Market power in a storable-good market: Theory and applications to carbon and sulfur trading

Matti Liski
Helsinki School of Economics, MIT-CEEPR, and HECER

and

Juan-Pablo Montero
Catholic University of Chile and MIT-CEEPR

Discussion Paper No. 91
December 2005

ISSN 1795-0562
Market power in a storable-good market: Theory and applications to carbon and sulfur trading*

Abstract

We consider a market for storable pollution permits in which a large agent and a fringe of small agents gradually consume a stock of permits until they reach a long-run emissions limit. The subgame-perfect equilibrium exhibits no market power unless the large agent's share of the initial stock of permits exceeds a critical level. We then apply our theoretical results to a global market for carbon dioxide emissions and the existing US market for sulfur dioxide emissions. We characterize competitive permit allocation profiles for the carbon market and find no evidence of market power in the sulfur market.

JEL Classification: L51; Q28.

Keywords: storage, market power, pollution permits.

Matti Liski
Department of Economics, Helsinki School of Economics
P.O. Box 1210
00101 Helsinki
FINLAND
e-mail: liski@hse.fi

Juan-Pablo Montero
Department of Economics
Catholic University of Chile
Vicuna Mackenna 4860
Santiago
CHILE
e-mail: jmontero@faceapuc.cl

* Montero is currently visiting the Harvard Kennedy School of Government under a Repsol-YPF Research Fellowship. Both authors are also Research Associates at the MIT Center for Energy and Environmental Policy Research. We thank Bill Hogan, Juuso Välimäki, Ian Sue-Wing and seminar participants at Harvard University, Helsinki School of Economics, Université Catholique of Louvain-CORE and Yale University for many useful comments. Liski acknowledges funding from the Academy of Finland and Nordic Energy Research Program and Montero from Fondecyt (grant # 1030961) and BBVA Foundation.
1 Introduction

Markets for trading pollution rights or permits have attracted increasing attention in the last two decades. A common feature in most existing and proposed market designs is the future tightening of emission limits accompanied by firms’ possibility to store today’s unused permits for use in later periods. The US sulfur dioxide trading program with its two distinct phases is a salient example but global trading proposals to dealing with carbon dioxide emissions share similar characteristics. In anticipation to a tighter emission limit, it is in the firms’ own interest to store permits from the early permit allocations and build up a stock of permits that can then be gradually consumed until reaching the long-run emissions limit. This build-up and gradual consumption of a stock of permits give rise to a dynamic market that shares many, but not all, of the properties of a conventional exhaustible-resource market (Hotelling, 1931).

As with many other commodity markets, permit markets have not been immune to market power concerns (e.g., Hahn, 1984; Tietenberg, 2006). Following Hahn (1984), there is substantial theoretical literature studying market power problems in a static context but none in the dynamic context we just described. This is problematic because static markets, i.e., markets in which permits must be consumed in the same period for which they are issued, are rather the exception. Taking a game-theoretical approach, in this paper we study the properties of the equilibrium path of a dynamic permit market in which there is a large polluting agent — that can be either a firm, country or cohesive cartel — and a competitive fringe of many small polluting agents. Agents receive a very generous allocation of permits for a few periods and then an allocation equal, on aggregate, to the long-term emissions goal established by the regulation. We are particularly interested in estimating the effect on the market outcome from allocating a significant fraction of the stock (i.e., early allocations) in the hands of the large agent.

---

1As documented by Ellerman and Montero (2005), during the first five years of the U.S. Acid Rain Program constituting Phase I (1995-99) only 26.4 million of the 38.1 million permits (i.e., allowances) distributed were used to cover sulfur dioxide emissions. The remaining 11.65 million allowances were saved and have been gradually consumed during Phase II (2000 and beyond).

2We wrote an earlier note on this problem but only considered the extreme cases of pure monopoly and perfect competition (Liski and Montero, 2005). It should also be mentioned that we are not aware of any empirical, as opposed to numerical, analysis of market power in pollution permit trading.

3Already in the very early programs such as the U.S. lead phasedown trading program and the U.S. EPA trading program firms were allowed to store permits under the so-called “banking” provisions – provisions that were extensively used (Tietenberg, 2006).

4The properties of the perfectly competitive equilibrium path are well understood (e.g., Rubin, 1996; Schennach, 2000).

5Following discussions on how to incorporate a large developing country such as China into global
Approaching this market power problem was not obvious to us for basically two reasons. The first reason comes from the extra modelling complexity. Agents in our model not only decide on how to sell the stock over time, as in any conventional exhaustible resource market, but also on how to consume it as to cover their own emissions. In addition, since permits can be stored at no cost agents are free to either deplete or build up their own stocks. The second reason is that the existing literature provides conflicting insights into what the equilibrium outcome might look like.

On one hand, one can conjecture from the literature on market power in a depletable-stock market, pioneered by Salant (1976), that the large firm restricts its permit sales during the first years so as to take over the entire market after fringe members have totally exhausted their stocks. Motivated by the structure of the oil market, Salant showed that no matter how small the initial stock of the large firm compared to that of the fringe is, the equilibrium path consists of two distinct phases. During the first or so-called competitive phase, both the fringe and the large agent sell to the market and prices grow at the rate of interest (note that there are no extraction costs). During the second or monopoly phase, which begins when the fringe’s stock is exhausted, the large agent’s marginal (net) revenues grow at the rate of interest, so prices grow at a rate strictly lower than the rate of interest. These results would suggest that market manipulation in a dynamic permit market would occur independently of the initial allocations, as the large firm would always deplete its stock of permits at a rate strictly lower than that under perfect competition.

On the other hand, one might conjecture from the literature on market power in a static permit market, pioneered by Hahn (1984), that the large agent’s market power does depend on its initial allocation. Hahn showed that a large polluting firm fails to exercise market power only when its permit allocation is exactly equal to its emissions in perfect competition. If the permit allocation is above (below) its competitive emission level, then, the large firm would find it profitable to restrict its supply (demand) of permits in order to move prices above (below) competitive levels. Based on these results one could argue then that the type of market manipulation in a dynamic permits market is dependent on the initial allocations in that the rate at which the large firm would (strategically) deplete its stock of permits can be either higher or lower than the perfectly competitive

rate. One can even speculate that there could be cases in which the large agent may want to increase its stock of permits via purchases in the spot market during at least the first few years.

The properties of our subgame-perfect equilibrium solve the conflict between the two conjectures. Our equilibrium outcome is qualitatively consistent with the Salant’s (1976) path if and only if the fraction of the initial stock allocated to the large firm is above a (strictly positive) critical level; otherwise, firms follow the perfectly competitive outcome (i.e., prices rising at the rate of interest up to the exhaustion of the entire permits stock). Consistent with Hahn (1984), the critical level is exactly equal to the fraction of the stock that the large firm would have needed to cover its emissions along the competitive path. But unlike Hahn (1984), the large firm cannot manipulate prices below competitive levels if its stock allocation is below the critical level. We believe that this explicit link between the stock allocation and market power has practical value, since evaluating the critical share of the stock for each participant of the trading program is not as difficult as one could have thought.

The reason our equilibrium can exhibit a sharp departure from the predictions of Salant (1976) and Hahn (1984) is because the large firm must simultaneously solve for two opposing objectives (revenue maximization and compliance cost minimization) and at the same time face a fringe of members with rational expectations that force it to follow a subgame-perfect path. Unlike in Salant (1976) and Hahn (1984), the opposing objectives problem arises because our large firm must decide on two variables at each point in time: how many permits to bring to the spot market and how much stock to leave for subsequent periods, or equivalently, how many permits to use for its own compliance. Fringe members clear the spot market and decide their remaining stocks based on what they (correctly) believe the market development will be.

When the initial stock allocated to the large firm is generous enough (i.e., above the critical level), the equilibrium path is governed by two equilibrium conditions: the large firm’s marginal net revenue from selling and marginal cost rise at the rate of interest. As the large firm’s initial stock decreases, these two conditions start conflicting with each other.

---

7 We say “qualitatively consistent” because our approach is very different from Salant’s in that we view firms as coming to the market in each period instead of making a one-time quantity-path announcement at the beginning of the game.

8 In a Salant (1976) period-by-period game, the large firm’s decision at any period would reduce to the amount of oil to bring to the market in that period. In Hahn’s (1984) static setting, the large firm’s decision also reduces to one variable: the amount of permits to bring to the market (emission abatement is not an independent decision variable but derives directly from the permit sales decision).
other and they no longer hold when the initial stock is below the critical level. It only holds that marginal costs grow at the rate of interest. More precisely, when the stock is smaller than the critical level, the large agent has no means to credibly commit to a purchasing profile that would keep prices below their competitive levels throughout. Any effort to depress prices below competitive levels would make fringe members to maintain a larger stock in response to their (correct) expectation of a later appreciation of permits. And such off-equilibrium effort would be suboptimal for the large agent, i.e., it is not the large agent’s best response to fringe members’ rational expectations.\footnote{Note that the depleteable nature of the permit stocks makes this time-inconsistency problem faced by the large agent similar to that of a durable-good monopolist (Coase, 1972; Bulow, 1982).}

We then apply our results to two permit markets: the carbon market that may eventually develop under the Kyoto Protocol and beyond and the existing sulfur market of the US Acid Rain Program. Motivated by the widespread concern about Russia’s ability to exercise market power, in the carbon application we illustrate how such ability greatly diminishes when countries affected by the Protocol are expected to store a significant fraction of early permits in anticipation of tighter emission constraints and higher prices in later periods. The reason is that Russia would not only hold a large stock, which is built during the first periods, but would also consume a large amount during later periods.

For the sulfur application, we use publicly available data on sulfur dioxide emissions and permit allocations to track down the actual compliance paths of the four largest players in the market, which together account for 43% of the permits allocated during the generous-allocation years, i.e., 1995-1999. We show that the behavior of these players, taken either individually or as a cohesive group, can only be consistent with perfect competition. The fact that these players appear as heavily borrowers of permits during and after 2000 rules out, according to our theory, any possibility of market power.

Although understanding the exercise of market power in pollution permit trading has been our main motivation, it is worth emphasizing that the properties of our equilibrium solution apply equally well to any conventional exhaustible resource market in which the large agent is in both sides of the market. The international oil market would be a good example if the oil domestic consumption of OPEC cartel members were large enough. The rest of the paper is organized as follows. The model is presented in Section 2. The characterization of the properties of our equilibrium solution are in Section 3. Extensions of the basic model that account for trends in permit allocations and emissions and long-run market power are in Section 4. The applications to carbon and sulfur trading are in
Section 5. Final remarks are in Section 6.

2 The Model

We are interested in pollution regulations that become tighter over time. A flexible way to achieve such a tightening is to use tradable pollution permits whose aggregate allocation is declining over time. When permits are storable, i.e., unused permits can be saved and used in any later period, a competitive permit market will allocate permits not only across firms but also intertemporally such that the realized time path of reductions is the least cost adjustment path to the regulatory target.

