The Timing of Patent Infringement and Litigation: Sequential Innovation, Damages and the Doctrine of Laches

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Abstract

Often, firms infringe patents when developing their own innovations. I analyze the implications of the doctrine of laches in a model where a follow-on innovation infringes a previous patent. This doctrine penalizes a patentholder who delayed enforcing her patent once infringement has been detected: she does not obtain damages for infringement that occurred in the delay period. However, the patent remains enforceable. There are two periods, there is exogenous uncertainty regarding the profitability of the follow-on innovation and litigation is costly. As a result, the infringer can invest before or after uncertainty is resolved and the patentholder can litigate before or after as well. I show that the doctrine can spur or deter investment. It can also speed-up investment or delay it. It can hurt the infringer though it is intended to protect him. The effect of the patentholder’s compensation via damages is also analyzed. An increase in this compensation can speed-up or delay investment, and it can paradoxically make the patentholder worse-off.

JEL Classification: O31, O32, K42

Keywords: patent, litigation, reasonable royalty damages, doctrine of laches, investment under uncertainty.

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1 Introduction

This paper analyzes the incentive effects of the level of damages and the doctrine of laches in a model of patent dispute over sequential innovations. When innovation is sequential, the owner of a patent over the first innovation is often entitled to collect revenues from the second (follow-on) innovation. This occurs for example when the second innovation is an application of the first one. The patentholder can litigate and collect damages to be compensated for infringement, and then negotiate with the infringer to obtain royalties if the infringer wants to continue exploiting the patent. The doctrine of laches punishes the patentholder if she delayed litigation after infringement has begun: the patentholder is not entitled to get damages for infringement that occurred during the delay period. However, the patentholder can still enforce her patent and thus collect licensing revenues if the infringer wishes to continue exploiting the patent. I propose a model which incorporates these features. I investigate the effects of the damages and the doctrine of laches on the timing of investment by the infringer and the timing of litigation by the patentholder. I show that the doctrine not only affects the timing of litigation, but also and perhaps most importantly, the timing of investment in the follow-on innovation. I also derive, inter alia, two counterintuitive results: first, an increase in the level of compensatory damages can hurt the patentholder and second, the doctrine of laches, meant to protect the infringer, can hurt him. Overall, my analysis suggests that it is worthwhile to deepen our understanding of legal mechanisms that play a role in patent disputes when innovation is sequential: in fine, these mechanisms impact innovation incentives.

1The “doctrine of laches” differs from the “doctrine of estoppel” analyzed in a companion paper in two ways. First, the application of the doctrine of estoppel has more requirements than a mere delay. Second, if these requirements of the “doctrine of estoppel” are fulfilled, the patent is completely unenforceable: the patentholder cannot collect any revenue from the infringer. Under the requirements of the “doctrine of laches”, the patent remains enforceable: the patentholder does not collect revenues for infringement that occurred during the delay period (damages) but she collects revenues for future act of infringement (licensing revenues if the infringer wants to continue using the patented invention).

2The rationale for these results differ from similar results obtained in a companion paper about the doctrine of estoppel.
It has long been acknowledged that innovations build on previous ones. Consider for example the biotechnology and pharmaceutical industries. Medicines are often developed by using previously patented innovations, such as the PCR technology for replicating DNA in test tubes (see Schankerman and Scotchmer (2001) for an extensive list of such “research tools”). The software industry also illustrates this phenomenon. Bessen and Maskin (2002) argue that previously patented technologies required to develop a follow-on technology hinder innovation in industries where innovation is complementary and sequential. The reason is that the follow-on innovator typically needs to obtain the right to use the previously patented innovation. When such a right is not secured by a licensing agreement prior to engaging in research and development, the infringer may find himself involved in a legal dispute ex-post. Indeed, the patentholder is entitled to litigate and collect damages to be compensated for infringement. It is not surprising that a follow-on innovator refrains from engaging in ex-ante licensing agreements with patent-holders. Chang (1995) and Denicolò (2002) make this assumption, as I do. One reason is that there are several patents that may be infringed and it is too costly (both in terms of time and money) to secure a license for each patent before any success in research and development (patent pools try to alleviate this problem). Another reason is that the follower is unwilling to engage in a costly ex-ante bargaining process because he expects to be able to “invent around” the patent when conducting R&D. Finally, by not signing an ex-ante licensing agreement, the follower avoids to disclose his idea to the patentholder who may otherwise be able to steal it and bring a product to the market first. The patent literature dealing with sequential innovation often abstracts from specific legal factors affecting patent disputes. By focusing on some of these determinants (the doctrine of laches, the level of compensatory damages and litigation costs), this paper aims at filling this gap.

In this paper, a firm has an idea which can be developed into a commercializable product at a given (sunk) cost. Development requires using a previously patented technology and ex-ante agreements with the paten-

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3As such, this does not explain that a licensing agreement is not signed: the easier it is for the infringer (follow-on innovator) to invent around the patent, the higher his share of the surplus in the bargaining agreement. However, if this bargaining process is costly, the infringer may prefer to avoid it.

4Chang (1995) also discusses other reasons why ex-ante agreements may not be signed.

5Exceptions are Schankerman and Scotchmer (2001) or Llobet (2003).
I

tholder are ruled out. If this firm (called the infringer) invests and infringes the patent, the patentholder can litigate and collect damages ex-post. Notice that infringement does not reduce the patentholder’s profit. However, its patent allows the patentholder to collect part of the revenues earned by the infringer. Litigation is costly as well, for both the infringer and the patentholder. Given this basic set-up, I introduce uncertainty regarding the demand for the innovation that the infringer wishes to develop. There are two periods and uncertainty is revealed at the end of the first period: with a given probability, a demand exists for the innovation and revenues are generated from which the patentholder can collect damages. With the complementary probability there is no demand and no revenues: the patentholder do not collect any damages (I rule out punitive damages). The infringer is the leader and decides when to invest: before uncertainty is resolved (at the beginning of period 1) or after (at the beginning of period 2). The patentholder is a follower and litigates only if infringement occured. If the infringer invested at the beginning of period 1, the patentholder decides whether she litigates immediately (i.e before uncertainty is resolved) or she delays until period 2 (when uncertainty is resolved). This delayed litigation is punished if the doctrine of laches applies: the patentholder cannot get damages for infringement that occured in the first period. However, she still can get licensing revenues from the second period profit if the infringer continues to produce his infringing product.

My main results are:

• Counterintuitively, an increase in the patentholder’s compensation can make her worse-off. Also counter-intuitively, the doctrine of laches can hurt the infringer, although it is designed to protect him.

• The doctrine of laches triggers earlier litigation and decreases the likelihood of litigation.

• An increase in the patentholder’s compensation can delay or speed-up investment.

• The doctrine of laches can have opposite effects on the timing of the infringing investment. Depending on the model’s parameters, it can speed-up or delay investment. It can also deter or spur investment.
The doctrine of laches in brief: legal requirements.

The doctrine of laches is a "defense" available to the infringer. That means that the infringer can invoke the doctrine to defend himself if the patentholder litigates him. To be successful with this defense, the infringer needs to show that the patentholder delayed litigation and that this delay caused a prejudice. If the Court is convinced, the punishment for the patentholder is simple: she cannot obtain damages for infringement that occurred during the delay period. However, the patent is still enforceable. Thus, the patentholder can collect damages for infringement occuring after litigation started, and she can collect licensing revenues if the infringer wants to continue producing his infringing product. Legal information about the doctrine of laches can be found from various sources. A particularly clear and well illustrated paper is Szendro (2002). As emphasized by Szendro (2002), "patentees against whom the laches defense has been successfully invoked are barred from collecting only those damages that accrued prior to filing suit. Patentees may recover damages flowing from infringing activity conduct that takes place after commencement of an infringement action, even where the accused infringer successfully invokes the laches defense. Accordingly, interposition of laches does not permit the alleged infringer to lawfully continue the infringing conduct. Continued infringement remains the subject of litigation that may require settlement, entering into licensing agreements that require the payment of royalties to the patentee (...)."

Related literature. To the best of my knowledge, my paper is the first to investigates the joint effect of the level of damages and the doctrine of laches on the incentives to infringe and litigate. Schankerman and Scotchmer (2001) analyze the doctrine of laches but assume that the doctrine prevents a patentholder from obtaining an injunction (they do not investigate the doctrine in the case where the patentholder is compensated by damages). This is at odds with the facts: the doctrine of laches *allows the patentholder to get an injunction to prevent future infringement*. Its role is only to prevent the patentholder from collecting damages for infringement that occured during the delay period. My model is also related to Choi (1998). As in Choi, both the timing of infringement and the timing of litigation are endogenized. Otherwise, my approach is substantially different in the issues investigated and the results obtained. In Choi (1998), there is an incumbent patentholder and two entrants. Entry reduces the profit of the patentholder. The first litigation reveals whether the patent is valid or not. As a result, a
waiting game can arise where the two entrants expect the other one to pay the cost of entry first (the other one entering only if the patent is invalid). But a "preemption game" can arise as well, because for some parameter values, the patentholder has an incentive not to litigate the first entrant in order not to reveal validity information to the second entrant. By contrast, in my model, infringement creates new revenues to be shared between the patentholder and the infringer (there is no profit erosion). The timing of litigation is driven by litigation costs and uncertainty regarding the profitability of the infringing innovation. The timing of infringement is affected by the sunk investment cost and uncertainty regarding the profitability of the innovation. The revelation of patent validity plays no role. Hence, the dynamics of my model do not rely on the same economic forces as in Choi (1998). Most importantly, my inquiry focuses on the doctrine of laches. I solve the model under two regimes, one where the doctrine applies and one where it does not, and I analyze the effect of the doctrine on players’ welfare, on litigation timing and infringement timing. This is not the focus of Choi who assumes away the application of the doctrine of laches. More broadly, my paper is related to a mushrooming literature which attempts to deepen our understanding of patent disputes by analyzing the economic impacts of various doctrines: Lanjouw and Lerner (2001) (the doctrine of "preliminary injunctions"), Schankerman and Scotchmer (2001) (the doctrines of "unjust enrichment", "lost profit" and "laches"), Llobet (2003) (the "doctrine of equivalents"), Anton and Yao (2004) (the doctrine of "lost profit"), Aoki and Small (2004) (the doctrine of "essential facilities"), Langinier and Marcoul (2005) (the doctrine of "contributory infringement").

