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Abstract

We propose a new bivariate autoregressive probit model for binary time series. The model nests various special cases, such as two separate univariate models, for which a LM test against the unrestricted bivariate model is developed. The parameters of the model are estimated by the method of maximum likelihood and forecasts can be computed by using explicit formulae. The model is applied to predict the current state of the U.S. business cycle and growth rate cycle recessions. Evidence of predictability of both recession periods is obtained by using financial variables as predictors. The bivariate model is found to outperform the univariate models built separately for each cycle indicator.

JEL Classification: C32, C35, E32

Keywords: Bivariate probit model, Autoregressive model, Business cycle, Growth rate cycle, Recession

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1 Introduction

In the previous literature on time series models for binary dependent variables, the models have typically been univariate. Given the importance of vector autoregressive models for continuous dependent variables, it is of interest to study multivariate binary time series models, where the probabilities of different binary outcomes are modeled jointly.

In this paper, we present a bivariate autoregressive probit model as an extension to the univariate autoregressive probit model of Kauppi and Saikkonen (2008). The model can also be seen as an extension of the "static" bivariate probit model of Ashford and Sowden (1970), where the dependence between two binary time series is modeled by using a bivariate cumulative normal distribution function. In our bivariate model, the static model is extended by the inclusion of the autoregressive model structure.

In the previous literature, only few bivariate and multivariate models have been considered. Those models have mainly been based on the latent variable formulation, where the values of binary time series are realizations of corresponding continuous latent variables (see, e.g., Chib and Greenberg, 1998; Mosconi and Seri, 2006). In this paper, the latent variable approach is not used. An advantage of our model is that parameter estimation can conveniently be carried out by the method of maximum likelihood and forecasts can be computed using explicit formulae. This is not typically the case in dynamic models based on the latent variables, such as the dynamic univariate model by Chauvet and Potter (2005) and the qual VAR model of Dueker (2005). Our bivariate model is somewhat similar to the model proposed by Anatolyev (2009), but we model the dependence between the two binary time series in a different way.

As an empirical application, we consider several alternative specifications of the proposed bivariate autoregressive probit model to nowcast the current state of the U.S. economy. We measure the state of the economy in terms of recession periods defined by the business cycle and the growth rate cycle indicators. Predicting business cycle recession periods with univariate probit models has attracted considerable attention in the literature (see, e.g., Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998; Chauvet and Potter, 2005), where the growth rate cycle indicator has hardly been considered at all. Given the fact that some economic slowdown periods do not turn into business cycle recessions, it is also of interest to consider the binary growth rate cycle indicator. To the best of our knowledge, this type of bivariate framework of two cycle indicators has not been considered in the previous literature.

A growth rate cycle is defined in terms of periods of increasing and decreasing growth rate in economic activity (see details in Banerji and Hiris, 2001; Osborn, Sensier and van Dijk, 2004). While "classical" business cycle recession periods are associated with the level of economic activity, a growth rate recession may occur without a decline in the level of economic activity. Therefore, growth rate cycle recessions are more numerous than classical business cycle recessions, but from the viewpoint of economic policy, they may be at least equally important and informative. For example, monetary policy decisions made by central banks are based on the real time assessment of the current, and also expected future, economic conditions using the data available at the time the decision is made. As Osborn *et al.* (2004) point out, growth rate cycles are closely related to the estimated output gap, which is supposedly an important variable affecting monetary policy decisions.

In this study, we concentrate on the predictive power of financial variables for business cycle and growth rate cycle recessions. The advantage of those variable is that those are available on a continuous basis without revisions. A difficulty with macroeconomic predictive variables, such as initial estimates of the real GDP or the estimated output gap, in contrast, is that they face substantial revisions during subsequent months and observations of some variables are not even available on a monthly basis. These properties of macroeconomic variables, which ultimately determine the values of both cycle indicators, also mean that the real-time state of the economy is always uncertain to some extent. Therefore, nowcasting the business cycle and growth rate cycle indicators is of interest, and the real-time availability supports financial variables as predictors.

Our results demonstrate the advantages of modeling the probabilities of business cycle and growth rate cycle recessions jointly. As a matter of fact, among the considered univariate and bivariate specifications, the proposed unrestricted bivariate autoregressive probit model yields the best in-sample, but also out-of-sample predictions. The lagged first difference of the Federal funds rate and monthly stock market returns turn out to be the best predictive variables for the U.S. growth rate cycle. As suggested in many previous studies, the U.S. term spread is an important predictive variable for predicting business cycle recessions, but its predictive power for growth rate cycle periods is limited.

The remainder of the paper is organized as follows. The bivariate autoregressive probit model is introduced in Section 2. Issues of parameter estimation, testing, and forecasting are discussed in Section 3. Section 4 presents the empirical results. Section 5 concludes.

2 Bivariate Autoregressive Probit Model

Consider two binary time series, y_{1t} and y_{2t}, $t = 1, 2, ..., T$. Let us assume that conditional on information set Ω_{t-1}, the random vector (y_{1t}, y_{2t}) follows a bivariate Bernoulli distribution,

$$(y_{1t}, y_{2t})|\Omega_{t-1} \sim B_2(P_{11,t}, P_{10,t}, P_{01,t}, P_{00,t}), \tag{1}$$

where

$$P_{ij,t} = P_{t-1}(y_{1t} = i, y_{2t} = j), \quad i, j = 0, 1, \tag{2}$$

and

$$P_{11,t} + P_{10,t} + P_{01,t} + P_{00,t} = 1. \tag{3}$$

Hence, the conditional marginal probabilities of the separate outcomes $y_{1t} = 1$ and $y_{2t} = 1$ are equal to

$$P_{1t} = P_{11,t} + P_{10,t}, \tag{4}$$

and

$$P_{2t} = P_{11,t} + P_{01,t}, \tag{5}$$

respectively.

A bivariate probit model was first proposed by Ashford and Sowden (1970) for analyzing cross-sectional data. In their model the joint probabilities for different outcomes of the vector (y_{1t}, y_{2t}) are determined as

$$\begin{aligned} P_{11,t} &= P_{t-1}(y_{1t}=1, y_{2t}=1) = \Phi_2(\pi_{1t}, \pi_{2t}, \rho), \\ P_{10,t} &= P_{t-1}(y_{1t}=1, y_{2t}=0) = \Phi_2(\pi_{1t}, -\pi_{2t}, -\rho), \\ P_{01,t} &= P_{t-1}(y_{1t}=0, y_{2t}=1) = \Phi_2(-\pi_{1t}, \pi_{2t}, -\rho), \\ P_{00,t} &= P_{t-1}(y_{1t}=0, y_{2t}=0) = \Phi_2(-\pi_{1t}, -\pi_{2t}, \rho), \end{aligned} \quad (6)$$

where $\Phi_2(\cdot)$ is the cumulative distribution function of the bivariate normal distribution with zero means, unit variances and correlation coefficient ρ, $|\rho| < 1$, and π_{1t} and π_{2t} are assumed to be linear functions of variables $\boldsymbol{x}_{1,t-k}$ and $\boldsymbol{x}_{2,t-k}$ included in the information set Ω_{t-1}, respectively. The sign changes in the arguments of the bivariate cumulative normal distribution function are needed to guarantee that condition (3) holds (see, for example, Greene, 2000, 849–850).

To complete the bivariate probit model, a parametrization for π_{1t} and π_{2t} needs to be specified. Ashford and Sowden (1970) introduced the following parametrization,

$$\begin{bmatrix} \pi_{1t} \\ \pi_{2t} \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{x}'_{1,t-k} & 0 \\ 0 & \boldsymbol{x}'_{2,t-k} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{bmatrix}, \quad (7)$$

where ω_1 and ω_2 are constant terms and $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ are coefficient vectors of the lagged explanatory variables included in the vectors $\boldsymbol{x}_{1,t-k}$ and $\boldsymbol{x}_{2,t-k}$, respectively. Note that using the same lag k in all explanatory variables is only for notational convenience and can easily be relaxed in practice. Equations (6) and (7) together define the static bivariate probit model.[1]

Dynamic extensions of the static model (7) can be obtained in various ways. In this paper, we propose the following "bivariate autoregressive probit model",

$$\begin{bmatrix} \pi_{1t} \\ \pi_{2t} \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \pi_{1,t-1} \\ \pi_{2,t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{x}'_{1,t-k} & 0 \\ 0 & \boldsymbol{x}'_{2,t-k} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{bmatrix}, \quad (8)$$

where π_{1t} and π_{2t} are specified as linear functions of their lags and the lagged values of the explanatory variables included in the vectors $\boldsymbol{x}_{1,t-k}$ and $\boldsymbol{x}_{2,t-k}$. Model (8) can compactly be written as

$$\boldsymbol{\pi}_t = \boldsymbol{\omega} + \mathbf{A}\boldsymbol{\pi}_{t-1} + \boldsymbol{x}'_{t-k}\boldsymbol{\beta}, \quad (9)$$