We start by defining the competitive benchmark model of such a dynamic market. Let I denote a continuum of heterogeneous pollution sources. Each source \( i \in I \) is characterized by a permit allocation \( a_i^t \geq 0 \), unrestricted emissions \( u_i^t \geq 0 \),\(^{10}\) and a strictly convex abatement cost function \( c_i(q_i^t) \), where \( q_i^t \geq 0 \) is abatement. Sources also share a common discount factor \( \delta \in (0, 1) \) per regulatory period \( t = 0, 1, 2, \ldots \) (a regulatory period is typically a year). The aggregate allocation \( a_t \) is initially generous but ultimately binding such that \( u_t - a_t > 0 \), where \( u_t \) denotes the aggregate unrestricted emissions (no index \( i \) for the aggregate variables). Without loss of generality,\(^{11}\) we assume that the aggregate allocation is generous only in the first period \( t = 0 \) and constant thereafter:

\[
a_t = \begin{cases} 
  s_0 + a & \text{for } t = 0 \\
  a & \text{for } t > 0,
\end{cases}
\]

where \( s_0 > 0 \) is the initial 'stock' allocation of permits that introduces the intertemporal gradualism into polluters' compliance strategies. Note that \( a \geq 0 \) is the long-run emissions limit (which could be zero as in the U.S. lead phasedown program). Assume for the moment that none of the stockholders is large; thus, we do not have to specify how the stock is allocated among agents. Aggregate unrestricted emissions are assumed to be constant over time, \( u_t = u > a \).\(^{12}\) While the first period reduction requirement may or

\(^{10}\)A firm's unrestricted emissions — also known as baseline emissions or business as usual emissions — are the emissions that the firm would have emitted in the absence of environmental regulation.

\(^{11}\)In Section 4, we allow for trends in allocations and unrestricted emissions. In particular, there can be multiple periods of generous allocations leading to savings and endogenous accumulation of the stock to be drawn down when the annual allocations decline. Permits will also be saved and accumulated if unrestricted emissions sufficiently grow, that is, if marginal abatement costs grow faster than the interest rate in the absence of saving. None of these extensions change the essence of the results obtained from the basic model.

\(^{12}\)Again, this will be relaxed in Section 4.
may not be binding, we assume that \( s_0 \) is large enough to induce savings of permits.

Let us now describe the competitive equilibrium, which is not too different from a Hotelling equilibrium for a depletable stock market.\(^{13}\) First, trading across firms implies that in all periods \( t \) marginal costs equal the price,

\[
p_t = c_i'(q^i_t), \forall i \in \mathcal{I}.
\]

Second, since holding permits across periods prevents arbitrage over time, equilibrium prices are equal in present value as long as some of the permit stock is left for the next period, \( s_{t+1} > 0 \). Exactly how long it takes to exhaust the initial stock depends on the tightness of the long-run target \( u - a > 0 \), and the size of the initial stock \( s_0 \). Let \( T \) be the equilibrium exhaustion period. Then, \( T \) is the largest integer satisfying (1) for all \( t \), and

\[
\begin{align*}
p_t &= \delta p_{t+1}, \quad 0 \leq t < T \quad (2) \\
q_T &\leq q_{T+1} = u - a \quad (3) \\
s_0 &= \sum_{t=0}^{T} (u - a - q_t). \quad (4)
\end{align*}
\]

These are the three Hotelling conditions that in exhaustible-resource theory are called the arbitrage, terminal, and exhaustion conditions, respectively. Thus, while (1) ensures that polluters equalize marginal costs across space, the Hotelling conditions ensure that firms reach the ultimate reduction target gradually so that marginal abatement costs are equalized in present value during the transition. Note that the terminal condition can also be written as

\[
p_T \geq \delta p_{T+1},
\]

where the inequality follows because of the discrete time; in general, stock \( s_0 \) cannot be divided between discrete time periods such that the boundary condition holds as an equality (when the length of the time period is made shorter, the gap \( p_T - \delta p_{T+1} \) vanishes).

Throughout this paper, we mean to model situations where the stock is large relative

---

\(^{13}\)While we will discuss the differences between the dynamic permit market and exhaustible-resource markets, it might be useful to note two main differences here. First, the permit market still exists after the exhaustion of the excessive initial allocations while a typical exhaustible-resource market vanishes in the long run. This implies that long-run market power is a possibility in the permit market, which, if exercised, affects the depletion period equilibrium. Second, the annual demand for permits is a derived demand by the same parties that hold the stocks whereas the demand in an exhaustible-resource market comes from third parties. This affects the way the market power will be exercised, as we will discuss in detail below.
to the period length so that the competitive equilibrium prices are almost continuous in present value between $T$ and $T + 1$.\footnote{To give the reader an idea why we are emphasizing this potential last period jump in present-value prices, we note that we will be making efficiency statements where it is important that the dominant agent in the market does not have much scope in moving the last period price. If the integer problem discussed above is severe, market power will be built into the model through the discrete time formulation that creates the last period problem. But when the stock is large relative to the period length (i.e., the stock is consumed over a span of several periods), the integer problem vanishes and with that any market power associated to this source.}

We are interested in the effect of market power on this type of equilibrium. To this end, we isolate one agent, denoted by the index $m$, from $I$ and call it large agent (or leader in the Stackelberg game below).\footnote{The terms large agent and leader will be used interchangeably.} The remaining agents $i \in I$ are studied as a single competitive unit, called the fringe, which we will denote by the index $f$. In particular, the stock allocation for the large agent, $s^m_0 = s_0 - s^f_0$, is now large compared to the holdings of any of the other fringe members. The annual allocations $a^m$ and $a^f$ are constants, as well as the unrestricted emissions $u^m$ and $u^f$, and still satisfying

$$u - a = (u^m + u^f) - (a^m + a^f) > 0.$$  

The fringe’s aggregate cost is denoted by $c_f(q^f_t)$, which gives the minimum cost of achieving the total abatement $q^f_t$ by sources in $I$. This cost function is strictly convex, as well as the cost for the leader, denoted by $c_m(q^m_t)$.

We look for a subgame-perfect equilibrium for the following game between the large polluter and the fringe. At the beginning of each period $t = 0, 1, 2, ...$ all agents observe the stock holdings of both the large polluter, $s^m_t$, and the fringe, $s^f_t$. We let the large agent be a Stackelberg leader in the sense that it first announces its spot sales of permits for period $t$, which we denote by $x^m_t > 0$ ($< 0$, if the leader is buying permits). Having observed stocks $s^m_t$ and $s^f_t$ and the large agent’s sales $x^m_t$, fringe members form rational expectations about future supplies by the leader and make their abatement decision $q^f_t$ as to clear the market, i.e., $x^f_t = -x^m_t$, at a price $p_t$. In equilibrium $p_t$ is such that it not only eliminates arbitrage possibilities across fringe firms at $t$, $p_t = c'_f(q^f_t)$, but also across periods, $p_t = \delta p_{t+1}$, as long as some of the fringe stock is left for the next period, that is

$$s^f_{t+1} = s^f_t + a^f - u^f + q^f_t - x^f_t > 0.$$  

It is clear that the fringe abatement strategy depends on the observable triple $(x^m_t, s^m_t, s^f_t)$, so we will write $q^f_t = q^f_t(x^m_t, s^m_t, s^f_t)$. Note that we assume that the fringe does not observe
and $q^m_t$ before abating at $t$, so the decisions on abatement are simultaneous.\footnote{There are three basic reasons for these timing assumptions. First, not observing $q^m_t$ is the most realistic assumption because this information becomes publicly available only at the closing of the period as firms redeem permits to cover their emissions of that period. Second, without the Stackelberg timing for $x^m_t$ we would have to specify a trading mechanism for clearing the spot market. In a typical exhaustible-resource market the problem does not arise since buyers are third party consumers. And third, assuming the Stackelberg timing not only for $x^m_t$ but also for $q^m_t$ does not change the results (Appendices A-B can be readily extended to cover this case).}

At each $t$ and given stocks $(s^m_t, s^f_t)$, the leader chooses $x^m_t$ and decides on $q^m_t$ knowing that the fringe can correctly replicate the leader’s problem in the subgame starting at $t + 1$. Let $V^m_t(s^m_t, s^f_t)$ denote the leader’s payoff given $(s^m_t, s^f_t)$. The equilibrium strategy $(x^m_t(s^m_t, s^f_t), q^m_t(s^m_t, s^f_t))$ must then solve the following recursive equation:

\[ V^m_t(s^m_t, s^f_t) = \max_{(x^m_t, q^m_t)} \{ p_t x^m_t - c_m(q^m_t) + \delta V^m_{t+1}(s^m_{t+1}, s^f_{t+1}) \} \]

where

\[
\begin{align*}
  s^m_{t+1} &= s_t^m + a_t^m - u_t^m + q_t^m - x_t^m, \\
  s^f_{t+1} &= s_t^f + a_t^f - u_t^f + q_t^f - x_t^f, \\
  x_t^f &= -x_t^m, \\
  q_t^f &= q_t^f(x_t^m(s_t^m, s_t^f)), \\
  p_t &= c_f(q_t^f),
\end{align*}
\]

and $q_t^f(x_t^m, s_t^m, s_t^f)$ is the fringe strategy. While individual $i \in \mathcal{I}$ takes the equilibrium path $\{x^m_t, s^m_t, s^f_t\}_{t \geq t}$ as given, aggregate $q^f_t$ for all $i \in \mathcal{I}$ can be solved from the allocation problem that minimizes the present-value compliance cost for the nonstrategic fringe as a whole. Letting $V^f_t(x^m_t, s^m_t, s^f_t)$ denote this cost aggregate given the observed triple $(x^m_t, s^m_t, s^f_t)$, we can find $q^f_t(x^m_t, s^m_t, s^f_t)$ from

\[ V^f_t(x^m_t, s^m_t, s^f_t) = \min_{q^f_t} \{ c_f(q^f_t) + \delta V^f_{t+1}(\tilde{x}^m_{t+1}, \tilde{s}^m_{t+1}, s^f_{t+1}, \tilde{s}^f_{t+1}) \} \]

where $\tilde{x}^m_{t+1}$ and $\tilde{s}^m_{t+1}$ are taken as given by equilibrium expectations. Although fringe members do not directly observe leader’s abatement $q^m_t$, they form (rational) expectations about the leader’s optimal abatement $q^m_t = q^m_t(s_t^m, s_t^f)$, which together with $x^m_t$ is then used in (6) to predict the leader’s next period stock $\tilde{s}^m_{t+1}$. The expectation of $\tilde{s}^m_{t+1}$ is thus independent of what fringe members are choosing for $q^f_t$. In contrast, the expectation of $\tilde{x}^m_{t+1}$ must be such that solving $q^f_t$ and $s^f_{t+1}$ from (11) and (7) fulfills...
this expectation, that is, \( \tilde{x}^m_{t+1} = x^m_{t+1}(s^m_{t+1}, s^f_{t+1}) \). In this way current actions are consistent with the next period subgame that the fringe members are rationally expecting. This resource-allocation problem is the appropriate objective for the nonstrategic fringe, because whenever market abatement solves (11) with equilibrium expectations, no individual \( i \in I \) can save on compliance costs by rearranging its plans.\(^{17}\)

### 3 Characterization of the Equilibrium

We solve the game by backward induction, so it is natural to consider what happens in the long run, i.e., when both stocks \( s^m_0 \) and \( s^f_0 \) have been consumed. Since our main motivation is to consider how large can be the transitory permit stock for an individual polluter without leading to market power problems, we do not want to assume market power through extreme annual allocations that determine the long-run trading positions. From Hahn (1984), we know that market power after the depletion of the stocks can be ruled out by assuming annual allocations \( a^{m*} \) and \( a^{f*} \) such that the long-run equilibrium price is\(^{18}\)

\[
\overline{p} = c'_f(q^f_t = u^f - a^{f*}) = c'_m(q^m_t = u^m - a^{m*}).
\]

Under this allocation the leader chooses not to trade in the long-run equilibrium because the marginal revenue from the first sales is exactly equal to opportunity cost of selling. In other words, \( c'_f(q^f_t) - c'_f(q^f_t = u^f - a^{f*}) = c'_m(q^m_t = u^m - a^{m*}) \) holds when \( x^m_t = 0 \).

