Discussion Papers

Political Parties and Network Formation

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Abstract

The main point of this paper is that anti-corruption laws may provide an efficiency rationale for why political parties should meddle in the distribution of political nominations and government contracts. Anti-corruption laws forbid trade in spoils that politicians distribute. However, citizens may pay for gaining access to politicians and, thereby, to become potential candidates for nominations. Such rent-seeking results in excessive network formation. Political parties may reduce wasteful network formation, thanks to their ability to enter into exclusive membership contracts. This holds even though anti-corruption laws bind also political parties.

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1 Introduction

Anti-corruption laws forbid trading attractive nominations made by politicians. But even though interested citizens are not able to buy such spoils, it pays off to be in good terms with the politicians. Citizens buy tickets to fund-raising events and spend time with politicians to be remembered when nominations are made. If there are no restrictions on whom politicians can allocate jobs to, citizens can gain by rubbing shoulders with several politicians. This is time-consuming both for citizens and for politicians and results in wasteful network formation. Here, political parties can step in.

Political parties are powerful gatekeepers in modern democracies. First, citizens looking for positions of trust, and politicians allocating these, cannot belong to more than one party. Second, political parties can require their politician members to pass the nominations to other party members. Thus, parties can reduce wasteful network formation between citizens and politicians thanks to their ability to sign exclusive membership contracts. On the other hand, there is an additional linking cost when parties are present: parties also need to build links to politicians.

We compare network formation with and without the role of the political parties. We take as our starting point that political parties exist and that politicians distribute non-ideological spoils, such as make nominations to positions of trust or hire civil servants who do not make political decisions. We ask whether the gatekeeping role of the political parties improves efficiency in the distribution of such spoils. We also ask who gains and who loses from the parties meddling in the network formation.

In our model, the party has full control of the network formation within the party. We focus on the networks used to allocate non-ideological spoils; for concreteness think of positions of trust in a municipality. In addition to providing exclusivity, political parties allocate each joining member to a unique politician belonging to the party. Political parties first pay politicians for joining and then sell links to politicians to rent-seeking citizens. In equilibrium, parties, politicians, and citizens have rational expectations about the network structure. Equilibrium prices are determined by these expectations and inter-party competition.

Also in the no-party equilibrium, expectations about the equilibrium network structure are rational. The equilibrium prices are determined by competition between politicians on one side and citizens on the other side of the market. Given prices, politicians and citizens choose the number of links they sell and buy, respectively.

The more time citizens or politicians spend rubbing shoulders, the higher the opportunity cost of time spent doing so. Thus, linking costs are convex and, in equilibrium, the prices of links are determined by marginal costs.

We first derive the equilibrium networking structure and the equilibrium payoffs in the no-party and in the party equilibrium. We show that a no-party equilibrium always exists and that the existence of a party equilibrium depends on the parameters of the model. We then study the welfare properties of the equilibria. Finally, we study the
incentive compatibility of the party equilibrium by analyzing whether a single politician
would prefer not to join a party but rather to sell links directly to citizens without party
control.\footnote{1}

We find that the net effect of intermediaries on social welfare depends on networking
costs, the value of lucrative nominations, and the relative numbers of politicians, citi-
zens and parties. Therefore, welfare-maximizing government may well find it optimal
to promote the involvement of political parties in some cases, and actively discourage
it in others.

Notice that allocating one rent-seeking citizen to each politician would minimize the
cost of network formation. Hypothetically, parties have two ways to accomplish this:
either by selling upcoming nominations within the party or by allowing in only one citi-
zen per each nominating politician. Yet, so as to the selling policy, anti-corruption laws
bind also political parties. At the same time, pre-commitment not to take additional
members is prohibited since this would violate equal rights to political participation.\footnote{2}

We also find that politicians would be better off if political parties were not involved
in the distribution of rents while citizens may gain from their presence, despite the fact
that parties charge membership fees from citizens and transfer money to politicians!
Even so, a party equilibrium may be incentive-compatible in that no single politician
would find it optimal not to link with the party, provided that others do.

Our model has common features with several strands of literature. First, we suppose
that for a citizen to receive a rent from a project initiated by a politician, a connection
must be established between the two. This relates the current paper to the literature
on cooperative networks, pioneered by Jackson and Wolinsky (1996). However, in our
model agents may trade in the right to control network formulation. This differentiates
our model from Jackson and Wolinsky (1996).

Second, in our model, the citizens compete for rents initiated by politicians as
in rent-seeking and lobbying contests literature (Tullock (1967, 1980), Bernheim and
Whinston (1986), Baye, Kovenock and de Vries (1993) and Grossman and Helpman
(1994)). Yet, there are two major differences. In our model, links are costly for both
and endogenous, and require mutual consent. Payments are made in exchange for
establishing links. In rent-seeking and lobbying literature, links are exogenous and
costless, and payments are bids in an auction or in a contest. Throughout the analysis,
we assume that anti-corruption laws work\footnote{3}. Therefore, we have implicitly in mind
a modern democracy with relatively low level of corruption. Previous literature on
1We refer only to the dimension of allocating the non-ideological spoils. Indeed, the role of the
political parties in distributing positions of trust and governments jobs has varied both between
countries and over time.
2We could generate the motivation not to restrict network formation by assuming that some citizens
want to join parties to seek for rents, while others for ideological reasons. Those joining parties for
ideological reasons favor a no-restriction policy.
3The inability of politicians and citizens to trade nominations when these arise could result from
outside monitoring or from there being a fraction of honest citizens and politicians who would report
asking or offering bribes, provided that punishments for corruption are sufficiently high.
contests has already analyzed extensively the case where anti-corruption laws can be circumvented.

Third, our model is related to two strands of intermediation literature, the middlemen literature (Rubinstein and Wolinsky (1987)) on the one hand, and the literature on two-sided markets (Rochet and Tirole (2003), Armstrong (2004)) on the other hand. In our model, the intermediaries, or the platforms, are the political parties. Our model differs from these models most importantly in that the intermediary plays a useful role by restricting the activity between the two sides of its market.

Fourth, our explanation complements previous efficiency rationales for the prominent role of political parties. For example, Alesina (1988) and Alesina and Spear (1988) find that political parties may reduce policy fluctuations, compared with the case in which subsequent cohorts of competing politicians with different preferences would engage in one-shot electoral competition. Caillaud and Tirole (2002) show that political parties may make up voters’ information deficit by designing and endorsing electoral platforms. These previous contributions leave the puzzle of why intermediaries arise also in cases where they do not reduce the time spent on searching, provide additional information, or solve various commitment problems. For example, it is questionable to what extent a political party would provide new information when filling positions of trust or public jobs in a small municipality. Yet, even these positions and many other jobs are typically earmarked to different political parties. Our explanation for the role of a political party applies even in cases where these explanations fail.

There are two features which separate politics and political parties from other areas of intermediation. The first one concerns the legal framework. Anti-corruption laws restrict the ability of politicians and citizens to enter even into informal contracts on allocating projects in exchange of payments. In other areas of intermediation (like the market of goods and ideas) the two sides are typically allowed to enter into private contracts on the underlying goods or services, rather than just on linking together. The second difference is in the services that the intermediary provides. In previous literature, intermediaries are used only if these provide additional services in reducing search costs between the two sides of market, information revelation of the underlying good or economizing transaction costs. Political parties need not provide any of these services; the service they provide is exclusivity - one cannot be a member of several political parties.