\textbf{A roadmap.} In section 2, I present the main assumptions of the model (players, actions, payoffs and timing). In section 3, I conduct the equilibrium analysis. I solve the game under two legal regimes: a regime where the doctrine of laches does not apply and a regime where the doctrine applies. This enables me to propose a comparison of the two regimes in a later section. Despite the conceptual simplicity of the model, the equilibrium analysis is long and sometimes cumbersome. This is because there are several "scenarios" to analyze, depending on the parameters of the model. In order to streamline the display of the investigation, I relegate many steps of this analysis to the Appendix. From this analysis, I am able to characterize the equilibrium outcomes of the game under both regimes. I use graphics that illustrate the different equilibrium outcomes. In the last two sections, I use
these figures to derive economic insights: In section 4, I analyze the effect of strengthening the patentholder’s compensation. In section 5, I analyze the effects of the doctrine of laches compared to a regime where it does not apply. Section 6 concludes.

2 Model setting

- Players, actions, payoffs.

I consider two players, a patentholder (she) labelled $H$ and a potential infringer (he) labelled $I$. At the outset, the patentholder has a patent on an innovation $A$ and the potential infringer is able to develop an innovative product $B$. I do not consider investment in obtaining innovation $A$ and simply assume that a patent exists\(^6\). The previously patented innovation is required as an input in the development of the new product $B$. This product, if developed by the infringer, does not compete away the patentholder’s profit. However, because of her patent, the patentholder can collect damages. Such a situation of ”sequential innovations” is common in practice and has been extensively scrutinized in the economic literature (Matutes, Regibeau and Rockett (1996); Schankermann and Scotchmer (2001)). For example, think of the patentholder as a biotechnology firm owning a patent on a research tool like a gene sequence ($A$), and the infringer as a pharmaceutical company contemplating developing a new drug ($B$) against a specific disease. If the development of this drug requires the use of the gene sequence, there is a risk of infringement. Like Chang (1995) or Denicolo (2000), I assume that ex-ante licensing is impossible. For example, the infringer could fear that the patentholder would ”steal” his idea.

There are two periods. To simplify the problem, I assume no discounting between periods. At the beginning of period 1, there is uncertainty regarding whether innovation $B$ will generate any profit. Specifically, with probability $\alpha$ the profit from $B$ will be $\pi$ (in both periods), whereas with probability

\(^6\)This is a common feature of many models of patent litigation. For some exceptions, see Llobet (2003) or Aoki and Hu (2003).
1 − α, the profit will be zero (in both periods). Uncertainty is resolved at the end of the first period.

<table>
<thead>
<tr>
<th></th>
<th>probability</th>
<th>period 1</th>
<th>period 2</th>
</tr>
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<tbody>
<tr>
<td>profit from product B</td>
<td>α</td>
<td>π</td>
<td>π</td>
</tr>
<tr>
<td>profit from product B</td>
<td>(1 − α)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
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• **The infringer** is the leader in the game. He has to decide whether to invest before uncertainty is resolved (i.e. at the beginning of period 1) or after (i.e. at the beginning of period 2). Investment is a sunk cost $K \in [0, +\infty)$. Delaying has a cost: if the infringer prefers to delay investment until uncertainty is resolved, and the venture turns out to be profitable, first period profit is foregone. But delaying also has a benefit: if the venture is unprofitable (which occurs with probability 1 − α), $K$ is ”saved”. Hence, the infringer faces a simple problem of investment under uncertainty.

• **The patentholder** is a follower. Conditional on observing infringement of her patent, she can litigate to obtain damages that will compensate her for the loss of licensing revenues she would have obtained, had an ex-ante licensing agreement been signed. The Court decides whether the patent is valid. It is valid with exogenous probability $\theta$. Then, damages can be awarded. I assume the calculation of damages goes as follows. The Court allows the patentholder to collect a share of the profit earned by the infringer during the period of infringement. In practice, this method of compensation is called compensation by ”lost profit” or ”reasonable royalty” (see *Georgia-Pacific Corp. v. United States Plywood Corp.*, 38 F. Supp. 1116, 1970). The idea is to give the patentholder a level of royalty that she would have gotten, had an ex-ante licensing agreement been signed. The share of $\pi$ awarded by the Court is denoted $\rho$. Thus the patentholder gets $\rho\pi$ as damages for infringement in a given period. This way of modeling damages can be found in Langinier and Marcoul (2005) (who investigates the doctrine of contributory infringement). Schankerman and Scotchmer (2001) or Anton and Yao (2004) also analyze the ”reasonable royalty” rule.

In practice, once the Court has calculated damages for infringement that occurred prior to litigation, the patentholder and the infringer are free to bar-
gain to share future revenues. Indeed, it is in the best interest of both parties that the "infringer" continues to produce his innovation, since it generates a profit that can be shared. I assume that if the infringer invested in period 1 while the patentholder litigated in period 2, then the patentholder gets $\rho \pi$ as damages for period 1 infringement and $\rho \pi$ as licensing revenues for the second period as well.\footnote{Ift he doctrine of laches does not apply.}

I define $\theta \rho \triangleq \delta$ and call $\delta$ the "compensatory rule". More accurately, it is the "expected compensatory rule", since $\theta$ is the probability that the patent is valid (and so the probability that the patentholder gets compensated).

Given this compensatory rule, the patentholder has to decide whether he litigates. Litigation costs $c$ for both players. The allocation of litigation costs follows the American rule whereby each party pays its own expenditures for litigation. If the infringer invested in period 1, the patentholder herself faces a "real option problem". She can litigate immediately (i.e. in period 1) before uncertainty is resolved, or she can delay litigation until period 2. If she delays litigation until period 2, she obtains damages for infringement that occurred in period 1 only if the doctrine of laches does not apply. I assume that the profit from the innovation is high enough compared to the cost of litigation: $\pi \geq 6c$. This is a simplifying assumption aiming at reducing the number of scenarios to analyze. Indeed, there is already an important number of scenarios. Increasing this number would hardly yield additional economic insights but it would considerably increase the length of the analysis.

\section*{Legal regimes: the "no laches" and the "laches" regimes}

I solve the game under two alternative regimes. In the first regime (the "no laches regime", labelled $N$), the doctrine of laches does not apply. In the second regime (the "laches regime", labelled $L$), the doctrine of laches applies. The difference between these two regimes matters only when the infringer invested in period 1 and the patentholder delayed litigation until period 2. In that case:

\begin{itemize}
  \item In the "no laches regime": the infringer cannot invoke the doctrine of laches and so the patentholder gets damages for period 1 infringement (she is not punished for having delayed litigation).
  \item In the "laches regime": the infringer can invoke the doctrine of laches. If he does so, the patentholder is punished for having delayed litigation.
\end{itemize}
and cannot obtain damages for period 1 infringement. However, she can get licensing revenues for future exploitation of her patent. Under the assumptions of the model, she does not get damages \(\rho t\) for infringement in period 1, but she gets \(\rho t\) as licensing revenues to compensate for exploitation of her patent in period 2.

Notice that this way of modeling the doctrine of laches is consistent with Szendro’s definition of the doctrine\(^8\). In particular, the doctrine of laches does not make the patent unenforceable. It punishes the patentholder who delayed simply by preventing her to recover damages for the delay period.

■ Timing of the game

The timing in period 1 is as follows:
1) The potential infringer decides whether to invest in period 1 or to delay his decision until uncertainty is resolved.
2) If the infringer invested in period 1, the patentholder decides whether to litigate early or to wait until demand uncertainty is resolved.
3) Uncertainty is resolved.

In period 2, the timing of the game depends on period 1’s actions:
If the infringer delayed investment until date 2, he will invest whenever the demand turns out to be high enough. Conditional on infringement, the patentholder litigates in period 2 or renunciates.
If the infringer invested in period 1 but the patentholder delayed litigation, she can litigate at the beginning of period 2.

This game is represented in extensive form in Figure 1.

\(^8\)proposed in the introduction.
Infringer I

Patentholder H

Period 1

Period 2

α

1 − α

α

1 − α

Invests in period 1

Does not invest in period 1

Litigates in period 1

Does not litigate in period 1

Invests in period 2

Does not invest in period 2

Figure 1: Game tree

Notation.

I denote by $U^r_i(a)$ player $i$’s payoff (for $i = H, I$) at time $t \in \{1, 2\}$ in regime $r \in \{N, L\}$ when action $a \in A_i$ is chosen. The infringer’s action set if given by $A_I = \{i, n\}$ and the patentholder’s action set is $A_H = \{l, nl\}$. $i$ means ”investment”, $n$ means ”no investment”, $l$ means ”litigation” $nl$ means ”no litigation”.

3 Equilibrium analysis

This two-period game is solved by backward induction and the solution concept is the subgame perfect equilibrium. First, in section 3.1, I analyze
the infringer’s ”defense strategy” if the patentholder litigates. By ”defense
strategy”, I mean whether the infringer invokes the doctrine of laches or not.
Then, moving one step backward, I investigate in section 3.2 the patent-
holder’s litigation decision. This decision depends, inter alia, on the period
in which the infringer invested. Facing infringement, the patentholder has to
decide whether and when to litigate. Finally, in section 3.3, I analyze the
infringer’s investment decision. He himself has to decide whether and when
to invest.

Section 3 is mainly concerned by the technical analysis of the model.
Because this analysis turns out to be cumbersome, many analytical steps are
proposed in the Appendix.