[1] The corresponding multivariate model is considered by Ashford and Sowden (1970), and Chib and Greenberg (1998), among others.

where $\boldsymbol{\pi}_t = \begin{pmatrix} \pi_{1t} & \pi_{2t} \end{pmatrix}'$, $\boldsymbol{x}_{t-k} = \text{diag}\begin{pmatrix} \boldsymbol{x}'_{1,t-k} & \boldsymbol{x}'_{2,t-k} \end{pmatrix}'$, $\boldsymbol{\omega} = \begin{pmatrix} \omega_1 & \omega_2 \end{pmatrix}'$, $\boldsymbol{\beta} = \begin{pmatrix} \boldsymbol{\beta}'_1 & \boldsymbol{\beta}'_2 \end{pmatrix}'$, and

$$\mathbf{A} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}.$$

In this bivariate autoregressive model we explicitly allow for the possibility that different explanatory variables can be included in $\boldsymbol{x}_{1,t-k}$ and $\boldsymbol{x}_{2,t-k}$. If the parameter matrix $\mathbf{A}$ is unrestricted, both π_{1t} and π_{2t} can depend on the lagged values of π_{1t} and π_{2t}. Thus, even if $\rho = 0$ in (6), the coefficients α_{12} and α_{21} provide a linkage between the variables π_{1t} and π_{2t} in model (8). Note that the static model is obtained from (9) with the restriction $\mathbf{A} = \mathbf{0}$. Furthermore, it is only in the special case where $\rho = 0$ and $\alpha_{12} = \alpha_{21} = 0$ that our bivariate autoregressive probit model reduces to two independent univariate autoregressive probit models.

Model (8) is somewhat similar to the multivariate dynamic binary model of Anatolyev (2009). The main difference is that Anatolyev (2009) suggests using the so called "dependence ratios" (cf. Ekholm, Smith, and McDonald, 1995) between the dependent variables to construct the conditional joint probabilities of the different outcomes of (y_{1t}, y_{2t}). In parameter estimation the dependence ratios and marginal probabilities for the variables y_{1t} and y_{2t} are handled separately by using a logistic function. In our model, the dependence between y_{1t} and y_{2t} is instead modeled by using the autoregressive specification (8) and the bivariate cumulative normal distribution function, where the correlation coefficient ρ is allowed to be nonzero. In addition, in the bivariate autoregressive probit model, parameter estimation can be carried out within the same system without dependence ratios.

Note that if the roots of $\det(I_2 - \mathbf{A}z)$ lie outside the unit circle, we obtain by recursive substitution of (9) the following representation,

$$\boldsymbol{\pi}_t = \sum_{j=1}^{\infty} \mathbf{A}^{j-1}\boldsymbol{\omega} + \sum_{j=1}^{\infty} \mathbf{A}^{j-1}\boldsymbol{x}'_{t-k-j+1}\boldsymbol{\beta}. \tag{10}$$

This shows that in bivariate autoregressive probit model (8) π_{1t} and π_{2t} depend on the whole infinite history of the explanatory variables in a parsimonious way and, therefore, the model can be interpreted as an "infinite order" extension of the static model (7). Furthermore, assuming that the explanatory variables included in $\boldsymbol{x}_{t-k}$

are stationary, also $\boldsymbol{\pi}_t$ is stationary.

It is worth noting that because of the characteristics of our empirical application (see Section 4.2), the lagged values of y_{1t} and y_{2t} included in Anatolyev's (2009) model are excluded. However, that would be a possible extension of model (8). This extension can be based on the univariate model of Kauppi and Saikkonen (2008), where the lag y_{t-1} is also included in the right hand side of the model.

3 Parameter Estimation, Testing and Forecasting

3.1 Maximum Likelihood Estimation

As in corresponding univariate models, parameter estimation in the bivariate autoregressive model defined by (6) and (8), as well as its special cases, can conveniently be carried out by the method of maximum likelihood (ML). Using the conditional probabilities in (6), one can write the likelihood function and obtain the maximum likelihood estimate by using numerical methods.

Following Greene's (2000, 849–850) notation, the log-likelihood function can be constructed as follows. Define $q_{jt} = 2y_{jt} - 1$ and $\mu_{jt} = q_{jt}\pi_{jt}$, $j = 1, 2$, so that

$$q_{jt} = \begin{cases} 1 & \text{if } y_{jt} = 1, \\ -1 & \text{if } y_{jt} = 0, \end{cases}$$

and

$$\mu_{jt} = \begin{cases} \pi_{jt} & \text{if } y_{jt} = 1, \\ -\pi_{jt} & \text{if } y_{jt} = 0. \end{cases}$$

Furthermore, set

$$\rho_t^* = q_{1t}q_{2t}\rho.$$

The conditional probabilities of the different outcomes of (y_{1t}, y_{2t}) can be expressed as

$$P_{t-1}(y_{1t}, y_{2t}) = \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*).$$

Let $\boldsymbol{\theta} = \begin{pmatrix} \text{vec}(\mathbf{A})' & \boldsymbol{\omega}' & \boldsymbol{\beta}' & \rho \end{pmatrix}'$ denote the vector of the parameters of the bivariate autoregressive probit model. The log-likelihood function, conditional on initial values, is the sum of the individual log-likelihood functions $l_t(\boldsymbol{\theta})$,

$$\begin{aligned} l(\boldsymbol{\theta}) &= \sum_{t=1}^{T} l_t(\boldsymbol{\theta}) = \sum_{t=1}^{T} \log\Big(\Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)\Big) \\ &= \sum_{t=1}^{T} \Big(y_{1t}y_{2t}\log(P_{11,t}) + y_{1t}(1-y_{2t})\log(P_{10,t}) \\ &\quad + (1-y_{1t})y_{2t}\log(P_{01,t}) + (1-y_{1t})(1-y_{2t})\log(P_{00,t})\Big). \end{aligned} \tag{11}$$

It is worth noting that if the correlation coefficient ρ in (6) is zero, the conditional probabilities in (6) are products of the marginal probabilities (4) and (5). For instance, in that case the conditional probability of the outcome $(y_{1t} = 1, y_{2t} = 1)$ is

$$P_{11,t} = P_{t-1}(y_{1t} = 1, y_{2t} = 1) = P_{t-1}(y_{1t})P_{t-1}(y_{2t}) = \Phi(\pi_{1t})\Phi(\pi_{2t}). \tag{12}$$

The score vector of the log-likelihood function (11) is

$$s(\boldsymbol{\theta}) = \frac{\partial l(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \sum_{t=1}^{T} s_t(\boldsymbol{\theta}) = \sum_{t=1}^{T} \frac{\partial l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}, \tag{13}$$

where

$$\frac{\partial l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \frac{1}{\Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)} \frac{\partial \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}{\partial \boldsymbol{\theta}}.$$

At this point it is convenient to split the parameter vector into three disjoint components, namely $\boldsymbol{\theta} = (\boldsymbol{\theta}_1' \quad \boldsymbol{\theta}_2' \quad \rho)'$, where the parameters in $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are related to the specifications of π_{1t} and π_{2t}, respectively. The score vector can be partitioned accordingly as

$$s_t(\boldsymbol{\theta}) = \Big(s_{1t}(\boldsymbol{\theta}_1)' \quad s_{2t}(\boldsymbol{\theta}_2)' \quad s_{3t}(\rho)\Big)'. \tag{14}$$

The first component of $s_t(\boldsymbol{\theta})$ can be written as

$$\begin{aligned} s_{1t}(\boldsymbol{\theta}_1) = \frac{\partial l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_1} &= \frac{1}{\Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)} \frac{\partial \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}{\partial \boldsymbol{\theta}_1} \\ &= \frac{1}{\Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)} \frac{\partial \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}{\partial \mu_{1t}} \frac{\partial \mu_{1t}}{\partial \boldsymbol{\theta}_1} \\ &= \frac{1}{\Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)} \phi(\mu_{1t}) \Phi\Big(\frac{\mu_{2t} - \mu_{1t}\rho_t^*}{\sqrt{1-\rho_t^{*2}}}\Big) q_{1t} \frac{\partial \pi_{1t}}{\partial \boldsymbol{\theta}_1}, \end{aligned}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the density function and the cumulative distribution function of the standard normal distribution, respectively. The value of $s_{1t}(\boldsymbol{\theta}_1)$ depends on the realized values of y_{1t} and y_{2t}. For instance, if ($y_{1t} = 1$, $y_{2t} = 1$), then by the definitions of μ_{jt} and q_{1t},

$$s_{1t}(\boldsymbol{\theta}_1) \quad = \quad \frac{1}{\Phi_2(\pi_{1t}, \pi_{2t}, \rho)}\phi(\pi_{1t})\Phi\Big(\frac{\pi_{2t} - \pi_{1t}\rho}{\sqrt{1-\rho^2}}\Big)\frac{\partial \pi_{1t}}{\partial \boldsymbol{\theta}_1}.$$

