On the Relationship Between Stock Option Compensation and Equity Values: a Note

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Abstract

We present a model where increasing employee participation in stock option scheme leads to higher performance but with a cost to shareholders. We show that firms with higher market values per employee are more likely to have option schemes and they offer stock options to a broader group of employees. The model yields empirical predictions that are consistent with the stock option boom of late 1990s and their reduced popularity after the stock market downfall.

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1 Introduction

The use of equity compensation schemes, such as employee stock ownership plans or broad-based stock options, has increased substantially during the past 20 years or so (Blasi et al., 2003). Consequently, there has appeared a number of papers to explain the firm-level determinants of the use of these schemes (e.g. Kruse, 1996; Core and Guay, 2001; Ittner et al., 2003). In this note, we contribute to this literature by proposing a novel hypothesis on how the market value of the firm relative to the number of employees is related to the allocation of equity incentives.

Our argument is that stock options are less expensive to use in firms where the market value per employee is high. The model is based on the idea that even when employee participation in equity compensation schemes may be good for performance, increased participation also implies increased dilution costs to shareholder, and therefore it may be in the interest of shareholders to limit participation in the program. However, with the same relative increases in share prices, firms with higher absolute levels of market value per employee are able to offer the same incentives with lower dilution than those with lower levels of market value per employee. This hypothesis has testable empirical implications both at the level of the firm and the stock exchange.

2 The model

There are two time periods in the model, 0 and 1. \( MV_0 \) and \( MV_1 \) denote market values of equity at time 0 and 1, respectively, \( S_0 \) and \( S_1 \) denote the number of shares at respective points of time, and \( Opt_0 \) denotes the number of options the firm distributes at time 0. The options are distributed free of charge. There are two types of players in the model. Share holders own \( S_0 \) shares (all the shares at time 0) and control the decision-making, so an option scheme diluting their ownership share can only be made with their consent. The option holders are a subset of employees, to whom to share holders decide to grant options. The option holders can exchange one option for one share at time 1 for a price of \( \frac{MV_0}{S_1} \), which is paid to the shareholders. The number of shares at time 1 equals the initial number of shares plus the shares subscribed by options if the market value at time 1 exceeds the market value at time 0 (\( S_1 = S_0 + Opt_0 \text{ if } MV_1 > MV_0 \)). In the case where the market value at time 1 equals or is lower than market value at time 0 (\( MV_1 \leq MV_0 \)), option holders do not want to exercise their options and the number of shares is the same at both times (\( S_1 = S_0 \)).
In the case where the market value increases \((MV_1 > MV_0)\), the pay-off for option holders collectively is \(\frac{S_1 - S_0}{S_1} (MV_1 - MV_0)\). The shareholders, in turn, receive \(\frac{S_0}{S_1} (MV_1 - MV_0)\). If \(MV_1\) equals \(MV_0\) or is lower, then the option holders do not exercise their options and their gain is 0.

In our model we assume that higher participation rates are related to higher levels of performance with a concave function. To motivate the concavity of the performance function, suppose that the stock option schemes are always filled in the order of hierarchical position, beginning from the top of the organization. The concavity implies that the marginal performance impact decreases when new, lower-level employees enter the scheme. The concave shape of the function is supported by the standard notion that stock option schemes provide stronger incentives to managers whose actions can have higher impact on share prices.

We assume that shareholders expect higher employee participation rates to be associated with higher performance, measured by the growth of the firm market value. The participation rate \((\pi)\) is the ratio between the number of persons participating to the scheme \((N)\) and the total number of employees \((L)\) and is thus given by the expression:

\[
\pi = \frac{N}{L}
\]  

In the following, we are going to assume that the number of employees is fixed, so that the choice variable is the number of persons participation to the scheme \((N)\).

Assume that the higher performance from higher participation rate \((\pi)\) translates into higher share returns \((r)\), so that the participation rate \(\pi\) is one of the arguments determining the growth of the market value. The increase in share price during this period includes a random component \(\epsilon\) and a component attributable to a higher performance due to the stock option scheme, depending on the participation rate \(\pi\). The relationship between the growth of the market value and participation rate is represented by a concave function:

\[
MV_1 - MV_0 = r(\pi, \epsilon)MV_0, r_\pi > 0, r_\epsilon < 0
\]  

We augment expression (2) by noting that the growth of the market value is not going to depend only on the existence of the option plan, but the plan has to be sizable enough to give meaningful incentives to each employee. We assume that for each employee there is a minimum threshold of option incentives that has to be reached before they have any effect. To keep things simple, we assume that once this threshold is reached firm market value increases by a certain amount as stipulated in expression (2) and that further increases in option incentives do not increase
market value. Equity incentives below the threshold do not produce any effect on market value. The optimal amount of stock options for a given individual is either equal to the threshold or zero.

