M}{L^*}) = \varepsilon_L^L \frac{\gamma M}{w_L L^*} \bar{\varepsilon}_L^L > 0$ the relationship between the low-skilled wage formation and outsourcing can be written as follows

$$\frac{dw_L^*}{dM} = -\frac{w_L^* \frac{\gamma}{L^*}}{\bar{\varepsilon}_L^L - 1 + \gamma \frac{M}{L^*} \varepsilon_L^L} = -\frac{w_L^* \gamma}{(\beta - 1)L^* + (1 + \beta)\varepsilon_L^L \gamma M} < 0 \tag{21}$$

so that higher outsourcing will decrease the wage of low-skilled workers. This lies in conformity with empirics, which we have already mentioned earlier.

Differentiating the implicit wage formation (19) with respect to the productivity of the outsourced low-skilled labour input relative to the domestic low-skilled labour input and low-skilled wage formation gives

$$\left(1 - \frac{\left[(\bar{\varepsilon}_L^L - 1) \frac{\partial \bar{\varepsilon}_L^L}{\partial w_L} - \bar{\varepsilon}_L^L \frac{\partial \bar{\varepsilon}_L^L}{\partial w_L} \right]}{(\bar{\varepsilon}_L^L - 1)^2} \hat{b}_L \right) dw_L^* = \frac{\left[(\bar{\varepsilon}_L^L - 1) \frac{\partial \bar{\varepsilon}_L^L}{\partial \gamma} - \bar{\varepsilon}_L^L \frac{\partial \bar{\varepsilon}_L^L}{\partial \gamma} \right]}{(\bar{\varepsilon}_L^L - 1)^2} \hat{b}_L d\gamma \tag{22}$$

which can be expressed also by using $\hat{b}_L = \frac{w_L(\bar{\varepsilon}_L^L - 1)}{\bar{\varepsilon}_L^L}$ as

$$\frac{dw_L^*}{d\gamma} = -\frac{w_L^* \frac{M}{L^*}}{\left[\bar{\varepsilon}_L^L - 1 + \gamma \frac{M}{L^*} \varepsilon_L^L \right]} = -\frac{w_L^* M}{(\beta - 1)L^* + (1 + \beta)\varepsilon_L^L \gamma M} < 0 \tag{23}$$

where $\frac{\partial \bar{\varepsilon}_L^L}{\partial \gamma} = \beta \left[\frac{L^* M - \gamma M L_\gamma^*}{L^{*2}} \right] = \frac{M}{L^*} \beta (1 + \gamma \frac{M}{L^*}) = \frac{M}{L^*} \bar{\varepsilon}_L^L > 0$. Higher productivity of the outsourced low-skilled labour input relative to the domestic low-skilled labour input will have a wage moderating effect concerning low-skilled workers' wage. Moreover, and importantly, equations (21) and (23) jointly with equation (13) imply $\frac{dw_H^*}{dM} > 0$ and $\frac{dw_H^*}{d\gamma} > 0$ so that higher outsourcing and higher productivity of the outsourced low-skilled labour input will have positive effects on the domestic high-skilled labour wage.

In terms of comparative statics of wage tax, tax exemption and outside option we have (see Appendix B)

$$\frac{dw_L^*}{dt_L} = \left(\frac{\bar{\varepsilon}_L^L}{\bar{\varepsilon}_L^L - 1 + \gamma \frac{M}{L^*} \varepsilon_L^L} \right) \frac{b_L - e}{(1-t_L)^2} = \frac{\beta (L^* + \gamma M)}{(\beta - 1)L^* + (1+\beta)\varepsilon_L^L \gamma M} \frac{b_L - e}{(1-t_L)^2} > 0 \text{ as } b_L - e > 0 \tag{24a}$$

$$\frac{dw_L^*}{de} = -\left(\frac{\bar{\varepsilon}_L^L}{\bar{\varepsilon}_L^L - 1 + \gamma \frac{M}{L^*} \varepsilon_L^L} \right) \frac{t_L}{(1-t_L)} = -\frac{\beta (L^* + \gamma M)}{(\beta - 1)L^* + (1+\beta)\varepsilon_L^L \gamma M} \frac{t_L}{(1-t_L)} < 0 \tag{24b}$$

$$\frac{dw_L^*}{db_L} = \left(\frac{\bar{\varepsilon}_L^L}{\bar{\varepsilon}_L^L - 1 + \gamma \frac{M}{L^*} \varepsilon_L^L} \right) \frac{1}{(1-t_L)} = \frac{\beta (L^* + \gamma M)}{(\beta - 1)L^* + (1+\beta)\varepsilon_L^L \gamma M} \frac{1}{(1-t_L)} > 0 \tag{24c}$$

According to (24a-24c) the effects of wage tax, tax exemption and outside option for unemployment benefit on low-skilled wage formation are qualitatively the same with and without outsourcing because $\left.\frac{dw_L^*}{dt_L}\right|_{M=0} = \frac{\beta}{(\beta-1)} \frac{b_L - e}{(1-t_L)^2} > 0$, $\left.\frac{dw_L^*}{de}\right|_{M=0} = -\frac{\beta}{(\beta-1)} \frac{t_L}{(1-t_L)} < 0$ and $\left.\frac{dw_L^*}{db_L}\right|_{M=0} = \frac{\beta}{(\beta-1)} \frac{1}{(1-t_L)} > 0$. Of course, in the absence of outsourcing the mark-up between outside option and wage formation is $A|_{M=0} = \frac{\beta}{\beta - 1} = \frac{1}{\rho(1-a)} > 1$, which is higher than in the presence of outsourcing. Therefore the effects of wage tax, tax exemption and outside option for unemployment benefit are smaller in the presence of international outsourcing. Therefore, equations (24a-c) imply jointly with equation (13) that $\frac{dw_H^*}{dt_L} < 0$, $\frac{dw_H^*}{de} > 0$ and $\frac{dw_H^*}{db_L} < 0$.

Finally, differentiating the implicit wage formation (19) with respect to the wage of low-skilled workers and the payroll tax gives

$$\left(1 - \frac{\left[(\bar{\varepsilon}_L^L - 1)\frac{\partial \bar{\varepsilon}_L^L}{\partial w_L} - \bar{\varepsilon}_L^L \frac{\partial \bar{\varepsilon}_L^L}{\partial w_L} \right]}{(\bar{\varepsilon}_L^L - 1)^2} \hat{b}_L \right) dw_L^* = \frac{\left[(\bar{\varepsilon}_L^L - 1)\frac{\partial \bar{\varepsilon}_L^L}{\partial s} - \bar{\varepsilon}_L^L \frac{\partial \bar{\varepsilon}_L^L}{\partial s} \right]}{(\bar{\varepsilon}_L^L - 1)^2} \hat{b}_L ds, \quad (25)$$

which can be expressed as follows

$$\frac{dw_L^*}{ds} = -\frac{w_L^* \frac{\varepsilon \gamma M}{L^*(1+s)}}{\left[\bar{\varepsilon}_L^L - 1 + \gamma \frac{M}{L^*} \varepsilon_L^L \right]} = -\frac{w_L^* \varepsilon \gamma M}{(1+s)\left[(\beta - 1)L^* + (1+\beta)\varepsilon_L^L \gamma M \right]} < 0 \quad (26)$$

where $\frac{\partial \bar{\varepsilon}_L^L}{\partial s} = \frac{\beta \gamma M}{L^*(1+s)} \left(-\frac{L_s^*(1+s)}{L^*} \right) = \frac{\varepsilon}{(1+s)} \frac{\gamma M}{L^*} \bar{\varepsilon}_L^L > 0$. Therefore, the payroll tax will have a wage moderating effect concerning the low-skilled workers' wage, because the payroll tax will have a positive effect on the wage elasticity. But in the absence of outsourcing it will have no effect because $\left. \frac{\partial \bar{\varepsilon}_L^L}{\partial s} \right|_{M=0} = 0$.