Having defined the efficient annual allocations, \( a^{m*} \) and \( a^{f*} \), it is natural to define next the corresponding stock allocations which have the same conceptual meaning as the efficient annual allocations: these endowments are such that no trading is needed for efficiency during the stock depletion phase. We denote the efficient stock allocations by \( s^{m*}_0 \) and \( s^{f*}_0 \). Then, if the leader and fringe choose socially efficient abatement strategies for all \( t \geq 0 \), their consumption shares of the given overall stock \( s_0 \) are exactly \( s^{m*}_0 \) and \( s^{f*}_0 \). We call such shares of the stock Hotelling shares. The socially efficient abatement pair \( \{q^{m*}_t, q^{f*}_t\}_{t \geq 0} \) is such that \( q_t = q^{m*}_t + q^{f*}_t \) satisfies both \( c'_f(q^{f*}_t) = c'_m(q^{m*}_t) \) and the

\(^{17}\)We emphasize that (11) characterizes efficient resource allocation, constrained by the leader’s behavior, without any strategic influence on the equilibrium path.

\(^{18}\)Alternatively, we can assume that the long-run emissions goal is sufficiently tight that the long-run equilibrium price is fully governed by the price of backstop technologies, denoted by \( \overline{p} \). This seems to a be a reasonable assumption for the carbon market and perhaps so for the sulfur market after recent announcements of much tighter limits for 2010 and beyond. In any case, we allow for long-run market power in Section 4. The relevant question there is the following: how large can the transitory stock be without creating market power that is additional that coming from the annual allocations.
Hotelling conditions (2)-(4) ensuring efficient stock depletion. Since we shall show that the Hotelling share $s_0^{m\ast}$ is the critical stock needed for market manipulation, we define it here explicitly for future reference.

**Definition 1** The Hotelling consumption shares of the initial stock, $s_0$, are defined by

$$
    s_0^{m\ast} = \sum_{t=0}^{T} (u_m^m - q_t^{m\ast} - a_t^m),
$$

$$
    s_0^{f\ast} = \sum_{t=0}^{T} (u_f^f - q_t^{f\ast} - a_t^f),
$$

where the pair $\{q_t^{m\ast}, q_t^{f\ast}\}_{t\geq 0}$ is socially efficient.

Salant’s (1976) equilibrium for our depleteable-stock market is a natural point of departure, although the subgame-perfect equilibrium of our game cannot be found by directly invoking his case since he considers a Cournot equilibrium for a different application (we will be specific about the differences below). Nevertheless, since the large permit seller’s problem of allocating its initial holding across several spot markets is not too different from that of a large oil seller facing a competitive fringe, it seems likely that the problems have similar solutions. Figure 1 depicts the manipulated equilibrium, as described by Salant, and the competitive equilibrium (the competitive price is denoted by $p^\ast$). The manipulated price is initially higher than the competitive price and growing at the rate of interest as long as the fringe is holding some stock. After the fringe period of exhaustion, denoted by $T^f$, the manipulated price grows at a lower rate because the leader is the monopoly stock-holder equalizing marginal revenues rather than prices in present value until the end of the storage period, $T^m$. The exercise of market power implies extended overall exhaustion time, $T^m > T$, where $T$ is the socially optimal exhaustion period for the overall stock $s_0$, as defined by conditions (2)-(4).

*** INSERT FIGURE 1 HERE OR BELOW ***

We will show that the subgame-perfect equilibrium of our game is qualitatively equivalent to the Salant’s equilibrium in the case where the leader moves the market. The equilibrium conditions that support this outcome are the following. First, as long as the fringe is saving some stock for the next period, $s_{t+1}^f > 0$, prices must be equal in present value, $p_t = \delta p_{t+1}$, implying that the market-clearing abatement for fringe $q_t^f (x_t^m, s_t^m, s_t^f)$ must satisfy

$$
    p_t = c_f(q_t^f) = \delta c_f(q_{t+1}^f) \text{ for all } 0 \leq t < T^f.
$$

(13)
Second, the leader’s equilibrium strategy is such that the gain from selling a marginal permit should be the same in present value for different periods. In Salant, the gain from selling at $t$ is the marginal revenue. In our context it is less clear what is the appropriate marginal revenue concept, since the leader is selling to other stockholders who adjust their storage decisions in response to sales while in Salant this ruled out by the Cournot timing assumption. However, the storage response will not change the principle that the present-value marginal gain from selling should be the same for all periods. Because in any period after the fringe exhaustion this gain is just the marginal revenue without the storage response, it must be the case that the subgame-perfect equilibrium gain from selling a marginal unit at any $t < T_f$ is equal, in present value, to the marginal revenue from sales at any $t > T_f$. The condition that ensures this indifference is the following:

$$c'_f(q^f_t) - x^m_t c''_f(q^f_t) = \delta [c'_f(q^f_{t+1}) - x^m_{t+1} c''_f(q^f_{t+1})]$$  \hspace{1cm} (14)

for all $0 \leq t < T^m$.

Third, the leader must not only achieve revenue maximization but also compliance cost minimization which is obtained by equalizing present-value marginal costs and, therefore,

$$c'_m(q^m_t) = \delta c'_m(q^m_{t+1})$$  \hspace{1cm} (15)

must hold for all $0 \leq t < T^m$. Finally, the leader’s strategy in equilibrium must be such that the gain from selling a marginal permit equals the opportunity cost of selling, that is,

$$c'_f(q^f_t) - x^m_t c''_f(q^f_t) = c'_m(q^m_t),$$  \hspace{1cm} (16)

must hold for all $t$.

In the Salant’s description of market power the competitive phase gets longer and monopoly phase shorter as the large agent’s share of the stock decreases; in the limit, the equilibrium becomes competitive (in terms of Figure 1, $T_f$ increases and $T^m$ decreases, and in competitive equilibrium, they meet at $T = T_f = T^m$ so that the competitive phase extends to the very end). However, qualitatively the equilibrium has the two phases as long as the large agent has some stock. In contrast with this, in our case the description of the subgame-perfect equilibrium depends critically on whether the large agent’s initial stockholding exceeds the Hotelling share of the overall stock which is, in general, not any marginal quantity. If $s^m_0 > s^{m*}_0$, the Salant’s description holds, otherwise it fails.

**Proposition 1** If $s^m_0 > s^{m*}_0$, then the subgame-perfect equilibrium has the competitive
and monopoly phases and satisfies the Salant’s conditions (13)-(16).

Proof. See Appendix A. ■

The proof is based on standard backward induction arguments. It determines for any given remaining stocks \((s^m_t, s^f_t)\) the number of periods (stages) it takes for the leader and fringe to sell their stocks such that at each stage the stocks and leader’s optimal actions are as previously anticipated. For initial stocks \((s^m_0, s^f_0)\), the number of stages is \(T^f\) for the fringe and \(T^m\) for the leader. If for some reason the stocks go off the equilibrium path, the number of stages needed for stock depletion change, but the equilibrium is still characterized as above.

Before explaining how the equilibrium looks like for \(s^m_0 \leq s^m_{*0}\), we need to discuss why the above subgame-perfect path is characterized by the Salant’s conditions, although he derived them in Cournot game with path strategies. To this end, note that the marginal revenue for the leader is

\[
MR_t = p_t + x^m_t \frac{\partial p_t}{\partial q^f_t} \frac{\partial q^f_t}{\partial x^m_t} = c'_f(q^f_t) + x^m_t c''_f(q^f_t) \frac{\partial q^f_t}{\partial x^m_t} \tag{17}
\]

where, in general, \(\frac{\partial q^f_t}{\partial x^m_t} > -1\), if \(s^f_{t+1} > 0\). This follows since the fringe responses to increase in supply by allocating more of its stock to the next period. The first-order condition for choosing sales, \(x^m_t\), using (5), equates the marginal revenues and the opportunity cost of selling:

\[
MR_t = -\delta \frac{\partial V^m_{t+1}(s^m_{t+1}, s^f_{t+1})}{\partial s^m_{t+1}} \frac{\partial s^m_{t+1}}{\partial x^m_t} \tag{18}
\]

\[
= c'_m(q^m_t)
\]

\[
-\delta \frac{\partial V^m_{t+1}(s^m_{t+1}, s^f_{t+1})}{\partial s^f_{t+1}} \left[ \frac{\partial s^f_{t+1}}{\partial q^f_t} \frac{\partial q^f_t}{\partial x^m_t} + \frac{\partial s^f_{t+1}}{\partial x^m_t} \right] \Delta_t
\]

\[
= \Delta_t
\]

The first term on the RHS is the opportunity cost from not being able use the sold permits for own compliance, which equals the marginal abatement cost. The second term is the opportunity cost from the fringe storage response, which is also positive but drops out as soon as the fringe exhausts its stock.\(^\dagger\) In Fig. 2, we show the marginal revenue and the full opportunity cost. Note that because \(c'_m(q^m_t)\) grows at the rate of interest, the

\[^{\dagger}\text{The term } \frac{\partial s^f_{t+1}}{\partial q^f_t} \frac{\partial q^f_t}{\partial x^m_t} + \frac{\partial s^f_{t+1}}{\partial x^m_t} \text{ is zero for } t \geq T^f, \text{ because } \frac{\partial s^f_{t+1}}{\partial q^f_t} = 1 = \frac{\partial s^f_{t+1}}{\partial x^m_t} \text{ and } \frac{\partial q^f_t}{\partial x^m_t} = -1.\]
net marginal revenue $MR_t - \Delta_t$ is also growing at this rate. Because of the fringe storage response, the leader’s sales become fungible across spot markets as long as the fringe is holding a stock, implying that selling a marginal unit today is like selling this unit to the fringe exhaustion period. But that period is the first period without the storage response and, therefore, $\partial q_{fT}^{f}/\partial x_{m}^{f} = -1$ and expression (16) must hold at $T^{f}$. Since the leader is indifferent between putting a marginal unit on the market today $t < T^{f}$ or at $t = T^{f}$, both sides of the expression must grow at the rate of interest. Hence, the stated equilibrium conditions must hold.

*** INSERT FIGURE 2 HERE OR BELOW ***

The above description of the market power is qualitatively consistent not only with Salant but also with Hahn (1984) in the sense that the large market participant having more than the competitive share (the Hotelling share in our case) of the overall allocation moves the market as a seller. However, when the large agent’s allocation falls below the Hotelling share both connections are broken.