3.1 The defense decision

I start by analyzing the condition for the infringer to invoke the doctrine of
laches. It is possible to invoke the doctrine only when the infringer invested in
period 1 and the patentholder delayed litigation until period 2. The doctrine
of laches will bar the patentholder from collecting period 1’s damages. The
infringer invokes the doctrine provided his payoff is higher than his payoff
without invoking it.\textsuperscript{9}

\[
\begin{align*}
\text{if doctrine of laches invoked} & \quad \pi + \theta(1-\rho)\pi + (1-\theta)\pi \geq \theta(1-\rho)\pi + (1-\theta)\pi + \theta(1-\rho)\pi + (1-\theta)\pi. \\
\text{if doctrine of laches not invoked} & \quad \pi + \theta(1-\rho)\pi + (1-\theta)\pi \geq \theta(1-\rho)\pi + (1-\theta)\pi + \theta(1-\rho)\pi + (1-\theta)\pi.
\end{align*}
\]

If the doctrine is invoked (left-hand side), the infringer keeps the first
period profit but if the patent is valid (with probability \(\theta\)), he gives a share
\(\rho\) from the second period profit to the patentholder. If the doctrine is not
invoked (right-hand side), he gives a share \(\rho\) from both period 1 and period
2 profits, if the patent is valid. Given that \(\delta \equiv \theta\rho\), this expression can be
rewritten as:

\[
\begin{align*}
\text{if doctrine of laches invoked} & \quad (2-\delta)\pi \geq (2-2\delta)\pi \\
\text{if doctrine of laches not invoked} & \quad (2-\delta)\pi \geq (2-2\delta)\pi.
\end{align*}
\]

Clearly this inequality always holds.

\textsuperscript{9}abstracting from the litigation costs which could be added to both sides of the in-
equality.
Lemma 1 Invoking the doctrine of laches as a defense argument is a dominant strategy for the infringer.

Hence the infringer invokes the doctrine of laches whenever feasible. Thus, the patentholder cannot obtain compensatory damages for infringement that occurred in period 1. Given this intuitive result, I now analyze the patentholder’s litigation decision.

3.2 Litigation timing by the patentholder

The patentholder observes that infringement has occurred and has to decide whether she litigates. There are two possibilities. First, the infringer delayed investment until uncertainty was resolved and invested in period 2. In that case, the patentholder decides in period 2 whether she litigates or not. The other possibility is that the infringer invested in period 1. In that case, the patentholder decides whether to litigate immediately, i.e. in period 1, or to delay litigation until period 2. The main benefit of delaying litigation is that litigation costs are saved if the infringing venture does not generate any demand. Assuming the infringer invested in period 1, I analyze in section 3.2.1 the patentholder’s decision to litigate in the ”no laches regime”. In section 3.2.2, I analyze her decision in the ”laches regime”. Finally, in section 3.2.3, I analyze her decision if the infringer invested in period 2. In the latter case, there is no distinction between the ”laches” and the ”no laches” regimes.

3.2.1 The patentholder’s decision in the ”no laches regime”

Suppose the infringer invested in period 1. Observing infringement, the patentholder has to decide whether she litigates and when she litigates.

Suppose the patentholder delayed litigation until period 2. She litigates in period 2 provided infringement of her patent generated revenues from which she can obtain royalties as damages. This occurs with probability $\alpha$. Since the doctrine of laches does not apply, she is not ”punished” for her delay and, if her patent is valid, she collects damages for infringement that
occured in period 1, in addition to damages for period 2 infringement. Given the litigation cost $c$, her payoff is\(^{10}\):

$$U^N_{H,2}(l) = \theta (\rho \pi + \rho \pi) - c = 2\delta \pi - c. \quad (1)$$

With probability $\theta$ the patent is valid and the Court awards a share $\rho$ of both period 1 and period 2 profits to the patentholder. Notice that $U^N_{H,2}(l)$ is increasing in $\delta$ (the compensatory rule) and decreasing in $c$ (the litigation cost). It follows that there exists a value $\delta$ above which it is profitable to litigate. Denoting $\overline{\delta}_N$ this value (the subscript $N$ referring to the "no laches regime"):

$$U^N_{H,2}(l) = 2\delta \pi - c \begin{cases} < 0 & \text{if } \delta < \overline{\delta}_N \triangleq \frac{c}{2\pi} \\ \geq 0 & \text{if } \delta \geq \overline{\delta}_N \triangleq \frac{c}{2\pi}. \end{cases} \quad (2)$$

In period 1, the patentholder computes her payoff if she does not litigate immediately, anticipating her period 2 net payoff\(^{11}\):

$$U^N_{H,1}(nl) = \begin{cases} 0 & \text{if } \delta < \overline{\delta}_N \triangleq \frac{c}{2\pi} \\ \alpha (2\delta \pi - c) & \text{if } \delta \geq \overline{\delta}_N \triangleq \frac{c}{2\pi}. \end{cases} \quad (3)$$

The expression (3) is derived from expression (2). Indeed, if $\delta < \overline{\delta}_N$, the patentholder would not litigate in period 2 (otherwise her net payoff would be negative according to (2)). As a result, if she does not litigate in period 1 either, she gets 0. And if $\delta \geq \overline{\delta}_N$, she would litigate in period 2 if and only if there is a demand for the infringer’s product, which occurs with probability $\alpha$. In that case, she gets $2\delta \pi - c$.

In period 1, the patentholder also computes her payoff if she litigates immediately:

$$U^N_{H,1}(l) = -c + \theta [\alpha (2\rho \pi)] \triangleq -c + \alpha 2\delta \pi. \quad (4)$$

Indeed, she has to pay the litigation cost $c$ and if her patent is valid (with probability $\theta$), she gets a share $\rho$ from both period 1 and period 2 profits. To determine the timing of litigation, I compare (3) and (4). Obviously, if $\delta \geq \overline{\delta}_N$, $U^N_{H,1}(nl) \geq U^N_{H,1}(l)$: the patentholder is strictly better-off if she delays

\(^{10}\)In expression (1), I use the notation defined in section 2. Hence, $U^N_{H,2}(l)$ denotes the net payoff for the patentholder (denoted by subscript $H$), in period 2 (subscript 2), in the "no laches regime" (superscript $N$). $l$ means that the patentholder’s action is litigation.

\(^{11}\)Again, I follow the notation defined in section 2. $U^N_{H,1}(nl)$ is the patentholder’s expected payoff in the no laches regime if she does not litigate ($nl$) in period 1.
litigation as she "saves" litigation costs in case the infringing product does not generate any revenue. If \( \delta < \delta_N \), litigating in period 2 is unprofitable since \( 2\delta\pi - c < 0 \). This implies that \( \alpha2\delta\pi - c < 0 \) as well, since \( \alpha \in [0, 1] \). By (4), this last inequality means that litigation in period 1 is unprofitable as well.

I summarize this analysis by the following lemma.

**Lemma 2 (litigation timing in the "no laches regime" when the infringer invested in period 1).** For \( \delta \in [\delta_N, 1] \), the patentholder delays litigation. For \( \delta \in [0, \delta_N] \), litigation is unprofitable, either in period 1 or in period 2.

Still assuming that the infringer invested in period 1, I now turn to the case where the doctrine of laches applies. This means that the patentholder is punished if she delays litigation until period 2: she does not obtain damages for infringement that occurred in the first period. This should affect the timing of litigation, compared to the previous section.

### 3.2.2 The patentholder’s decision in the "laches regime"

Suppose again that the infringer invested in period 1. In solving the patentholder’s problem, I use the same methodology I used for the "laches regime" above.

Suppose first the patentholder delays litigation until uncertainty is resolved. She litigates in period 2 provided this is profitable. Since the doctrine of laches applies, she obtains royalties for period 2, but no damage royalties for period 1. Her net litigation payoff in period 2 is thus:

\[
U_{H,2}(l) = \theta \rho \pi - c = \delta \pi - c.
\]

Indeed, with probability \( \theta \) the patent is valid and the Court allows the patentholder to get a share \( \rho \) of the forthcoming second period profit. Notice that this net payoff differs from the net payoff found in equation (2) where the doctrine of laches did not apply. When the doctrine of laches applies, the net payoff from delayed litigation is lower since the first-period damages are forgone. This can be seen by comparing (1) and (5). \( U_{H,2}(l) \) is increasing in
δ and decreasing in c. Hence, there exists a value δ above which litigation is profitable. Denoting \( \delta_L \) this value (the subscript \( L \) referring to the "laches regime"):

\[
U^L_{H,2}(l) = \begin{cases} 
< 0 & \text{if } \delta < \delta_L \triangleq \frac{\xi}{\pi} \\
\delta \pi - c & \geq 0 & \text{if } \delta \geq \delta_L \triangleq \frac{\xi}{\pi}.
\end{cases}
\]  

(6)

In period 1, the patentholder computes her payoff if she does not litigate immediately, anticipating her period 2 net payoff:

\[
U^L_{H,1}(nl) = \begin{cases} 
0 & \text{if } \delta < \delta_L \triangleq \frac{\xi}{\pi} \\
\alpha(\delta \pi - c) & \text{if } \delta \geq \delta_L \triangleq \frac{\xi}{\pi}.
\end{cases}
\]  

(7)

The expressions in (7) come from (6). Indeed, if \( \delta < \delta_L \), she would not litigate in period 2 (since her net payoff would be negative according to (6)). Hence, if she does not litigate in period 1 either, she obtains 0. And if \( \delta \geq \delta_L \), she would litigate in period 2, but only if the infringing venture generates revenues, which occurs with probability \( \alpha \). In that case, she gets \( \delta \pi - c \).

The patentholder also computes her net payoff if she litigates immediately (i.e. in period 1):

\[
U^L_{H,1}(l) = -c + \theta [\alpha(2\rho \pi)] \triangleq -c + \alpha 2\delta \pi.
\]  

(8)

She has to pay the litigation cost \( c \) and if her patent is valid (with probability \( \theta \)) she gets a share \( \rho \) from both period 1 and period 2 profits.

The next step consists in determining a condition on \( \delta \) (the compensatory rule) such that \( U^L_{H,1}(l) \geq U^L_{H,1}(nl) \), where these net payoffs are given by (7) and (8). Because the net payoff \( U^L_{H,1}(nl) \) differs depending whether \( \delta < \delta_L \) or \( \delta \geq \delta_L \), I distinguish between these two cases. "Case 1" means that \( \delta \in [0, \delta_L] \) and "case 2" means that \( \delta \in [\delta_L, 1] \).

- **Case 1**: \( \delta \in [0, \delta_L] \). On this interval, given (7), \( U^L_{H,1}(nl) = 0 \). It follows that the condition \( U^L_{H,1}(l) \geq U^L_{H,1}(nl) \) is equivalent to \( U^L_{H,1}(l) \geq 0 \). Using (8), this condition holds if and only if \( -c + \alpha 2\delta \pi \geq 0 \) or:

\[
\delta \geq \frac{c}{2\alpha \pi} \triangleq \delta_L.
\]  

(9)

Notice that \( \overline{\delta}_L \leq \delta_L \) if and only if \( \alpha \geq \frac{1}{2} \). From that remark, I can conclude:
If $\alpha < \frac{1}{2}$, then $\overline{\delta}_L > \overline{\delta}_L$. This implies that for any $\delta \in [0, \overline{\delta}_L]$, $\delta < \overline{\delta}_L$. So, (9) is violated and $U_{H,1}^L(l) \geq 0$ does not hold: the patentholder does not litigate.