The paper is organized as follows. Section 2 presents the model. Section 3 analyzes equilibrium payoffs and linking in the absence of parties, and section 4 in their presence. Section 5 explores whether politicians prefer a network with or without parties, and whether an eventual party equilibrium is incentive compatible. Section 6 presents a welfare comparison, and section 7 concludes.
2 Model

There are \( q \) agents of type \( w \), and there are three types \( w \in \{A, B, C\} \). Type \( A \) is called a politician, \( i = 1, ..., n_A \), type \( B \) is called a citizen, \( j = 1, ..., n_B \) and type \( C \) is called a party boss, \( k = 1, ..., n_C \). The politician receives a project with probability \( p \). The project generates a surplus \( s \) to the citizen who receives it. This surplus may be fixed by law, like salaries in political nominations and compensations for positions of trust.\(^4\)

Each politician is indifferent to which citizen to pass the project. Also, each citizen is indifferent from which politician she receives the project. Yet, for politician \( i \) to be able to pass the project to citizen \( j \), there has to be a direct link between them. Party bosses can connect to citizens indirectly, through politicians, but they need direct links to politicians. Note that our framework allows analyzing politics at different levels. At the national level, party bosses would be leaders of the national parties. At municipal level, they would be local leaders and politicians could be then, for example, members of the municipal council. Whichever the level, politicians require having a direct access to their party boss. We take the identity of party bosses as exogenous\(^5\). Citizens can be interpreted as individuals interested in positions distributed by elected politicians or they can be interpreted as representatives of corporations interested in public projects the allocation of which depends on decisions made by politicians.

The strength of a link between agents \( a \) and \( a' \) is denoted \( m_{aa'} \). Obviously, the strength of the link from \( a \) to \( a' \) must equal the strength of the link from \( a' \) to \( a \), thus, \( m_{aa'} = m_{a'a} \). Maintaining a link requires time, and may require other costs. A decreasing marginal productivity in other activities, or an increasing marginal utility of leisure, implies that the marginal cost of time spent on networking is increasing. Furthermore, networking with competing agents simultaneously may pose additional challenges. For example, a politician who seeks to extract contributions from competing contractors may need to spend more time in convincing these of the benefits of giving. To capture these features, we model the marginal costs of networking as increasing. A cost for \( i \) of maintaining number \( m \) of links is

\[
\frac{1}{2} cm^2
\]

where \( c \) is a positive cost parameter independent of player’s type.

The number of links in a network without political parties need not be an integer. Formally, the width of each link is between zero and one, zero implying no link at all and one implying a full link. The width of a link can be thought to be proportional to the time spent in maintaining the link. The probability weight that each link receives when politician allocates projects to citizens is proportional to the width of the link.

\(^4\)All the results would remain the same if also the politician would receive certain surplus, in case a project is fulfilled.

\(^5\)In a richer framework, we could model overlapping generations of politicians, with all young politicians being of type A and one of the old politicians becoming party boss in the second.
The width of potential links with political parties, on the other hand, is restricted to be either zero or one. This reflects the differences between the formal relationships with political parties and the informal relationships between citizens and politicians. We assume that the number of agents is sufficiently large that citizens never find it attractive to maintain full links to all politicians.

3 Networks without Political Parties

Assume that there are more citizens than politicians, \( 0 < n_A < n_B \). In this section, we assume that the political parties do not participate in the network formation between politicians and citizens looking for projects that they distribute. There are \( \gamma \) times more citizens than politicians, \( n_B = \gamma n_A, \gamma \in \{2,3,...\} \). If there are several citizens linked to a politician, we assume that the politician allocates the project randomly, so that the probability that each citizen is chosen is proportional to the width of the link. Thus the probability of getting the project from politician \( i \) equals

\[
p \frac{m_{ij}}{\sum_{\beta=1}^{n_B} m_{\beta i}}
\]

In the special case in which the width of each link is one, each citizen would then have an equal probability to receive the project.

A citizen has to pay politician \( i \) a reward, \( r_i \), for maintaining a full link. If the link is only partial, then the reward is reduced proportionally. Note that this is a gross price, and it has to compensate the politician for her marginal cost of linking. In addition to paying politicians \( r_i \) for maintaining links, citizens have to pay their own linking costs.

Citizens are able to pay politicians for networking even if they cannot pay for projects. For example, citizens interested in nominations can give campaign contributions to candidates or volunteer work. Explicitly requiring a politician to assign a nomination in exchange for such contribution, on the other hand, would be considered bribing and not lobbying. Citizens are only allowed to pay for access to the politician.

The expected payoff of a citizen reads\(^6\)

\[
\sum_j p \frac{m_{ij}}{\sum_i m_{ij}} s - \sum_i m_{ji} r_i - \frac{c}{2} (\sum_i m_{ji})^2
\]

The politician’s maximization problem is as follows

\[
\max_{m_{i1},...,m_{iB}} \left\{ \sum_j m_{ij} r_i - \frac{c}{2} (\sum_j m_{ij})^2 \right\}
\]

\(^6\)This formulation coincides with Tullock (1980) with \( R = 1 \). Yet, the decision variable is not cost of effort, but rather, the number of links and the cost of linking is not linear but convex in the number of links.
resulting in first order conditions
\[ r_i - c\left(\sum_j m_{ij}\right) = 0 \]
for \( j = 1, \ldots, n_B \). We assume that each citizen acts atomistically\(^7\) taking as given the reward \( r_i \) and the aggregate number of links that each politician has to citizens \( m_{iB} \) and maximizes
\[
\max_{m_i, \ldots, m_{nA}} \left\{ \sum_i p \frac{m_{ij}}{m_{iB}} s - \sum_i m_{ji} r_i - \frac{c}{2} \left(\sum_i m_{ji}\right)^2 \right\}
\]
Thus, the first order conditions write
\[
\frac{ps}{m_{iB}} - r_i - c \sum_i m_{ji} = 0. \tag{1}
\]
for \( i = 1, \ldots, n_A \).

It is easy to see that each politician is indifferent so as to which citizens are linked to her. She only cares about the aggregate amount of links to citizens, \( m_{iB} \). Also, as long as linking rewards and the aggregate amount of links, \( m_{iB} \), are equal across politicians, each citizen is indifferent so as to how to allocate the links between politicians. All that matters is her aggregate amount of links, \( m_{jA} \). Since linking cost functions are convex and they equal across politicians and citizens, the number of links equal across politicians, on the one hand, and across citizens, on the other hand. Equilibrium rewards equal the politicians’ marginal linking cost. Thus, citizens equate their own marginal linking cost and the equilibrium reward, \( c \sum m_{ji} + r^N_{BA} \), to the expected rent from linking, \( \frac{ps}{m_{AB}} \), where \( m^N_{AB} \) is the equilibrium mass of links from a politician to citizens and \( r^N_{BA} \) is the equilibrium reward. Furthermore, we denote \( m^N_{BA} \) the amount of links that a citizen has to politicians in equilibrium.