The expected gain of an option holder is the expected growth rate in market value \( E(r) \) times the market value at time 0, multiplied by the size of his holdings relative to the total stock outstanding \( \frac{S_i}{S_1} \). If \( E(r) \) is zero or negative then no option scheme will be launched.

Assume that the threshold level required for the incentive effects can be written as a fraction \( \beta_i \) of the wages, and \( \beta \) is assumed to be equal for all employees \( (\beta_i = \beta \forall i = 1, \ldots, N) \). Thus, we have the following expression:

\[
\frac{S_i}{S_1} E(r)MV_0 = \beta w_i, \quad E(r) > 0, \quad 0 < \beta < 1.
\]

To abstract from subjective valuations of options, assume that both employees and outside investors are risk neutral and share the same expectations about the development of share price. Next, we will derive the relationship between total dilution and participation rate.

Recall that employees will join the option scheme in order of their hierarchical position in firm organization. Assuming that there is a positive, monotonic relationship between wages and hierarchical position, we find that the sum of wages must be a concave function: when we start summing up wages from the top of the firm organization, \( W(N) \) increases, but each addition is smaller or equal than the previous one, since we add employees with lower hierarchical position. Although the number of employees is discrete, we can approximate the sum of wages as a continuous function and thus write it as an integral of individual wages:

\[
W(N) = \int_1^N w(x) dx, W_N > 0, W_{NN} < 0
\]

Using (3) and (4), we can now write the total dilution as a function of \( N \):

\[
\frac{S_1 - S_0}{S_1} = \frac{\beta W(N)}{E(r(\pi, \varepsilon))MV_0} \quad \text{where } E(r) > 0 \quad \text{and} \quad \frac{\partial}{\partial N} S_1 - S_0 > 0.
\]

Note that if the denominator would increase faster than the numerator in the expression (5), then increasing the number of participants in the stock option program would actually decrease the total dilution. In such a case, increasing participation rate would always be optimal. However, we
concentrate on the more interesting case and assume that the dilution is always strictly increasing in the number of participants.

The shareholders’ problem is then to choose the optimal participation rate \( \pi \) knowing that increasing the number of participants in the stock option program increases share returns but it also increases the costs of the program in the form of dilution. This can be written as

\[
\max_{\pi} \frac{S_0}{S_1} E(r(\pi, \varepsilon)MV_0)
\]  

(6)

Substituting from (5) we obtain

\[
\max_{\pi} E(r(\pi, \varepsilon))MV_0 - \beta W(N)
\]  

(7)

Taking the first order conditions of (7) with respect to \( \pi \) and rearranging we obtain

\[
E(r) \frac{MV_0}{L} = \beta W_N
\]  

(8)

The left-hand side of the expression (8) can be interpreted as the marginal benefit from increasing the participation rate, while the right-hand side may be interpreted as marginal cost of increasing the number of participants to the stock option scheme (dilution effect). Regarding the optimal participation rate \( \pi^* \), four different cases can be distinguished:

1) The marginal cost of including the first employee (top manager) to the scheme is larger than the marginal benefit, and the marginal cost of including additional employees to the scheme increases faster than the marginal benefit. More formally:

\[
E(r) \frac{MV_0}{L} < \beta W_N \text{ for } N = 1 \text{ and } E(r) \frac{MV_0}{L^2} < \beta W_{NN}
\]

In this case, the optimal participation rate is clearly zero, since it does not pay to include the first employee, and second derivative shows that the costs of the scheme exceed the benefits for all other employees as well.

2) The marginal cost of including the top manager to the scheme is larger than the marginal benefit, but the marginal benefits increase faster than marginal costs:

\[
E(r) \frac{MV_0}{L} < \beta W_N \text{ for } N = 1 \text{ and } E(r) \frac{MV_0}{L^2} > \beta W_{NN}
\]
In this case, the expression (8) does not give the maximum gain from an option scheme but rather the minimum where the losses from the scheme are largest. Thus initially the total cost of the scheme exceeds the total benefits. However, once the marginal benefit exceeds the marginal cost the net cost of the scheme starts to fall, and each additional employee has a positive impact on the profitability of the scheme (this follows from the assumptions of strict concavity). The optimal participation rate is 1 if the expected outcome under full participation is larger than expected outcome under no participation (i.e. \( E(r(L, \varepsilon)) MV_0 - \beta W(L) > E(r(0, \varepsilon) MV_0) \)), and 0 in the reverse case.