If $\alpha \geq \frac{1}{2}$, then $\overline{\delta}_L \leq \overline{\delta}_L$. This implies that (9) holds for some $\delta \in [0, \overline{\delta}_L]$. More accurately, for $\delta < \overline{\delta}_L$, (9) does not hold and the patentholder does not litigate. But for $\delta \in [\overline{\delta}_L, \overline{\delta}_L]$, (9) holds and so the patentholder litigates in period 1.

Case 2: $\delta \in [\overline{\delta}_L, 1]$. On this interval, using (7) and (8), $U_{H,1}^L(l) \geq U_{H,1}^L(nl)$ if and only if $-c + \alpha 2\delta \pi \geq \alpha (\delta \pi - c)$ or:

$$\delta \geq \frac{(1 - \alpha) c}{\alpha \pi} \triangleq \overline{\delta}_L.$$  \hspace{1cm}  (10)

Proceeding as I did above, notice that $\overline{\delta}_L \geq \overline{\delta}_L$ if and only if $\alpha \leq \frac{1}{2}$. From that remark, I can conclude:

- If $\alpha \leq \frac{1}{2}$, then $\overline{\delta}_L \geq \overline{\delta}_L$. This implies that for all $\delta \in [\overline{\delta}_L, 1]$, we have the following partition. If $\delta \in [\overline{\delta}_L, \overline{\delta}_L]$, condition (10) is violated and so the patentholder delays litigation. And if $\delta \in [\overline{\delta}_L, 1]$, condition (10) holds and the patentholder litigates in period 1.

- If $\alpha > \frac{1}{2}$, then $\overline{\delta}_L < \overline{\delta}_L$. This implies that for all $\delta \in [\overline{\delta}_L, 1]$, $\delta \geq \overline{\delta}_L$. So, condition (10) holds and the patentholder litigates in period 1.

The following lemma summarizes these findings:

**Lemma 3** (Litigation timing in the "laches regime" when the infringer invested in period 1).

- If the probability of commercial success is high ($\alpha \geq \frac{1}{2}$), the patentholder does not litigate when $\delta \in [0, \overline{\delta}_L]$ and litigates early for $\delta \in [\overline{\delta}_L, 1]$.  

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• If the probability of commercial success is intermediary \((\alpha \in [\frac{c}{c+\pi}, \frac{1}{2}])\), the patentholder does not litigate when \(\delta \in [0, \delta_L]\), delays litigation for \(\delta \in [\delta_L, \delta]\) and litigates early for \(\delta \in [\delta, 1]\).

• If the probability of success is low \((\alpha \in [0, \frac{c}{c+\pi})\), the patentholder does not litigate when \(\delta \in [0, \delta_L]\) and delays litigation when \(\delta \in [\delta_L, 1]\).

Finally, I analyze the case where the infringer invested in period 2.

### 3.2.3 Litigation when the infringer invested in period 2

Suppose now that the infringer delayed investment. Then the regime is irrelevant. The patentholder can only litigate in period 2 and uncertainty is resolved at that time. She litigates provided this is profitable i.e. provided \(\delta \pi \geq c\) or

\[
\delta \geq \frac{c}{\pi} = \delta_L.
\]

Notice that the cutoff value \(\delta\) above which litigation occurs is identical to the cutoff value identified in the "laches regime".

**Lemma 4 (Litigation timing when the infringer invested in period 2).** When the infringer delayed investment until period 2, the patentholder litigates if and only if the compensatory rule is high enough i.e \(\delta \geq \frac{c}{\pi} = \delta_L\).

The various equilibrium actions of the patentholder are represented in Figure 2. The thick solid lines represent boundaries between different regions where a particular litigation strategy occurs in equilibrium. The case where the infringer invested in period 2 is represented by the **right-hand-side graphic**. Following lemma 4, in the \((\alpha, \delta)\) space, the boundary value \(\delta_L\) separates a region where litigation occurs from a region where it does not occur. Intuitively, an increase in the compensatory rule \(\delta\) yields a switch from "no litigation" to "litigation", for a given level of the litigation cost. The case where the infringer invested in period 1 is represented by the **two left-hand-side graphics**. Notice that I distinguish between the "no laches regime"
The boundary $\bar{\alpha}_L$ is the inverse of $\bar{\delta}_L$ and $\bar{\alpha}_L$ is the inverse of $\bar{\delta}_L$. Comparing the bottom graphic with the top graphic shows the main effects of the doctrine of laches on the patent holder's litigation strategy. These effects are stated in proposition 1 below.

**Proposition 5** *(the doctrine of laches and litigation).* The doctrine of laches has two possible effects on litigation compared to a regime where it does not apply:

- first, it increases the likelihood of early litigation,

12Since $\bar{\delta}_L = \frac{c}{\delta L}$ it follows that $\bar{\alpha}_L = \frac{c}{\delta L}$. And since $\bar{\delta}_L = \frac{(1-\alpha)c}{\delta L}$ it follows that $\bar{\alpha}_L = \frac{c}{\delta L}$. Both $\bar{\alpha}_L$ and $\bar{\alpha}_L$ are decreasing in $\delta$ and they intersect at $\delta = \frac{c}{\delta L}$. 

Figure 2: Litigation in either regime
second, it decreases the likelihood of litigation.

Proof. The proof is straightforward upon inspection of the cutoff values.

Both results in proposition 1 are intuitive. The first result is the most expected: because the doctrine of laches punishes the patentholder who delays by reducing the amount of damages she can recover, it forces some patentholders to react in a timely manner (i.e. in period 1). The second result comes from the fact that litigation is costly and there is uncertainty about whether the infringing innovation will be profitable. The patentholder herself faces a real option problem: if she delays litigation, and infringement turns out to be unprofitable, she saves litigation costs. But the interposition of the doctrine of laches forces her to litigate earlier, that is, before uncertainty is resolved. For any given \( c \) and \( \alpha \), and a low enough damage \( D \), litigating early will be non profitable and so litigation will be deterred.

Now, I move one step backward and I investigate the infringer’s decision. He faces a ”real option” problem as well in the sense that he can invest in period 1 or in period 2 (or not at all). In making his decision, the infringer anticipates how the patentholder will react. It means that he anticipates whether the patentholder will litigate and if she does, in which period it happens.

3.3 Investment timing by the infringer

As for litigation, I distinguish between the two regimes. First, in section 3.3.1, I analyze the investment decision in the ”no laches regime”. Then, in section 3.3.2, I analyze the investment decision in the ”laches regime”. In both cases, different scenarios must be analyzed depending on the values of the parameters. Displaying the analysis for every scenarios in this section would be cumbersome and would only slow down the progression towards deriving economic insights. As a result, part of the necessary analytical steps of this section are given in Appendix A. Also, in section 3, I shall assume that the probability that the innovation is profitable is high enough,
namely $\alpha \geq \frac{1}{2}$. This is clearly a simplifying assumption. Like the previous simplifying assumption ($\pi \geq 6c$), it aims at reducing the number of scenarios to investigate. Notice that when $\alpha \geq \frac{1}{2}$, the two effects of the doctrine of laches on litigation are still captured: the doctrine induces earlier litigation or it deters litigation. This can be seen by comparing the two left-hand side graphics in Figure 2. Hence, the essential economic insights regarding the influence of these two effects on the timing of investment can be derived when $\alpha \geq \frac{1}{2}$.

3.3.1 The infringer’s decision in the ”no laches regime”

At the beginning of period 1, the infringer must decide whether and when he invests (and thereby infringes the patent). He knows that the doctrine of laches does not apply. He anticipates the patentholder’s litigation strategy if he invests in period 1 (represented by the left-hand-side ”bottom graphic” in Figure 2). He also anticipates the patentholder’s litigation strategy when he invests in period 2 (represented by the right-hand side graphic in Figure 2). Based on these two graphics, there are three scenarios to consider:

- **Scenario 1**: $\delta \in [\delta_L, 1]$. The infringer faces litigation in period 2 regardless of the timing of investment.

- **Scenario 2**: $\delta \in [\delta_N, \delta_L]$. The infringer will not face litigation if he invests in period 2. However, he will face litigation in period 2 if he invests in period 1.

- **Scenario 3**: $\delta \in [0, \delta_N]$. The infringer will never face litigation.

For each scenario, the infringer has to decide whether and when to invest. The methodology used to solve this problem is similar to that used for analyzing the patentholder’s litigation decision. Because I repeat the same analytical steps for all three scenarios, the details of the reasoning for scenarios 2 and 3 is reported in Appendix A.1. Here, I only report the detailed analysis for scenario 1. For each scenario, I conclude by a lemma where I summarize the infringer’s investment decision (lemmas 5, 6 and 7) Also, it is
useful to define here two values that play a role in the forthcoming analysis: \( \hat{\alpha} = \frac{\pi}{2(\pi - c)} \) and \( \hat{\alpha} = \frac{\pi}{2\pi - 3c} \).