This allows us to derive the no-party equilibrium\(^8\). We summarize it as Proposition 1:

**Proposition 1** In the no-party equilibrium, the supply of links by each politician is
\[
m^N_{AB} = \sqrt{\frac{\gamma ps}{\gamma + 1}} c; \tag{2}
\]
the demand for links by each citizen is
\[
m^N_{BA} = \sqrt{\frac{ps}{\gamma (\gamma + 1)c}}; \tag{3}
\]
\(^7\)Appendix A shows how the equilibrium derived here corresponds to an equilibrium of a dynamic game. Notice, that the atomisticity implies that there is no strategic own-side membership externality effect (see Rochet and Tirole, 2004) even if such an effect is present in our model non-strategically.

\(^8\)We use the superscript \( N \) for the equilibrium values of endogenous variables in the no-party equilibrium. Similarly, superscript \( P \) is used for the party equilibrium in section 4.
and the reward for links from citizen to politician is

\[ r^N = \sqrt{\frac{c\gamma ps}{(\gamma + 1)}}. \]  \hspace{1cm} (4)

**Proof.** See appendix. □

It is now straightforward to calculate the equilibrium surpluses. These are summarized in proposition 2:

**Proposition 2** The equilibrium payoff for the politician is

\[ \pi_A^N = \frac{\gamma ps}{2(\gamma + 1)} \]  \hspace{1cm} (5)

which is increasing in \( p, s \) and \( \gamma \).

The expected equilibrium payoff for the citizen is

\[ \pi_B^N = \frac{ps}{2\gamma(\gamma + 1)} \]  \hspace{1cm} (6)

which is increasing in \( p \) and \( s \) and decreasing in \( \gamma \).

**Proof.** See appendix. □

The politician benefits from the scarcity of politicians, whereas the opposite holds for the citizen. As the relative number of citizens per politician increases, the demand of links of each citizen decreases but the demand per politician increases. Thus, due to marginal cost pricing and convex costs, the profit of the politician increases. Yet, for the citizen the strength of the link per each politician decreases and, thus, so does the probability of getting the project. Yet, the reward that needs to be paid is higher. Hence, the payoff for the citizen decreases.

Surprisingly, the cost of maintaining links enters neither the politician’s surplus nor that of the citizen\(^9\). This implies that the efficiency gains due to reduction of the cost of maintaining links are wasted in additional network formation.

Notice that for any parameter values of the model the equilibrium number of links is positive and both the citizen and the politician receive a positive surplus. This implies the following corollary that establishes the existence of no-party equilibrium.

**Proposition 3** For any feasible parameter values of the model, there exists a no-party equilibrium.

\(^9\)This result is not in the core of our analysis. It may be due to functional forms and may not be robust to other specifications.
4 Networks with Political Parties

In this section, we introduce political parties as intermediaries that join together politicians and citizens. The service that the parties provide turns out to be exclusivity of links: no politician or citizen can link to a member of another party. This reduces wasteful multiplication of links to each politician. Yet, there are costs to this as well, since each citizen must now link to the political party in addition to the politicians.

We now have also C types who serve as intermediaries. Party bosses exert control rights over their party (or its local unit in the municipality interpretation). They maximize the party’s surplus, net of their own linking costs. We do not take stance whether party bosses would keep the surplus, or part of it, for private consumption, or if they use the surplus for ideological purposes. Assume that there are \( \phi \) politicians per each party boss, where \( \phi \in \{2, 3, \ldots\} \). Therefore, \( n_A = \phi n_C \) and as \( n_B = \gamma n_A \), \( n_B = \gamma\phi n_C \).

**Assumption 1.** Each citizen pays a reward, \( r_{BC} \), to the party boss and each politician gets a reward, \( r_{CA} \), from the party boss. The party boss receives the right to control and design the network of all the politicians and citizens linked with it on the condition that the party bears all the linking costs.\(^{10}\)

**Assumption 2.** Political parties, represented by party bosses, sign exclusive contracts that state that citizens linked to them are not able to sign up with other political parties. The political parties cannot commit not to sign contracts with additional politicians and citizens.

**Assumption 3.** When citizens make their linking decisions, they know how many politicians belong to each party.

**Assumption 4.** Keeping up the links between politicians and citizens requires effort or resources, like in the case with no political parties. Also, links between party bosses and politicians require maintaining.

**Assumption 5.** The party bosses play the active role, making take-it-or-leave-it offers to the politicians and citizens.

**Assumption 6.** There must be a direct link between the politician and the citizen who carries out the project initiated by the politician.

There is an indirect link between \( a \) and \( a' \), when there is a third agent \( a'' \) with whom both \( a \) and \( a' \) are linked to. We denote an indirect link between \( a \) and \( a' \) by \( \mu_{aa'} \). We focus on an equilibrium where all politicians and citizens are party members - every politician would have an incentive to require politicians to build more links \textit{ex post} than they have agreed \textit{ex ante}.

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\(^{10}\)If the party would not bear the linking costs, then party boss would have an incentive to require politicians to build more links \textit{ex post} than they have agreed \textit{ex ante}. 

8
politician and every citizen has a direct or an indirect link to a party. For notational simplicity, the number of links that a agent of type $t$ has to $t'$ types is denoted by the same variable for all agents of the same type. This is restrictive in general but, as in the no-party equilibrium, it turns out to be a a property of any equilibrium: all agents of the same type have equal number of links and pay and receive equal payments. There will be no direct payments between politicians and citizens since the party regulates links. Yet, by assumption 6, the structure of the network within a party always includes direct links between politicians and citizens.

The party $i$’s profits is

$$
\pi_C(m_{CA}, m_{CB}) = (\mu_{CB} + m_{CB})[r_{BC} - \frac{c}{2}(m_{BA} + m_{BC})^2] - (m_{CA} + \mu_{CA})[r_{CA} + \frac{c}{2}(m_{AB} + m_{AC})^2] - \frac{c}{2}(m_{CA} + m_{CB})^2
$$

By assumption, there must be a direct link between the politician and the citizen who carry out the project. This being the case, the optimal structure of the network is such that a party boss constructs a network where the party boss is directly linked to the politicians and each politician is linked directly to citizens. There are no direct links between the party boss and the citizens. Thus, (7) reduces to

$$
\pi_C(m_{CA}, m_{CB}) = \mu_{CB}[r_{BC} - \frac{c}{2}] - m_{CA}[r_{CA} + \frac{c}{2}(m_{AB} + 1)^2] - \frac{c}{2}(m_{CA})^2
$$

Here $r_{BC}$ and $r_{CA}$ are payments received or paid by the party for each link to citizens and to politicians. These may generally be positive or negative. A payment is made independently of the type of the link but a cost of link is born only from direct links. The party carries out the politician’s linking costs. This is the last term of the party’s payoff.

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11In section 5.2, we study whether a politician has an incentive not to join a party and link to citizens without the control of the party. We do not consider equilibria in which the value of the project is so low that not all citizens are willing to link to a party even if all politicians are party members.