**Proposition 2** If $s_{m}^{0} \leq s_{m}^{*}$, the subgame-perfect depletion path is efficient.

**Proof.** See Appendix B. ■

This result is central to our applications below. It follows, first, because one-shot deviations through large purchases that move the price above the competitive level are not profitable and, second, because the fringe arbitrage prevents the leader from depressing the price through restricted purchases. Moving the price up is not profitable since the fringe is free-riding on the market power that the leader seeks to achieve through large purchases; the gains from monopolizing the market spill over to the fringe asset values through the increase in the spot price, while the cost from materializing the price increase is borne by the leader only. Formally, if the large agent makes a purchase at $T - 1$ (one period before exhaustion) that is large enough to imply a permit holding in excess of the leader’s own demand at $T$, then the spot market at $T - 1$ rationally anticipates this, leading to a price satisfying

$$p_{T-1} = \delta p_{T} > \delta[c'_{f}(q_{T}^{f}) - x_{T}^{m}c''_{f}(q_{T}^{f})].$$

The equality is due to fringe arbitrage. It implies that the leader is paying more for the permits than the marginal gain from sales, given by the discounted marginal revenue from market $t = T$. This argument holds for any number of periods before the overall
stock exhaustion, implying that, if a subgame-perfect path starts with $s^m_0 \leq s^m_0^*$, the leader’s share of the stock remains below the Hotelling share at any subsequent stage.

The leader cannot depress the price as a large monopsonistic buyer either. At the last period $t = T$, because of the option to store, no fringe member is willing to sell at a price below $\delta p_{T+1}$ where $p_{T+1}$ is the price after the stock exhaustion (which is competitive). This argument applies to any period before exhaustion where the leader’s holding does not cover its future own demand along the equilibrium path; the fringe anticipates that reducing purchases today increases the need to buy more in later periods, which leads to more storage and, thereby, offsets the effect on the current spot price.

Further intuition for Proposition 2 can be provided with the aid of Figure 3. The perfectly competitive price path is denoted by $p^*$. Ask now, what would be the optimal purchase path for the leader if it can fully commit to it at time $t = 0$? Since letting the leader choose a spot purchase path is equivalent to letting it go to the spot market for a one-time stock purchase at time $t = 0$, conventional monopsony arguments would show that the leader’s optimal one-time stock purchase is strictly smaller than its purchases along the competitive path $p^*$. The new equilibrium price path would be $p^{**}$ and the fringe’s stock would be exhausted at $T^{**} > T$. The leader, on the other hand, would move along $c'_m$ and its own stock would be exhausted at $T^m < T^{**}$ (recall that all three paths $p^*$, $p^{**}$ and $c'_m$ rise at the rate of interest). But in our original game where players come to the spot market period after period, which is what happens in reality, $p^{**}$ and $c'_m$ are not time consistent (i.e., violate subgame perfection). The easiest way to see this is by noticing that at time $T^m$ the leader would like to make additional purchases, which would drive prices up. Since fringe members anticipate and arbitrate this price jump the actual equilibrium path would lie somewhere between $p^{**}$ and $p^*$ (and $c'_m$ closer to $p^*$). But the leader has the opportunity to move not twice but in each and every period, so the only time-consistent path is the perfectly competitive path $p^*$.

*** INSERT FIGURE 3 HERE ***

4 Extensions

4.1 Trends in allocations and emissions

In most cases the transitory compliance flexibility is not created by a one-time allocation of a large stock of permits but rather by a stream of generous annual allocations, as
in the U.S. Acid Rain Program (see footnote 1). In a carbon market, the emissions constraint is likely to become tighter in the future not only due to lower allocations but also to significantly higher unrestricted emissions prompted by economic growth. This is particularly so for economies in transition and developing countries whose annual permits may well cover current emission but not those in the future as economic growth takes place.

To cover these situations, let us now consider aggregate allocation and unrestricted emission sequences, \( \{a_t, u_t\}_{t \geq 0} \), such that the reduction target \( u_t - a_t \) changes over time in a way that makes it attractive for firms to first save and build up a stock of permits and then draw it down as the reduction targets become tighter.\(^{21}\) As long as the market is leaving some stock for the next period, the efficient equilibrium is characterized by the Hotelling conditions, with the exhaustion condition replaced by the requirement that aggregate permit savings are equal to the stock consumption during the stock-depletion phase.\(^22\)

Although the stock available is now endogenously accumulated, each agent’s Hotelling share of the stock at \( t \) can be defined almost as before: it is a stock holding at \( t \) that just covers the agent’s future consumption net of the agent’s own savings. Let us now consider the Hotelling shares for the leader and fringe, facing reduction targets given by \( \{a^m_t, u^m_t\}_{t \geq 0} \) and \( \{a^f_t, u^f_t\}_{t \geq 0} \). Then, the leader’s Hotelling share of the stock at \( t \) is just enough to cover the leader’s future own net demand:

\[
s^m_t = \sum_{\tau=t}^T (u^m_\tau - q^m_\tau - a^m_\tau),
\]

where \( q^m_\tau \) denotes the socially efficient abatement path for the leader. On the other

---

\(^{20}\)We continue assuming that \( \{a_t, u_t\}_{t \geq 0} \) is known with certainty. Uncertainty would provide an additional storage motive, besides the one coming from tightening targets, as in standard commodity storage models (Williams and Wright, 1991). It seems to us that uncertainty may exacerbate the exercise of market power, but the full analysis and the effect on the critical holding needed for market power is beyond the scope of this paper.

\(^{21}\)If the reduction target increases because of economic growth, as in climate change, it is perhaps not clear why the marginal costs should ever level off. However, the targets will also induce technical change, implying that abatement costs will also change over time (see, e.g., Goulder and Mathai, 2000). While we do not explicitly include this effect, it is clear that the presence of technical change will limit the permit storage motive.

\(^{22}\)Obviously, the same description applies irrespective of whether savings start at \( t = 0 \) or at some later point \( t > 0 \), or, perhaps, at many distinct points in time. The last case is a possibility if the trading program has multiple distinct stages of tightening targets such that the stages are relatively far apart, i.e., one storage period may end before the next one starts.
hand, the socially efficient stock holdings, which are denoted by

\[ \hat{s}_t^m = \sum_{\tau=0}^{t} (a_{\tau}^m - a_{\tau}^t + q_{\tau}^m), \]

will typically differ from \( s_t^{m*} \). It can nevertheless be established:

**Proposition 3** If \( \hat{s}_t^m \leq s_t^{m*} \) for all \( t \), the subgame-perfect equilibrium is efficient.

The formal proof follows the steps of the proof of Proposition 2 and is therefore omitted. During the stock draw-down phase it is clear that we can directly follow the reasoning of Proposition 2 because it does not make any difference whether the market participants’ permit holdings were obtained through savings or initial stock allocations. Since, by \( \hat{s}_t^m \leq s_t^{m*} \), the leader needs to be a net buyer in the market to cover its own future demand, we can consider two cases as in Proposition 2. First, the leader cannot depress the price path down from the efficient path through restricted purchases (and increased own abatement) because of the fringe arbitrage; the fringe can store permits and make sure that its asset values do not go below the long-run competitive price in present value. Second, the leader cannot profitably make one-shot purchases large enough to monopolize the market such that the leader would be a seller at some later point; the market would more than fully appropriate the gains from such an attempt. As a result, the leader will in equilibrium trade quantities that allow cost-effective compliance but do not move the market away from perfect competition. This same argument holds for dates at which the market is accumulating the aggregate stock, because the argument does not depend on whether the leader is a net saver or user at \( t \).

The implications of Proposition 3 can be illustrated with the following two cases. Consider first the case in which the leader’s cumulated efficient savings \( \hat{s}_t^m \) are non-negative for all \( t \). Then, it suffices to check at date \( t = 0 \) that the leader’s cumulative allocation does not exceed the cumulative emissions. That is, if it holds that

\[ \sum_{t=0}^{T} a_t^m \leq \sum_{t=0}^{T} (u_t^m - q_t^{m*}), \]  

\[ (19) \]

then, it is the case that \( \hat{s}_t^m \leq s_t^{m*} \) holds throughout the subgame-perfect equilibrium.

Consider now the case depicted in Figure 4 which shows the time paths for the leader’s allocation and socially efficient emissions. Suppose that the areas in the figure are such that \( B - A = C \), which implies that (19) holds as an equality at \( t = 0 \). But at \( t = t' \) Proposition 3 no longer holds because \( B > C \) (recall that the large agent has been buying from the market in order to cover its permits deficit \( A \)). The subgame-perfect
equilibrium associated with this allocation profile cannot be efficient, because assuming efficiency up to $t = t' - 1$ implies that the equilibrium of the continuation game at $t = t'$ is not competitive but characterized as in Proposition 1. Therefore, the equilibrium path starting at $t = 0$ must have the shape of the noncompetitive path depicted in Fig. 1.

It is easy to see that moving to the less competitive equilibrium only benefits the fringe but not the large agent. The large agent is forced to be a net buyer in subgame-perfect equilibrium (it follows a lower marginal abatement path). In other words, market power shifts the emission path $u^m_t - q^m_t$ to the right as shown in Figure 4, whereas in the competitive equilibrium net purchases are zero, i.e., $B - A = C$. It then follows directly from Proposition 2 that the net purchase is not profitable: the leader buys permits at higher than competitive prices and then sells them, on average, at lower prices. Thus the gains from market manipulation spill over to fringe asset values.

Although using future allocations for current compliance is ruled out by regulatory design, the leader can restore the competitive solution as a subgame-perfect equilibrium by swapping part of its far-term allocations for near-term allocations of competitive agents. To be more precise, the large agent would need to swap at the least an amount equal to area $A$ in Figure 4.24

*** INSERT FIGURE 4 HERE ***

4.2 Long-run market power

So far we have considered that after exhaustion of the overall stock firms follow perfect competition. This is the result of assuming either that the large agent’s long-run permit allocation is close to its long-run competitive emissions or that the long-run equilibrium price of permits is fully governed by the price of backstop technologies (see (12) and footnote 18). While the long-run perfect competition assumption is reasonable for both of our applications below, it is still interesting to explore the implications of long-run market power on the evolution of the permits stock. Since long-run market power is intimately related to the large agent’s long-run annual allocation relative to its emissions, it should be possible to make a distinction between the market power attributable to the long-run annual allocations and the transitory market power attributable to the stock

---

23 In all existing and proposed market designs firms are not allowed to “borrow” permits from far-term allocations to cover near-term emissions (Tietenberg, 2006).

24 Although not necessarily related to the market power reasons discussed here, it is interesting to note that swap trading is commonly used in the US sulfur market (see Ellerman et al., 2000).
allocations.\textsuperscript{25}

The first relevant case is that of long-run monopoly power, which following the equilibrium conditions of Propositions 1 and 2 is illustrated in Figure 5. For clarity, we assume that long-run allocations are constants. Then, the long-run market power coming from an annual allocation $a^m > a^{m*}$ implies a higher than competitive price $p^m > p^*$. Whether there is any further transitory market power coming from the stock allocation depends, as in previous sections, on the large agent’s share of the transitory stock. The equilibrium without transitory market power is characterized by a competitive storage period with a distorted terminal price at $p^m > p^*$, where the ending time is denoted by $T_0^f$ to reflect the fact that the fringe is holding a stock to the very end of the storage period. This path is depicted in Fig. 5 as $p_0^m$. The critical stock is defined by this path as the holding that just covers the leader’s own compliance needs without any spot trading additional to that prevailing after the stock exhaustion. Note that the overall stock is depleted faster than what is socially optimal, $T_0^f < T$, because the long-run monopoly power allows the leader to commit to consuming more than the efficient share of the available overall allocation.