It can be seen that the main difference between the score vector of the static model ($\mathbf{A} = \mathbf{0}$) and model (8) is in the derivative term $\partial\pi_{1t}/\partial\boldsymbol{\theta}_1$. In model (8),

$$\frac{\partial \pi_{1t}}{\partial \boldsymbol{\theta}_1} = \begin{pmatrix} \frac{\partial \pi_{1t}}{\partial \omega_1} \\ \frac{\partial \pi_{1t}}{\partial \alpha_{11}} \\ \frac{\partial \pi_{1t}}{\partial \alpha_{12}} \\ \frac{\partial \pi_{1t}}{\partial \boldsymbol{\beta}_1} \end{pmatrix} = \begin{pmatrix} 1 + \alpha_{11}\frac{\partial \pi_{1,t-1}}{\partial \omega_1} + \alpha_{12}\alpha_{21}\frac{\partial \pi_{1,t-2}}{\partial \omega_1} \\ \pi_{1,t-1} + \alpha_{11}\frac{\partial \pi_{1,t-1}}{\partial \alpha_{11}} + \alpha_{12}\alpha_{21}\frac{\partial \pi_{1,t-2}}{\partial \alpha_{11}} \\ \pi_{2,t-1} + \alpha_{11}\frac{\partial \pi_{1,t-1}}{\partial \alpha_{12}} + \alpha_{12}\alpha_{21}\frac{\partial \pi_{1,t-2}}{\partial \alpha_{12}} \\ \boldsymbol{x}_{1,t-k} + \alpha_{11}\frac{\partial \pi_{1,t-1}}{\partial \boldsymbol{\beta}_1} + \alpha_{12}\alpha_{21}\frac{\partial \pi_{1,t-2}}{\partial \boldsymbol{\beta}_1} \end{pmatrix},$$

whereas, in the static model, it reduces to $\begin{pmatrix}1 & \boldsymbol{x}_{1,t-k}\end{pmatrix}'$. The derivative $\partial l_t(\boldsymbol{\theta})/\partial\boldsymbol{\theta}_2$ is obtained in the same way by replacing $\partial\pi_{1t}/\partial\boldsymbol{\theta}_1$ in the definition of $s_t(\boldsymbol{\theta})$ by

$$\frac{\partial \pi_{2t}}{\partial \boldsymbol{\theta}_2} = \Big(\frac{\partial \pi_{2t}}{\partial \omega_2} \quad \frac{\partial \pi_{2t}}{\partial \alpha_{22}} \quad \frac{\partial \pi_{2t}}{\partial \alpha_{21}} \quad \frac{\partial \pi_{2t}}{\partial \boldsymbol{\beta}_2}\Big)'.$$

Given the result

$$\frac{\partial \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}{\partial \rho_t^*} = \phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*),$$

the derivative with respect ρ is

$$\frac{\partial \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}{\partial \rho} = \frac{\partial \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}{\partial \rho_t^*}\frac{\partial \rho_t^*}{\partial \rho} = \phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)q_{1t}q_{2t}.$$

Therefore, the score of the correlation coefficient ρ becomes

$$s_{3t}(\rho) = \frac{\partial l_t(\boldsymbol{\theta})}{\partial \rho} = \frac{1}{\Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}\frac{\partial \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}{\partial \rho_t^*}\frac{\partial \rho_t^*}{\partial \rho} = \frac{\phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}{\Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*)}q_{1t}q_{2t}.$$

The value of $s_{3t}(\rho)$ depends on realized values of the dependent variables. For example, if $y_{1t} = 1$ and $y_{2t} = 1$,

$$s_{3t}(\rho) = \frac{\phi_2(\pi_{1t}, \pi_{2t}, \rho)}{\Phi_2(\pi_{1t}, \pi_{2t}, \rho)},$$

and if $y_{1t} = 1$ and $y_{2t} = 0$,

$$s_{3t}(\rho) = -\frac{\phi_2(\pi_{1t}, -\pi_{2t}, -\rho)}{\Phi_2(\pi_{1t}, -\pi_{2t}, -\rho)}.$$

Maximization of the log-likelihood function (11) yields the maximum likelihood estimate $\hat{\boldsymbol{\theta}}$, which solves the first order condition $s(\hat{\boldsymbol{\theta}}) = 0$. Under appropriate regularity conditions, including the stationarity of explanatory variables and the correctness of the probit model specification, the conventional large sample theory of ML estimation gives the usual asymptotic distribution,

$$T^{1/2}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \xrightarrow{L} N(\mathbf{0}, \mathcal{I}(\boldsymbol{\theta})^{-1}), \tag{15}$$

where $\mathcal{I}(\boldsymbol{\theta}) = \text{plim}\, T^{-1}\partial^2 l(\boldsymbol{\theta})/\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'$.

A practical difficulty with the bivariate autoregressive probit model (8) is that the number of parameters can become large if many explanatory variables are included. ML estimation is considerably simplified if the correlation coefficient ρ is restricted to zero because then the bivariate probabilities in the log-likelihood function (11) factor into products of marginal probabilities, as in (12). Thus, it is of interest to test for the hypothesis $\rho = 0$. In the next section, a LM test for this purpose is developed.

3.2 LM Test for the Correlation Coefficient

For testing the significance of the correlation coefficient, the Lagrange multiplier test is attractive because it only requires ML estimation under the null hypothesis $\rho = 0$. Kiefer (1982) has proposed a corresponding LM test for the static bivariate probit model (7). In this section, the test is extended to the bivariate autoregressive model (8).

Let $\tilde{\boldsymbol{\theta}} = (\tilde{\boldsymbol{\theta}}_1 \quad \tilde{\boldsymbol{\theta}}_2 \quad 0)$ be the restricted ML estimate of $\boldsymbol{\theta}$ obtained by assuming

$$H_0 : \rho = 0. \tag{16}$$

The general form of the LM test statistic (see, for example, Engle, 1984) is

$$LM = s(\tilde{\boldsymbol{\theta}})'\tilde{\mathcal{I}}(\tilde{\boldsymbol{\theta}})^{-1}s(\tilde{\boldsymbol{\theta}}), \tag{17}$$

where $\tilde{\mathcal{I}}(\tilde{\boldsymbol{\theta}})$ is a consistent estimate of the information matrix $\mathcal{I}(\boldsymbol{\theta})$ and s$(\tilde{\boldsymbol{\theta}})$ is the score vector (13) evaluated at the restricted ML estimates $\tilde{\boldsymbol{\theta}}$. Under the null hypothesis (16) the test statistic has an asymptotic χ^2_1 distribution.

Due to the complexity of the second derivatives of the log-likelihood function (11), the outer-product of the score is an attractive estimator of the information matrix $\mathcal{I}(\boldsymbol{\theta})$. The resulting test statistic is

$$LM^{\rho} = \boldsymbol{\iota}' S(\tilde{\boldsymbol{\theta}})\Big(S(\tilde{\boldsymbol{\theta}})' S(\tilde{\boldsymbol{\theta}})\Big)^{-1} S(\tilde{\boldsymbol{\theta}})'\boldsymbol{\iota}, \tag{18}$$

where $\boldsymbol{\iota}$ is a vector of ones and the matrix $S(\tilde{\boldsymbol{\theta}})$ is given by

$$S(\tilde{\boldsymbol{\theta}}) = \Big(s_1(\tilde{\boldsymbol{\theta}}) \quad s_2(\tilde{\boldsymbol{\theta}}) \dots s_T(\tilde{\boldsymbol{\theta}})\Big)^{'}.$$

As in (14), the score vector $s_t(\tilde{\boldsymbol{\theta}})$, evaluated at $\tilde{\boldsymbol{\theta}}$, consists of three components. The bivariate densities and probabilities factor into products of marginals and, consequently, the components of the score reduce to

$$\begin{aligned} s_{1t}(\tilde{\boldsymbol{\theta}}_1) &= \frac{\phi(\tilde{\mu}_{1t})}{\Phi(\tilde{\mu}_{1t})} q_{1t} \frac{\partial \tilde{\pi}_{1t}}{\partial \boldsymbol{\theta}_1}, \\ s_{2t}(\tilde{\boldsymbol{\theta}}_2) &= \frac{\phi(\tilde{\mu}_{2t})}{\Phi(\tilde{\mu}_{2t})} q_{2t} \frac{\partial \tilde{\pi}_{2t}}{\partial \boldsymbol{\theta}_2}, \end{aligned}$$

and

$$s_{3t}(0) = \frac{\phi(\tilde{\mu}_{1t})\phi(\tilde{\mu}_{2t})}{\Phi(\tilde{\mu}_{1t})\Phi(\tilde{\mu}_{2t})} q_{1t} q_{2t},$$

where "$\sim$" on the right hand side means that the quantities are evaluated at $\boldsymbol{\theta} = \tilde{\boldsymbol{\theta}}$. The derivatives $\partial \pi_{1t}/\partial \boldsymbol{\theta}_1$ and $\partial \pi_{2t}/\partial \boldsymbol{\theta}_2$ depend on the considered specifications of π_{1t} and π_{2t} (see Section 3.1).