12Note that often citizens pay to the party in the form of volunteer work. Our framework could be generalized to allow for this, without changing the qualitative results. Then the interpretation would be the following. Each citizen pays to the party in the form of work up to the point in which the marginal cost for citizen equals the marginal benefit for the party. The difference between the monetarized value of this efficient work effort and the equilibrium payment is settled in money. Party bosses may then let citizens (or part of them) to work directly for the politician, thus transferring part of the compensation to the politician in kind.
Equally, the citizen’s expected payoff is
\[ \pi_B = p_B s - r_{BC} \]

The probability that a given link results in a project is
\[ p_B = p \frac{m_{CA}}{m_{CB}} \]

Finally, a politician linked to a party receives a surplus equalling
\[ \pi_A = r_{CA}. \]

In the party equilibrium, each citizen and each politician is linked to a party. The next propositions 4 to 6 characterize party equilibria

**Proposition 4** In any party equilibrium, each party boss is linked with \( \phi \) politicians and \( \phi \gamma \) citizens. The number of direct links are \( m_{AB}^P = \gamma, m_{BA}^P = 1, m_{AC}^P = 1, m_{CA}^P = \phi, m_{BC}^P = 0 = m_{CB}^P. \)

**Proof.** In the appendix. ■

Also, we can derive the equilibrium rewards paid by the party and the citizen and the equilibrium probability that a citizen gets the project.

**Proposition 5** In any party equilibrium, each party pays for each politician a reward
\[ r_{CA}^P = \frac{c(\gamma^2 + \gamma - 2\phi - 2)}{2} \]

and each citizen pays the party a reward
\[ r_{BC}^P = c(\gamma + 2) \]

for the right to link with a politician. The equilibrium probability of getting the project is
\[ p_B^P = \frac{p}{\gamma} \]

**Proof.** In the appendix. ■

The equilibrium network structure is based on two principles. First, the party is forced to build up direct links between the citizens and the politicians. This being the case, it is less costly to the party boss to build his own links to the citizens via a politician rather than directly. Moreover, allocating equitable numbers of citizens to politicians minimizes the cost of linking.

Second, competition drives the benefit from an additional link equal to its marginal cost. Having a unique market reward and inequitable number of links would violate the
condition of zero marginal net benefit. The one with less links can apply the cheapest network structure described above and get the same reward with a lower marginal cost. Hence, number of links for any two agents of the same kind in the market must be the same. The rewards are such that the parties are indifferent between selling an additional link to a citizen, or buying an additional link to a politician, and sticking to the equilibrium number of links.

Notice yet, that if there is no linking cost between a citizen and a party boss, the same connections could be built with lower cost by linking the party bosses and the politicians via the citizens. Yet, in order to gain exclusive control over the links of the politician, the party boss first builds direct links to them.

Due to the marginal cost pricing and the fact that each politician is allocated an equal number of citizens, the reward paid by the citizen to the party increases in $\gamma$, the number of citizens per politician, and in $c$, the linking cost parameter, (9).

Again, due to marginal cost pricing, the reward that a party pays to the politician equals the rewards from $\gamma$ citizens linked to the marginal politician, less the marginal politician’s costs of linking to the $\gamma$ citizens and the cost of those citizens to link to the politician, less the party’s marginal cost of linking to the politicians (29). The surplus per politician, $\gamma r_{BC}^P - \frac{c}{2}(\gamma + 1)^2 - \frac{\gamma c}{2}$, increases in $\gamma$ and in $c$. On the other hand, the party’s marginal cost of linking to the politicians increases in $c$ and in $\phi$ since each party gets $\phi$ politicians in equilibrium. Hence, the equilibrium reward to politician decreases in $\phi$ and increases in $\gamma$. As both the gross surplus per each politician and the marginal cost of linking to them increase in $c$, its net effect on the reward that political parties pay politicians is open, a priori.

Next, we characterize the equilibrium payoffs.

**Proposition 6** In any party equilibrium, the equilibrium payoffs are

$$\pi_C^P = \frac{c\phi(\phi + 1)}{2}$$  \hspace{1cm} (10)$$

$$\pi_A^P = \frac{c}{2}(\gamma^2 + \gamma - 2\phi - 2)$$  \hspace{1cm} (11)$$

$$\pi_B^P = \frac{ps}{\gamma} - c(\gamma + 2)$$  \hspace{1cm} (12)$$

**Proof.** These follow from inserting the equilibrium demands and rewards into the surplus expressions. !

The comparative statics of these equilibrium payoffs are straightforward
Proposition 7  The comparative statics of the equilibrium payoffs are

\[
\frac{\partial \pi_C^P}{\partial c} > 0, \quad \frac{\partial \pi_C^P}{\partial \phi} > 0, \\
\frac{\partial \pi_A^P}{\partial c} > 0, \quad \frac{\partial \pi_A^P}{\partial \phi} < 0, \quad \frac{\partial \pi_A^P}{\partial \gamma} > 0, \\
\frac{\partial \pi_B^P}{\partial c} < 0, \quad \frac{\partial \pi_B^P}{\partial \gamma} < 0.
\]

The following corollary identifies cases where the citizen’s party equilibrium payoff is higher than her no-party equilibrium payoff

Corollary 1  The citizen’s party equilibrium payoff is higher than her no-party equilibrium payoff if

\[
\frac{ps}{c} \geq \frac{2(\gamma + 2)(\gamma + 1)\gamma}{(2\gamma + 1)}
\]  \hspace{1cm} (13)

We found two countervailing effects of the linking cost \( c \) on the party’s equilibrium payoff. Proposition 7 shows that the effect of linking cost on politicians’ equilibrium payoff is positive. The effect through the higher rewards paid by the citizens dominates the effect of higher marginal linking cost to politicians.

Proposition 7 reveals an interest conflict concerning the cost parameter. Politicians and political parties actually prefer a higher cost of networking, while citizens prefer a lower one. The seemingly counterintuitive result that some agents benefit from an increasing transaction cost follows from competition between politicians. When selling links to the citizens, the political parties charge a reward, \( r_{BC}^P \), equal to the marginal cost of adding one additional link. A lower value of the cost parameter would result in fiercer competition between political parties, and such reductions in rents from politicians to political parties would also result in lower equilibrium payments to them. This suggests, that parties and politicians might object technological innovations that might reduce the costs of networking - such as political participation via internet.

What remains is to characterize the existence of equilibria:13

Proposition 8  There is a party equilibrium with \( r_{CA} \geq 0 \) where each citizen and each politician links to a unique party if and only if

\[
\frac{ps}{c} \geq \gamma(\gamma + 2) \geq 2\phi + 2 + \gamma.
\]  \hspace{1cm} (14)

13We suppose that in equilibrium the parties do not receive payments from the politicians, \( r_{CA} < 0 \). The politicians would be willing to pay such a reward only if they can be forced not to stay out of the parties, since staying out and letting any partner link to them for free would give them a higher payoff.
that is if and only if the relative number of citizens per politician is not too small or too large, the linking cost and the number of politicians per party boss is sufficiently small, and the expected rent is sufficiently large.

**Proof.** These follow from the assumption that \( r_{CA} \geq 0 \) and from the requirement that \( \pi_{st}^P \geq 0 \). ■

The politicians’ and citizens’ individual rationality constraints create bounds for the number of citizens per politician. The condition \( \gamma^2 + \gamma \geq 2\phi + 2 \) is a requirement that when a politician links with a party, the reward that the party pays to the politician

\[
\gamma^2 + \gamma \geq 2\phi + 2
\]

is a requirement that when a politician links with a party, the reward that the party pays to the politician

\[
r_{CA} = \frac{c(\gamma^2 + \gamma - 2\phi - 2)}{2}.
\]  

Such a condition is in line with what we observe in politics: political parties typically pay their politicians in the form of campaign contributions etc. rather than the other way round, and members pay parties membership fees rather than parties paying members.