The transitory market power, that arises for holdings above the critical level, leads to an equilibrium price path $p_{m1}$ with a familiar shape. This path reaches price $p^m$ at $t = T^m$, which can be smaller or larger than $T$ depending on whether the long-run shortening effect is greater or smaller than the transitory extending effect.

*** INSERT FIGURE 5 HERE ***

The second relevant case is that of long-run monopsony power, which is illustrated in Figure 6. Here, the equilibrium path without transitory market power, which is denoted by $p_0^m$, stays below the socially efficient path throughout ending at $p^m < p^*$. The time of overall stock depletion is extended, i.e., $T_0^f > T$, because the long-run monopsonist restricts purchases and is thereby able to depress the price level throughout the equilibrium. Again, this path defines the critical stock for the transitory market power as the holding that allows compliance cost minimization without adding to the long-run trading activity. Quite interestingly, for stockholdings above this critical level,

\textsuperscript{25}Note that in the presence of long-run market power we may no longer treat the stock depletion game as a strictly finite-horizon game for the case in which the large agent is not a single firm but a cartel of two or more firms. One could argue, for example, that the (subgame-perfect) threat of falling into the (long-run) noncooperative equilibrium may even allow firms to sustain monopsony power during the stock depletion phase.
the large agent has more than its own need during the transition, so that the agent is first a seller of permits but later on becomes a buyer of permits. The price path with transitory market power is denoted by $p_t^m$ which ends at $t = T^m$ and intersects the marginal cost $c'_m(q^m_t)$ at the point where $x^m_t = 0$, so that this intersection identifies the precise moment at which the large agent start coming to the market to buy permits (while continue consuming from its own stock). Note the transitory motive to keep marginal net revenues equalized in present value extends the overall depletion period further in addition to the extension coming from the long-run monopsony power and, therefore, $T^m$ is unambiguously greater than $T$.

*** INSERT FIGURE 6 HERE ***

5 Applications

We illustrate the use of our theoretical results with two very different applications: the carbon market that may eventually develop under the Kyoto Protocol and beyond and the existing (multibillion dollar) sulfur market of the U.S. Acid Rain Program of the 1990 Clean Air Act Amendments (CAAA).

5.1 Carbon trading

Motivated by the widespread concern about Russia’s ability to exercise market power (e.g., Bernard et al., 2003; Manne and Richels, 2001; Hagem and Westskog, 1998), the purpose of this first application is to illustrate whether and to what extent Russia’s ability to manipulate the carbon market is ameliorated when we take into consideration the possibility of storing today’s permits for future use. Except for the permit allocations established by the Kyoto Protocol for the period 2008-2012, the input data we used in this exercise come from work done at the MIT Joint Program on the Science and Policy of Global Change. Since the computable general equilibrium model developed by MIT — known as the MIT-EPPA model (Babiker et al., 2001) — aggregates all Former Soviet Union (FSU) countries into one region, we take the FSU region as our large agent. Unrestricted emissions for years 2010-2050 and different Kyoto regions come from the MIT-EPPA runs reported in Bernard et al. (2003). Marginal cost curves are borrowed

\footnote{We thank John Reilly of MIT for sharing the background data of this paper with us. Besides FSU, the remaining Kyoto regions of the MIT-EPPA model are Japan (JPN), The European Union (EEC), other
from Ellerman and Decaux (1998). Post-Kyoto permits allocations are constructed upon the architecture discussion provided by Jacoby et al. (1999). From their discussion one can envision that countries accepting binding commitments will see their future permits allocations declining at some rate between 1 and 2% per year until they reach some minimum level that we believe should not be much lower than 50%. Carbon permit prices are capped by the existence of backstop technologies available at prices in the range of $400-500 per ton of carbon (all prices are 1995 prices). We also allow for some Clean Development Mechanism (CDM) credits, i.e., voluntarily-generated credits outside the Kyoto regions, following the approach of Bernard et al. (2003). Finally, we adopt the commonly used discount rate of 5% (e.g., Jacoby et al., 1999).

Let us first consider market equilibrium outcomes when the Kyoto commitment period, 2008-2012, is taken in isolation from future commitment periods (results are presented only for 2010, which is taken as the representative year for this five-year period). The first row of Table 1 presents expected unrestricted emissions, i.e., emissions in the absence of regulation. The next two rows show, respectively, the perfectly competitive equilibrium solution and the outcome when FSU exercises maximum market power. FSU emissions in perfect competition are 75% of its permit allocation, which is an indication of potential for market power. In implementing the market-power outcome FSU abates no emissions and restricts its supply of hot air (permits above unrestricted emissions) in 18 million tons of carbon (mtC) from its total of 186 mtC almost doubling the equilibrium price.

*** INSERT TABLE 1 HERE OR BELOW ***

Let us now extend the regulatory window beyond Kyoto while keeping the same permit trading regulatory approach. In Table 2 we present an equilibrium path in which

---

27 At least this is consistent with the recent announcement of the UK government setting itself the target of cutting CO2 emissions to 60% below 1990 levels by 2050. For more go to http://www.dti.gov.uk/energy/whitepaper/index.shtml.

28 This is not only realistic, but as mentioned in footnote 17, it simplifies the analysis because the presence of backstop technologies eliminates any (long-run) market power considerations after the exhaustion of the stock of permits.

29 Unless otherwise indicated we use $\alpha = 300$ for the supply coefficient in the CDM equation of Bernard et al. (2003). We cap CDM credits at 1000 million tons of carbon (mtC) to keep them close to the total allocation of Kyoto regions other than the FSU’s. Note that CDM credits can be interpreted more generally as the inclusion of non-Kyoto countries into binding commitments as their income per capita increases.
FSU fails to exercise market power. FSU cumulative emissions over 2010-2050 are equal to its cumulative allocation of permits during that time (following the Kyoto design, post-Kyoto commitment periods are also taken as five-year periods). Following on our input data discussion above, this competitive outcome is constructed upon several assumptions. We let the permit allocation of Kyoto regions to remain equal to the Kyoto allocation for next two commitment periods and then decline at an annual rate of 2% for the remaining periods. The stock of permits is totally exhausted at year 2050 when carbon prices approach backstop technology prices around 450 $/tC.

*** INSERT TABLE 2 HERE OR BELOW ***

Despite the equilibrium path of Table 2 does not exhibit market power, it is important to understand that agents’ subgame perfect equilibrium strategies do impose restrictions on the maximum amount of stock-holdings that the large agent can have at any point in time before constituting a (costly) deviation. Such amounts are the Hotelling shares $s^m_t$ identified in Section 4.1 and are shown in the penultimate column of Table 2. Note that because FSU cumulative emissions are equal to its cumulative allocation, its stock-holdings $\hat{s}^m_t$ coincide with the Hotelling shares at all times. Obviously, FSU can sell part or all of its stock $\hat{s}^m_t$ in the market without altering the competitive equilibrium.

Let us now introduce some market power by changing two assumptions: reduce the CDM supply coefficient by half and let future permits allocations of Kyoto regions decline by 1% instead of 2%. In Table 3 we present the competitive market equilibrium. Since cumulative permits over the period 2010-2050 are now above cumulative emissions, there is certainly room for the exercise of market power; unless the large agent faces the stock-holding limits dictated by the Hotelling shares of the last column. Since stock-holdings cannot go negative by regulatory design, the negative numbers of the first two commitment periods should be interpreted as zero stock-holdings during that time. Alternatively, we could require the large agent to dispose part of its permit allocation by taking short positions in the forward market. In any case, when market power is believed to be a problem, our framework should prove useful in understanding how the imposition of stock-holding limits can make an early generous allocation of permits be fully compatible with competitive behavior. This seems to be particularly relevant for discussions on how to incorporate large developing countries such as China under a global carbon trading regime.
5.2 Sulfur trading

Our second application differs from the carbon application in important ways. Unlike the carbon market, the market for sulfur dioxide (SO\textsubscript{2}) emissions has been operating since the early 90s; right after the 1990 CAAA allocated allowances/permits to electric utility units for the next 30 years in designated electronic accounts.\textsuperscript{30} So, we can make use of agents’ actual behaviors, as opposed to hypothetical ones, to test for the existence, or more precisely, for the no existence of market power during the evolution of the permit stock. The data we use for our test, which is publicly available, comprises electric utility units’ annual SO\textsubscript{2} emissions and allowance allocations from 1995 — the first year of compliance with SO\textsubscript{2} limits — through 2003. We purposefully exclude 2004 numbers because of the four-fold increase in SO\textsubscript{2} allowance prices during 2004-05 in response to the proposed implementation of the Clean Air Interstate Rule, which would effectively lower the SO\textsubscript{2} limits established in the original regulatory design by two-thirds in two steps beginning in 2010. Although this recent price increase provides further evidence that in anticipation of tighter limits firms do respond by building up extra stocks (or by depleting exiting stocks less intensively), we concentrate on firms’ behavior under the original regulatory design where we have nine years of data and can therefore, make reasonable projections as needed. The long-term emissions goal under the original design is slightly above 9 million tons of SO\textsubscript{2}.

Following our theory, the test consists in identifying potential strategic players and check whether or not the necessary condition for market power (that initial allocations be above perfectly competitive emissions, i.e., \( s^m_0 > s^{m*}_0 \) holds. The potential strategic players in our analysis, acting either individually or as a cohesive group, are assumed to be the four largest permit-stock holding companies — American Electric Power, Southern Company, FirstEnergy\textsuperscript{31} and Allegheny Power — that together account for 42.5% of the permits allocated during Phase I of the Acid Rain Program, i.e., 1995-1999, which corresponds to the ”generous-allocation” phase.\textsuperscript{32} While \( s^m_0 \) is readily obtained from

\textsuperscript{30}For details in market design and performance see Ellerman et al. (2000) and Joskow et al. (1998).
\textsuperscript{31}Note that FirstEnergy was the result of mergers in 1997 and 2001 but for the purpose of this analysis we make the conservative assumption that all mergers were consummated by 1995.
\textsuperscript{32}Their individual shares of Phase I permits are 13.2, 13.5, 9.3 and 6.5%, respectively. The next permit-stock holder is Union Electric Co. with 4.2% of the permits. Neither was Tennessee Valley
agents’ cumulative permit allocations, calculating $s_0^{m*}$ would seem to require a more elaborate procedure based, perhaps, on some abatement cost estimates. But unlike the carbon application, this is not necessarily so because we have actual emissions data.\footnote{Note that our focus is on transitory market power, i.e., market power during the evolution of the permit stock. We believe long-run market power to be less of a problem because strategic players’ long-run allocations are greatly reduced in relative terms (the largest player receives less than 8% and the Group of Four 23%). Even if we want to test for long-run market power, we cannot do so before observing the actual behavior in long-run equilibrium. We come back to this issue at the end of this section.}

Table 4 presents a summary of compliance paths for the two largest strategic players, the Group of Four, as well as for all firms. The noticeable discontinuities in 2000 — the first year of Phase II — are due to both a significant decrease in permit allocations and the entry of a large number of previously unregulated sources.\footnote{Some of these unregulated sources voluntarily opted in earlier into Phase I and received permits under the so-called Substitution Provision. Since with very few exceptions opt-in sources have helped utilities to increase their permit stocks (Montero, 1999), for the purpose of our analysis we treat these sources (with their emissions and allocations) as Phase I sources.} Precisely because of this discontinuity in the regulatory design firms had incentives to build a large stock of permits during Phase I, which reached an aggregate peak of 11.65 million allowances by the end of 1999. Although strategic players, either individually or as a group, present a significant surplus of permits by 1999 that may be indicative of possible market power problems,\footnote{In reality their actual stocks may be larger or smaller than these figures depending on firms’ market trading activity. Our theoretical predictions, however, are independent of trading activity as long as it is observed, which in this particular case can be done with the aid of the U.S. EPA allowance tracking system. We will come back to the issue of imperfect observability in the concluding section.} it is also true that these players are rapidly depleting their stocks from the simple fact that their annual emissions are above their annual permit allocations. By 2003, the last year for which we have actual emissions, the stock of the Group of Four is already reduced to 1.11 million allowances while the aggregate stock is still significant at 6.47 million allowances.