3.3 Forecasting

As shown by Kauppi and Saikkonen (2008), explicit formulae can be used to obtain one-period and multiperiod forecasts in the case of the univariate model. The obtained forecasts are probability forecasts for different outcomes of (y_{1t},y_{2t}). In the following we show that the same principles can also be applied in the proposed bivariate model.

In the mean-square sense, the optimal h-period forecast based on the given information available at time $t-h$, $h \geq 1$, is the conditional expectation

$$E_{t-h}(y_{1t}, y_{2t}) = \Phi_2(\mu_{1t}, \mu_{2t}, \rho_t^*). \tag{19}$$

For example, the forecast for outcome ($y_{1t} = 1$, $y_{2t} = 1$) is given by

$$E_{t-h}(y_{1t} = 1, y_{2t} = 1) = \Phi_2(\pi_{1t}^{(h)}, \pi_{2t}^{(h)}, \rho),$$

where, as shown in (10), by recursive substitution the bivariate system (9) can be written as

$$\boldsymbol{\pi}_t^{(h)} = \mathbf{A}^h \boldsymbol{\pi}_{t-h} + \sum_{j=1}^{h} \mathbf{A}^{j-1}\Big(\boldsymbol{\omega} + \boldsymbol{x}_{t-k-j+1}^{'}\boldsymbol{\beta}\Big), \tag{20}$$

where $\boldsymbol{\pi}_t^{(h)} = \left(\pi_{1t}^{(h)} \quad \pi_{2t}^{(h)}\right)'$ and the vector $\boldsymbol{\pi}_{t-h}$ is a function of values of the explanatory variables and the initial values of $\pi_{1,0}$ and $\pi_{2,0}$. In addition, the condition $k \geq h$ for all predictors included in $\boldsymbol{x}_{t-k}$ must hold indicating that the employed lags of the predictive variables are always tailored to match the information available at the time of forecasting. The usual case is obtained by selecting $k = h$. The right hand side of (20) gives the h step forecast for the outcome ($y_{1t} = 1, y_{2t} = 1$) "directly" using the information up to the forecast time $t - h$. Forecasts for other outcomes of vector (y_{1t}, y_{2t}) are obtained by imposing necessary sign changes in bivariate normal cumulative distribution function (see (6)).

Both in-sample and out-of-sample predictive performance of the employed models can be evaluated with goodness-of-fit measures commonly used for binary dependent variables. Comparisons between different models can be based on the value of the maximized log-likelihood function (11), denoted by logL below. It can also be used to compute values of model selection criteria, such as the Schwarz information criterion (Schwarz, 1978) defined as

$$BIC = -\text{logL} + K\,\frac{\log(T)}{2}, \tag{21}$$

where K is the number of parameters in $\boldsymbol{\theta}$ and T is the number of observations. Another goodness-of-fit measure is the quadratic probability score, QPS, suggested by Diebold and Rudebusch (1989). Using the marginal conditional probability forecasts P_{1t} and P_{2t} (see (4) and (5)), the quadratic probability score for variable y_{jt} is

$$QPS_j = \frac{1}{T}\sum_{t=1}^{T} 2\Big(y_{jt} - P_{jt})\Big)^2, \tag{22}$$

where $j = 1, 2$. The values of the QPS_j lie on the interval [0,2] with the value 0 indicating a perfect fit. It can be seen as a counterpart of the mean square error used with models for continuous variables.

Because of the binary nature of the dependent variable, the percentage of correct predictions (CR) is a natural measure of predictive performance. However, a threshold value must be specified that translates the probability forecasts into signal forecasts ($y_{jt} = 1$ or $y_{jt} = 0$, $j = 1, 2$). The most commonly used and natural threshold value is 0.50, which is also used in this paper. When the signal forecasts are constructed, a test proposed by Pesaran and Timmermann (1992) is available for the evaluation of the directional predictive performance of a model. The null hypothesis of the test is that the value of the correct prediction ratio does not differ significantly from the ratio that would be obtained in the case of no-predictability, where the forecasts and realized values of y_t are independent. Under the null hypothesis of no-predictability, the test statistic has an asymptotic standard normal distribution.

4 Empirical Application: Predicting the Current State of the U.S. Economy

We apply different bivariate probit model specifications to predict the state of the U.S. economy. In this application, we predict, or more specifically "nowcast", values of the dependent U.S. business cycle and growth cycle indicators, to be discussed in more detail in Section 4.1, using the real-time information on financial predictive variables. The monthly sample size covers the period from January 1971 to December 2005.

Knowledge of the current state of aggregate economic activity is important for many economic agents in business and finance, as well as for policymakers, such as central banks and government organizations. However, because of informational lags and revisions of important macroeconomic variables, such as the real GDP, the current state of the economy is always uncertain to some extent. In our nowcasting exercise, the forecast horizon will therefore be one month, $h = 1$. Thus we are interested in predicting the probabilities of business cycle and growth cycle recessions for month t using the information up to the end of the previous month $t - 1$. In other words, the nowcasts are constructed at the beginning of month t.

4.1 Binary Indicators for the Business and Growth Rate Cycles

Forecasting the recession periods of the economy with various univariate binary time series models has attracted considerable attention in many previous studies (see, among others, Estrella and Mishkin, 1998; Chauvet and Potter, 2005; Kauppi and Saikkonen, 2008; Nyberg, in press). These recession periods are related to business cycle fluctuations defined in terms of the level of economic activity. Thus our first binary recession indicator is

$$y_{1t} = \begin{cases} 1, & \text{if the economy is in a recession at time } t, \\ 0, & \text{if the economy is in an expansion at time } t. \end{cases} \tag{23}$$

The best-known indicator for the U.S. is that one provided by the National Bureau of Economic Research (NBER). It is based on the definition of a recession as "a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales." [2] It is important to note that the NBER uses a broader array of economic indicators than just, say, the real GDP, to determine the recession periods.

In this study, we are mainly interested in predicting the growth rate cycles of the U.S. economy, and, to the best of our knowledge, only Osborn *et al.* (2004) have so far studied these cycle periods by means of binary time series models. In contrast to classical business cycles characterized by the recession indicator (23), growth rate cycles are related to the growth rate of aggregate economic activity. We adopt the growth rate cycle periods defined by the Economic Cycle Research Institute (ECRI). Based on their definition of "periods of cyclical upswings and downswings in growth", we introduce the binary indicator

$$y_{2t} = \begin{cases} 1, & \text{if the growth rate cycle is in a downswing state at time } t, \\ 0, & \text{if the growth rate cycle is in an upswing state at time } t. \end{cases} \tag{24}$$

In this paper, the "downswing state" of the growth rate indicator ($y_{2t} = 1$) is referred to as a "growth rate recession".[3]

ECRI determines the turning points in the growth rate cycle in a way analogous to the "NBER approach", where the co-movements and cyclical turns in various measures of the aggregate macroeconomic activity are taken into account. Banerji and Hiris (2001) provide a more detailed discussion on the business cycle and growth rate cycle

[2] See details on http://www.nber.org/cycles/recessions.html.

[3] Osborn *et al.* (2004) refer these periods as "growth regimes".

periods (see also Layton and Moore, 1989). It is expected that the growth rate cycle indicator y_{2t} exhibits more regime switches than the business cycle indicator y_{1t}. The reason is that a period with a lower growth rate may be classified as a growth rate recession, but it is not necessarily defined as a business cycle recession.

Figure 1 depicts the ECRI growth rate recession periods along with the NBER recession periods. Table 1 shows the cross-tabulation of the realized values of y_{1t} and y_{2t} defined in terms of these periods. As expected, growth rate cycles are more numerous than ”classical” business cycles. All slowdowns in the growth of economic activity do not involve business cycle recessions. On the other hand, the growth rate cycle recessions seem to lead the business cycle recessions: the growth rate recession has typically started a few months before a business cycle recession period. Furthermore, it should be pointed out that the rare outcome ($y_{1t} = 1, y_{2t} = 0$), i.e. the economy is in a business cycle recession, but at the same time in a growth rate expansion, is also possible. Table 1 shows that this outcome has been occurred in five months in our data set. When taking a closer look at the turning point chronologies of the NBER and ECRI, it can be seen that those periods have been related, as expected, to the endpoints of the business cycle recessions ($y_{1t} = 1$), where the growth rate expansion ($y_{2t} = 0$) has started before the business cycle expansion ($y_{1t} = 0$).

4.2 Data Set and Predictive Models

In addition to y_{1t} and y_{2t}, our data set consists of a number of financial variables, such as interest rates and stock market returns, which are used as predictors in bivariate probit models.[4] Both levels and first differences of various interest rates are considered. Assuming that monetary policy has an impact on real economic activity and its growth rate, it is of interest to study which interest rate variable is the most informative predictor. The Federal funds rate (FF_t) is closely related to the monetary policy in the U.S., so that it is a natural candidate variable (see, e.g., Bernanke and Blinder, 1992).