As long as the number of citizens per politician is sufficiently high relative to the number of politicians per party, \( \gamma(\gamma + 1) \geq 2\phi + 2 \), the party pays a reward to the politician and carries the politician’s linking costs. The politician gets a positive payoff anyway. All we have to worry about is citizen’s surplus. The expression \( \frac{ps}{c} - \gamma(\gamma + 2) \) reflects the surplus of the citizen in the party equilibrium for a given number of citizens per politician (see equation (10)). Increasing the number of citizens per politician sufficiently makes the citizen’s expected surplus negative.

Combining the existence conditions with (13), we notice that it is possible that the citizen prefers the no-party equilibrium to the party equilibrium or vice versa. Another point of interest, which we report as a corollary to proposition 8 is that, in the party equilibrium, the number of citizens per politician is smaller whenever the party equilibrium exists.

**Corollary 2** If the party equilibrium exists, then the number of citizens per politician is smaller in the party equilibrium than in the no-party equilibrium

**Proof.** By proposition 4, the number of citizens per politician in the party equilibrium equals \( \gamma \). By proposition 2, the number of citizens per politician in the no-party equilibrium is \( \sqrt{\frac{ps}{(\gamma + 1)c}} \). Thus, the claim amounts to

\[
\gamma < \sqrt{\frac{ps}{(\gamma + 1)c}}
\]

which is equivalent to

\[
\gamma(\gamma + 1) < \frac{ps}{c}
\]

which holds if the party equilibrium exists by proposition 8. ■
As in the economy as a whole, also in each party, there are $\gamma$ times more citizens than politicians by proposition 4. Thus, if the party let its politicians and citizens freely interact with the only restriction that none of its politicians or citizens could interact with non-members, the equilibrium payoffs would not be affected vis-à-vis the no-party equilibrium. Yet, the party enforces the network design within the party so that each citizen is linked to one politician only. The number of links per politician $\gamma$ in the party-equilibrium is always smaller than the number of links per politician in the no-party-equilibrium, $\sqrt{\frac{\gamma p_s}{(\gamma + 1)c}}$.

5 Incentive Compatibility and the Politicians’ Preference for the Party Equilibrium

In the equilibrium derived above, the party provides linking services for the citizens and the politicians. The individual rationality constraints guarantee that each politician and each citizen rather links to the party than remains inactive. Yet, on the one hand, we do not know whether the politicians prefer the party equilibrium to the no-party equilibrium. If the politicians were able to make a collective binding decision not to link with any political parties, could they gain? On the other hand, given that all the other politicians are linked to a party, each politician might prefer not linking with a party. Requiring that no politician prefers not linking with a party unilaterally provides an incentive compatibility condition for the party equilibrium.

In this section, we study whether the politicians prefer linking directly to citizens. We first study the profitability of a coordinated and collective shift by the politicians from the party equilibrium to the no-party equilibrium. Second, we study the profitability of a unilateral deviation of a single politician from linking with a party to linking with citizens directly when all other politicians remain linked with the political parties.

Even if the politicians prefer not to link with the political parties, the political parties might be able to make it unattractive for the politicians not to link to them. If not one politician alone but all together prefer linking to political parties, the structure is similar to that of a prisoner’s dilemma game: even though the utility of a deviating politician exceeds that of politicians sticking to using political parties, the politicians’ payoff in the party equilibrium exceeds their utility in the no-party equilibrium. We show that such dilemma structure never arises.

5.1 Politicians’ preferences concerning the equilibria

We next identify which of the equilibria is the politicians’ preferred equilibrium. Politicians prefer the no-party equilibrium to the party equilibrium if the surplus in the former, (5), is greater than the surplus in the latter, (11). Hence, the condition that each politician prefers the no-party equilibrium is
When (16) holds, the no-party equilibrium is preferred by the politicians. The next proposition shows that the politicians prefer the no-party equilibrium.

**Proposition 9** The politicians prefer the no-party equilibrium.

**Proof.** In the appendix. ■

This result is not too surprising in the light of corollary 2. The number of links per politician \( \gamma \) in the party-equilibrium is always smaller than the number of links per politician in the no-party-equilibrium, \( \sqrt{\frac{\gamma_{PS}}{1+\gamma}} \). But, when competing for the politicians, the parties pay politicians a reward equaling the marginal benefit of that politician to the party which on the other hand is just \( \gamma \) times the profit that it makes per each citizen that becomes a member of the party. This latter, on the other hand, is smaller than what the politician makes for each citizen in the no-party equilibrium since, as shown above, the number of links per politician is smaller in the party-equilibrium since.

### 5.2 Incentive compatible party equilibrium

If a politician cannot be prevented by other means from not linking with a party and linking with citizens directly, the equilibrium surplus for a politician has to be greater than or equal to the surplus if the politician links directly with citizens. Studying a politician’s incentives to remain linked with a party when all other politicians are linked with a party is the focus in this subsection. We assume that a deviating politician is able to make take-it-or-leave-it offers to potential partners. She asks a reward that citizens must pay to connect to her. The politician is unable to commit not to sell additional links. Therefore, she sells links until an additional link would give a negative payoff to her. This determines the number of citizens that will connect. In equilibrium, the citizens anticipate the number of links that a deviating politician would like to sell. We show that there are cases where the party equilibrium is incentive compatible and others where it is not.

**Proposition 10** There are feasible parameter values for which the party equilibrium is incentive compatible and others for which it is not.

**Proof.** The proof is in the appendix. ■

### 6 Welfare

The aggregate surplus in the no-party equilibrium, defined as \( W^N \), equals

\[
W^N = n_{APS} - n_B \frac{1}{2 \gamma_1} \frac{\gamma_{PS}}{\gamma + 1} - n_A \frac{1}{2 (\gamma + 1)}.
\]
In the party equilibrium, the aggregate surplus $W^P$ equals

$$W^P = n_A ps - n_A \frac{c}{2} (\gamma + 1)^2 - n_B \frac{c}{2} - n_C \frac{c}{2} \phi^2.$$ 

The next proposition gives a necessary and a sufficient condition for the party equilibrium to create a larger surplus

**Proposition 11** $W^P \geq W^N$ if and only if $\gamma^2 + 3\gamma + \phi + 1 \leq \frac{ps}{c}$

**Proof.**

$$n_A ps - n_A \frac{c}{2} (\gamma + 1)^2 - n_B \frac{c}{2} - n_C \frac{c}{2} \phi^2 \geq n_A ps - n_B \frac{1}{2} \frac{ps}{\gamma(\gamma + 1)} - n_A \frac{1}{2} \frac{\gamma ps}{\gamma + 1}$$

$$n_A \frac{c}{2} (\gamma^2 + 2\gamma + 1) + n_B \frac{c}{2} + n_C \frac{c}{2} \phi^2 \leq n_A \frac{1}{2} \frac{ps}{\gamma + 1} + n_A \frac{1}{2} \frac{\gamma ps}{\gamma + 1}$$

$$n_C [\phi \frac{c}{2} (\gamma^2 + 2\gamma + 1) + \gamma \phi \frac{c}{2} + \frac{c}{2} \phi^2] \leq n_C \phi \frac{ps}{2}$$

$$\gamma^2 + 3\gamma + \phi + 1 \leq \frac{ps}{c}$$

When (i) the number of politicians per party, $\phi$, (ii) the number of citizens per politician, $\gamma$, and (iii) the linking costs, $c$, are sufficiently small vis-à-vis the citizen’s expected share of the project, $ps$, then the intermediation is socially preferred.