**Scenario 1.** \( \delta \in [\delta_L, 1] \).

Suppose the infringer delays investment until uncertainty is resolved. He invests in period 2 provided that there is a demand for his product. This occurs with probability \( \alpha \). If he invests in period 2, his net payoff is:

\[
U_{N,2}^I(i) = -K - c + \theta(1 - \rho)\pi + (1 - \theta)\pi \triangleq K - c + \pi(1 - \delta). \tag{11}
\]

Because the infringer knows that the patentholder will litigate in period 2 he will face litigation cost \( c \) in addition to the sunk investment cost \( K \). With probability \( \theta \) the patent is valid and the patentholder collects a share \( \rho \) of second period profit \( \pi \). With probability \( 1 - \theta \) the patent is invalid. Notice that \( U_{N,2}^I(i) \) is increasing in \( \pi \) and \( 1 - \delta \) but is decreasing in \( K \). Hence, there exists a value of \( K \) below which investment is profitable. Denoting \( K_{N,1} \) this value (\( N \) is for the "no laches regime" and 1 refers to scenario 1) it follows that:

\[
U_{N}^I(i) = -K - c + \pi(1 - \delta) \begin{cases} < 0 & \text{if } K > K_{N,1} \triangleq \pi(1 - \delta) - c \\ \geq 0 & \text{if } K \leq K_{N,1} \triangleq \pi(1 - \delta) - c. \end{cases} \tag{12}
\]

In period 1, the infringer can compute his payoff if he does not invest in period 1:

\[
U_{I,1}(i) = \begin{cases} 0 & \text{if } K > K_{N,1} \triangleq \pi(1 - \delta) - c \\ \alpha[\pi(1 - \delta) - c - K] & \text{if } K \leq K_{N,1} \triangleq \pi(1 - \delta) - c. \end{cases} \tag{13}
\]

If \( K > K_{N,1} \), the infringer would not invest in period 2. So, if he does not invest in period 1, he gets 0. If \( K \leq K_{N,1} \), the infringer would invest in period 2 if he does not invest in period 1, provided the demand for his product exists. This occurs with probability \( \alpha \).

The payoff from investing in period 1 is:

\[
U_{I,1}^N(i) = -K + \alpha[\theta 2\pi(1 - \rho) + (1 - \theta)2\pi - c] \triangleq K + \{\alpha[2\pi(1 - \delta) - c]\}. \tag{14}
\]

Indeed, if he invests in period 1, the infringer faces litigation in period 2, provided the demand for the infringing products exists. This occurs with
probability $\alpha$. Then, with probability $\theta$ the patent is valid and the patent-holder gets a share $\rho$ of both period 1 and period 2 profits (the sum being $2\pi$). With probability $1 - \theta$ the infringer keeps the sum of the profits for himself. In any case, he has to pay the litigation cost $c$.

The next step consists in determining a condition on $K$ such that $U_{N,1}^N(i) \geq U_{I,1}^N(n)$, i.e., such that the infringer invests in period 1. These two net payoffs are given by (13) and (14). Because $U_{I,1}^N(n)$ differs depending whether $K > K_{N,1}$ or $K \leq K_{N,1}$, I distinguish between these two cases. "Case 1" means that $K > K_{N,1}$ and "Case 2" means that $K \leq K_{N,1}$.

□ Case 1. If $K > K_{N,1}$, delaying investment is never profitable. Investing today is profitable as long as $U_{N,1}^N \geq 0$ which is equivalent to

$$K \leq 2\alpha\pi(1 - \delta) - \alpha c \triangleq K_{N,1}.$$

□ Case 2. If $K \leq K_{N,1}$, delaying yields a non-negative profit. As a result, the infringer will invest today if and only if $U_{I,1}^N(i) \geq U_{I,1}^N(n)$ or

$$K \leq \frac{\alpha}{1-\alpha}\pi(1 - \delta) \triangleq K_{N,1}.$$

From this analysis, I derive the timing of investment by the infringer when the compensatory rule $\delta$ belongs to the interval $[\delta_L, 1]$ (scenario 1). To do so, I analyze in more depth the respective positions of $K_{N,1}$, $\overline{K}_{N,1}$, and $\overline{K}_{N,1}$. This is done in Appendix A.1 and I obtain the following result:

**Lemma 6** Under scenario 1, in the "no laches" regime, for all $K \leq \overline{K}_{N,1}$, the infringer invests in period 1 and for all $K > \overline{K}_{N,1}$, he does not invest.

The next steps consist in repeating this analysis for the two other intervals: $\delta \in [\delta_N, \delta_L]$ (scenario 2) and $\delta \in [0, \delta_N]$ (scenario 3).

□ Scenario 2. $\delta \in [\delta_N, \delta_L]$.
In Appendix A.1, I detail the analysis of this scenario. The methodology is identical to that used for scenario 1 above, but the payoffs, and thus the "boundary" values $K_{N,2}$, $\overline{K}_{N,2}$ and $\bar{K}_{N,2}$, are different:

\[
\left\{ \begin{array}{l}
K_{N,2} \triangleq \pi \\
\overline{K}_{N,2} \triangleq 2\alpha\pi(1-\delta) - \alpha c \\
\bar{K}_{N,2} \triangleq \frac{\alpha}{1-\alpha}[\pi(1-2\delta) - c].
\end{array}\right.
\] (17)

As shown in Appendix A.1, it is necessary to define two values. First, the function $\dot{\delta} = \frac{2\alpha - a - \alpha c}{2\alpha}$ such that $\dot{\delta} \in [\delta_N, \delta_L]$. Then the kinked curve $K = \overline{K}_{N,1}$ if $\delta \in [\delta_N, \dot{\delta}]$ and $K = \overline{K}_{N,2}$ if $\delta \in [\dot{\delta}, \delta_L]$. I show in Appendix A.1 that the following lemma holds:

**Lemma 7** Under scenario 2, in the "no laches" regime, there are three possibilities depending on the value of the probability $\alpha$ that the innovation is profitable.

- If $\alpha \in (\frac{1}{2}, \hat{\alpha}]$, the infringer invests in period 1 if $K \leq \overline{K}_{N,2}$. He delays investment if $K \in (\overline{K}_{N,2}, \underline{K}_{N,2}]$ and he does not invest if $K \geq \underline{K}_{N,2}$.
- If $\alpha \in (\hat{\alpha}, \hat{\alpha}]$, the infringer invests in period 1 if $K \leq \dot{K}$, delays investment if $K \in [K, \overline{K}_{N,2}]$ and does not invest if $K \geq \dot{K}$ and $K \geq \underline{K}_{N,2}$.
- If $\alpha \in (\hat{\alpha}, 1]$, the infringer invests in period 1 if $K \leq \underline{K}_{N,1}$. Otherwise, he does not invest.

**Scenario 3.** $\delta \in [0, \delta_N]$.

The analysis of this scenario is detailed in Appendix A.1. For the same reason as in scenario 2, I report here the three boundaries:

\[
\left\{ \begin{array}{l}
\overline{K}_{N,3} \triangleq \pi \\
\underline{K}_{N,3} \triangleq 2\alpha\pi \\
\bar{K}_{N,3} \triangleq \frac{\alpha}{1-\alpha}\pi.
\end{array}\right.
\] (18)
Lemma 8 Under scenario 3, in the "no laches" regime, the infringer invests if $K \leq K_{N,3}$. Otherwise he does not invest.

Combining the results concerning the timing of investment (lemmas 5 to 7) with those concerning litigation (lemma 2 and 4), I obtain different equilibrium outcomes as summarized in proposition 2.$^{13}$

Proposition 9 (Equilibrium outcomes when the doctrine of laches does not apply). When the probability of commercial success is high ($\alpha \geq \frac{1}{2}$), there are four possible equilibrium outcomes depending on the parameters of the model:

- The infringer invests in period 1 and the patentholder does not litigate (EN).
- The infringer invests in period 1 and the patentholder litigates in period 2 (ED).
- The infringer invests in period 2 and the patentholder does not litigate (DN).
- The infringer does not invest (NO).

The rationale behind the names given to each outcome is as follows. The first block letter refers to the infringer’s action: $E$ means early investment (period 1) and $D$ means delayed investment (period 2). The second block letter refers to the patentholder’s action: $E$ means early litigation (period 1), $D$ means delayed litigation (period 2), and $N$ means no litigation. Finally, $NO$ means no investment (and so no litigation). To help figuring out the different equilibrium outcomes in the "no laches regime", I present three figures $N1$ to $N3$. The label $N$ refers to the "no laches" regime. These figures represent the equilibrium outcomes of the game in the $(K,\delta)$ space. $K$ is the sunk investment cost born by the infringer and $\delta$ is the compensation.

$^{13}$In this proposition, I do not detail the exact parameters values for which a particular equilibrium outcome occurs. The exposition would be tedious otherwise. This is done in Appendix A.1
rule which governs the share of the profit obtained by the patentholder. There are three figures because, when \( \delta \in [\delta_N, \delta_L] \), the equilibrium outcomes are affected by the value of \( \alpha \). There are three intervals to consider for \( \alpha \geq \frac{1}{2} \): \( \alpha \in \left[ \frac{1}{2}, \hat{\alpha} \right] \), \( \alpha \in (\hat{\alpha}, \tilde{\alpha}] \), \( \alpha \in (\tilde{\alpha}, 1] \). This comes from lemma 6. These figures will be analyzed more in-depth in sections 4 and 5. However, notice that the higher the sunk cost \( K \) and the higher the patentholder’s compensation (i.e. the higher is \( \delta \)), the less often investment occurs. This is intuitive: a higher \( K \) renders investment more costly and a higher \( \delta \) reduces the share obtained by the infringer (for all \( K \)), thereby making investment less attractive.

Figure N1: \( \alpha \in \left[ \frac{1}{2}, \hat{\alpha} \right] \)
Figure N2: $\alpha \in [\delta, 1]$. 

$K$ 

$2\alpha\pi$ 

$\pi$ 

$\delta = 1 - \frac{c}{2\pi}$ 

$\bar{K}_{N,3} = 2\alpha\pi$

Figure N3: $\alpha \in [\delta, 1]$. 

$K$ 

$2\alpha\pi$ 

$\pi$ 

$\delta = 1 - \frac{c}{2\pi}$ 

$\bar{K}_{N,3} = 2\alpha\pi(1 - \delta) - \alpha c$

early investment, no litigation. 

early investment, delayed litigation. 

delayed investment, no litigation. 

No investment.

early investment, no litigation. 

early investment, delayed litigation. 

delayed investment, no litigation. 

No investment.

EN

ED

DN

NO

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3.3.2 The infringer’s decision in the ”laches regime”

Again, at the beginning of period 1, the infringer has to decide whether and when he invests. Contrary to the previous section, the doctrine of laches applies. In making his decision, the infringer anticipates the patentholder’s reaction if he invests in period 1 (represented by the left-hand-side top graphic in Figure 2). He also anticipates the patentholder’s reaction if he invests in period 2 (represented by the right-hand side graphic in Figure 2). Based on the observation of these two graphics, there are three scenarios to consider in the ”laches regime”:

- **Scenario 1**: $\delta \in [\delta_L, 1]$. The infringer faces litigation in the period of investment.