*** INSERT TABLE 4 HERE OR BELOW ***

Taking a linear extrapolation of aggregate emissions from its 2003 level of 10.60 million tons to the long-run emissions limit of 9.12 million tons, we project the aggregate stock of permits to be depleted by 2012, which is very much in line with the more elaborated
projections of Ellerman and Montero (2005). Assuming that the share of emissions for the projected years is the same as during 2000-2003, the numbers in the last row of Table 4 show that the compliance paths followed by the potential strategic players, taken either individually or collectively, can only be consistent with perfect competition. As established by Propositions 2 and 3, a necessary condition for a large agent, whether a firm or a cartel, to exercise market power is that of being a net seller of permits. But the net sellers in this market are many of the smaller players, not the large players.

Our focus has been on transitory market power, i.e., market power during the evolution of the permit stock. Testing for the long-run market power discussed in Section 4.2 is not feasible without having data on actual long-run behavior. We believe, however, long-run market power to be less of a problem because large players’ long-run allocations are greatly reduced in relative terms. The largest player (Southern Company) receives less than 8% of the total allocation and the Group of Four only 23%. Any larger coalition of players would be hard to imagine. Moreover, it is quite possible that the long-run market equilibrium would have been dictated by the price of scrubbing technologies capable of removing up to 95% of SO\textsubscript{2} emissions.

6 Concluding Remarks

We developed a model of a market for storable pollution permits in which a large polluting agent and a fringe of small agents gradually consume a stock of permits until they reach a long-run emissions limit. We characterized the properties of the subgame-perfect equilibrium for different permit allocations and found the conditions under which the large agent fails to exercise any degree of market power. These findings have practical implications that go from the diagnosis of market-power (as seen in the sulfur application) to the design of ways in which large unregulated sources with low abatement costs can be brought to the regulation via generous permit allocations that induce participation without market-power side effects. This latter issue was illustrated in the carbon application.

In view of the different type of market transactions that we observe in the US sulfur

\footnote{This is a reasonable assumption in the sense that the extra reduction needed to reach the long-run limit is moderate and not much larger than the reduction that has already taken place in Phase II. In addition, since we know that all firms move along their marginal cost curves at the (common) discount rate regardless of the exercise of market power, their emission shares should not vary much if we believe their marginal cost curves have similar curvatures in the relevant range.}

\footnote{The same argument applies if the overall stock is expected to be depleted much earlier, say, in 2009.
market (see Ellerman et al., 2000), it is natural to ask whether and how our equilibrium solution would change if we extend agents’ action space to stock and forward transactions. Since stock transactions are observable (as spot transactions are), allowing agents to trade large chunk of permits does not introduce any changes in our equilibrium solution. Our definition of spot transaction implicitly allows for this interpretation. The same does not apply to forward trading, however. A large agent with an initial stock large enough to move the market wants to avoid forward transactions.\textsuperscript{38} By selling part of its stock forward, the large agent introduces a time-inconsistency problem that makes it worse off. Fringe members correctly anticipate that right after the leader has committed part of its stock forward, the continuation game becomes more competitive (Liski and Montero, 2006a).\textsuperscript{39}

Our model also assume that agents’ stock-holdings are observable at the beginning of each period. While the EPA allowance tracking system may significantly facilitate keeping track of agents’ stock-holdings in the US sulfur market,\textsuperscript{40} it is still interesting to ask what would happen to our equilibrium solution if we let stock-holdings be somewhat private information (or alternatively, assume that large stockholders can use third parties, e.g., brokers, to hide their identities). Lewis and Schmalensee (1982) have already identified this incomplete information problem for a conventional nonrenewable resource market where agents’ reserves are only imperfectly observed. They argue that Salant’s (1976) solution no longer holds: the large agent could increase profits (above Salant’s) by covertly producing either more or less than its Salant equilibrium output. We see the exact same problems affecting our equilibrium solution. Unfortunately Lewis and Schmalensee (1976) do not offer much insight as to what the new equilibrium conditions might look like. We think this is an interesting topic for future research.

Uncertainty is another ingredient absent in our model. This may be particularly relevant for the carbon application that shows time-horizons of several decades. There are multiple sources of uncertainty related to different aspects of the problem such as technology innovation, economic growth, future permit allocations, timing and extent of participation of non-Kyoto countries, etc. How these uncertainties, acting either individually or collectively, could affect the essence of our equilibrium solution is not immediately

\textsuperscript{38}Even in the absence of uncertainty it is possible that agents in oligopolistic markets get engaged in forward trading for pure strategic reasons (Allaz and Vila, 1993; Liski and Montero, 2006b).

\textsuperscript{39}If the large agent is a cartel of firms, one could argue that firms may need to resort to forward trading in order to sustain tacit collusion. But if anything, we suspect that this should be done through long positions, i.e., by buying forward contracts (Liski and Montero, 2006b).

\textsuperscript{40}For a description of the EPA tracking system go to http://www.epa.gov/airmarkets/tracking/.

26
obvious to us because of the irreversibility associated to the build-up and depletion of the permits stock. Tackling these issues would require to put together the strategic elements found in this paper with those of the literature of investment under uncertainty (Dixit and Pindyck, 1994).

Although one can view our sulfur application as the first attempt at empirically estimating the extent of market power in pollution permit trading, we were somehow fortunate that our test ended with the finding that the necessary conditions for market power did not hold. Had we found the opposite, that cumulative permit allocations of the large players were above their cumulative emissions, we would have needed to develop an econometric procedure that, among other things, had enabled us to recover firm cost data from public observables like emissions and prices. It is not clear to us how the econometric tools from the new empirical industrial organization literature could be applied to such endeavour because of an identity problem. Unlike conventional markets, in permit markets we cannot tell ex-ante the buyers from the sellers. Identities are endogenous to the trading activity, which in turn, depends on whether market power is being exercised or not.

Appendix A: Proof of Proposition 1

Proof. The proof has two main parts. First, we show that working backwards from the leader’s exhaustion date \( T^m \) to the fringe exhaustion date \( T^f \), and finally to initial time \( t = 0 \), leads to equilibrium conditions (13)-(16). Second, we show that conditions (13)-(16) pin down unique \( T^m \) and \( T^f \) such that \( T^m > T^f \) if \( s^m_0 > s^*_0 \).

First part. The leader’s problem starting at the fringe exhaustion date \( T^f \) is a monopoly decision problem whose solution maximizes the value \( V^{m}_{T^f}(s^m_{T^f}, s^f_{T^f}) \) which was defined in (5) and is restated here for convenience

\[
V^{m}_{t}(s^m_{t}, s^f_{t}) = \max_{\{x^m_{t}, q^m_{t}\}} \left\{ p_t x^m_{t} - c_m(q^m_{t}) + \delta V^{m}_{t+1}(s^m_{t+1}, s^f_{t+1}) \right\}.
\]  

(20)

At any stage \( t \in \{T^f, ..., T^m - 1\} \), the optimal choice for \( x^m_{t} \) and \( q^m_{t} \) must satisfy

\[
c'_{f}(q^f_{t}) + x^m_{t} c''_{m}(q^m_{t}) \frac{\partial q^m_{t}}{\partial x^m_{t}} + \delta \frac{\partial V^{m}_{t+1}(s^m_{t+1}, 0)}{\partial s^m_{t+1}} \frac{\partial s^m_{t+1}}{\partial x^m_{t}} = 0
\]

\[
- c'_{m}(q^m_{t}) + \delta \frac{\partial V^{m}_{t+1}(s^m_{t+1}, 0)}{\partial s^m_{t+1}} \frac{\partial s^m_{t+1}}{\partial q^m_{t}} = 0.
\]

Note that since the fringe is not storing permits, its response to sales satisfies \( \partial q^f_t / \partial x^m_t = -1 \) (see (7) with \( s^f_{t+1} = 0 \)). This together with \( \partial s^m_{t+1} / \partial x^m_t = -1 \) and \( \partial s^m_{t+1} / \partial q^m_t = 1 \)
implies that the first-order conditions can be combined to yield

\[ c'_f(q^f_t) - x^m_t c''_f(q^f_t) = c'_m(q^m_t), \]  

which is the condition (16) in the text. Applying the envelope theorem to (20) shows that for any \( t \in \{T^f, ..., T^m - 1\}, \)

\[ \frac{\partial V^m_t(s^m_t, 0)}{\partial s^m_t} = \frac{\partial V^m_{t+1}(s^m_{t+1}, 0)}{\partial s^m_{t+1}}. \]  

(22)

For \( t = T^m \), we must have

\[ \frac{\partial V^m_{T^m}(s^m_{T^m}, 0)}{\partial s^m_{T^m}} = c'_f(q^f_{T^m}) - x^m_{T^m} c''_f(q^f_{T^m}) \]

\[ = c'_m(q^m_{T^m}) \]

\[ \geq \delta c'_m(u^m - a^m). \]  

The first equality follows from (22) and (23). The second follows by applying the Envelope Theorem to \( V^m_{T^m}(s^m_{T^m}, 0) \) and using the fact that \( s^f_{T^m+1} = 0 \).

The following Lemma states the relevant information needed about the monopoly phase for the analysis of subgames preceding the fringe exit.

**Lemma 1** Suppose \( T^f \) is the fringe exhaustion date in the subgame-perfect equilibrium. Then, the leader’s value function at \( T^f \) satisfies

\[ \frac{\partial V^m_{T^f}(s^m_{T^f}, s^f_{T^f})}{\partial s^m_{T^f}} = c'_f(q^f_{T^f}) - x^m_{T^f} c''_f(q^f_{T^f}) \]

\[ \frac{\partial V^m_{T^f}(s^m_{T^f}, s^f_{T^f})}{\partial s^f_{T^f}} = -x^m_{T^f} c''_f(q^f_{T^f}). \]

**Proof.** The first equality follows from (22) and (23). The second follows by applying the Envelope Theorem to \( V^m_{T^f}(s^m_{T^f}, s^f_{T^f}) \) and using the fact that \( s^f_{T^f+1} = 0 \).