Notice, that we have cases when the party equilibrium creates a smaller surplus even if it exists and cases where the party equilibrium creates a larger surplus and, yet, the market agents may coordinate on the no-party equilibrium. To see this, suppose that the second inequality in the existence condition for the party equilibrium, (14) holds, that is $\gamma(\gamma + 2) \geq 2\phi + 2 + \gamma$. This can be achieved, for instance, by setting $\gamma \geq \phi = 2$. The first inequality in (14) sets a lower bound for $n_A ps$. This inequality is far less stringent than the condition of proposition 11. When the latter holds, so does the lower bound for $\frac{ps}{c}$ in (14). Thus, if we choose large enough $\frac{ps}{c}$, the party equilibrium exists and generates a larger surplus than the no party equilibrium. Yet, when we choose $\frac{ps}{c}$ such that $2\phi + 2 + \gamma \leq \gamma(\gamma + 2) \leq \frac{ps}{c} < \gamma^2 + 3\gamma + \phi + 1$, the party equilibrium exists but generates a lower surplus than the no-party equilibrium. Thus, depending on the parameter values of the model, intervention may be needed in order to enforce the no-party equilibrium or in order to enforce the party equilibrium depending on the parameter values of the model.

The politicians prefer the no-party equilibrium, and they have an incentive to bring about institutions to guarantee its emergence. Sometimes, the interest of the social planner coincides with the interest of the politicians and promoting the emergence of such institutions may be in the planners interest. Yet, when the party equilibrium is preferable, the social planner should make every effort to prevent the emergence of such institutions. Furthermore, even the politician’s individual incentives may render the party equilibrium unstable for instance if the linking costs are sufficiently low.


7 Conclusion

In this paper, we suggest a novel explanation for the role of political parties: politicians distribute rents to other party members only, thus cancelling out the incentive of non-members to seek rents and spend resources in constructing links to the parties’ politicians. This reduces link formation and enables the party to create value for the party members.

We take as our starting point anti-corruption laws which forbid citizens from paying politicians directly for nominations or publicly commissioned projects. These laws still allow citizens (or firms) to pay for gaining access to politicians, for example by buying tickets to fund-raising events, by volunteering, and by making campaign contributions. Competition for access and politicians’ attention results in wasteful network formation, which political parties may alleviate, when citizens can belong to only one party. Political parties can reduce network formation costs by attaching each party member to a given politician, rather than allowing them to build links to several politicians. Similarly, political parties require politicians to pass projects to party members attached to them.

It should be highlighted that anti-corruption laws restrict also the activities of political parties. These are not allowed to trade in nominations or projects directly, but only to receive membership payments and allocate funds to politicians’ campaigns. Even political parties are unable to fully eliminate wasteful network formation, as they cannot commit to restricting the number of citizens they admit as members.\footnote{Allowing political parties to pre-commit not to take additional members would disenfranchise those citizens not belonging to the selected few from fully participating in the political life.}

Even though political parties may save network formation costs, they need not. There are two reasons for this result which may appear at the first glance counterintuitive. First, the use of political parties typically necessitates more formal network structure (there are only two degrees of strengths of the link, either there is a link or there is not). Politicians and citizens, on the other hand, may have more informal links with varying degrees of strength of the link. Second, maintaining links to the political parties is costly. When parties are present, the network must cover the parties in addition to politicians and citizens, creating a need for additional links.

The use of political parties improves welfare whenever the linking costs, the number of politicians per party boss and the number of citizens per politician are sufficiently low and the expected rent for the citizen is sufficiently large. Yet, when the number of citizens per politicians or the number of politicians per party boss is high, but not too high to prevent the party equilibrium from emerging, the no-party equilibrium is socially preferred.

We also find that politicians would be better off without political parties while citizens may gain from their presence, despite the fact that parties charge membership fees from citizens and transfer money to politicians. This surprising result arises as
payments by citizens might be even higher without parties. Even so, a party equilibrium may be incentive-compatible in that no single politician would find it optimal to deviate from it, provided that others do not. If a party equilibrium results in higher welfare, the problem of the social planner is to prevent the emergence of politicians’ collective bodies that exist to guarantee the coordination on the no-party equilibrium. If the party equilibrium results in higher welfare and is not incentive compatible, then the society may change this by increasing the costs of individual politicians deviating from it. This may explain, for example, why public money to political campaigns is often channelled through political parties, rather than directly to politicians.

Our framework raises several topics for further research. First, we could relax the assumption of efficient anti-corruption laws. Previous literature on rent-seeking has assumed that lobbyists can make politicians payments in exchange for certain policies. This leaves for future research the intermediate case in which links are the only channels through which payments can be made to influence policies. Second, we could endogenize the identity of politicians in the citizen-candidate tradition pioneered by Osborne and Slivinski (1996) and Besley and Coate (1997). Similarly, we could endogenize the identity of party bosses by presenting an overlapping generations framework in which party bosses arise from senior politicians. Finally, Persson and Tabellini (2003) show that electoral rules have significant consequences on the organization of political parties and on economic policy. To what extent do these differences arise through the role that political parties play in network formation?

8 Appendix

8.1 Foundation for the atomistic approach

We suppose that in the no-party equilibrium, citizens and politicians decide their optimal demands given the market price and in addition citizens do not take into account the effect of their demand to the amount of links that the politician has. In this appendix we give a game theoretic foundation for this approach.

We suppose that the links are created before the politician’s term of office or the election. We let the effective strength of the link, \( m_{ji}^e \), be a function not only of the strength, \( m_{ij} \), but also of the timing of the creation of the link. It is more likely that the project is passed to someone whom the politician has known for a longer time. If the finite but continuous time interval during which links are built is normalized to one, let \( \delta_{ij} \) be the fraction of time that elapses before link between \( i \) and \( j \) is created. Then the effective strength of the link is \( m_{ji}^e = m_{ij}(1 - \delta_{ij}) \).