The term spread (SP_t) between the long-term interest rate and the short-term interest rate has often been found the most important predictor of business cycle

[4] The variables are described in more detail in Table 2.

recessions (see, e.g., Estrella and Mishkin, 1998). Hence, it is of interest to examine whether the term spread is also an important predictor for the growth rate cycle periods. Furthermore, as a forward-looking variable and incorporating expectations of future dividends and profitability of firms, stock market returns (r_t) should also have predictive power.

Osborn *et al.* (2004) found the output gap between the potential and realized level of output an important predictive variable for growth rate cycle periods in European countries. According to their results, the long-term and short-term interest rates and stock market returns also have some predictive power, but their importance turned out to be secondary to the output gap. However, as they pointed out, the final estimate of the output gap, which was employed as a predictor, is not available on a real-time basis (see, e.g., Orphanides and van Norden, 2002). This is also the case for some other macroeconomic variables. For example, initial GDP estimates are revised later on. Therefore, in this paper, we concentrate on the potential predictive information of financial variables, which are available with no revisions or informational lags at the monthly frequency.

In addition to the real-time availability of predictive variables, another issue that should be taken into account in the specification of the predictive model is the fact that the values of the NBER business cycle phases (y_{1t}), and apparently also the growth cycle periods defined by the ECRI (y_{2t}), become available with very long delays. We call these delays as "publication lags". Without explicit assumptions concerning the publication lags it is difficult to use lagged values of the cycle indicators in the predictive model. Overall, the publication lags are typically so long that it is likely the lagged values of the indicators are statistically insignificant in estimated models (see the evidence on univariate models in Nyberg, in press).[5] Therefore, we only consider models excluding the lagged values of the indicators y_{1t} and y_{2t}.

We consider four different model specifications obtained from the bivariate au-

[5] The most recent publication lags of the NBER have varied from five up to twenty months (see http://www.nber.org/cycles/cyclesmain.html).

toregressive probit model given in (6) and (8). The models are defined as follows.

$$\begin{aligned}\text{Model 1} &: \alpha_{12}=0, \alpha_{21}=0, \rho=0,\\ \text{Model 2} &: \alpha_{12}=0, \alpha_{21}=0,\\ \text{Model 3} &: \rho=0,\\ \text{Model 4} &: \text{unrestricted model.}\end{aligned}$$

Model 1 consist of two independent autoregressive probit models. Model 2 is obtained from Model 1 by removing the restriction that the correlation coefficient ρ is zero. Note that Models 1 and 2 are already extensions of the static bivariate model (7) because both π_{1t} and π_{2t} follow univariate autoregressive models. Model 4 is the bivariate autoregressive probit model (8) without any restrictions whereas only the correlation coefficient ρ in (6) is restricted to zero in Model 3.

4.3 Model Selection and In-Sample Results

Model selection considered in this section is based on models estimated over the entire sample period from January 1971 to December 2005. The first 12 observations are used as initial values in estimation. This section is also a starting point for out-of-sample forecasts for the U.S. for the 2006–2008 period considered in Section 4.5.

As mentioned in Section 4.1, unlike the business cycle recession periods, there are few previous results on the predictability of growth rate cycle periods. In model selection, therefore, we concentrate on predicting the growth rate cycle recessions with various financial variables. For simplicity, we employ the same explanatory variables as Nyberg (in press) in the predictive models of the U.S. business cycle recession periods, i.e.,

$$\boldsymbol{x}_{1,t-k} = \begin{pmatrix} SP_{t-6} & r_{t-1} & SP^{GE}_{t-6} \end{pmatrix}', \tag{25}$$

where SP_t is the U.S. term spread, r_t is the monthly stock market return, and SP^{GE}_t is the German term spread. Here the employed lags are the same as in Nyberg (in press) and, because the forecast horizon is one month ($h = 1$), the condition $k \geq h$ (see Section 3.3) is satisfied.

We apply the following model selection procedure concerning the variables included in $\boldsymbol{x}_{2,t-k}$. First, we estimate univariate autoregressive probit models with one predictor. The considered predictive variables are listed in Table 2. When the best single predictor based on BIC is found, it will be retained in the model and different models with two predictors are estimated. We mainly restrict ourselves to models with two predictors, but make some experiments with models containing the third predictor as well. Finally, we consider bivariate models with the predictors selected at the first stage.

Table 3 shows values of the Schwarz information criterion (21) for Model 1 with different explanatory variables, when the employed lag varies from one ($k = 1$) to six ($k = 6$).[6] Especially in more general bivariate probit models, such as Model 4, the number of parameters is quite large, indicating that ML estimation may become difficult. Therefore, we prefer parsimonious models which makes BIC a suitable model selection criterion.

According to Table 3, the best single predictive variable seems to be the first difference of the Federal funds rate lagged by two or three months (ΔFF_{t-2} or ΔFF_{t-3}). The first difference of the short-term interest rate (Δi_{t-k}) also performs quite well. Furthermore, as seen from Table 4, when models with two predictive variables are considered, the lagged stock market return r_{t-1} combined with ΔFF_{t-2} provides the lowest value of the BIC. Thus the vector $\boldsymbol{x}_{2,t-k}$ is selected as

$$\boldsymbol{x}_{2,t-k} = \begin{pmatrix} \Delta FF_{t-2} & r_{t-1} \end{pmatrix}'. \tag{26}$$

This selection is also meaningful from the viewpoint of the predictive ability of financial markets because variables reflecting the effect of both the monetary policy (ΔFF_{t-2}) and the stock market returns (r_{t-1}) are now included in the model.

According to Tables 3 and 4, the U.S. term spread has some ability to predict the growth rate cycle periods. However, it is outperformed by several other variables. The term spread is also found to be a statistically insignificant predictor when used a third predictor in Models 1–4 together with the explanatory variables given in (26).

Table 5 shows the estimation results for Models 1–4 with the explanatory variables given in (25) and (26). The signs of the estimated coefficients are as expected. In the case of the growth rate cycle periods, increasing values of the differenced Federal

[6] It appears that the evidence of predictive power of different explanatory variables is the same when goodness-of-fit measures other than the BIC are used.

funds rate and negative stock market returns increase the probability of growth rate recession. The results thus indicate that the U.S. monetary policy has a statistically significant predictive impact for the current state of the growth rate cycle via the first difference of the Federal funds rate. Stock market returns also have predictive power for the growth rate cycles as well as for business cycle recession periods. In addition, the U.S. term spread (SP_{t-6}) and the German term spread (SP_{t-6}^{GE}) are also statistically significant predictors.

In Model 2, the estimate of the correlation coefficient ρ is statistically significant. The LM^{ρ} test based on the restricted Model 1 yields the same conclusion. A positive estimate of ρ is obtained, as expected, since the correlation between the dependent variables is positive. Further, in Model 3, the estimates of the off-diagonal elements of the matrix $\mathbf{A}$, α_{12} and α_{21}, for $\pi_{2,t-1}$ and $\pi_{1,t-1}$ are statistically significant, and according to the values of the BIC, Model 3 outperforms Model 2 as an extension of Model 1. However, both the LM^{ρ} test based on Model 3 and the Wald test based on Model 4 point to a nonzero value of the correlation coefficient ρ. Thus Model 4 clearly yields the best in-sample fit. This is especially the case when the models for the growth rate cycle periods are compared with the values of QPS_2 and CR_2.

The fact that Model 4 outperforms alternative bivariate probit models reflects the fact that recession probabilities of the two cycle indicators are dependent on both $\pi_{1,t-1}$ and $\pi_{2,t-1}$. For instance, a positive estimate of α_{12} indicates that the probability of a business cycle recession is high when the lagged probability of growth rate recession is high (high value of π_{2t}). This is in line with the fact that the growth rate recession appears to precede occurred business cycle recession periods. Note that in Models 3 and 4, the U.S. term spread, which is a statistically significant predictor for business cycle recession periods, has also an effect on the growth rate cycle recession probability via the coefficient α_{21} for $\pi_{1,t-1}$ in π_{2t}.

As an extension of the results presented in Table 5, we consider the possibility that there has been a structural break in the data generating process. In the previous literature, it has been suggested that there might have been a structural change in the U.S. economic activity in the mid-1980s. For example, McConnell and Perez-Quiros (2000) and Blanchard and Simon (2001) have documented that the variability of output, and also the variability of inflation, have declined after the mid-1980s.[7] Sensier and van Dijk (2004) also provide evidence that there have been structural breaks in the unconditional volatility of many U.S. macroeconomic time series around the years 1984 to 1986.

In our application, a parsimonious way to allow for the potential effect of a structural break is the inclusion of an additional dummy variable in the model. The dummy variable, denoted by $\mathbf{1}_{71-84}$, takes the value one before the beginning of the year 1985. The results for the augmented models are presented in Table 6, where the dummy variable turns out to be a statistically significant predictor. The models are also superior to their counterparts in Table 5 according to BIC values. Model 4 augmented with the dummy variable $\mathbf{1}_{71-84}$ yields clearly the best in-sample predictions, especially for growth rate cycle recessions.