The total effect of the payroll tax on the high-skilled workers' wage is the following

$$\frac{dw_H^*}{ds} = \underbrace{\frac{\partial w_H^*}{\partial s}}_{-} + \underbrace{\frac{\partial w_H^*}{\partial w_L^*}}_{-} \underbrace{\frac{dw_L^*}{ds}}_{-} \quad (27)$$

and using equations (13), (14) and (26) this can be expressed as

$$\frac{dw_H^*}{ds} = -\frac{\varepsilon}{\varepsilon_H^H}\frac{w_H^*}{(1+s)} + \frac{\varepsilon_L^H}{\varepsilon_H^H}\frac{w_H^*\varepsilon\gamma M}{(1+s)\left[(\beta-1)L^* + (1+\beta)\varepsilon_L^L\gamma M\right]}$$

$$= -\frac{\varepsilon w_H^*}{\varepsilon_H^H(1+s)}\left[1 - \frac{\varepsilon_L^H\gamma M}{\left[(\beta-1)L^* + (1+\beta)\varepsilon_L^L\gamma M\right]}\right]$$

$$= -\frac{w_H^*\varepsilon}{\varepsilon_H^H(1+s)\left[(\beta-1)L^* + (1+\beta)\varepsilon_L^L\gamma M\right]}\left((\beta-1)L^* + ((1+\beta)\varepsilon_L^L - \varepsilon_L^H))\gamma M\right) < 0 \tag{28}$$

which is also negative because $(\beta-1)L^* + ((1+\beta)\varepsilon_L^L - \varepsilon_L^H)\gamma\, M > 0$, where $(1+\beta)\varepsilon_L^L - \varepsilon_L^H = \varepsilon_L^L - \varepsilon_L^H + \beta\varepsilon_L^L = 1 + \beta\varepsilon_L^L > 0$.

We can now summarize our findings in terms of the low-skilled wage formation in the presence of outsourcing as follows.

Proposition 3 *In the presence of outsourcing*

(a) the higher share of outsourced production and a higher productivity of outsourced production will decrease the wage for the low-skilled labour and increase the wage for the high-skilled labour, thereby inducing higher wage dispersion, and

(b) the higher low-skilled wage tax will increase the wage for the low-skilled labour and decrease the wage for high-skilled labour and a higher low-skilled wage tax exemption will decrease the wage for the low-skilled labour and will increase the wage for the high-skilled labour, and these qualitative results are also similar but higher in the absence of outsourcing, and

(c) the higher payroll tax for the firms will decrease the wage for the low-skilled and high-skilled labour. In the absence of outsourcing, a higher payroll tax for the firms will decrease the wage for the high-skilled labour, but has no effect on the wage of low-skilled labour.

The first part of Proposition 3 reveals political economy considerations related to outsourcing and taxation. An increased outsourcing benefits high-skilled workers, but hurts low-skilled workers. Such a result is perfectly in line with the fact that the outsourced input is a substitute to the low-skilled labor, and a complement to high-skilled labor. Nonetheless, the conventional analysis has focused on competitive labour markets.

The second part of Proposition 3 reports that the qualitative effects of wage taxes and tax exemption for the low-skilled workers are not changed by outsourcing. A higher wage tax for the low-skilled labour will encourage labour unions to increase their wage demand, which pushes for higher low-skilled unemployment and lower high-skilled wages. A higher tax exemption for the low-skilled, on the other hand, reduces the wage demand by the labour union. This, in turn, results in a lower unemployment for the low-skilled, and increases the wages for the high-skilled.

The third part of Proposition 3 reveals that outsourcing may change qualitatively how wage demands by labour unions respond to payroll taxes. In the absence of outsourcing, a higher payroll tax has no effect on the wage of low-skilled labour that the labour unions set. With outsourcing, labour unions cut their wage demand when the payroll tax is increased. The wage for the high-skilled is decreasing in the payroll tax rate, both with and without outsourcing.

V. Optimal Committed Outsourcing Before Wage Formation and Domestic Labour Demand

We now turn to explore the stage, where the firm commits itself to the outsourcing activity M prior to the determination of wages and domestic employment. We characterize how the labour market imperfection and tax parameters affect the equilibrium production mode. It is assumed that the long-run production mode decision internalizes the effect of the share of outsourced production on the low-skilled wage and also on the high-skilled wage in different directions.

The firm determines the magnitude of outsourcing so as to maximize its profit. It is assumed that the firm has rational expectations regarding the subsequent outcomes with respect to the high-skilled and low-skilled wage and employment so that the production mode internalizes the effects of the share of outsourced production on wages and employment. The production mode is determined by the following optimization problem in the presence of linearly progressive wage taxation and proportional payroll taxation

$$\underbrace{Max}_{(M)}\ \pi = \left[H^{*a}(L^{*}+\lambda\, M)^{1-a}\right]^{\rho} - \tilde{w}_{H}H^{*} - \tilde{w}_{L}L^{*} - 0.5cM^{2} \qquad (29)$$

$$\text{s.t. } V_{L}=0,\ \pi_{L}=0 \text{ and } H^{*}=H^{s}$$

where $\left[H^{*a}(L^{*}+\lambda\, M)^{1-a}\right]^{\rho}=F$, $\tilde{w}_{H}=(1+s)w_{H}$ and $\tilde{w}_{L}=(1+s)w_{L}$.

By applying the envelope theorem we get the following first-order condition for the optimal amount of committed outsourcing associated with the optimization problem (29) by taking tax parameters as given[8]

$$\pi_{M} = F_{M} \underbrace{-\frac{dw_{L}^{*}}{dM}(1+s)L^{*}}_{+} \underbrace{-\frac{\partial w_{H}^{*}}{\partial w_{L}^{*}}\frac{dw_{L}^{*}}{dM}(1+s)H^{*}}_{-} - cM = 0 \qquad (30)$$

where $\frac{dw_{L}^{*}}{dM} = -\frac{w_{L}^{*}\gamma}{(\beta-1)L^{*}+(1+\beta)\varepsilon_{L}^{L}\gamma\, M} < 0$ (see equation 21) and

$$F_{M} = \gamma\, F_{L} = \gamma(1+s)w_{L}^{*} \qquad (31a)$$

$$\frac{\partial w_{H}^{*}}{\partial w_{L}^{*}}\frac{dw_{L}^{*}}{dM} = \frac{\varepsilon_{L}^{H}}{\varepsilon_{H}^{H}}\frac{w_{H}^{*}\frac{\gamma}{L^{*}}}{\left[\bar{\varepsilon}_{L}^{L}-1+\gamma\frac{M}{L^{*}}\varepsilon_{L}^{L}\right]} = \frac{\varepsilon_{L}^{H}}{\varepsilon_{H}^{H}}\frac{w_{H}^{*}\gamma}{(\beta-1)L^{*}+(1+\beta)\varepsilon_{L}^{L}\gamma\, M} > 0 \qquad (31b)$$

Hence, in addition to the direct marginal cost cM and the direct marginal profit F_{M} introducing outsourcing will decrease the wage cost of the domestic low-skilled labour,

8 Outsourcing does not have a direct effect on the high-skilled wage, but only via the effect of low-skilled wage, see equation (12).

because these are substitutes, but will increase the market equilibrium wage cost of domestic high-skilled labour, because these are complements. Therefore, according to (30) the presence of domestic labour market imperfection increases the returns from outsourcing because it has an aggregate wage-moderating effect, but also decreases the returns due to wage increasing effect of high-skilled labour.[9]