- **Scenario 2**: $\delta \in [\bar{\delta}_L, \delta_L]$. The infringer will not face litigation if he delays investment. However, he will face litigation in period 1 if he invests in period 1.

- **Scenario 3**: $\delta \in [0, \bar{\delta}_L]$. The infringer will never face litigation.

For each scenario, the infringer decides whether and when to invest. Again, the methodology used to solve this timing problem is identical to that used in the previous sections. Here, I detail only the first scenario. This enables me to stress the difference with the ”no laches regime”. The detailed analysis of scenarios 2 and 3 is reported in Appendix A.2. Also it is useful to define here two values that play a role in the analysis below: $\bar{\alpha} = \frac{\pi + 2c}{2\pi}$ and $\tilde{\alpha} = \frac{\pi + c}{2(\pi - c)}$.

**Scenario 1.** $\delta \in [\delta_L, 1]$.

Suppose the infringer delays investment until period 2. He invests in period 2 provided that there is a demand for his product. This occurs with probability $\alpha$. If he invests in period 2, his net payoff is:

$$U_{1,2}^L(i) = -K - c + \theta(1 - \rho)\pi + (1 - \theta)\pi \triangleq -K - c + \pi(1 - \delta)$$

(19)

This is unchanged compared to the ”no laches regime”. Indeed, the regime does not matter when the infringer delays investment until period
2. As noticed for the "no laches regime", $U_{I,2}(i)$ is increasing in $\pi$ and $1 - \delta$ but it is decreasing in $K$. Hence, there is a value $K$ below which the infringer would invest in period 2. Denoting $K_{L,1}$ this value ($L$ referring to the "laches regime" and 1 to scenario 1), it follows that:

$$U_{I,2}(i) = -K - c + \pi(1 - \delta) \begin{cases} < 0 & \text{if } K > K_{L,1} \equiv \pi(1 - \delta) - c \\ \geq 0 & \text{if } K \leq K_{L,1} \equiv \pi(1 - \delta) - c. \end{cases} \quad (20)$$

In period 1, the infringer can compute his payoff if he does not invest in period 1:

$$U_{I,1}(n) = \begin{cases} 0 & \text{if } K > K_{L,1} \\ \alpha [-K - c + \pi(1 - \delta)] & \text{if } K \leq K_{L,1}. \end{cases} \quad (21)$$

This is still identical to the "no laches regime".

Also, in period 1, the infringer computes his net payoff if he invests immediately. This payoff differs from the "no laches regime":

$$U_{I,1}(i) = -K - c + 2\alpha\pi(1 - \delta). \quad (22)$$

Here, the doctrine of laches encourages the patentholder to litigate in period 1 if the infringer invests in period 1. As a result, the infringer faces litigation costs $c$ in period 1, before uncertainty is resolved. In the "no laches regime", the situation was different: the patentholder preferred to delay litigation until period 2 and, as a result, the infringer faced litigation costs only if a demand for the infringing product turned out to exist, i.e. only with probability $\alpha$. This difference plays a crucial role in the analysis of the doctrine of laches in section 5.

The next step consists in determining a condition on $K$ such that $U_{I,1}(i) \geq U_{I,1}(n)$, that is: such that the infringer prefers to invest in period 1. These two net payoffs are given by (21) and (22). Because $U_{I,1}(n)$ differs depending whether $K > K_{L,1}$ or $K \leq K_{L,1}$, I distinguish between these two cases. "Case 1" means that $K > K_{L,1}$ while "Case 2" means that $K \leq K_{L,1}$.

\[ \square \text{ Case 1: } K > K_{L,1}. \text{ Delaying investment is not profitable.} \]

The infringer invests today if and only if $U_{I,1}(i) \geq 0$ or

$$K \leq 2\alpha\pi(1 - \delta) - c \equiv K_{L,1}. \quad (23)$$

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Case 2: \( K \leq \mathcal{K}_{L,1} \). Delaying investment yields a non-negative payoff. The infringer invests today if and only if \( U_{I,1}(i) \geq U_{I,1}(n) \) or
\[
K \leq \frac{\alpha \pi (1 - \delta) + \alpha c - c}{1 - \alpha} \triangleq \mathcal{K}_{L,1}. \tag{24}
\]

Lemma 10 Under scenario 1, in the laches regime, the infringer invests in period 1 if \( K \leq \mathcal{K}_{L,1} \). If \( K > \mathcal{K}_{L,1} \) he does not invest.

Scenario 2. \( \delta \in [\bar{\delta}_L, \bar{\delta}_L] \).
In Appendix A.2, I detail the analysis corresponding to this scenario. The methodology is identical to that used for scenario 1 above, but the payoffs, and thus the “boundaries” functions \( \mathcal{K}_{L,2}, \mathcal{K}_{L,2}, \) and \( \mathcal{K}_{L,2} \), are different:
\[
\begin{cases}
K_{L,2} \triangleq \pi \\
\mathcal{K}_{L,2} \triangleq 2\alpha \pi (1 - \delta) - c \\
\mathcal{K}_{L,2} \triangleq \frac{\alpha \pi (1 - 2\delta) - c}{1 - \alpha}.
\end{cases} \tag{25}
\]

As shown in Appendix A.2, it is necessary to define two values. First, the function \( \delta = \frac{2\alpha \pi - c - \pi}{2\alpha \pi} \) such that \( \delta \in [\bar{\delta}_L, \bar{\delta}_L] \). Then the kinked curve \( K = \mathcal{K}_{L,1} \) if \( \delta \in [\bar{\delta}_L, \bar{\delta}_L] \) and \( K = \mathcal{K}_{L,2} \) if \( \delta \in [\bar{\delta}_L, \bar{\delta}_L] \). I show in Appendix A.2 that:

Lemma 11 Under scenario 2, in the laches regime, there are three possibilities depending on the probability \( \alpha \) that the innovation is profitable:

- If \( \alpha \in \left[\frac{1}{2}, \tilde{\alpha}\right] \), the infringer invests in period 1 if \( K \leq \mathcal{K}_{L,2} \), delays investment for \( K \in [\mathcal{K}_{L,2}, \mathcal{K}_{L,2}] \) and does not invest if \( K \geq \mathcal{K}_{L,2} \).

- If \( \alpha \in \left[\tilde{\alpha}, \bar{\alpha}\right] \), the infringer invests in period 1 if \( K \leq \tilde{K} \). If \( K \in [\tilde{K}, \mathcal{K}_{L,2}] \), the infringer delays investment. If \( K > \tilde{K} \) and \( K > \mathcal{K}_{L,2} \), the infringer does not invest.
• If $\alpha \in \left[\alpha, 1\right]$, the infringer invests in period 1 if $K \leq \bar{K}_{L,1}$. He does not invest if $K > \bar{K}_{L,1}$.

Scenario 3. $\delta \in [0, \bar{\delta}_L]$.
For these values of the compensatory rule $\delta$, the patentholder does not litigate. Hence, the analysis is formally equivalent to the no laches case. The three boundaries on $K$ are given in (18). Lemma 7 applies here.

Now, I can combine the results concerning the timing of investment (lemmas 7, 8 and 9) with those concerning litigation (lemmas 3 and 4). I obtain the different equilibrium outcomes as summarized in proposition 3. As for the "no laches" regime, I do not detail the exact parameters' values for which a particular outcome occurs: the exposition would be tedious. This is done in Appendix A.2.

Proposition 12 (Equilibrium outcomes under the doctrine of laches).
When the probability of commercial success is high ($\alpha \geq \frac{1}{2}$), there are four equilibrium outcomes depending on the parameters of the model:

• The infringer invests in period 1 and the patentholder does not litigate (EN).
• The infringer invests in period 1 and the patentholder litigates in period 1 (EE).
• The infringer invests in period 2 and the patentholder does not litigate (DN).
• The infringer does not invest (NO).

The logic behind the names of the outcomes is similar to that in the "no laches" regime. I present figures L1 to L3 to illustrate the different equilibrium outcomes of the game in a livelier manner. There are three figures because, when the compensatory rule $\delta$ belongs to $[\bar{\delta}_L, \bar{\delta}_L]$, the timing of investment depends on $\alpha$: three intervals must be considered separately on
$\alpha \in \left[ \frac{1}{2}, 1 \right]$. At first sight, these figures are quite similar to the ones representing the equilibrium regions in the "no laches" regime. In fact, the main differences come from the respective position of the boundaries between the regions. This will be analyzed in section 5.

Figure L1: $\alpha \in \left[ \frac{1}{2}, \frac{\pi}{\delta} \right]$. 

Figure L2: $\alpha \in \left[ \frac{\pi}{\delta}, \frac{\pi}{\delta} \right]$. 

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Now that I have derived the equilibrium outcomes in both regimes, I turn to analyzing the economic insights of the model. In section 4, I investigate the effects of the compensatory rule $\delta$, in both the "no laches" and the laches regimes. In section 5, I analyze the effect of the doctrine of laches, compared to the situation where it does not apply.

4 Effect of the compensatory rule on investment and players’ welfare

Lemmas 2, 3 and 4 constituted a first step into analyzing the effect of the compensatory rule $\delta$ on litigation. Now, I analyze how this rule affects the timing of investment and players’ welfare. The main results are captured by propositions 4 and corollary 1 below.
Proposition 13 In either regime, an increase in the compensatory rule $\delta$ can

- delay investment or
- speed-up investment.

The first result in this proposition is intuitive. An increase in the compensatory rule $\delta$ reduces the infringer’s gross payoff (since it increases the patentholder’s gross payoff). As a result, for any $c$ (cost of litigation), $\alpha$ (probability of success) and $K$ (sunk cost) given, this payoff reduction encourages the infringer to delay investment: this is consistent with the basic insight from a real option setting. Consider that in each period the infringer gets a gross payoff $\Pi(\delta)$ which decreases with $\delta$. If he invests in period 1, he gets $-K + \alpha \left[ \Pi(\delta) \right]_{\text{period 1}}$. If he delays investment, he gets $\alpha \left[ -K + \Pi(\delta) \right]_{\text{period 2}}$. Delaying is preferrable if and only if $K \geq \frac{\alpha}{1-\alpha} \Pi(\delta)$. The threshold value is decreasing in $\delta$ meaning that delaying becomes the preferred option for a wider range of $K$ values.