Although Model 4 seems to outperform its special cases, the estimated coefficients of the explanatory variables are almost equal in all models in Tables 5 and 6. These findings confirm our earlier results that the changes in the Federal funds rate and stock market returns appear to be the main predictive variables for growth rate cycles also in the more general bivarite probit models.

Figures 2 and 3 depict the in-sample fitted values from Models 1 and 4 in Table 6. As the estimation results in Tables 5 and 6 suggest, the business cycle recession periods predicted by these two models are almost identical. For the growth rate recession periods, Model 4 gives somewhat more precise signals around the years from 1984 to 1987 and before the year 2000.

In conclusion, especially the general bivariate autoregressive model (Model 4), but also its special cases (Models 2 and 3), are superior to the independent univariate autoregressive models for both cycle indicators (Model 1). These findings indicate that superior predictive power can be found by using bivariate models instead of univariate models.

[7] This time period after the mid-1980s is often referred to the "Great Moderation" period.

4.4 Out-of-Sample Performance

In this section, we examine the out-of-sample predictive performance of different models. The first out-of-sample nowcasts are made for January 2000 and the last ones for December 2005. Notice that the out-of-sample period contains three growth rate cycle recession periods, but only one business cycle recession in 2001 (see Figure 1). In addition to these out-of-sample nowcasts, in Section 4.5, the best bivariate probit models, according to model selection criteria and in-sample predictions provided in Section 4.3, are used to assess the state of U.S. economy from 2006 to 2008.

Different predictive models are estimated using the data up to December 1999 assuming that the state of the economy is known at that time. To emulate real-time forecasting, we should take into account the fact that the latest values of the dependent variables are unknown at the time the forecast is made (see Section 4.2). Thus the out-of-sample exercise is carried out without updating the parameter estimates using the sample period up to December 1999. The same framework is also applied in Section 4.5. It turns out that the model selection procedure employed in Section 4.3, using the data set up to December 1999, yields the same conclusions as obtained when using the whole sample in estimation. Therefore, the differenced Federal funds rate and the stock market returns have predictive power for the growth rate cycles also in this estimation period.

Table 7 shows the out-of-sample performance of the employed models. Out-of-sample forecast accuracy is evaluated by the QPS and the percentage of correct forecasts (CR). The first four models also include the dummy variable $\mathbf{1}_{71-84}$. As in Section 4.3, the bivariate autoregressive probit model (Model 4) yields the best out-of-sample predictions although Model 3 also produces good forecasts. Based on the predictability test of Pesaran and Timmermann (1992) the reported percentages of correct forecasts are statistically significant at the 1 % level in models including the $\mathbf{1}_{71-84}$ variable. Without the additional dummy variable the percentages of correct forecasts are lower and, consequently, the p-values are higher and statistically insignificant at 5 % level.

Although the dummy variable $\mathbf{1}_{71-84}$ is useful in predicting the growth rate cycle recessions, this is not the case for nowcasts of the business cycle recession periods.

However, the more general Models 3 and 4 outperform Models 1 and 2 even in this case. Note that the percentages of correct forecasts for business cycle periods are statistically significant in all models presented in Table 7.

Figure 4 illustrates the out-of-sample performance of the unrestricted Model 4. It appears that the recession probabilities match well the realized values of both cycle indicators. As depicted in the left panel, Model 4 predicts the business cycle recession in 2001 really well, but the increase in the recession probability in 2002 weakens the performance of the model in terms of the values of the statistical goodness-of-fit measures reported. The out-of-sample nowcasts depicted in the right panel match the growth rate cycle periods well. For instance, when the 50 % threshold value for probabilitity forecasts to construct signal forecasts for growth rate recessions and expansions is used, Model 4 gives the correct signal forecast with approximately 85 % accuracy ($CR = 0.847$).

In summary, the results confirm that the proposed general bivariate autoregressive model (8) outperforms its restricted versions and the values of both cycle indicators appear to be predictable also out of sample.

4.5 Predictions for 2006–2008

There has been great uncertainty about the state of the economy in the United States during the last couple of years. Therefore, it is of great interest to consider the probabilities of the business and growth rate cycle periods during 2006–2008. Business cycle recession probabilities are of special interest due to the fact that the NBER Business Cycle Dating Committee declared that a peak in the U.S. economic activity occurred in December 2007 indicating that the value of the recession indicator (23) has been one, at least in some months from January 2008.

In this section, we examine nowcasts of the state of the economy using the best models found in Sections 4.3 and 4.4. The first predictions are made for January 2006 and the last ones for November 2008. According to the recent announcement of the NBER, the U.S. economy has been in business cycle recession since the beginning of the year 2008. In the case of the growth rate cycle periods, it is, however, not evident that there has been a constant downswing state (i.e. a growth recession, $y_{2t} = 1$) after

January 2006. Thus, at the time this paper is written, the realized values of y_{2t} are not known after January 2006.

Figures 5 and 6 depict the nowcasts from Models 1 and 4. The probabilities in Figure 6 are based on the models including the additional dummy variable $\mathbf{1}_{71-84}$. The evidence appears to be ambiguous between different models. It seems that Model 1 produces greater business cycle recession probabilities than Model 4 at the beginning of the recession in January 2008 (see Figure 5). On the other hand, in both cases, Model 1 gives higher "false" recession risks than model 4 for some months before the recession started. Overall, the employed models seem to nowcast the beginning of the recession period at 2008 reasonably well.

As discussed above, the latest values of the growth rate cycle indicator are unknown, that it is difficult to make comparisons between different models with the currently available information. All in all, it seems that the growth rate recession probability have been decreasing from mid-2006. As the business cycle recession started at the beginning of 2008, it is likely that the U.S. economy has been in a growth recession some months before. For those months, the predicted probabilities in Figure 5 are quite high and exceed the 50% threshold value, indicating a growth rate cycle recession. Note also that a decreasing probability of growth rate recession affects the business cycle recession probability in Model 4. This is a potential reason why the business cycle recession probabilities in the left panels of Figures 5 and 6 are lower than in the independent univariate autoregressive model (Model 1).

5 Conclusions

In this paper, we introduce a new bivariate time series model for binary dependent variables. The bivariate autoregressive probit model is a bivariate extension of the univariate autoregressive model of Kauppi and Saikkonen (2008), but is can also be considered a dynamic extension of the static bivariate probit model of Ashford and Sowden (1970).

The bivariate autoregressive probit model, and its special cases, are applied to predict the current state of the U.S. economy using binary indicator variables for the level and the growth rate of the U.S. economic activity. The proposed bivariate model framework extends the traditional univariate analysis of business cycle recession periods examined in the previous literature, where only business cycle recessions have been considered.

We found strong in-sample and out-of-sample evidence in favor of the bivariate autoregressive probit model proposed in the paper. The results suggest that it is possible to gain additional predictive power by modeling the recession probabilities of the two cycle indicators jointly instead of considering two independent univariate autoregressive probit models. The lagged first difference of the Federal funds rate is the most useful single predictor of the state of the growth rate cycle, but also monthly stock market returns turn out to be statistically significant predictors for both cycle indicators. As suggested in previous studies, a term spread between the long-term and short-term interest rates is an important explanatory variable for business cycle recession periods, but it turned out not to be the best predictive variable for the growth rate cycle periods. We also found evidence that the probability of growth rate recession was systematically higher in the 1971–1984 period than after the mid-1980s.

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Tables and Figures

Table 1: Dependent variables and the cross-tabulation of realized values.

		y_{2t}	
		0	1
y_{1t}	0	162	205
	1	5	48

Notes: U.S. business cycle periods y_{1t} (recession/expansion) and growth rate cycle periods y_{2t} (growth recession/ growth rate expansion) are obtained from http://www.nber.org/cycles/cyclesmain and http://www.businesscycle.com. The sample is 1972 M1–2005 M12.

Table 2: Explanatory variables.

r_t	Stock market return, log-difference of the S&P500 index
FF_t	Federal funds rate
i_t	The 3-month Treasury bill rate, secondary market
R_t	The 10-year Treasury bond yield rate, constant maturity
SP_t	The term spread, $R_t - i_t$
ΔFF_t	The first difference in the Federal funds rate
Δi_t	The first difference in the 3-month Treasury bill rate
ΔR_t	The first difference in the 10-year Treasury bond yield
SP_t^{GE}	The German term spread between the long-term and short-term interest rate.

Notes: Interest rates are from Federal Reserve Statistical Release Historical Data (http://www.federalreserve.gov/releases/h15/data.htm). S&P500 stock market index is taken from Yahoo Finance (http://finance.yahoo.com) and from http://www.econstats.com. German term spread is constructed as the difference between 10-year Federal security (series WZ9826, the missing values between 1971 M1-1972 M9 are replaced by the OECD 10-year interest rate) and the three-month money market rate (series su0107, see http://www.bundesbank.de/statistik/statistik).