By using equations (21) and (31b) we have

$$-\frac{dw_L^*}{dM}(1+s)L^* = \frac{w_L^* \gamma (1+s)L^*}{(\beta-1)L^* + (1+\beta)\varepsilon_L^L \gamma M} > 0 \qquad (32a)$$

and

$$-\frac{\partial w_H^*}{\partial w_L^*}\frac{dw_L^*}{dM}(1+s)H^* = -\frac{\varepsilon_L^H}{\varepsilon_H^H}\frac{w_H^* \gamma (1+s)H^*}{(\beta-1)L^* + (1+\beta)\varepsilon_L^L \gamma M} < 0 \qquad (32b)$$

Incorporating (31a), (32a) and (32b) into the first-order condition (30) we can now re-express it in the following way (see Appendix C)

$$\pi_M = \frac{\gamma \varepsilon_L^L (1+s) w_L^*}{\varepsilon_H^H}\left(\frac{L^* + g\gamma M}{(\beta-1)L^* + (1+\beta)\varepsilon_L^L \gamma M}\right) - cM = 0 \qquad (33)$$

where $g = (1+\beta)\varepsilon_H^H - \varepsilon_H^L / \varepsilon_L^L > 0$.

Now we analyze the effects of wage tax, tax exemption, unemployment benefit as well as the effect of payroll tax on the optimal outsourcing. Using the notation $(\beta-1)L^* + (1+\beta)\varepsilon_L^L \gamma\, M = X$ the second-order condition is $\pi_{MM} = -\frac{\gamma^2 \varepsilon_L^L (1+s) w_L^*}{\varepsilon_H^H X^2}\left[L^*((1+\beta) - \frac{\varepsilon_L^H \varepsilon_H^L}{\varepsilon_L^L \varepsilon_H^H})) + \gamma M (g(2-\beta) + (1+\beta)\varepsilon_L^L\right] - c < 0$. Using $c = \frac{\gamma \varepsilon_L^L (1+s) w_L^*}{\varepsilon_H^H M}\left(\frac{L^* + g\gamma M}{X}\right)$ from (33) the second-order condition can be written as

[9] This lies in conformity with empirics, see e.g. Braun and Scheffel (2007), Egger and Egger (2006), Feenstra and Hanson (1999, 2001), Hijzen, Görg and Hine (2005), Geishecker and Görg (2008), Hijzen (2007), Munch and Skaksen (2005) and Yan (2006).

$$\pi_{MM} = -\frac{\gamma^2 \varepsilon_L^L (1+s) w_L^*}{\varepsilon_H^H X^2}\left[L^*((1+\beta) - \frac{\varepsilon_L^H}{\varepsilon_H^H}\frac{\varepsilon_H^L}{\varepsilon_L^L})) + \gamma\, M(g(2-\beta) + (1+\beta)\varepsilon_L^L) \right] -$$
$$\frac{\gamma \varepsilon_L^L (1+s) w_L^*}{\varepsilon_H^H M X^2}\left[(L^* + g\gamma\, M) X \right] = -\frac{\gamma^2 \varepsilon_L^L (1+s) w_L^*}{\varepsilon_H^H M X^2} Z < 0 \tag{34}$$

where $Z = L^*((1+\beta - \frac{\varepsilon_L^H}{\varepsilon_H^H}\frac{\varepsilon_H^L}{\varepsilon_L^L})) + \gamma\, M(g(2-\beta) + (1+\beta)\varepsilon_L^L) + \frac{(L^* + g\gamma M)X}{\gamma M} > 0$.

In terms of the wage tax rate t_L the first-order condition (33) changes as

$$\pi_{Mt_L} = \frac{\gamma \varepsilon_L^L}{\varepsilon_H^H}(1+s)\frac{dw_L^*}{dt_L}\left(\frac{L^* + g\gamma M}{X} \right) +$$
$$\frac{\gamma \varepsilon_L^L}{\varepsilon_H^H}(1+s) w_L^* \left(\frac{X - (L^* + g\gamma M)(\beta - 1)}{X^2} \right) L^*_{w_L^*} \frac{dw_L^*}{dt_L} \tag{35}$$

Using $L^*_{w_L^*} = -\frac{\varepsilon_L^L (L^* + \gamma M)}{w_L^*}$, (35) can be written after the calculations as (Appendix C)

$$\pi_{Mt_L} = \frac{\gamma \varepsilon_L^L (1+s)}{\varepsilon_H^H X^2} \underbrace{\frac{dw_L^*}{dt_L}}_{+} T > 0 \quad as \quad b_L - e > 0 \tag{36}$$

where $T = X(L^* + g\gamma M) - \left(X - (L^* + g\gamma\, M)(\beta - 1) \right) \varepsilon_L^L (L^* + \gamma\, M) > 0$. Using (34) and (36) we have

$$\frac{\partial M}{\partial t_L} = -\frac{\pi_{Mt_L}}{\pi_{MM}} = \underbrace{\frac{dw_L^*}{dt_L}}_{+} \frac{M}{w_L^* \gamma} \frac{T}{Z} > 0 \text{ as } b_L - e > 0 \tag{37}$$

In terms of tax exemption and higher unemployment benefit we will get in the similar way the following results: By using the cross-derivative for tax exemption $\pi_{Me} = \frac{\gamma \varepsilon_L^L (1+s)}{\varepsilon_H^H X^2} \underbrace{\frac{dw_L^*}{de}}_{-} T < 0$ gives

$$\frac{\partial M}{\partial e} = -\frac{\pi_{Me}}{\pi_{MM}} = \underbrace{\frac{dw_L^*}{de}}_{-} \frac{M}{w_L^* \gamma} \frac{T}{Z} < 0 \tag{38}$$

and using the cross-derivative $\pi_{Mb_L} = \frac{\gamma \varepsilon_L^L (1+s)}{\varepsilon_H^H X^2} \underbrace{\frac{dw_L^*}{db_L}}_{+} T > 0$ gives

$$\frac{\partial M}{\partial b_L} = -\frac{\pi_{Mb_L}}{\pi_{MM}} = \underbrace{\frac{dw_L^*}{db_L}}_{+} \frac{M}{w_L^* \gamma} \frac{T}{Z} > 0 \tag{39}$$

(see equations (24a-24c) concerning $\frac{dw_L^*}{dt_L}$, $\frac{dw_L^*}{de}$ and $\frac{dw_L^*}{db_L}$). Therefore, both higher domestic low-skilled wage tax and higher unemployment benefit increase optimal outsourcing, while higher tax exemption, ceteris paribus, decreases optimal outsourcing, when we have also allowed the effects of these policy parameters via the wage of the high-skilled workers.