The next Lemma builds upon Lemma 1 to derive an equilibrium condition that will readily lead to the verification of the Salant’s conditions characterizing the subgame-perfect equilibrium path.
Lemma 2 Let \( t \in \{0, ..., T^f - 1\} \) be some date such that the fringe is holding a stock for the next period in the subgame-perfect equilibrium. Then, the following must hold:

\[
c'_f(q^f_t) + x^m_t c''_f(q^f_t) \frac{\partial q^f_t}{\partial x^m_t} = \delta [c'_f(q^f_{t+1}) + x^m_{t+1} c''_f(q^f_{t+1}) \frac{\partial s^m_{t+1}}{\partial q^f_t} \frac{\partial q^f_t}{\partial x^m_t}], \tag{24}
\]

**Proof.** Sales at any \( t \in \{0, ..., T^f - 1\} \) must satisfy

\[
c'_f(q^f_t) + x^m_t c''_f(q^f_t) \frac{\partial q^f_t}{\partial x^m_t} + \delta \frac{\partial V^m_{t+1}(s^m_{t+1}, s^f_{t+1})}{\partial s^m_{t+1}} \frac{\partial s^m_{t+1}}{\partial x^m_t} + \delta \frac{\partial V^m_{t+1}(s^m_{t+1}, s^f_{t+1})}{\partial s^f_{t+1}} \left( \frac{\partial s^f_{t+1}}{\partial q^f_t} \frac{\partial q^f_t}{\partial x^m_t} + \frac{\partial s^f_{t+1}}{\partial x^m_t} \right) = 0.
\]

In particular, at \( t = T^f - 1 \), this first-order condition leads to (24) by Lemma 1. By the fact that the fringe is holding permits between \( T^f - 1 \) and \( T^f \), we have

\[p_{T^f-1} = c'_f(q^f_{T^f-1}) = \delta c'_f(q^f_{T^f}) = \delta p_{T^f},\]  

which together with (24) implies

\[x^m_{T^f-1} c''_f(q^f_{T^f-1}) = \delta x^m_{T^f} c''_f(q^f_{T^f}).\]  

Next we again apply the envelope theorem to (20) at \( t < T^f \) to obtain

\[
\frac{\partial V^m_t(s^m_t, s^f_t)}{\partial s^m_t} = \left\{ x^m_t c'_f(q^f_t) + \delta \frac{\partial V^m_{t+1}(s^m_{t+1}, s^f_{t+1})}{\partial s^f_{t+1}} \frac{\partial s^f_{t+1}}{\partial q^f_t} \frac{\partial q^f_t}{\partial s^m_t} \right\} \frac{\partial q^f_t}{\partial s^m_t} \tag{27}
\]

and

\[
\frac{\partial V^m_t(s^m_t, s^f_t)}{\partial s^f_t} = \left\{ x^m_t c'_f(q^f_t) + \delta \frac{\partial V^m_{t+1}(s^m_{t+1}, s^f_{t+1})}{\partial s^f_{t+1}} \frac{\partial s^f_{t+1}}{\partial q^f_t} \frac{\partial q^f_t}{\partial s^f_t} \right\} \frac{\partial q^f_t}{\partial s^f_t} \tag{28}
\]

Consider now these expressions at \( t = T^f - 1 \). Using the second equality of Lemma 1
together with (26), implies

\[
\frac{\partial V_{m,T-1}}{\partial s_{m,T-1}} = \delta \frac{\partial V_{m,T}}{\partial s_{m,T}} \quad \text{(29)} \\
\frac{\partial V_{m,T-1}}{\partial s_{T,T-1}} = \delta \frac{\partial V_{T,T}}{\partial s_{T,T}} \quad \text{(30)}
\]

We have now enough material for the recursion establishing (24) for all \( t < T_f \). At \( t = T_f - 2 \), the first-order condition for \( x_{m,T-2} \) can be written, using (29)-(30), as follows:

\[
c^*_f(q_{T-2}^f) + x_{T-2}^m c''_f(q_{T-2}^f) \frac{\partial q_{T-2}^f}{\partial x_{T-2}^m} + \delta^2 [c'_f(q_{T}^f) - x_{T}^m c''_f(q_{T}^f)] \frac{\partial s_{T+1}^m}{\partial x_{T}^m} + \delta^2 [-x_{T}^m c''_f(q_{T}^f)] \frac{\partial s_{T+1}^m}{\partial q_{T}^f} + \delta^2 \frac{\partial s_{T+1}^m}{\partial x_{T}^m} = 0
\]

which, by (25) and (26), shows that (24) holds between \( T_f - 2 \) and \( T_f - 1 \). Repeating the above steps for all periods \( t = \{T_f - 3, T_f - 4, ..., 0\} \) completes the proof.

We can now complete the first part of the proof. The fringe arbitrage condition \( p_t = \delta p_{t+1} \) and Lemma 2 imply that

\[
x_t^m c''_f(q_t^f) = \delta x_{t+1}^m c''_f(q_{t+1}^f)
\]

holds for all \( t \in \{0, ..., T_f - 1\} \). Therefore, we have

\[
c'_f(q_t^f) - x_t^m c'_f(q_t^f) = \delta [c'_f(q_{t+1}^f) - x_{t+1}^m c'_f(q_{t+1}^f)]
\]

for all \( t \in \{0, ..., T_f - 1\} \), which is the condition (14) in the text.

Second Part. We explain next that, if \( s_0^m > s_0^{m*} \), conditions (13)-(16) provide enough information to pin down unique time periods \( T_f \) and \( T_m \) such that \( T_m > T_f \). For ease of exposition, let us first consider the case \( s_0^f = 0 \). Then, condition (13) drops out, and we only need to solve the difference equation (15) \( x_t^m \) will be chosen to satisfy eq. (16), so eq. (14) will automatically hold, given the path for \( q_t^m \). We can find the initial value \( q_0^m \) and terminal time \( T_m \) \( (T_f = 0 \text{ by } s_0^f = 0) \) using the boundary condition

\[
q_0^m \leq q_{T_m+1}^m = u^m - a^{m*},
\]
and the stock exhaustion condition

\[ s_0^m = \sum_{t=0}^{T_m} (u^m - a^{m*} - q_t^m + x_t^m). \]

Consider now the case \( s_0^f > 0 \). The solution is found as above but we have to solve two difference equations, (13) and (15); again, \( x_t^m \) is defined through (16). There are now four unknowns: \( q_0^m, q_0^f, T^m, \) and \( T^f \). We can use

\[ q_{T^f}^f \leq q_{T^f+1}^f \]

as an additional boundary condition and

\[ s_0^f = \sum_{t=0}^{T^f} (u^f - a^{f*} - q_t^f - x_t^m) \]

as an additional exhaustion condition.

To complete the proof we need to demonstrate that the above solution has the following property

\[ s_0^m > s_0^{m*} \implies T^m > T^f. \] (31)

Note that

\[ s_0^m = s_0 \implies T^m > T^f = 0, \] (32)

\[ s_0^m = s_0^{m*} \implies T^m = T^f = T, \] (33)

where \( s_0^m = s_0 \) is the monopoly case and \( s_0^m = s_0^{m*} \) is the Hotelling share (\( T \) is the competitive exhaustion time). From (32), it is clear that property (31) holds if \( s_0^m \) is close to the monopoly share. Property (33) follows since the system (13)-(16) has a unique solution with \( x_t^m = 0 \) and \( T^m = T^f = T \) if the leader’s share of the stock equals the Hotelling share, \( s_0^m = s_0^{m*} \). We need to show that

\[ s_0^m - s_0^{m*} = \varepsilon > 0 \implies T^m > T^f \] (34)

where \( \varepsilon \) is small. We have assumed that the Hotelling prices are almost continuous in present value between \( T \) and \( T+1 \), i.e., \( p_T \approx \delta p_{T+1} \). More precisely, let \( \eta = p_T - \delta p_{T+1} > 0 \) denote the gap in the Hotelling prices due to the integer problem caused by discrete time. We claim that for any given \( \varepsilon > 0 \), there exists \( \eta > 0 \) small enough such that the property (34) holds. For given \( \varepsilon \), assume that the equilibrium satisfies \( T^m = T^f = T \) and let \( \eta \to 0. \)
This leads to a contraction since, by \( \varepsilon > 0 \), it is an option for the leader to comply without trading, i.e., equalize marginal costs in present value by using permits from the stock \( s_m^0 \). But this implies that the leader has not used \( \varepsilon \) when period \( t = T \) arrives. Selling \( \varepsilon \) at \( t = T \) leads to \( p_T < \delta p_{T+1} \), which is the contradiction. Reoptimizing at the last period implies that it becomes optimal to extend the overall depletion period, \( T^m > T^f \). ■

Appendix B: Proof of Proposition 2

**Proof.** The result is proved by backward induction as follows. First, at the exhaustion period \( T \) the equilibrium is shown to be efficient if \( s^m_T \leq s^m_T^* \) (\( s^m_T^* \) denotes the Hotelling share at \( t \)). Second, at \( T - 1 \) the equilibrium is efficient if \( s^m_{T-1} \leq s^m_{T-1}^* \). In particular, the leader’s spot market transaction at \( T - 1 \) is such that the exhaustion period stock is also competitive, \( s^m_T \leq s^m_T^* \). Finally, the argument extends to any number of periods to go, implying that the efficiency follows from \( s^0_m \leq s^0_m^* \).

Consider now period \( T \) and let \( s^m_T \leq s^m_T^* \). Suppose first that the leader is a seller at the exhaustion period, \( x^m_T > 0 \). Then, the leader’s spot market transaction must be such that the following hold:

\[
\begin{align*}
    p_T &= c'_f(q^m_T) \geq \delta c'_f(q^T_{T+1}) = \delta p_{T+1} \quad (35) \\
    c'_f(q^m_T) - x^m_T c''_f(q^m_T) &= c'_m(q^m_T) \quad (36) \\
    \quad \geq \delta c'_m(q^m_{T+1}) = \delta p_{T+1}, \quad (37)
\end{align*}
\]

where, by the fact that \( T \) is the exhaustion period,

\[
\begin{align*}
    q^f_T &= u^f - a^f - s^f_T + x^m_T, \\
    q^m_T &= u^m - a^m - s^m_T - x^m_T, \\
    q^f_{T+1} &= u^f - a^f^*, \\
    q^m_{T+1} &= u^m - a^m^*.
\end{align*}
\]

By the definition of \( s^m_T^* \), \( s^m_T = s^m_T^* \) implies that condition (36) holds without trading, i.e., \( x^m_T = 0 \). Therefore, choosing \( x^m_T > 0 \) when \( s^m_T \leq s^m_T^* \) implies \( c'_m(q^m_T) > p_T = c'_f(q^m_T) \), which contradicts profit maximization.