3) The marginal cost of including the top manager to the scheme is smaller than the marginal benefit, but the marginal costs increase faster than marginal benefits:

\[
E(r) \frac{MV_0}{L} > \beta W_N \text{ for } N = 1 \text{ and } E(r) \frac{MV_0}{L^2} < \beta W_{NN}
\]

This is the case where an interior solution to the equality (8) is likely to appear, unless the marginal benefit of including the last employee exceeds marginal cost (i.e. \( E(r) \frac{MV_0}{L} > \beta W_N \) for all \( N \leq L \)), in which case the optimal participation rate is 1. Thus the optimal participation rate in this case is between \( \frac{1}{L} \) and 1 (endpoints included).

4) The marginal cost of including the top manager to the scheme is smaller than the marginal benefit, and the marginal benefits increase faster than marginal costs:

\[
E(r) \frac{MV_0}{L} > \beta W_N \text{ for } N = 1 \text{ and } E(r) \frac{MV_0}{L^2} > \beta W_{NN}
\]

For reasons stated above, it is clear that in this case the optimal participation rate is 1.

The different cases are summarised in Table 1. It is noteworthy that an interior solution is observed only in the case 3). However, since we often observe interior solutions in practice, this is perhaps the most relevant case.

**TABLE 1 AROUND HERE**

Finally, we show that higher market values per employee increase participation rates.

Using the implicit function rule \( \frac{\partial \pi^*}{\partial \frac{MV_0}{L}} = -\frac{F_{MV_0}}{F_{\pi}} \) we obtain from (8)
Let us first discuss the case where $\pi^*$ has an interior solution. We know that this can only happen when the marginal cost increases faster than the marginal benefit, so the denominator in (9) must be negative and therefore the expression (9) is positive. Thus, an increase in market value per employee increases the optimal participation rate $\pi^*$.

If $\pi^* = 0$ (case 1 or 2), an increase in $\frac{MV_0}{L}$ increases the probability that it pays to include the top manager into the scheme (i.e. $E(r)_x \frac{MV_0}{L} > \beta W_N$ for $N = 1$), so that the regime would shift from 1 to 3 and from 2 to 4. In other words, the likelihood that a firm not having an option scheme would adopt one increases when $\frac{MV_0}{L}$ increases. Moreover, in the case 2 an increase in $\frac{MV_0}{L}$ would reduce $\pi^*$. Since $\pi^*$ is in this case a minimum, this means that the total gains from option scheme would increase and the likelihood of adopting a scheme would increase, even if the firm remains at regime 2.

Finally, if $\pi^* = 1$ (case 4) an increase in $\frac{MV_0}{L}$ cannot increase the participation rate since it is already at maximum.

3 Conclusion
Our results suggest that the use of option compensation should be related to the firm market value per employee. This finding has interesting empirical implications. First, at the firm level those firms having higher market value per employee are more likely to use stock options in their compensation package. They are also more likely to target options to broader group of employees. Second, our model fits well the common observation that broad-based stock options became common during the stock market upheaval of the late 1990s. It also predicts that during stock market downturns, firms target their stock options to a more select group of employees and some firms cease to issue options altogether. There is preliminary evidence from the Finnish stock market supporting these hypotheses (Jones et al., 2004). Our results suggest that future research would benefit by paying
more attention to the role of equity values as potential determinants of the use of equity compensation.

References


Table 1. The determination of optimal participation ratio

<table>
<thead>
<tr>
<th>$E(r) \frac{MV_0}{L^2} &lt; \beta W_{NN}$</th>
<th>$E(r) \frac{MV_0}{L^2} &gt; \beta W_{NN}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r) \frac{MV_0}{L} &lt; \beta W_N$ for $N = 1$</td>
<td>$\pi^* = 0$</td>
</tr>
<tr>
<td>$E(r) \frac{MV_0}{L} &gt; \beta W_N$ for $N = 1$</td>
<td>$\pi^* = \left[ \frac{1}{L}, 1 \right]$</td>
</tr>
</tbody>
</table>

Notice that in differentiating the wage function we used the expression (1) and the chain rule $\frac{dW}{d\pi} = (\frac{dW}{dN}) \ast (\frac{dN}{d\pi})$. 