In terms of payroll tax rate s the first-order condition (33) will change in the more complicated way via three different aspects in the following way

$$\begin{aligned}\pi_{Ms} = &\frac{\gamma \varepsilon_L^L}{\varepsilon_H^H} (w_L^* + (1+s)\frac{dw_L^*}{ds}) \left(\frac{L^* + g\gamma M}{X} \right) + \\ &\frac{\gamma \varepsilon_L^L (1+s) w_L^*}{\varepsilon_H^H} \frac{\left[X - (L^* + g\gamma M)(\beta - 1) \right]}{X^2} \left(L_{w_L^*}^* \frac{dw_L^*}{ds} + L_s^* \right)\end{aligned} \tag{40}$$

which can be expressed as follows (see Appendix C)

$$\pi_{Ms} = \underbrace{\frac{\gamma \varepsilon_L^L w_L^*}{\varepsilon_H^H X^2}}_{+} \left(\left[(L^* + g\gamma M)(X - \varepsilon\gamma M) \right] - \left[1 - \frac{(L^* + g\gamma M)(\beta - 1)}{X} \right] \left[\varepsilon (L^* + \gamma M)(X - \varepsilon_L^L \gamma M) \right] \right)$$

(40’)

By using the cross-derivative π_{Ms} gives

$$\frac{\partial M}{\partial s} = -\frac{\pi_{Ms}}{\pi_{MM}} = \frac{M}{(1+s)\gamma} \frac{V}{Z} \tag{41}$$

This is positive, if $V = \left[(L^* + g\gamma M)(X - \varepsilon\gamma M)\right] > \left[1 - \frac{(L^* + g\gamma M)(\beta - 1)}{X}\right]\left[\varepsilon(L^* + \gamma M)(X - \varepsilon_L^L \gamma M)\right]$, where

$\left[1 - \frac{(L^* + g\gamma M)(\beta - 1)}{X}\right] = \left[\frac{\gamma M(1+\beta)\varepsilon_L^L - (\beta - 1)g}{X}\right] < 1$ because $(1+\beta)\varepsilon_L^L > (\beta - 1)g$.

Under reasonable assumptions this is the case. In this case higher payroll tax increases optimal outsourcing.

We can now summarize our findings in terms of optimal outsourcing as follows.

Proposition 4 *Optimal committed outsourcing will affect by the policy parameters as follows*

(a) the higher domestic low-skilled wage tax and a higher unemployment benefit increases optimal outsourcing, while a higher tax exemption, ceteris paribus, decreases optimal outsourcing, whereas

(b) the higher payroll tax for the firms under reasonable assumptions increases optimal outsourcing.

Proposition 4 reports that in the presence of outsourcing, higher marginal tax on the low-skilled workers tends to increase optimal outsourcing. The same holds for a higher unemployment benefit, while higher tax exemption on the low-skilled labour decreases outsourcing. The intuition for these results is the following: In the absence of a change in outsourcing, higher marginal tax rate, higher unemployment benefit, or lower tax exemption would each encourage the labor union to increase its wage demand. This would, in turn, increase the optimal level of outsourcing. Anticipating the policy response by the labour unions, firms increase the amount of outsourcing**.** If payroll tax changes, it is reasonably the optimal response of the labour union.

VI. Determinants of Equilibrium Unemployment by Low-Skilled Workers

VI.1. Outsourcing and Equilibrium Unemployment

We now move on to explore the determinants of equilibrium unemployment of low-skilled workers in dual labour markets, when there is both unionized and competitive determination of wages in the home country. First we analyze the effect of outsourcing given labour tax parameters and second we study the effects of labour taxation parameters on equilibrium unemployment both via wage and outsourcing changes. According to (19) the wage formation for low-skilled workers in industry i is of the form $w_L^* = A\,\hat{b}_L$, where $\hat{b}_L = \frac{b_L - t_L e}{1 - t_L}$ is in the presence of linearly progressive wage taxation and the mark-up factor is $A = \frac{\beta(L^* + \gamma M)}{(\beta - 1)L^* + \beta\gamma M} > 1$. This mark-up factor is, in principle, industry-specific. In a general equilibrium the term b_L should be re-interpreted as the endogenous outside option, which we specify in a conventional way as

$$b_L = (1 - u_L)w_L + u_L \bar{b}_L \tag{42}$$

where u_L is the unemployment rate, $\bar{b}_L$ captures the unemployment benefit and w_L denotes the wage formation in all identical industries (see e.g. Nickell and Layard (1999), p. 3048-3049 for a further discussion). Assuming a constant benefit-replacement ratio $0 < q = \bar{b}_L / w_L^* < 1$ so that by using (42) we have $\hat{b}_L = \frac{b_L - t_L e}{1 - t_L} = \frac{w_L^* + u_L(q-1)w_L^* - t_L e}{1 - t_L}$ and $w_L^* = A\,\hat{b}_L$ can be written as $(1 - t_L)w_L^* = A(w_L^* - t_L e) + Au_L(q-1)w_L^*$ and in this case the equilibrium low-skilled unemployment can be presented

$$u_L = \frac{1}{(1-q)}\left(\frac{A-1+t_L(1-\frac{Ae}{w_L^*})}{A}\right) = \frac{1}{(1-q)}G \qquad (43)$$

where $G \equiv \left(1-\frac{1}{A}\frac{b_L(1-t_L)}{(b_L-t_Le)}\right)$. According to (43) in the presence of a constant benefit-replacement ratio $q=\bar{b}_L/w_L^*$ the impact of outsourcing on equilibrium unemployment under both progressive wage labour taxation and proportional payroll labour taxation comes via the mark-up $A=\frac{\beta(L^*+\gamma M)}{(\beta-1)L^*+\beta\gamma M}$ in (43). In the presence of outsourcing the mark-up depends positively on low-skilled labour demand, i.e. $A_L=\frac{\beta\gamma M}{\left[(\beta-1)L^*+\beta\gamma M\right]^2}>0$, and negatively on outsourcing, i.e. $A_M=-\frac{\gamma A}{\left[(\beta-1)L^*+\beta\gamma M\right]}<0$.

In terms of outsourcing we have

$$\frac{dG}{dM}=\underbrace{\frac{\frac{dA}{dM}}{A^2}}_{-}\underbrace{\left(\frac{b_L(1-t_L)}{(b_L-t_Le)}\right)}_{+}<0 \qquad (44)$$

where outsourcing will have both the direct negative effect and the indirect positive effect via the wage on the mark-up, but the direct effect dominates as $\frac{dA}{dM}=A_M+A_{w_L^*}\frac{dw_L^*}{dM}=-\frac{\gamma A}{\left[(\beta-1)L^*+\beta\gamma M\right]}\left(1-\frac{\varepsilon_L^L\gamma M}{(\beta-1)L^*+(1+\beta)\varepsilon_L^L\gamma M}\right)<0$ by using $A_M=-\frac{\gamma A}{\left[(\beta-1)L^*+\beta\gamma M\right]}<0$, $A_{w_L^*}=-\frac{\gamma\,\varepsilon_L^L\gamma MA}{w_L^*\left[(\beta-1)L^*+\beta\gamma M)\right]}<0$ and $\frac{dw_L^*}{dM}=-\frac{w_L^*\gamma}{(\beta-1)L^*+(1+\beta)\varepsilon_L^L\gamma M}<0$ (see (21)). Therefore by combining (44) and (43) gives $\frac{du_L}{dM}=\frac{1}{1-q}\frac{dG}{dM}<0$ so that higher outsourcing also in the presence of progressive wage taxation and proportional payroll taxation will decrease equilibrium unemployment when the benefit-replacement ratio is fixed and less than one.

We can now summarize this finding as.

> **Proposition 5:** *A production mode with a higher amount of outsourced production, ceteris paribus, will reduce equilibrium unemployment of low-skilled workers both in the presence and in the absence of progressive wage taxation and proportional payroll taxation.*

Proposition 5 reports very importantly the negative relationship between outsourcing and equilibrium unemployment of low-skilled workers, i.e. only concerning the relationship between higher wage elasticity of low-skilled labour demand and outsourcing, which leads to wage moderation of low-skilled workers and thereby smaller unemployment. Of course if there would be wage rigidity, then higher outsourcing would increase unemployment due to a decrease in domestic low-skilled labour demand.