The second result in proposition 4 is less intuitive. The reason is that, in a real option setting, one expects a decrease in the investor’s payoff to delay investment, as explained above. But in the present setting, one needs to consider the effect of an increase in the compensatory rule $\delta$ on the patentholder’s behavior. The basic reason behind the second insight of proposition 4 is the possibility of an equilibrium switch due to an increase in $\delta$. To see this, consider the increase from $\delta_2$ to $\delta_3$ in Figure 3 below. This figure concerns the "no laches" regime but the same rationale applies to the laches regime. When $\delta = \delta_2$ the infringer delays and the patentholder does not litigate. Suppose now that $\delta = \delta_3$ and $\delta_3 > \delta_2$. If the infringer were to delay (i.e stick to the same strategy), he would now face litigation. This is because the increase in the compensatory rule from $\delta_2$ to $\delta_3$ provides incentives for the patentholder to litigate. As a result, the infringer’s gross payoff from delaying is lower when $\delta = \delta_3$ than when $\delta = \delta_2$ (since he faces litigation for $\delta = \delta_3$ but not for $\delta = \delta_2$). Ceteris paribus, this implies that
investing early (in period 1), becomes more attractive. So, an increase in the compensatory rule can indeed speed-up investment.

Corollary 14 An increase in the compensatory rule $\delta$ can make the patentholder worse-off in both regimes.

To see this, consider Figure 3 above. Again, it concerns the "no laches" regime but the same rationale applies in the laches regime. Focus now on an increase from $\delta_1$ to $\delta_2$. This induces a switch from an equilibrium with early investment and delayed litigation ($ED$) to an equilibrium with delayed investment and no litigation ($DN$). Clearly the patentholder is worse-off as
she does not litigate anymore under DN and so she is not compensated. To understand this insight, one needs to remember the first effect derived in proposition 4: an increase in the compensatory rule $\delta$ incites the infringer to delay investment. It implies that for a given cost of litigation $c$, litigation becomes less attractive for the patentholder: when investment is delayed ($\delta = \delta_2$), she obtains compensation only from the second-period profit ($\delta_2 \pi$) whereas when investment is not delayed ($\delta = \delta_1$) she obtains compensation from both period 1- and period 2- profits ($\delta_1 \pi + \delta_1 \pi$). For low enough values of $\delta_2$ the increase in $\delta$ does not compensate the decrease in the "pie" that the two players must share (this "pie" decreases from $\pi + \pi$ to $\pi$). And so the patentholder is worse-off.\textsuperscript{14}

Summarizing, there are two main results. First, a decrease in the infringer’s gross payoff (through an increase in the patentholder’s compensation) can speed-up investment. This differs from the standard implication of the real option set-up. Second, an increase in the patentholder’s compensation can make her worse-off.\textsuperscript{15} I now turn to the analysis of the doctrine of laches.

5 Regime comparison

In section 4, I focused on the effect of the compensatory rule, in either regime. In this section, I compare the laches and the "no laches" regimes, for any

\textsuperscript{14}In a companion paper on the doctrine of estoppel, I show that a higher probability of a valid patent can hurt the patentholder. However, the argument in the present paper is totally different: the patentholder’s payoff is reduced when $\delta$ increases due to a change in the timing of investment by the infringer.

\textsuperscript{15}Two of Choi (1998)’s results echo these findings. However the underlying economic explanations are totally different. He shows that an increase in patent validity can “accelerate” entry. This is because for some parameters values, there is no “room” for two infringers: being the second entrant is unprofitable and there is a race to be the first one. Choi also shows that an increase in patent validity can reduce the patentholder’s payoff. This is because the first entry is accommodated and occurs immediately due to the "pre-emption" race: the patentholder’s profit is reduced because entry is "accelerated". In my model, an increase in the compensation rule reduces the patentholder’s profit, not because it generates earlier infringement, but because it delays infringement (see the interpretation of corollary 1). Hence, the explanation is the opposite of Choi’s.
given level of the compensatory rule. The question is: what are the qualitative effects induced by the doctrine of laches, compared to a regime where it does not apply? Proposition 1 (where I focus on litigation) constituted a first step into this comparative analysis. Now, the idea is to investigate how the implementation of a laches defense affects:

- the timing of investment into the infringing activity,
- the equilibrium outcomes of the infringement-litigation game and
- players’ welfare,

for any given level of the parameters of the model.

Comparing the regimes implies to consider separately five different cases, depending on the magnitude of \( \alpha \). To see that, consider figures N1 to N3 and figures L1 to L3: the timing of investment changes depending on cutoff values for \( \alpha \), which are not the same in the laches and in the "no laches" regimes. As a result, five intervals must be considered: \( \alpha \in \left[ \frac{1}{T}, \hat{\alpha} \right], \ \alpha \in \left[ \hat{\alpha}, \overline{\alpha} \right], \ \alpha \in \left[ \hat{\alpha}, \overline{\alpha} \right], \ \alpha \in \left[ \overline{\alpha}, 1 \right] \). Appendix B.1 shows that the cutoff values are indeed ranked in this way. To be as accurate as possible, the comparative analysis is conducted for each interval. Yet, it turns out that no additional insight is obtained by considering intervals others than \( \alpha \in \left[ \frac{1}{T}, \hat{\alpha} \right] \). Hence, in what follows, I concentrate on the interval \( \alpha \in \left[ \frac{1}{T}, \hat{\alpha} \right] \) which exhibits all the possible effects induced by introducing a defense of laches. The four other cases are treated in Appendix B.2. In section 5.1 I analyze the effect of the doctrine on the occurrence and the timing of investment in the follow-on innovation. In section 5.2, I investigate the effect of the doctrine of players’ welfare.

5.1 Investment and equilibrium outcomes

Figure 4 below illustrates the effect of the doctrine of laches on the equilibrium outcomes. The figure is obtained by superposing Figure N1 and Figure L1. A dotted line represents a boundary under the doctrine of laches while a solid line represents the same boundary in the no laches regime. As shown in the figure, the doctrine of laches induces a change of the equilibrium outcome.
for six parameters configurations denoted $I, I', J, M, O$ and $P$. It does so essentially, but not only, by modifying some boundaries between the equilibrium regions. This is why it is important to compare analytically these boundaries. It can be seen that the doctrine of laches affects the boundaries $\overline{K}_{N,1}$ and $\overline{K}_{N,2}$, but leaves both $\overline{K}_{N,3}$ and $\overline{K}_{N,2}$ unchanged. The following lemma summarizes how $\overline{K}_{N,1}$ and $\overline{K}_{N,2}$ are modified.

**Lemma 15** i) $\overline{K}_{N,1}(D) \geq \overline{K}_{L,1}(D)$ and ii) $\overline{K}_{N,2}(D) \geq \overline{K}_{L,2}(D)$.

**Proof.** For i), we have $\overline{K}_{N,1}(D) = 2\alpha\pi(1 - D) - \alpha c \geq \overline{K}_{L,1}(D) = 2\alpha\pi(1 - D) - c$ since $\alpha \leq 1$ and by the same token, $\overline{K}_{N,2}(D) = \frac{\alpha\pi(1 - 2D) - \alpha c}{1 - \alpha} \geq \overline{K}_{L,2}(D) = \frac{\alpha\pi(1 - 2D) - c}{1 - \alpha}$. □

In Figure 4, it is clear that $\overline{K}_{N,1}$ is above $\overline{K}_{L,1}$ and $\overline{K}_{N,2}$ is above $\overline{K}_{L,2}$.
Figure 4: Effect of a switch from the "no laches" regime to the laches regime, when $\alpha \in \left[\frac{1}{2}, \tilde{\alpha}\right]$.

I now discuss specifically the effect attached to each of these six configurations. The main insight from the following analysis (summarized by proposition 5), is that the doctrine of laches can have opposite effects on the occurrence and the timing of investment, depending on the parameters of the model.

- **Configuration I**: The doctrine of laches induce may a switch from an equilibrium where investment occurs in period 1 and litigation is delayed ($ED$), to an equilibrium where investment does not occur at all ($NO$).
For these values of the compensatory rule ($\delta \geq \delta_L$), the infringer does not benefit from delaying investment: the patentholder’s compensation is too high for the infringer to sacrifice period-1 profit. Hence, the infringer’s trade-off is between investing in period 1 and not investing at all. The doctrine of laches has a time-inconsistency effect: by lemma 1, we know that if the doctrine of laches is available, the infringer will always invoke it as a defense argument when the patentholder litigates. Anticipating that, as shown in proposition 1, the patentholder may litigate earlier, that is: before uncertainty is fully resolved. As a result, the infringer would face litigation costs with probability 1 if he were to invest in period 1. On the contrary, in the absence of the doctrine, the infringer would face litigation only with probability $\alpha$, as the patentholder would delay litigation until uncertainty is resolved and litigate only when demand is high (which occurs with probability $\alpha$). The prospect of being involved in patent litigation at an early stage can discourage the infringer to invest, although he would have invested in the absence of the doctrine. This effect is illustrated in Figure 4: for $\delta \geq \delta_L$, the boundary $\overline{K}_{N,1}$ that separates investment and no investment in a ”no laches” regime switches to $\overline{K}_{L,1}$ under the doctrine of laches. Because $\overline{K}_{L,1} \leq \overline{K}_{N,1}$, it follows that for all $K \in [\overline{K}_{L,1}, \overline{K}_{N,1}]$, investment does not occur anymore. The first effect of the doctrine of laches is identified: it may deter investment in the follow-on innovation.

- **Configuration I’**: The doctrine of laches may induce a switch from an equilibrium where investment occurs in period 1 and litigation is delayed ($ED$) to an equilibrium where both occur in period 1 ($EE$).

Section 3 analyzed and explained the intuition for this change in litigation behavior: the doctrine of laches encourages the patentholder not to delay precisely because delay is punished by a reduction of the damages collected. Notice that for the values of $K$ such as in configuration $I'$, the infringer still invests (in period 1), despite early litigation.