Thus, the leader must be a buyer, \( x^m_T \leq 0 \), when the stock is below the Hotelling share, \( s^m_T \leq s^m_T^* \). When there is monopsony power, conditions (35) and (37) must still
hold but (36) must be replaced by
\[
\max\{\delta p_{T+1}, c'_f(q_T) - x^m_T c''_f(q_T)\} = c'_m(q_T^m)
\]
since the fringe arbitrage implies that their permit values cannot go below \(\delta p_{T+1} = \delta c'_f(q_{T+1}) = \delta c'_m(q_{T+1}^m)\). This in turn implies that the most the leader can do is to depress the last period price to make the arbitrage condition in the last period, condition (35), to hold as an equality. We have assumed that the Hotelling prices are almost continuos in present value between \(T\) and \(T+1\), i.e., \(p_T \approx \delta p_{T+1}\). Since the true price at \(t = T\) is sandwiched between the Hotelling price (achieved exactly when \(s_T^m = s_{T-1}^{m*}\)) and \(\delta p_{T+1}\), we can ignore the leader’s ability to depress the period \(T\) price.

Consider now period \(T-1\) and let \(s_{T-1}^m \leq s_{T-1}^{m*}\). Again, selling at \(T-1\) from a stock below the Hotelling share leads to a contradiction with profit maximization because this would imply
\[
p_{T-1} < c'_m(q_{T-1}) = \delta c'_m(q_{T}^m),
\]
where the equality follows from the fact that the leader has an option to store permits. The leader is thus a buyer at \(T-1\) and has two options. First, he can buy a quantity that keeps the stock below the Hotelling share,
\[
0 \geq x_{T-1}^m \geq s_{T-1}^m - s_{T-1}^{m*}.
\]
(38)

Period \(T-1\) price cannot be depressed by restricting the purchase, since the fringe arbitrage implies
\[
p_{T-1} = \delta p_T
\]
where the equality follows from the fact that the fringe is storing permits. The equilibrium is therefore competitive at \(T-1\) as long as (38) holds. Second, the leader can buy a quantity that raises the holding above the Hotelling share
\[
x_{T-1}^m < s_{T-1}^m - s_{T-1}^{m*}.
\]
(39)

Now, the leader is a seller at \(T\) and conditions (35)-(37) must hold at \(T\), which is anticipated by the market at \(T-1\). Having observed (39), the fringe arbitrage leads to
\[
p_{T-1} = \delta p_T > \delta^2 p_{T+1},
\]
\footnote{See footnote 14.}
where the inequality follows from (35)-(36) together with $x_T^m > 0$. However, the leader is now buying with a price higher than the marginal benefit from buying,

$$p_{T-1} = \delta p_T > \delta \{ c'_f(q_f^T) - x_T^m c^m_f(q_f^T) \} = \delta c'_m(q_T^m).$$

This shows that keeping the stock below the Hotelling share, strategy (38), is optimal at $T - 1$. Therefore, the equilibrium is competitive at $T - 1$, if $s_{m}^T_{T-1} \leq s_{m}^a_{T-1}$.

To complete the proof, we note that the argument for stage $T - 1$ can be repeated for any stage between $t = 0$ and $t = T - 1$, implying that the subgame-perfect equilibrium is competitive if $s_{m}^T_{t} \leq s_{m}^a_{t}$. The equilibrium abatement path is unique, although any actual trading path $x_t^m$ that is consistent with the per-period compliance requirement and that keeps the stock below the Hotelling share qualifies as an equilibrium path. ■

References


FIGURE 1: Salant's equilibrium path
FIGURE 2: Market power and storage response
FIGURE 3: Equilibrium path for a stock below the critical value
FIGURE 4: Allocation path that leads to unwanted market power
FIGURE 5: Long-run monopoly power
FIGURE 6: Long-run monopsony power
Table 1: Static equilibrium in 2010

<table>
<thead>
<tr>
<th>Prices ($/tC)</th>
<th>Emissions (mtC)</th>
<th>Permits (mtC)</th>
<th>FSU sales (mtC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OKR</td>
<td>FSU</td>
<td>Total</td>
</tr>
<tr>
<td>No regulation</td>
<td>2,153</td>
<td>818</td>
<td>2,970</td>
</tr>
<tr>
<td>Perfect competition</td>
<td>11.0</td>
<td>2,027</td>
<td>749</td>
</tr>
<tr>
<td>Maximum market-power</td>
<td>20.1</td>
<td>1,975</td>
<td>818</td>
</tr>
</tbody>
</table>

OKR: other Kyoto regions; FSU: Former Soviet Union; CDM: Clean Development Mechanism.

Table 2: Perfectly competitive equilibrium path

<table>
<thead>
<tr>
<th>Year</th>
<th>Prices ($/tC)</th>
<th>Annual Emissions (mtC)</th>
<th>Annual Permits (mtC)</th>
<th>Stocks (mtC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>OKR</td>
<td>FSU</td>
<td>Total</td>
</tr>
<tr>
<td>2010</td>
<td>62</td>
<td>1,816</td>
<td>654</td>
<td>2,470</td>
</tr>
<tr>
<td>2015</td>
<td>79</td>
<td>1,877</td>
<td>714</td>
<td>2,591</td>
</tr>
<tr>
<td>2020</td>
<td>101</td>
<td>1,909</td>
<td>754</td>
<td>2,663</td>
</tr>
<tr>
<td>2025</td>
<td>129</td>
<td>1,941</td>
<td>786</td>
<td>2,728</td>
</tr>
<tr>
<td>2030</td>
<td>164</td>
<td>1,970</td>
<td>816</td>
<td>2,785</td>
</tr>
<tr>
<td>2035</td>
<td>210</td>
<td>2,046</td>
<td>856</td>
<td>2,901</td>
</tr>
<tr>
<td>2040</td>
<td>268</td>
<td>2,112</td>
<td>890</td>
<td>3,002</td>
</tr>
<tr>
<td>2045</td>
<td>342</td>
<td>2,161</td>
<td>912</td>
<td>3,072</td>
</tr>
<tr>
<td>2050</td>
<td>436</td>
<td>2,191</td>
<td>922</td>
<td>3,113</td>
</tr>
</tbody>
</table>

TOTALS 18,021 7,304 25,325 5,852 12,168 7,305 25,325

OKR: other Kyoto regions; FSU: Former Soviet Union; CDM: Clean Development Mechanism.

Table 3: Perfectly competitive equilibrium path with stock-holding limits

<table>
<thead>
<tr>
<th>Year</th>
<th>Prices ($/tC)</th>
<th>Annual Emissions (mtC)</th>
<th>Annual Permits (mtC)</th>
<th>Stocks (mtC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>OKR</td>
<td>FSU</td>
<td>Total</td>
</tr>
<tr>
<td>2010</td>
<td>66</td>
<td>1,805</td>
<td>650</td>
<td>2,455</td>
</tr>
<tr>
<td>2015</td>
<td>84</td>
<td>1,865</td>
<td>709</td>
<td>2,574</td>
</tr>
<tr>
<td>2020</td>
<td>107</td>
<td>1,895</td>
<td>748</td>
<td>2,643</td>
</tr>
<tr>
<td>2025</td>
<td>136</td>
<td>1,926</td>
<td>780</td>
<td>2,705</td>
</tr>
<tr>
<td>2030</td>
<td>174</td>
<td>1,952</td>
<td>808</td>
<td>2,760</td>
</tr>
<tr>
<td>2035</td>
<td>222</td>
<td>2,025</td>
<td>847</td>
<td>2,872</td>
</tr>
<tr>
<td>2040</td>
<td>283</td>
<td>2,089</td>
<td>880</td>
<td>2,969</td>
</tr>
<tr>
<td>2045</td>
<td>361</td>
<td>2,135</td>
<td>901</td>
<td>3,035</td>
</tr>
<tr>
<td>2050</td>
<td>461</td>
<td>2,161</td>
<td>910</td>
<td>3,071</td>
</tr>
</tbody>
</table>

TOTALS 17,851 7,233 25,084 3,538 13,463 8,083 25,084

OKR: other Kyoto regions; FSU: Former Soviet Union; CDM: Clean Development Mechanism.
Table 4: Evolution of largest holding companies’ compliance paths in the sulfur market

<table>
<thead>
<tr>
<th>Year</th>
<th>American Elec. Power</th>
<th>Permits</th>
<th>Emissions</th>
<th>Southern Company</th>
<th>Permits</th>
<th>Emissions</th>
<th>Group of Four</th>
<th>Permits</th>
<th>Emissions</th>
<th>All firms</th>
<th>Permits</th>
<th>Emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1,194,410</td>
<td>739,322</td>
<td>1,079,502</td>
<td>534,392</td>
<td>3,607,506</td>
<td>2,049,809</td>
<td>8,694,296</td>
<td>5,298,617</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td></td>
<td>883,634</td>
<td>723,589</td>
<td>991,297</td>
<td>614,790</td>
<td>3,001,809</td>
<td>2,088,510</td>
<td>6,991,170</td>
<td>4,944,666</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td></td>
<td>663,514</td>
<td>1,136,095</td>
<td>734,464</td>
<td>1,048,296</td>
<td>2,121,591</td>
<td>3,307,858</td>
<td>9,714,830</td>
<td>11,202,052</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>663,514</td>
<td>979,653</td>
<td>734,464</td>
<td>959,338</td>
<td>2,119,625</td>
<td>3,059,693</td>
<td>9,282,297</td>
<td>10,175,057</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td></td>
<td>653,062</td>
<td>1,039,413</td>
<td>728,778</td>
<td>988,245</td>
<td>2,103,487</td>
<td>3,161,696</td>
<td>9,123,376</td>
<td>10,595,945</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td></td>
<td>653,062</td>
<td>1,017,878</td>
<td>728,778</td>
<td>969,568</td>
<td>2,103,487</td>
<td>3,096,652</td>
<td>9,123,376</td>
<td>10,432,326</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td>653,062</td>
<td>890,164</td>
<td>728,778</td>
<td>847,915</td>
<td>2,103,487</td>
<td>2,708,114</td>
<td>9,123,376</td>
<td>9,123,376</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTALS</td>
<td>Cumulative</td>
<td>5,027,741</td>
<td>4,220,420</td>
<td>5,132,478</td>
<td>2,947,783</td>
<td>16,204,259</td>
<td>10,939,725</td>
<td>38,098,555</td>
<td>26,449,572</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTALS</td>
<td>Cumulative</td>
<td>7,671,345</td>
<td>8,374,201</td>
<td>8,064,648</td>
<td>6,901,534</td>
<td>24,668,587</td>
<td>23,559,684</td>
<td>75,526,623</td>
<td>69,053,969</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>diff. 2003</td>
<td>-702,856</td>
<td>1,163,114</td>
<td>7,108,052</td>
<td>2,119,625</td>
<td>3,059,693</td>
<td>9,282,297</td>
<td>10,175,057</td>
<td>10,595,945</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOTALS</td>
<td>Cumulative</td>
<td>13,548,903</td>
<td>16,960,388</td>
<td>14,623,650</td>
<td>15,080,208</td>
<td>43,599,970</td>
<td>49,681,131</td>
<td>157,637,007</td>
<td>157,054,629</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>diff. 2012</td>
<td>-3,411,485</td>
<td>-456,558</td>
<td>6,081,161</td>
<td>6,081,161</td>
<td>582,378</td>
<td>582,378</td>
<td>582,378</td>
<td>582,378</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>