VI.2. Labour Tax Instruments and Equilibrium Unemployment

Next we analyze the effect of labour tax parameters on equilibrium unemployment of low-skilled workers. According to Proposition 4 higher domestic low-skilled wage tax and lower wage tax exemption increases optimal outsourcing, and
low-skilled wage tax and lower wage tax exemption increases optimal outsourcing, and
higher payroll tax also affects outsourcing positively under reasonable assumptions.

Concerning labour tax parameters following the time sequence of decisions, presented in Figure 1, the total wage effects of tax policy instruments consists both of the direct effects and of the indirect effects via the impact these instruments have on the strategic outsourcing decision of firms and thereby also on the wage rate. The total effect of the wage tax is $\frac{dw_L^*}{dt_L} = \underbrace{\frac{\partial w_L^*}{\partial t_L}}_{+} + \underbrace{\frac{\partial w_L^*}{\partial M}\frac{\partial M}{\partial t_L}}_{-}$. The direct wage effect is positive, and the indirect effect via outsourcing is negative, because the wage tax makes outsourcing more attractive which lowers the benefit of the wage increase for the monopoly labour union. By using equations (21), (24a) and (37) we can rewrite it as follows

$$\frac{dw_L^*}{dt_L} = \frac{\partial w_L^*}{\partial t_L} + \frac{\partial w_L^*}{\partial M}\frac{\partial M}{\partial t_L} = \frac{\partial w_L^*}{\partial t_L}\left[1 + \frac{\partial w_L^*}{\partial M}\frac{MT}{w_L^*\gamma Z}\right] = \underbrace{\frac{\partial w_L^*}{\partial t_L}}_{+}\left[1 - \frac{MT}{XZ}\right] \tag{45}$$

In what follows we assume that the direct effect dominates the indirect effect, which is a reasonable assumption. We make the same assumption also in the case of tax exemption e and payroll tax s.

By differentiating the mark-up of (43) in terms of wage tax t_L which gives via w_L^* and M

$$\frac{dG}{dt_L} = \underbrace{\frac{\frac{dA}{dt_L}}{A^2}\left(\frac{(b_L(1-t_L))}{(b_L - t_L e)}\right)}_{-} + \underbrace{\frac{b_L(b_L - e)}{A(b_L - t_L e)^2}}_{+} = ? \text{ as } b_L > e \tag{46}$$

where $\frac{dA}{dt_L} = \frac{\beta\gamma ML_{w_L^*}^* \frac{dw_L^*}{dt_L} - \beta\gamma(L^* + \gamma M)\frac{\partial M}{\partial t_L}}{\left[(\beta-1)L^* + \beta\gamma M\right]^2} = \frac{\beta\gamma\left(ML_{w_L^*}^* \frac{dw_L^*}{dt_L} - (L^* + \gamma M)\frac{\partial M}{dt_L}\right)}{\left[(\beta-1)L^* + \beta\gamma M\right]^2} < 0$ as $b_L - e > 0$ as $\frac{dw_L^*}{dt_L} = \underbrace{\frac{\partial w_L^*}{\partial t_L}}_{+} + \underbrace{\frac{\partial w_L^*}{\partial M}\frac{\partial M}{\partial t_L}}_{-} = \frac{\partial w_L^*}{\partial t_L}\left[1 - \frac{MT}{XZ}\right] > 0$. Therefore by combining (46) and (43) gives $\frac{du_L}{dt_L} = \frac{1}{1-q}\frac{dG}{dt_L} = ?$ so that higher wage tax in the presence of outsourcing will have an ambiguous effect on equilibrium unemployment when the benefit-replacement ratio is fixed and less than one. This is because the total effect of higher wage tax on wage of low-skilled workers is negative and thereby increases the wage elasticity and lowers the mark-up because of lower labour demand and higher outsourcing. But there is also the positive direct effect of wage tax on G due to $\frac{d}{dt_L}\left(\frac{b_L(1-t_L)}{b_L-t_Le}\right)<0$. In terms of tax exemption e we have

$$\frac{dG}{de}=\underbrace{\frac{\frac{dA}{de}}{A^2}\left(\frac{(b_L(1-t_L)}{(b_L-t_Le)}\right)}_{+}\underbrace{-\frac{t_L(b_L(1-t_L))}{A(b_L-t_Le)^2}}_{-}=? \tag{47}$$

where $\frac{dA}{de}=\frac{\beta\gamma\left(ML^*_{w^*_L}\frac{dw^*_L}{de}-(L^*+\gamma M)\frac{\partial M}{\partial e}\right)}{\left[(\beta-1)L^*+\beta\gamma M\right]^2}>0$ as $\frac{dw^*_L}{de}=\underbrace{\frac{\partial w^*_L}{\partial e}}_{-}+\underbrace{\frac{\partial w^*_L}{\partial M}\frac{\partial M}{\partial e}}_{+}=\frac{\partial w^*_L}{\partial e}\left[1-\frac{MT}{XZ}\right]<0$. Therefore by combining (47) and (43) gives $\frac{du_L}{de}=\frac{1}{1-q}\frac{dG}{de}=?$ so that higher tax exemption in the presence of outsourcing will also have an ambiguous effect on equilibrium unemployment. This is because the total effect of higher tax exemption on wage of low-skilled workers is positive and thereby decreases the wage elasticity and raises mark-up because of higher labour demand and lower outsourcing. But there is also the negative effect of tax exemption on G due to $\frac{d}{de}\left(\frac{(b_L(1-t_L)}{b_L-t_Le}\right)>0$.

Finally, by differentiating the mark-up of (43) in terms of payroll tax s gives

$$\frac{dG}{ds}=\frac{\frac{dA}{ds}}{A^2}\left(\frac{b_L(1-t_L)}{(b_L-t_Le)}\right)<0 \tag{48}$$

where $\frac{dA}{ds}=\frac{\beta\gamma\left(M(L^*_{w^*_L}\frac{dw^*_L}{ds}+L^*_s)-(L^*+\gamma M)\frac{\partial M}{\partial s}\right)}{\left[(\beta-1)L^*+\beta\gamma M\right]^2}<0$ and $\frac{dw^*_L}{ds}=\underbrace{\frac{\partial w^*_L}{\partial s}}_{-}+\underbrace{\frac{\partial w^*_L}{\partial M}\frac{\partial M}{\partial s}}_{-}<0$

(see Appendix C). Therefore by combining (48) and (43) gives $\frac{du_L}{ds} = \frac{1}{1-q}\frac{dG}{ds} < 0$ so that higher payroll tax in the presence of outsourcing will decrease equilibrium unemployment because it will decrease the wage of low-skilled workers both directly and indirectly via higher outsourcing and thereby it increases the wage elasticity and decreases the mark-up via $\frac{L^*}{\beta(L^* + \gamma M)}$.

We can now summarize our findings in terms of the effect of tax parameters as follows.

Proposition 6: *In the presence of outsourcing when the benefit-replacement ratio is fixed and less than one and concerning the assumption that the direct effects of tax parameters on wage formation dominate the indirect effect via outsourcing*

(a) the higher wage tax and the higher tax exemption will have an ambiguous effect on equilibrium unemployment, while

(b) the higher payroll tax will decrease equilibrium unemployment because it will decrease the wage of low-skilled workers and increase the wage elasticity and thereby decreases the mark-up.