Table 3: BIC values for Model 1 with different explanatory variables for the growth rate cycle indicator.

k	r_{t-k}	FF_{t-k}	i_{t-k}	R_{t-k}	SP_{t-k}	ΔFF_{t-k}	Δi_{t-k}	ΔR_{t-k}
1	308.77	312.76	315.26	331.25	300.52	335.83	335.83	335.41
2	314.75	312.20	314.66	330.95	300.51	280.41	286.39	335.48
3	319.02	314.11	314.92	330.27	304.30	280.34	282.19	336.05
4	321.64	317.55	318.36	330.21	313.58	289.94	286.34	335.90
5	327.01	321.22	320.82	330.55	318.26	298.25	286.87	319.66
6	331.01	323.71	323.53	330.80	323.52	304.70	308.30	325.52

Notes: Explanatory variable included in $\boldsymbol{x}_{2,t-k}$ and its lag k are mentioned in the first row and the first column of the table. Schwarz information criterion, BIC, is defined in (21).

Table 4: BIC values for Model 1 with the lagged first difference of the Federal funds rate and other explanatory variables for the growth rate cycle indicator.

	k	r_t	FF_t	i_t	R_t	SP_t	Δi_t	ΔR_t
ΔFF_{t-2}	1	269.75	277.56	277.49	276.91	283.39	281.06	278.26
	2	273.18	277.49	277.31	276.54	303.51	283.08	280.49
	3	278.27	277.49	277.25	276.26	283.28	282.00	282.94
	4	277.05	277.64	277.35	276.11	283.20	280.28	283.30
	5	278.38	277.88	277.57	276.12	283.11	276.31	282.87
	6	282.23	278.19	277.97	276.23	282.94	281.82	282.66
ΔFF_{t-3}	1	270.27	278.18	277.90	277.49	283.32	282.57	280.85
	2	274.09	278.43	278.08	277.22	303.44	281.84	281.95
	3	278.64	278.62	278.73	276.84	307.16	282.85	282.64
	4	278.61	278.62	278.30	276.83	282.92	283.23	283.24
	5	279.22	278.63	278.30	276.55	282.87	280.26	283.34
	6	282.07	278.78	278.51	276.53	282.72	282.28	282.93

Notes: The employed predictive variables in $\boldsymbol{x}_{2,t-k}$ are the second, or the third, lag of the first difference of the Federal funds rate and variable mentioned in the first row of the table. See also Notes to Table 3.

Table 5: Estimation results of different bivariate models.

model	variable	Model 1	Model 2	Model 3	Model 4
π_{1t}	constant$_1$	0.06	0.06	-0.07	-0.16
		(0.03)	(0.03)	(0.05)	(0.05)
	$\pi_{1,t-1}$	0.85	0.86	0.88	0.86
		(0.02)	(0.02)	(0.01)	(0.01)
	$\pi_{2,t-1}$			0.17	0.18
				(0.05)	(0.05)
	SP_{t-6}	-0.19	-0.16	-0.08	-0.06
		(0.03)	(0.04)	(0.03)	(0.02)
	r_{t-1}	-0.11	-0.10	-0.10	-0.10
		(0.02)	(0.02)	(0.02)	(0.02)
	SP^{GE}_{t-6}	-0.08	-0.07	-0.13	-0.11
		(0.03)	(0.02)	(0.03)	(0.03)
π_{2t}	constant$_2$	0.07	0.07	-0.02	-0.05
		(0.01)	(0.01)	(0.01)	(0.01)
	$\pi_{2,t-1}$	0.91	0.91	0.92	0.96
		(0.02)	(0.01)	(0.01)	(0.01)
	$\pi_{1,t-1}$			-0.03	-0.04
				(0.01)	(0.01)
	ΔFF_{t-2}	0.51	0.51	0.34	0.31
		(0.18)	(0.06)	(0.06)	(0.05)
	r_{t-1}	-0.03	-0.03	-0.06	-0.06
		(0.01)	(0.01)	(0.01)	(0.01)
	ρ		0.58		0.53
			(0.21)		(0.21)
logL		-242.70	-240.31	-223.38	-217.94
BIC		269.75	270.36	256.44	254.01
QPS_1		0.061	0.063	0.061	0.062
QPS_2		0.340	0.330	0.302	0.284
$CR_1^{50\%}$		0.963	0.961	0.961	0.963
$CR_2^{50\%}$		0.713	0.723	0.770	0.789
LM^{ρ}		8.58		27.21	
p-value		0.000		0.000	

Notes: The models are estimated using the data from 1971 M1 to 2005 M12. The first 12 observations are used as initial values. Standard errors of the estimated coefficients are given in parentheses. Estimated values of the log-likelihood function (11), logL, and Schwarz (1978) information criteria, BIC, are reported, as well as the values of quadratic probability scores, QPS_j, where $j = 1, 2$. Further, $CR_j^{50\%}$ indicate the ratio of correct prediction with using the 50% threshold value in the classification of probability forecasts. Lagrange Multiplier test statistics LM^{ρ} (see (18) for the null hypothesis (16) and the corresponding p-values are also reported.

Table 6: Estimation results of different models with the additional dummy variable $\mathbf{1}_{71-84}$.

model	variable	Model 1	Model 2	Model 3	Model 4
π_{1t}	constant$_1$	0.09	0.07	0.08	0.03
		(0.05)	(0.05)	(0.04)	(0.04)
	$\mathbf{1}_{71-84}$	-0.04	-0.00	-0.25	-0.21
		(0.06)	(0.05)	(0.09)	(0.08)
	$\pi_{1,t-1}$	0.85	0.86	0.89	0.88
		(0.02)	(0.02)	(0.01)	(0.01)
	$\pi_{2,t-1}$			0.11	0.11
				(0.04)	(0.03)
	SP_{t-6}	-0.20	-0.16	-0.11	-0.10
		(0.04)	(0.04)	(0.03)	(0.02)
	r_{t-1}	-0.11	-0.09	-0.09	-0.09
		(0.04)	(0.02)	(0.02)	(0.02)
	SP^{GE}_{t-6}	-0.08	-0.08	-0.12	-0.11
		(0.03)	(0.03)	(0.03)	(0.02)
π_{2t}	constant$_2$	0.06	0.06	-0.07	-0.09
		(0.01)	(0.01)	(0.02)	(0.02)
	$\mathbf{1}_{71-84}$	0.06	0.06	0.09	0.08
		(0.02)	(0.02)	(0.03)	(0.02)
	$\pi_{2,t-1}$	0.91	0.91	0.93	0.94
		(0.01)	(0.01)	(0.01)	(0.01)
	$\pi_{1,t-1}$			-0.04	-0.04
				(0.01)	(0.01)
	ΔFF_{t-2}	0.55	0.55	0.40	0.38
		(0.07)	(0.06)	(0.08)	(0.07)
	r_{t-1}	-0.03	-0.03	-0.08	-0.07
		(0.01)	(0.01)	(0.01)	(0.01)
	ρ		0.59		0.77
			(0.22)		(0.15)
logL		-235.15	-232.76	-198.63	-192.46
BIC		268.21	268.83	237.71	234.54
QPS_1		0.061	0.063	0.058	0.059
QPS_2		0.327	0.310	0.256	0.254
$CR_1^{50\%}$		0.963	0.961	0.958	0.958
$CR_2^{50\%}$		0.725	0.740	0.816	0.826
LM^ρ		8.14		26.21	
p-value		0.000		0.000	

Notes: See notes to Table 5. Variable $\mathbf{1}_{71-84}$ indicates a variable which takes value one at period from 1971 M1 to 1984 M12, and zero otherwise.

Table 7: Out-of-sample performance of different models.

	QPS_1	CR_1	QPS_2	CR_2
Model 1 with $\mathbf{1}_{71-84}$	0.395	0.722	0.395	0.708
Model 2 with $\mathbf{1}_{71-84}$	0.435	0.722	0.395	0.708
Model 3 with $\mathbf{1}_{71-84}$	0.167	0.889	0.303	0.833
Model 4 with $\mathbf{1}_{71-84}$	0.130	0.903	0.307	0.847
Model 1	0.179	0.833	0.446	0.625
Model 2	0.228	0.819	0.446	0.625
Model 3	0.094	0.931	0.446	0.625
Model 4	0.108	0.944	0.608	0.611

Notes: The out-of-sample values of QPS_j and CR_j, $j = 1, 2$, based on models on the left. Explanatory variables included in the model are the same as in the estimation results presented in Tables 5 and 6. Nowcasts from model 4 with the dummy variable $\mathbf{1}_{71-84}$ (the fourth model) are depicted in Figure 4.