If politicians post take-it or leave-it rewards at each point in time and the pairwise strengths of the links are settled based on the posted rewards, the emerging market equilibrium should correspond to the static equilibrium of section 3.
8.2 Proof of proposition 1

Proof. If all agents of the same type behave identically in aggregate demands and supplies in equilibrium, the supply of links equals the number agents times the number of links that each agent forms,

\[ S_m = n_A m^N_{AB}. \] (17)

On the other hand, the demand for links equals,

\[ D_m = n_B m^N_{BA}. \] (18)

In equilibrium, the supply of links by \(A\)'s has to equal demand by \(B\):s:

\[ n_A m^N_{AB} = n_B m^N_{BA}. \] (19)

Thus, acting atomistically, the probability that a citizen gets a project when she acquires a full link, \(m_i\), is \(p^N_i m_i\) where

\[ p^N_B = \frac{p n_A}{n_A m^N_{AB}} = \frac{p}{m^N_{AB}} \] (20)

is taken as given. Thus we can sum up the first order conditions and write the citizen’s first order condition as

\[ p^N_B s - r^N_{BA} - c m^N_{BA} = 0. \] (21)

In a similar manner, we can sum up the politician’s first order conditions to get

\[ r^N_{BA} - c m^N_{BA} = 0. \] (22)

Inserting (19), (20) and (22) into (21) gives

\[ \frac{p}{m^N_{AB}} s - c m^N_{AB} - c \frac{n_A m^N_{AB}}{n_B} = 0. \]

Inserting \(n_B = \gamma n_A\) gives

\[ \frac{p}{m^N_{AB}} s - c m^N_{AB} - c \frac{m^N_{AB}}{\gamma} = 0. \]

Hence, in equilibrium, a politician’s supply of links equals

\[ m^N_{AB} = \sqrt{\frac{\gamma p s}{(\gamma + 1)c}}. \] (23)

From (19), we can solve a citizen’s equilibrium demand for links,

\[ m^N_{BA} = \sqrt{\frac{p s}{\gamma (\gamma + 1)c}}. \] (24)

Therefore, by (22), the equilibrium reward for the link is

\[ r = \frac{c \gamma p s}{(\gamma + 1)}. \] (25)

\]
8.3 Proof of proposition

**Proof.** The equilibrium payoffs follow from plugging the equilibrium demand, supply and reward of the previous proof into the payoff functions. ■

8.4 Proof of proposition 4

**Proof.** 1) Let us first assume that each party has \( \gamma \) citizens for each politician linked to it. The equilibrium reward must be such that the party is indifferent on whether to sell one additional link or not. Selling one additional link would increase the linking costs of the politician to whom the citizen would be linked from \( \frac{c}{2}(\gamma + 1)^2 \) to \( \frac{c}{2}(\gamma + 2)^2 \). In addition to this, the party would have to pay \( \frac{c}{2} \) as the new citizen’s linking cost as we assume that the party bears all linking costs. The marginal increase in the linking costs then equals \( \frac{c}{2}(2\gamma + 4) = c(\gamma + 2) \). For any party, the net gain that a party would derive from selling a link to one more citizen cannot be positive, since then it would have an incentive to deviate and sell a link to an additional citizen. Hence, \( r_{BC}^P \leq c(\gamma + 2) \). On the other hand, it is not possible that the net gain is negative, \( r_{BC}^P < c(\gamma + 2) \), since then each party could increase the reward that a citizen has to pay up to \( c(\gamma + 2) \). This is because for every party \( r_{BC}^P \leq c(\gamma + 2) \) and hence no party strictly prefers offering a link to an additional citizen and the citizen can nothing but remain with her party even with the higher reward. Thus,

\[
r_{BC}^P = c(\gamma + 2)
\]

2) Let us now show that given that each party has \( m_{CA} \) politicians, the equilibrium number of citizens is \( m_{CA}\gamma \). Suppose that there are two political parties, \( C' \) and \( C'' \) such that the number of citizens linked to the two political parties are such that \( \frac{\nu_{CB}'}{m_{CA}'} < \frac{\nu_{CB}''}{m_{CA}''} \). Then, since all citizens and politicians are linked and \( n_B = \gamma n_A = \gamma \phi n_C \), we can choose two political parties so that \( \frac{\nu_{CB}''}{m_{CA}''} < \gamma < \frac{\nu_{CB}'}{m_{CA}'} \). But then using the cheapest structure described in point (1) of the proof, for all politicians linked to \( C'' \) the number of links \( m'' \) is smaller than or equal to \( \gamma + 1 \). Yet, for the party \( C'' \) there must be a politician for whom the number of links \( m'' \) is strictly greater than \( \gamma + 1 \). Hence,\n
\[
\frac{c}{2}(m'' + 1) \leq \frac{c}{2}(\gamma + 1) < \frac{c}{2}(m' + 1)
\]

The reward \( r_{BC}' \) of the party \( C' \) must be higher than or equal to \( 2m' + 2 \). Otherwise, the last additional link does not provide positive profit. But for \( C'' \) the marginal cost is lower and hence,\n
\[
r_{BC}'' \geq c(m' + 2) > c(m'' + 2)
\]

and hence party \( C'' \) makes profit by selling an additional link to a customer of \( C' \) with a cheaper price and the customer has a higher or equal probability of getting the project.
with $C''$ than with $C'$ and this cannot be an equilibrium. We have a contradiction. Hence, $\frac{\nu_{CB}^B}{m_{CA}} = \frac{\nu_{CB}^C}{m_{CA}} = \gamma$.

3) Let us now show that the equilibrium reward $r_{CA}$ satisfies

$$r_{CA}^P = \frac{c(\gamma^2 + \gamma - 1)}{2} - \frac{c}{2}(2m_{CA}^P + 1). \quad (28)$$

The benefits of the party are the payments from all citizens linked to the politicians, $m_{CA}^P \gamma r_{BC}^P$. The costs include the payment made to the politicians $m_{CA}^P r_{CA}^P$, the linking costs of politicians paid by political parties, $m_{CA}^P \xi(\gamma + 1)^2$, the linking costs of the citizens linked to the politicians of the party, $m_{CA}^P \gamma \frac{\xi}{2}$, and the party’s own linking costs to the politicians $\frac{\xi}{2}(m_{CA}^P)^2$. In equilibrium, the marginal benefit from linking to political parties must equal its marginal cost, that is

$$\gamma r_{BC}^P = \frac{c}{2} \gamma + r_{CA}^P + \frac{c}{2}(\gamma + 1)^2 + \frac{c(2m_{CA}^P + 1)}{2} \quad (29)$$

Plugging in from (26), the payment $r_{CA}^P$ is given by (28).

4) Let us now show that any network structure where for some party, $m_{CA} \neq \phi$, cannot be an equilibrium. Suppose that there are two political parties $C''$ and $C'$ with $m_{CA}^{C''} < m_{CA}^{C'}$. Then the party $C''$ is not willing to pay more than $r_{CA}^{C''} = \frac{c(\gamma^2 + \gamma - 1)}{2} - \frac{c}{2}(2m_{CA}^{C''} + 1)$ to the politicians linked to it. Otherwise, the last additional politician would deteriorate $C''$’s payoff. But $C'$ can buy a politician customer of $C''$ with positive profit, since

$$r_{CA}^{C'} \leq \frac{c(\gamma^2 + \gamma - 1)}{2} - \frac{c}{2}(2m_{CA}^{C''} + 1) < \frac{c(\gamma^2 + \gamma - 1)}{2} - \frac{c}{2}(2m_{CA}^{C'} + 1)$$

and $C'$ can afford paying $r_{CA}^{C''} + \varepsilon$ for $\varepsilon > 0$ sufficiently small. Hence, $m_{CA} \neq \phi$ cannot be an equilibrium. ■