Ambiguity associated with workers' taxation parameters is due to the facts that the total effect of higher wage tax (tax exemption) on wage of low-skilled workers is negative (positive) so that wage elasticity increases (decreases) and the mark-up lowers (raises), but there is also the positive (negative) direct effects of parameters. In the absence of outsourcing we $\left.\frac{dG}{dt_L}\right|_{M=0} = \underbrace{\frac{(\beta-1)b_L(b_L-e)}{\beta(b_L-t_Le)^2}}_{-} > 0$, $\left.\frac{dG}{de}\right|_{M=0} = \underbrace{-\frac{(\beta-1)t_Lb_L(1-t_L)}{\beta(b_L-t_Le)^2}}_{+} < 0$, and $\left.\frac{dG}{ds}\right|_{M=0} = 0$. Therefore, the effects tax parameters are different in the presence of absence of outsourcing, i.e. we have

Corollary: *In the absence of outsourcing*

(a) the higher wage tax and the higher tax exemption will have a positive effect on equilibrium unemployment, while

(b) the higher payroll tax will have no effect.

VI.3. Higher Degree of Tax Progression and the Low-Skilled Labour Demand

Finally, we analyze the effect of wage tax progression on wage formation by the low-skilled workers and labour demand. We assume that the tax reform will keep the relative tax burden per low-skilled worker constant, i.e. this means

$$t_L - \frac{t_L e}{w_L} = R \qquad (49)$$

The government can raise the degree of tax progression when it increases t_L and e such that $dR = 0$. Formally we have by using equations (37) and (38)

$$\left.\frac{de}{dt_L}\right|_{dR=0} = \frac{\left(w_L^* - e + \frac{t_L e}{w_L}\frac{dw_L^*}{dt_L}\right)}{\left(t_L - \frac{t_L e}{w_L}\frac{dw_L^*}{de}\right)} = \frac{\left(w_L^* - e + \frac{t_L e}{w_L}\frac{\partial w_L^*}{\partial t_L}(1-B)\right)}{\left(t_L - \frac{t_L e}{w_L}\frac{\partial w_L^*}{\partial e}(1-B)\right)} > 0 \qquad (50)$$

where $B = \frac{MT}{XZ} < 1$. Concerning the low-skilled wage effect of this tax reform we have $dw_L^* = (1-B)\left[\frac{\partial w_L^*}{\partial t_L}dt_L + \frac{\partial w_L^*}{\partial e}de\right]$ and dividing by dt_L and substituting the RHS of (49) for de/dt_L gives (see Appendix D)

$$\left.\frac{dw_L^*}{dt_L}\right|_{dR=0} = \frac{(1-B)\left[\frac{\partial w_L^*}{\partial t_L} + \frac{(w_L^* - e)}{t_L}\frac{\partial w_L^*}{\partial e}\right]}{\left[1 - \frac{e}{w_L^*}\frac{\partial w_L^*}{\partial e}(1-B)\right]} < 0 \qquad (51)$$

so that a higher degree of tax progression, keeping the relative tax burden per worker constant, will decrease the low-skilled wage rate both in the presence and absence of outsourcing (when $B=0$). Finally, we characterize the low-skilled employment effect of this tax reform. By raising tax progression according to (50) we have $dL^* = L^*_{w_L^*}(1-B)\left[\frac{\partial w_L^*}{\partial t_L}dt_L + \frac{\partial w_L^*}{\partial e}de\right] - \gamma\left[M_{t_L}dt_L + M_e de\right]$ so that the first term indicates the effect on the wage rate on los-skilled labour demand and the second term indicates the induced outsourcing. Dividing this by dt_L and substituting the RHS of (50) for de/dt_L gives after calculations (see Appendix D)

$$\left.\frac{dL^*}{dt_L}\right|_{dR=0} = \underbrace{L^*_{w_L^*}\left.\frac{dw_L^*}{dt_L}\right|_{dR=0}}_{+} - \gamma\underbrace{\left.\frac{dM}{dt_L}\right|_{dR=0}}_{-} > 0 \qquad (52)$$

so that a higher degree of tax progression, keeping the relative tax burden per worker constant, will increase the low-skilled labour demand both in the presence and absence of outsourcing (when the second term is zero).

We can now summarize our findings as follows.

Proposition 7: *In the presence of outsourcing raising the wage tax and the tax exemption to keep the relative tax burden per worker constant, this higher degree of tax progression will decrease the wage rate and labour demand of low-skilled workers both in the presence and absence of outsourcing.*

VII. Conclusions

We have studied some new issues in the presence of international outsourcing and heterogeneous workers in the dual domestic labour markets when there is both unionized determination of wages of low-skilled workers and competitive determination of wages of high-skilled workers in the home country.

We have shown that the own wage elasticity, the cross wage elasticity and the outsourcing elasticity for the low-skilled labour demand depend positively on the amount of outsourcing, and it also depends positively on the payroll tax, whereas the own wage elasticity, the cross wage elasticity and the outsourcing elasticity for the high-skilled labour demand are independent of the amount of outsourcing and the payroll tax. In the presence of outsourcing the high-skilled wage formation by the monopoly labour union depends negatively on the low-skilled wage and the payroll tax, whereas the high-skilled wage is independent of the high-skilled wage tax parameters. In terms of low-skilled wage determination in the presence of outsourcing a higher share of outsourced production and a higher productivity of outsourced production will decrease the wage for the low-skilled labour and increase the wage for the high-skilled labour, thereby inducing higher wage dispersion, and a higher low-skilled wage tax will increase the wage for the low-skilled labour and a higher low-skilled wage tax exemption firms will decrease the wage for the low-skilled labour and increase the wage for the high-skilled labour, while a higher payroll tax for the firms will decrease the wage for the low-skilled labour and also under reasonable assumptions decrease the wage for the high-skilled labour.

In terms of optimal committed outsourcing it will affect by the policy parameters as follows: a higher domestic low-skilled wage tax and a higher unemployment benefit increases optimal outsourcing, while a higher tax exemption, ceteris paribus, decreases optimal outsourcing, whereas a higher payroll tax for the firms will have an ambiguous effect on optimal outsourcing.

Finally, in terms of the effects of outsourcing and some policy variables on equilibrium unemployment of low-skilled workers under alternative unemployment benefit specifications we have the following results: A production mode with a higher amount of outsourced production will reduce equilibrium unemployment of low-skilled workers both in the absence and presence of progressive wage taxation and proportional payroll taxation. In the presence of outsourcing concerning the assumption that the direct effect of tax parameters on wage formation dominates the indirect effect via outsourcing the higher wage tax and the higher tax exemption will have an ambiguous effect on equilibrium unemployment while the higher payroll tax will decrease equilibrium unemployment when the benefit-replacement ratio is fixed and less than one. In the absence of outsourcing the higher wage tax and the higher tax exemption will have a positive effect on equilibrium unemployment, while the higher payroll tax will have no effect. In the presence of outsourcing raising the wage tax and the tax exemption while keeping the relative tax burden per worker constant, this higher degree of tax progression will decrease the wage rate and labour demand of low-skilled workers. This result is qualitatively similar in the absence of outsourcing.

There are several new research topics associated with these issues. We have focused on the case where firms decide outsourcing before wage formation. But sometimes firms may be flexible to decide outsourcing activity after wage is set by the labour union. Other important issue is to study empirically the implications of labour taxation and labour tax reforms on optimal outsourcing. Finally, it is also important to do numerical simulations by checking the precise role of size of various parameters.

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Appendix A: Optimal Low-Skilled Labour Demand

Substituting the RHS of (4) for H into (3b) gives

$$\rho\left\{(\frac{w_L}{w_H})^a(\frac{a}{1-a})^a(L+\gamma M)^a(L+\gamma M)^{1-a}\right\}^{\rho-1}(1-a)(\frac{w_L}{w_H})^a(\frac{a}{1-a})^a(L+\gamma M)^a(L+\gamma M)^{-a} = \tilde{w}_L \tag{A1}$$

so that

$$\rho\left\{(\frac{w_L}{w_H})^a(\frac{a}{1-a})^a(L+\gamma\, M)\right\}^{\rho-1}(1-a)(\frac{w_L}{w_H})^a(\frac{a}{1-a})^a=\tilde{w}_L\,, \tag{A2}$$

which is equivalent to

$$(L+\gamma\, M)^{\rho-1}(\frac{w_L}{w_H})^{a\rho}(1-a)(\frac{a}{1-a})^{a\rho}=\rho^{-1}\tilde{w}_L\ . \tag{A3}$$

(A3) in its turn gives (5). QED.