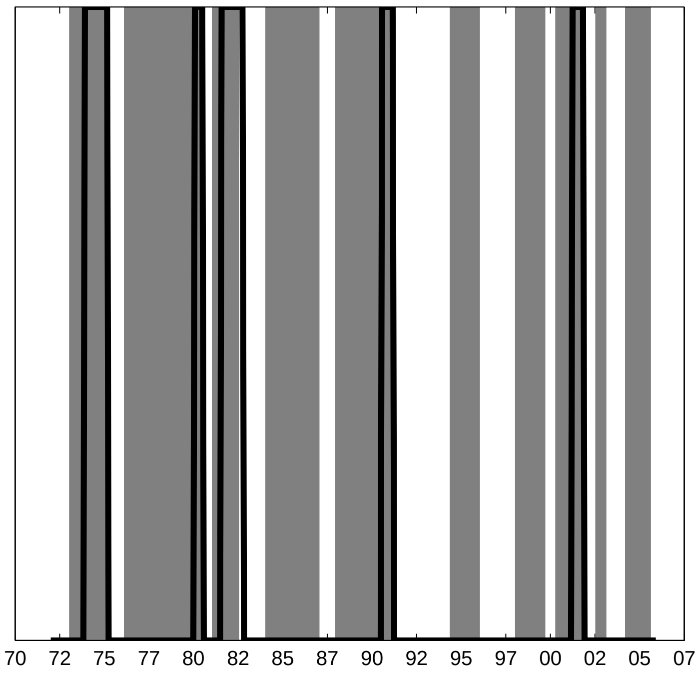

Figure 1: U.S. recession periods (line) since January 1972 until December 2005. Shaded areas indicate growth rate recessions (downswing periods in the growth rate cycle, i.e. $y_{2t} = 1$).

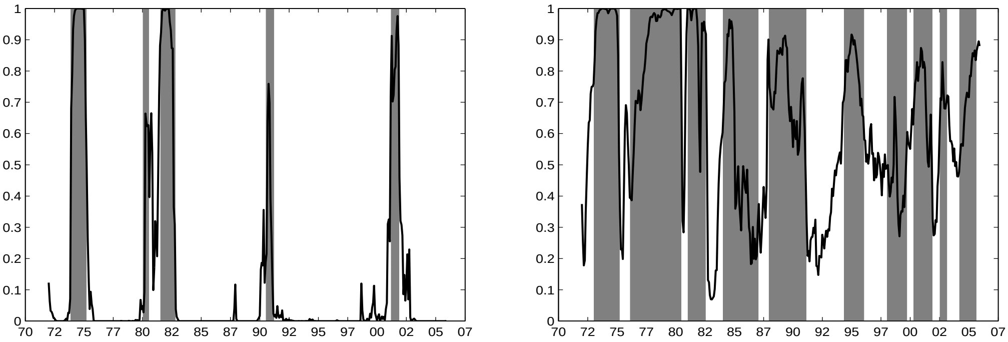

Figure 2: In-sample fitted values from the Model 1 presented in Table 6. Recession periods are depicted with shaded areas. Business cycle recession probabilities are presented in the left panel, probabilities for growth rate cycle periods in the right panel.

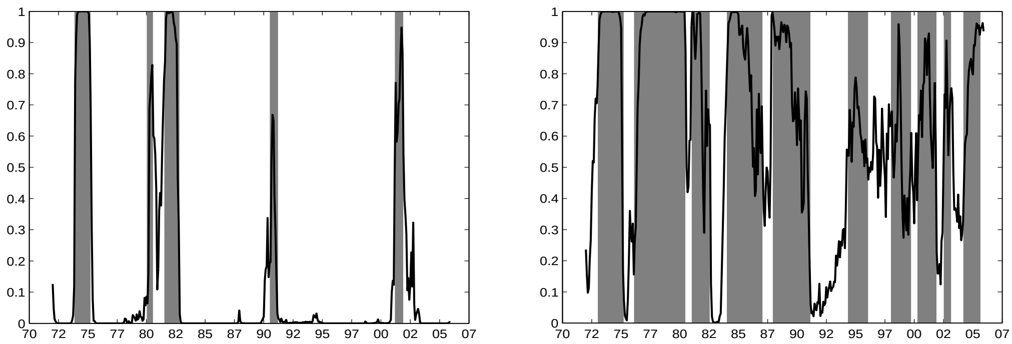

Figure 3: In-sample fitted values from the Model 4 presented in Table 6. Recession periods are depicted with shaded areas. Business cycle recession probabilities are presented in the left panel, probabilities for growth rate cycle periods in the right panel.

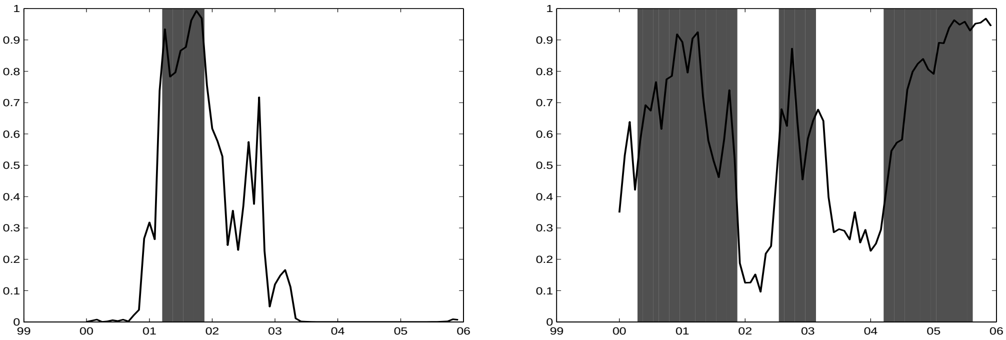

Figure 4: Out-of-sample nowcasts for business cycle recession (left panel) and growth rate cycle (right panel) recession periods (shaded areas) using the bivariate autoregressive probit model (Model 4) which contains also the dummy variable $\mathbf{1}_{71-84}$ given in Section 4.3.

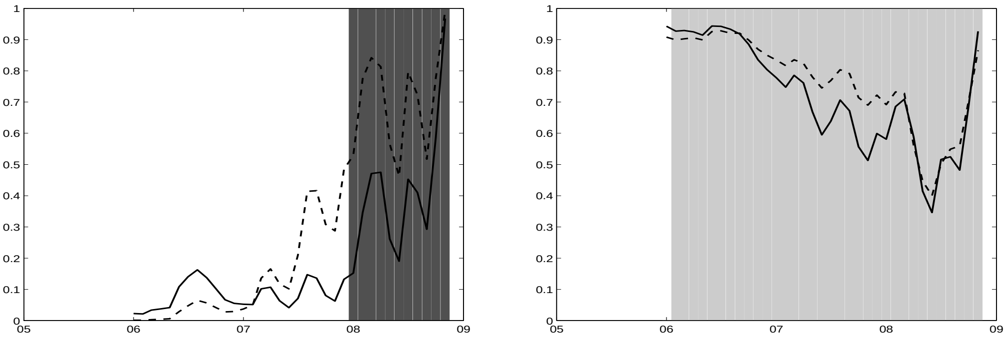

Figure 5: Real-time predictive probabilities from Model 1 (dashed line) and from Model 4 for the business cycle (left panel) and the growth rate cycle periods (right panel) using the models described in Table 5. In the left panel, the business cycle recession that began at 2008 is depicted with shaded area. In the right panel, the shaded area corresponds to the time period of unknown values of the growth rate cycle indicator y_{2t}.

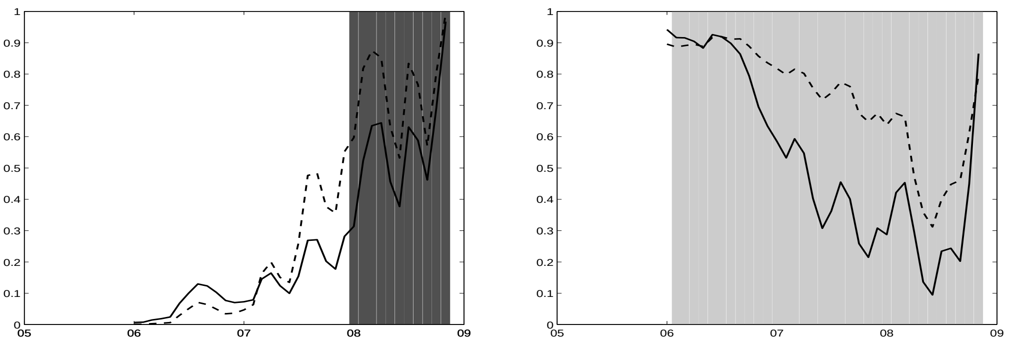

Figure 6: Real-time recession probabilities from Model 1 (dashed line) and from Model 4 for the business cycle (left panel) and the growth rate cycle periods (right panel) using the models described in Table 6. In the left panel the business cycle recession period that began at 2008 is depicted with shaded area. In the right panel, the shaded area corresponds to the time period of unknown values of the growth rate cycle indicator y_{2t}.