8.5 Proof of proposition 5

Proof. The equilibrium reward $r_{BC}^P$ follows immediately from part (i) of the proof of proposition 4 and is given by

$$r_{CA}^P = \frac{c}{2}(\gamma^2 + \gamma - 2\phi - 2) \quad (30)$$

now follows from plugging $m_{CA}^P = \phi$ into (29) in the proof of proposition 4 and rearranging. By plugging in, we get the equilibrium probability of getting the project

$$p_B^P = \frac{\phi}{\phi \gamma} = \frac{p}{\gamma}$$

■
8.6 Proof of proposition 9

**Proof.** The condition for politicians to have higher total surplus with political parties than without them is

\[
\frac{c}{2} (\gamma^2 + \gamma - 2\phi - 2) \geq \frac{\gamma ps}{2(\gamma + 1)} \tag{31}
\]

We can rewrite (31) as

\[
c(\gamma^2 + \gamma - 2\phi - 2) \geq \frac{\gamma ps}{(\gamma + 1)}
\]

\[
\frac{ps}{c} \leq \frac{(\gamma^2 + \gamma - 2\phi - 2)(\gamma + 1)}{\gamma}
\]

\[
\frac{ps}{c} \leq \gamma^2 + \gamma + 1 - 2\phi - 2 - \frac{2\phi + 2}{\gamma}
\]

As we simultaneously have the participation constraint for citizens to read as \(\gamma(\gamma + 2) \leq \frac{ps}{c}\), political parties prefer a feasible network structure with political parties to a situation without political parties if

\[
\gamma(\gamma + 2) \leq \frac{ps}{c} \leq \gamma^2 + 2\gamma + 1 - 2\phi - 2 - \frac{2\phi + 2}{\gamma} \tag{32}
\]

For this inequality to hold, it must be that

\[-1 - 2\phi - \frac{2\phi + 2}{\gamma} \geq 0\]

This is a contradiction. Therefore, the party equilibrium is not preferred by the politicians. \(\blacksquare\)

8.7 Proof of proposition 10

Suppose that \(\sqrt{\frac{ps}{c}} + 1 - 1\) is a positive integer. Let us assume that the links built by the deviating politician (henceforth, \(S\)) \(S\) have to be full links. \(S\) acts as a monopoly with respect to citizens, making them take-it-or-leave it offers on building links. Citizens correctly anticipate what is the number of links \(n\) that \(S\) is going to sell.

Anticipating that the number of links that \(S\) sells is \(n\), a citizen who is offered a link is willing to pay up to \(r\) satisfying

\[
\frac{ps}{n} = r + \frac{c}{2} (1 + 1)^2 - \frac{c}{2}
\]

simplifying as \(\frac{ps}{n} = r + \frac{3c}{2}\).

The politician’s surplus writes

\[
rn - \frac{cn^2}{2}
\]

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which is increasing in \( r \) for a given \( n \) and increasing in \( n \) if for a given \( r \) if \( r > \frac{\pi}{2}(2n+1) \). Thus the optimum must satisfy

\[
  r = \frac{ps - 3c}{n} - \frac{2}{2} \quad r \leq \frac{ps}{2}(2n+1)
\]

We assumed that parameters \( p, s, \) and \( c \) are such that \( n = \sqrt{\frac{ps}{c}} + 1 - 1 \) is an integer. However, it is easy to check that \( \sqrt{\frac{ps}{c}} + 1 - 1 \) is a solution to \( \frac{\pi}{2}(2n+1) = \frac{ps}{n} - \frac{3c}{2} \). Thus the optimal \( r \) and \( n \) are \( r^* = \frac{ps}{\sqrt{\frac{ps}{c} + 1} - 1} - \frac{3c}{2} \) and \( n^* = \sqrt{\frac{ps}{c} + 1} - 1 \).

As a consequence, the surplus for \( S \) satisfies

\[
  r^*n^* - \frac{c(n^*)^2}{2} = \frac{(ps - 3c)n^* - c(n^*)^2}{2} = \frac{c}{2}((n^*)^2 + 3n^*) = -\frac{c}{2}(\sqrt{\frac{ps}{c}} + 1 - 1)(\sqrt{\frac{ps}{c}} + 1 - 1 + 3) = -\frac{c}{2}(\frac{ps}{c} + 1 + \sqrt{\frac{ps}{c} + 1} - 2) = -\frac{c}{2}(\frac{ps}{c} - 1 + \sqrt{\frac{ps}{c} + 1}) = \frac{c}{2}(\frac{ps}{c} - 1 - \sqrt{\frac{ps}{c} + 1})
\]

Denote \( \frac{ps}{c} = x \). The IC condition is satisfied if and only if the politician’s party equilibrium payoff is greater than or equal to the payoff of remaining not linked to a party. That is

\[
  \frac{c}{2}(x + 1 - \sqrt{x + 1}) \leq \frac{c}{2}(\gamma^2 + \gamma - 2\phi - 2)
\]

Equivalent to

\[
  (x + 1 - \sqrt{x + 1}) \leq (\gamma^2 + \gamma - 2\phi - 2)
\]

Rearrange

\[
  (x + 1 - (\gamma^2 + \gamma - 2\phi - 2)) \leq \sqrt{x + 1}
\]

both sides are positive since \( x \geq \gamma^2 + \gamma - 2\phi - 2 \). Otherwise, citizens would not be willing to buy links in a party equilibrium. Square

\[
  (x + 1)^2 - 2(x + 1)(\gamma^2 + \gamma - 2\phi - 2) + (\gamma^2 + \gamma - 2\phi - 2)^2 \leq (x + 1)
\]

and collect the terms with \( x + 1 \)

\[
  (x + 1)[(x + 1) - 1 - 2(\gamma^2 + \gamma - 2\phi - 2)] + (\gamma^2 + \gamma - 2\phi - 2)^2 \leq 0
\]

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(x + 1)[x - (\gamma^2 + \gamma - 2\phi - 2)]
-(x + 1)(\gamma^2 + \gamma - 2\phi - 2)
+(\gamma^2 + \gamma - 2\phi - 2)^2
\leq 0

collect the last two terms with factor \((\gamma^2 + \gamma - 2\phi - 2)\)

(x + 1)[x - (\gamma^2 + \gamma - 2\phi - 2)] + [(\gamma^2 + \gamma - 2\phi - 2) - (x + 1)](\gamma^2 + \gamma - 2\phi - 2) \leq 0

\[
(x + 1)[x - (\gamma^2 + \gamma - 2\phi - 2)]
+[(\gamma^2 + \gamma - 2\phi - 2) - x](\gamma^2 + \gamma - 2\phi - 2)
-(\gamma^2 + \gamma - 2\phi - 2)
\leq 0
\]

collect the first two terms with factor \([x - (\gamma^2 + \gamma - 2\phi - 2)]\)

\[
[(x + 1) - (\gamma^2 + \gamma - 2\phi - 2)][x - (\gamma^2 + \gamma - 2\phi - 2)] - (\gamma^2 + \gamma - 2\phi - 2) \leq 0
\]

Now setting \(x = (\gamma^2 + \gamma - 2\phi - 2)\) will make the expression negative since \((\gamma^2 + \gamma - 2\phi - 2) > 0\). On the other hand, letting \(x \to \infty\) makes the expression positive.

References