Appendix B: Optimal Wage Setting under Progressive Wage Taxation and Proportional Payroll Taxation

The first-order condition associated with $\underbrace{\max}_{(w_L)}\ V=((1-t_L)w_L+t_Le-b_L)L$ s.t.

$\pi_L=0$ and $H^*=H^s$ can be written as follows

$$V_{w_L}=(1-t_L)w_L(1-(\hat{\varepsilon}_L^L+\hat{\varepsilon}_H^L\frac{dw_H}{dw_L}\frac{w_L}{w_H}))+(b_L-t_Le)(\hat{\varepsilon}_L^L+\hat{\varepsilon}_H^L\frac{dw_H}{dw_L}\frac{w_L}{w_H})$$

$$=(1-t_L)w_L(1-(\varepsilon_L^L+\varepsilon_H^L\frac{dw_H}{dw_L}\frac{w_L}{w_H})(1+\gamma\frac{M}{L^*}))+(b_L-t_Le)(\varepsilon_L^L+\varepsilon_H^L\frac{dw_H}{dw_L}\frac{w_L}{w_H})(1+\gamma\frac{M}{L^*})=0 \tag{B1}$$

where the own wage elasticity of labour demand is $\hat{\varepsilon}_L^L=\varepsilon_L^L\left(1+\gamma\frac{M}{L^*}\right)$ and the cross wage elasticity is $\hat{\varepsilon}_H^L=\varepsilon_H^L\left(1+\gamma\frac{M}{L^*}\right)$ and the labour demand under proportional payroll taxation is the following one $L^*=m\,w_L^{-\varepsilon_L^L}w_H^{-\varepsilon_H^L}(1+s)^{-\varepsilon}-\gamma\,M=L(\underbrace{w_L}_{-},\underbrace{w_H}_{-},\underbrace{M}_{-},\underbrace{\gamma}_{-},\underbrace{s}_{-})$**.** In the case of the Cobb-Douglas utility function we have

$$w_H=\left[\frac{\mu(1-a)}{ma}\right]^{-\frac{1}{\varepsilon_H^H}}w_L^{-\frac{\varepsilon_L^H}{\varepsilon_H^H}}(1+s)^{-\frac{\varepsilon}{\varepsilon_H^H}} \tag{B2}$$

so that $$\frac{dw_H}{dw_L}=-\frac{\varepsilon_L^H}{\varepsilon_H^H}\left[\frac{\mu(1-a)}{ma}\right]^{-\frac{1}{\varepsilon_H^H}}w_L^{-\frac{\varepsilon_L^H}{\varepsilon_H^H}-1}(1+s)^{-\frac{\varepsilon}{\varepsilon_H^H}}=-\frac{\varepsilon_L^H}{\varepsilon_H^H}\frac{w_H}{w_L}<0 \tag{B3}$$

Using (B2) and (B3) gives $\frac{dw_H}{dw_L}\frac{w_L}{w_H}=-\frac{\varepsilon_L^H}{\varepsilon_H^H}=-\frac{\rho(1-a)}{1-\rho(1-a)}<0$, which implies the equation (19) because

$$\varepsilon_L^L-\varepsilon_H^L\frac{\varepsilon_L^H}{\varepsilon_H^H}=\frac{\varepsilon_L^L\varepsilon_H^H-\varepsilon_H^L\varepsilon_L^H}{\varepsilon_H^H}=\frac{(1-\rho a)(1-\rho(1-a))-\rho a\rho(1-a)}{(1-\rho(1-a))(1-\rho)}=\frac{1}{1-\rho(1-a)}=\beta>1\,\text{QED}$$

Differentiating (19) in terms of low-skilled wage and wage tax rate gives

$$\left(1-\frac{\left[(\bar{\varepsilon}_L^L-1)\frac{\partial\bar{\varepsilon}_L^L}{\partial w_L}-\bar{\varepsilon}_L^L\frac{\partial\bar{\varepsilon}_L^L}{\partial w_L}\right]}{(\bar{\varepsilon}_L^L-1)^2}\hat{b}_L\right)dw_L^*=\frac{\bar{\varepsilon}_L^L}{(\bar{\varepsilon}_L^L-1)}\frac{b_L-e}{(1-t_L)^2}dt_L \qquad \text{(B4)}$$

and using $\hat{b}_L=\frac{w_L(\bar{\varepsilon}_L^L-1)}{\bar{\varepsilon}_L^L}$ (B4) can be expressed as

$$\left(1+\frac{\frac{\partial\bar{\varepsilon}_L^L}{\partial w_L}\frac{w_L}{\bar{\varepsilon}_L^L}}{(\bar{\varepsilon}_L^L-1)}\right)dw_L^*=\frac{\bar{\varepsilon}_L^L}{(\bar{\varepsilon}_L^L-1)}\frac{b_L-e}{(1-t_L)^2}dt_L \qquad \text{(B5)}$$

which gives (24a). Equations (24b) and (24c) can be derived in the similar way. QED.

Appendix C: Optimal Committed Outsourcing Before Wage Formation and Domestic Labour Demand

By using (31a), (32a) and (32b) the first-order condition (30) can be re-expressed as

$$\begin{aligned}\pi_M &= \gamma(1+s)w_L^*+\frac{\gamma(1+s)w_L^*L^*}{\left[(\beta-1)L^*+(1+\beta)\varepsilon_L^L\gamma M\right]}-\frac{\varepsilon_L^H}{\varepsilon_H^H}\frac{\gamma(1+s)w_H^*H^*}{\left[(\beta-1)L^*+(1+\beta)\varepsilon_L^L\gamma M\right]}-cM\\ &=\gamma(1+s)w_L^*\left(\frac{\beta L^*+(1+\beta)\varepsilon_L^L\gamma M}{(\beta-1)L^*+(1+\beta)\varepsilon_L^L\gamma M}\right)-\\ &\quad \gamma(1+s)w_H^*\frac{\varepsilon_L^H}{\varepsilon_H^H}\left(\frac{H^*}{(\beta-1)L^*+(1+\beta)\varepsilon_L^L\gamma M}\right)-cM \qquad \text{(C1)}\\ &=\left(\frac{\gamma(1+s)}{(\beta-1)L^*+(1+\beta)\gamma\varepsilon_L^L M}\right)\left(w_L^*(\beta L^*+(1+\beta)\varepsilon_L^L\gamma M)-w_H^*\frac{\varepsilon_L^H}{\varepsilon_H^H}H^*\right)-cM=0\end{aligned}$$

Using equation (4) we have $-w_H^*\frac{\varepsilon_L^H}{\varepsilon_H^H}H^*=-w_L^*\frac{\varepsilon_L^H}{\varepsilon_H^H}\frac{a}{1-a}(L^*+\gamma M)$ so that

$$w_L^*(\beta L^*+(1+\beta)\varepsilon_L^L\gamma M)-w_H^*\frac{\varepsilon_L^H}{\varepsilon_H^H}H^*=w_L^*\left[L^*(\beta-\frac{\varepsilon_L^H a}{\varepsilon_H^H(1-a)})+\gamma M((1+\beta)\varepsilon_L^L-\frac{\varepsilon_L^H a}{\varepsilon_H^H(1-a)})\right] \qquad \text{(C2)}$$

where $$\beta-\frac{\varepsilon_L^H a}{\varepsilon_H^H(1-a)}=\frac{1-\rho}{1-\rho(1-a)}=\frac{\varepsilon_L^L}{\varepsilon_H^H}>0 \qquad \text{(C3a)}$$

and $(1+\beta)\varepsilon_L^L-\frac{\varepsilon_L^H a}{\varepsilon_H^H(1-a)}=\frac{1}{\varepsilon_H^H}\left[(1+\beta)\varepsilon_L^L\varepsilon_H^H-\varepsilon_L^H\frac{a}{1-a}\right]=\frac{1}{\varepsilon_H^H}\left[(1+\beta)\varepsilon_L^L\varepsilon_H^H-\varepsilon_H^L\right]>0$.

(C3b)

Using (C3a) and (C3b) makes it possible to rewrite (C1) as equation (33). Concerning equation (36) one term in its numerator can be expressed as follows

$$\mathrm{T} = X(L^* + g\gamma M) - (X - (L^* + g\gamma M)(\beta - 1))\varepsilon_L^L (L^* + \gamma M) =$$
$$X(L^* + g\gamma M) - \gamma M\left[1 + \beta + \frac{\varepsilon_L^H \varepsilon_H^L}{\varepsilon_H^H}\right]\varepsilon_L^L (L^* + \gamma M) \quad \text{(C4)}$$

By using $X = (\beta - 1)L^* + (1+\beta)\varepsilon_L^L \gamma M$ this can be written as

$$T = (\beta - 1)L^{*2} + \left[(\beta - 1)g - \frac{\varepsilon_H^L \varepsilon_L^H}{\varepsilon_H^H}\right]\gamma M L^* + \left[(1+\beta)(\varepsilon_L^L g - 1) - \frac{\varepsilon_H^L \varepsilon_L^H}{\varepsilon_H^H}\right](\gamma M)^2 > 0 \quad \text{(C5)}$$

so that $\pi_{Mt_L} > 0$ as $b_L - e > 0$.

Concerning the payroll tax by using equation (26) we can rewrite one term in (40) as follows

$$\frac{\gamma \varepsilon_L^L}{\varepsilon_H^H}(w_L^* + (1+s)\frac{dw_L^*}{ds}) = \frac{\gamma \varepsilon_L^L w_L^*}{\varepsilon_H^H}\frac{[X - \varepsilon \gamma M]}{X} > 0 \quad \text{(C6)}$$

Concerning (40) by using $L^*_{w_L^*} = -\frac{\varepsilon_L^L (L^* + \gamma M)}{w_L^*}$, $L_s^* = -\frac{\varepsilon (L^* + \gamma M)}{1+s}$ and (26) gives

$$L^*_{w_L^*}\frac{\partial w_L^*}{\partial s} + L_s^* = -\frac{\varepsilon (L^* + \gamma M)}{1+s}(\frac{X - \varepsilon_L^L \gamma M}{X}) = -\frac{\varepsilon (L^* + \gamma M)\left[(\beta - 1)L^* + \beta \varepsilon_L^L \gamma M\right]}{(1+s)X} =$$
$$-\frac{\varepsilon (L^* + \gamma M)\left[X - \varepsilon_L^L \gamma M\right]}{(1+s)X} < 0. \quad \text{(C7)}$$

which gives (40'). Concerning (48) using this gives $M(L^*_{w_L^*}\frac{\partial w_L^*}{\partial s} + L_s^*) < 0$ and

$$ML^*_{w_L^*}\frac{\partial w_L^*}{\partial M}\frac{\partial M}{\partial s} - (L^* + \gamma M)\frac{\partial M}{\partial s} = \frac{\partial M}{\partial s}(L^* + \gamma M)\left(\frac{\varepsilon_L^L \gamma M}{X} - 1\right) < 0 \quad \text{(C8)}$$

This gives (48). QED.

Appendix D: Tax Progression and Los-Skilled Labour Demand

Substituting the RHS of (50) for de/dt_L into $dw_L^* = (1-B)\left[\frac{\partial w_L^*}{\partial t_L}dt_L + \frac{\partial w_L^*}{\partial e}de\right]$ implies

$$\left.\frac{dw_L^*}{dt_L}\right|_{dR=0} = \frac{\left(\frac{\partial w_L^*}{\partial t_L}(1-B)t_L(1-\frac{e}{w_L^*}\frac{\partial w_L^*}{\partial e}(1-B)) + \frac{\partial w_L^*}{\partial e}(w_L^*-e)(1-B) + \frac{\partial w_L^*}{\partial e}(1-B)\frac{t_L e}{w_L^*}\frac{\partial w_L^*}{\partial t_L}(1-B)\right)}{t\left[1-\frac{e}{w_L^*}\frac{\partial w_L^*}{\partial e}(1-B)\right]} \tag{D1}$$

which gives (51), where the denominator is positive. Concerning $\frac{\partial w_L^*}{\partial t_L} + \frac{(w_L^*-e)}{t_L}\frac{\partial w_L^*}{\partial e}$ in (51) we obtain that it is negative, i.e.

$$\frac{\partial w_L^*}{\partial t_L} + \frac{(w_L^*-e)}{t_L}\frac{\partial w_L^*}{\partial e} = \frac{Y}{(1-t_L)^2}(b_L - w_L^*) < 0 \tag{D2}$$

where $Y = \frac{\beta(L^* + \gamma M)}{(\beta-1)L^* + (1+\beta)\varepsilon_L^L \gamma M} > 0$. The total labour demand effect $dL^* = L_{w_L^*}^*(1-B)\left[\frac{\partial w_L^*}{\partial t_L}dt_L + \frac{\partial w_L^*}{\partial e}de\right] - \gamma\left[M_{t_L}dt_L + M_e de\right]$ can be expressed as

$$\left.\frac{dL^*}{dt_L}\right|_{dR=0} = L_{w_L^*}^*(1-B)\left[\frac{\partial w_L^*}{\partial t_L} + \frac{\partial w_L^*}{\partial e}\left.\frac{de}{dt_L}\right|_{dR=0}\right] - \gamma\left[M_{t_L} + M_e\left.\frac{de}{dt_L}\right|_{dR=0}\right] \tag{D3}$$

where $\frac{\partial w_L^*}{\partial t_L} + \frac{\partial w_L^*}{\partial e}\left.\frac{de}{dt_L}\right|_{dR=0} = \left.\frac{dw_L^*}{dt_L}\right|_{dR=0} < 0$ (equation (51)) and by using $M_e = \frac{M_{t_L}}{\frac{\partial w_L^*}{\partial t_L}}\frac{\partial w_L^*}{\partial e}$ from (37) and (38) gives

$$\left.\frac{dM}{dt_L}\right|_{dR=0} = \left[M_{t_L} + M_e\left.\frac{de}{dt_L}\right|_{dR=0}\right] = M_{t_L} + M_e\frac{\left[\frac{w_L^*-e}{t_L} + \frac{e}{w_L^*}\frac{\partial w_L^*}{\partial t_L}(1-B)\right]}{\left[1-\frac{e}{w_L^*}\frac{\partial w_L^*}{\partial e}(1-B)\right]} = \tag{D4}$$

$$\frac{M_{t_L}}{\frac{\partial w_L^*}{\partial t_L}}\frac{\left[\frac{\partial w_L^*}{\partial t_L} + \frac{(w_L^*-e)}{t_L}\frac{\partial w_L^*}{\partial e}\right]}{\left[1-\frac{e}{w_L^*}\frac{\partial w_L^*}{\partial e}(1-B)\right]} < 0$$

which gives (52). QED.