- **Configuration J**: The doctrine may induce a switch from an equilibrium where investment occurs in period 1 and litigation is delayed ($ED$) to an equilibrium where investment is delayed and litigation does not occur ($DN$).
Notice first that for $\delta \in [\bar{\delta}_L, \bar{\delta}_L]$, delaying investment can be profitable. If the doctrine of laches applies and the infringer invests in period 1, he faces litigation in period 1 (see Figure 2). If the doctrine of laches does not apply and the infringer invests in period 1, he faces litigation in period 2 only if the investment turns out to be profitable, i.e. with probability $\alpha$ (see Figure 2 as well). Hence, the ”real” cost of investing in period 1 is higher under the doctrine of laches ($K + c$) than in the ”no laches” regime ($K + \alpha c$). Consequently, everything else equal, this higher cost implies that delaying investment becomes more attractive, for some values of $K$. Configuration $J$ illustrates this effect. This is the second effect of the doctrine: it may delay investment.

- **Configuration $M$** : The doctrine induces a switch from an equilibrium where investment occurs in period 1 and litigation is delayed ($ED$) to an equilibrium where investment occurs in period 1 and litigation is deterred ($EN$).

Here, the doctrine of laches does not affect the timing of investment. But, as stated in lemma 3, it deters litigation. More precisely, for all $\delta \in [\bar{\delta}_N, \bar{\delta}_L]$, (delayed) litigation occurs in a ”no laches” regime but does not occur in a laches regime. I provided an explanation for this effect in section 3.

- **Configuration $O$** : The doctrine may induce a switch from an equilibrium where investment is delayed and litigation is deterred ($DN$) to an equilibrium where investment occurs in period 1 and litigation is still deterred ($EN$).

As for configuration $M$, the doctrine of laches deters litigation for these parameters values, if the infringer invests in period 1. But if he invests in period 1 in the absence of the doctrine, he faces litigation in period 2. The prospect of not being litigated under the doctrine of laches increases
the expected reward from investment, for all values of $\alpha$ and $K$ (due to the absence of litigation costs and damages). This incites the infringer to invest in period 1 instead of delaying. In the absence of the doctrine, if he were to invest in period 1, he would face litigation in period 2 while if he were to delay, he would not face litigation. This latter effect dominates in the absence of the doctrine, and the infringer has an incentive to delay investment. This is the third effect of the doctrine: it may "speed-up" investment.

- **Configuration $P$**: The doctrine may induce a switch from an equilibrium where investment is deterred ($NO$) to an equilibrium where it occurs in period 1 and litigation is deterred ($EN$).

As for configurations $M$ and $O$, the doctrine of laches deters litigation. Here, and for the same reasons advanced to explain the qualitative changes for configuration $O$, the prospect of not being litigated encourage the infringer to invest (and to invest early). On the contrary, in the "no laches" regime, anticipating litigation, the infringer was deterred from investing for these high values of $K$. This is the fourth effect of the doctrine: it spurs investment.

Proposition 3 summarizes the analysis conducted for each parameters configuration.

**Proposition 16 (the doctrine of laches and the investment into the infringing activity).** The doctrine of laches has four possible effects:

- it can deter investment (configuration $I$),
- it can delay investment (configuration $J$),
- it can speed-up investment (configuration $O$),
- it can spur investment (configuration $P$).

Having analyzed how the doctrine of laches affects the incentives to invest in the follow-on innovation, I turn to investigating how it affects players’ payoffs.
5.2 Players’ welfare

I can now investigate more precisely how the doctrine of laches affects players’ welfare. There are two results. First, the doctrine of laches can make both players worse off (proposition 6). This is a straightforward implication of the effect isolated in proposition 3 for configuration I: the fact that the doctrine deters investment. Second, the doctrine can leave the players indifferent or make the patentholder worse off and the infringer better off (proposition 7).

Proposition 17 When the probability of success is high (\(\alpha \geq \frac{1}{2}\)) and patent protection is strong (\(\delta \geq \bar{\delta}_L\)) or intermediate (\(\delta \in [\underline{\delta}_L, \bar{\delta}_L]\)), a regime where the doctrine of laches applies can make both the patentholder and the infringer worse off compared to a regime where it does not apply.

Proof.

- Consider first strong patent protection (\(\delta \geq \bar{\delta}_L\)). From proposition 5, we know that for configuration I investment is deterred in a laches regime, while it would occur in a ”no laches” regime, implying that both players are worse-off with the doctrine of laches. Consider then configuration I’. We can compute players’ payoffs in both regimes. First in the ”no laches” regime \(N\) (litigation is delayed):

\[
\begin{align*}
U^N_H &= -ac + 2\alpha \pi (1 - \delta) \\
U^N_I &= -K - ac + 2\alpha \pi (1 - \delta).
\end{align*}
\]

Then, in the laches regime \(L\) (litigation is not delayed):

\[
\begin{align*}
U^L_H &= -c + 2\alpha \pi (1 - \delta) \\
U^L_I &= -K - c + 2\alpha \pi (1 - \delta).
\end{align*}
\]

It follows that \(U^N_H \geq U^L_H\) and \(U^N_I \geq U^L_I\).

- Consider then intermediate patent protection (\(\delta \in [\underline{\delta}_L, \bar{\delta}_L]\)). Here the two relevant configurations are \(J\) and \(I'\). I compute players’ payoffs under either regime, for each configuration. Consider configuration \(J\). In the ”no laches” regime \(N:\)

\[
\begin{align*}
U^N_H &= -ac + 2\alpha \pi (1 - \delta) \\
U^N_I &= -K - ac + 2\alpha \pi (1 - \delta)
\end{align*}
\]
and in the laches regime $L$:

$$\begin{cases} U^N_H = 0 \\ U^I_L = \alpha(\pi - K). \end{cases}$$

Clearly, the patentholder is worse-off in regime $L$. The infringer is worse-off if and only if:

$$K \leq \frac{\alpha \pi (1 - 2\delta) - \alpha c}{1 - \alpha} = K_{N,2},$$

which holds for configuration $J$. Then, consider configuration $I'$. It has been proved above that for this configuration, players are better-off in a no laches regime.

This proposition states a counterintuitive result: the defense available to the defendant (infringer) can make him worse-off. To understand this point, consider simply the explanation for the equilibrium switch characterizing configuration $I$: the doctrine of laches deters investment compared to a regime where it does not apply and so leaves both the patentholder and the infringer worse-off.

**Proposition 18** When the probability of success is high ($\alpha \geq \frac{1}{2}$) and patent protection is low ($\delta \leq \delta_L$), a regime where the doctrine of laches applies can leave the patentholder indifferent or make her worse-off, compared to a regime where it does not apply. Also, it can leave the infringer indifferent or make him better-off.

**Proof.** For all $\delta \leq \delta_N$, litigation does not occur under either regime so that a regime change leaves the players indifferent. For $\delta \in [\delta_N, \delta_L]$, consider three configurations: $M, O$ and $P$. For configuration $M$; introducing a laches defense deters litigation and so makes the patentholder worse-off and the infringer better-off (he still invests in period 1 but does not pay damages). For configuration $O$, the patentholder would not litigate in either regime. So introducing a laches defense leaves her indifferent. But the infringer would invest earlier so that his expected payoff is $-K + 2\alpha \pi$ in a laches regime, and $\alpha(\pi - K)$ in the "no laches" regime. He is better-off in a laches regime since $-K + 2\alpha \pi \geq \alpha(\pi - K)$ if and only if $K \leq \frac{\alpha \pi}{1 - \alpha} = K_{L,3}$, which holds for configuration $O$. Finally, for configuration $P$, the patentholder is not affected.
by the introduction of a laches defense (in the laches regime, she does not litigate and in the "no laches" regime, there is no investment in the first place). But the infringer is clearly better-off as the defense of laches makes investment profitable.

The results in proposition 7 are more conform to the explicit objective of the doctrine of laches than the result derived in proposition 6: As a ”defense” argument, the doctrine is supposed to benefit the infringer (the ”defendant” in the trial) and sanction the patentholder (if she adopts the prohibited behavior). Here, in the particular economic context inquired, I show that indeed the doctrine improves the situation of the infringer and possibly penalizes the patentholder, but only for specific values of the parameters. The most interesting result is that the doctrine can hurt both players, as stated in proposition 6.

I now briefly discuss my results of this section. The main insight of the analysis is that the doctrine of laches has contrasted and possibly opposite effects on the incentive to invest in the infringing activity. It can spur or deter investment and it can delay investment or speed it up (proposition 5). The occurrence of one or the other of these outcomes depends very much on the parameters of the model. In particular, the model suggests that when patent protection is strong (because the patentholder’s compensation δ is high), then it might be better not to implement the doctrine of laches: indeed proposition 6 states that the doctrine hurts both the patentholder and the infringer since the infringer is deterred from investing. But it also hurts society which does not benefit from the follow-on innovation. Of course, this analysis is only a first investigation into the quantitative and qualitative effects of the doctrine of laches, within a stylized framework. I believe it is much too early to derive policy recommendations. However, the model shows that the doctrine is likely to have non-trivial effects on both the occurrence of investments into follow-on innovations and on the ”pace” of these investments.

6 Conclusion

In this paper, I analyzed the effects of the doctrine of laches and compensatory damages on the incentives to infringe a patent and to enforce this
patent. "Infringement" is equivalent to an investment in a follow-on innovation which requires the patented technology. Both the infringer and the patentholder have a "real option" problem. The profitability of the infringing product is initially uncertain. The infringer is the leader and can invest before or after uncertainty is resolved. The patentholder is the follower and, if the infringer invested before uncertainty was resolved, she herself can litigate before or after profitability becomes known. Litigation is costly for both players. Delayed litigation can be punished by the doctrine of laches which prevents the patentholder from getting damages for infringement that occurred during the delay period.

I show that the doctrine of laches can deter litigation or prompt earlier litigation (proposition 1). I also show that the doctrine has different effects on the timing of investment in the follow-on innovation depending on parameters values: it can deter or spur investment. It can speed-up or delay investment (proposition 5). It also affects players' welfare: although it is a defense argument, it can make the defendant (the infringer) worse-off (proposition 6). I show that an increase of the patentholder's compensation can delay or speed-up investment (proposition 4). It can make the patentholder worse-off (corollary 1).

The analysis of the model proposed is sometimes cumbersome. Yet, it is also useful because it yields counterintuitive conclusions. For example, increasing the level of damages does not necessarily benefit the patentholder. Regarding the doctrine of laches, I show that it not only affects the timing of litigation, but also and perhaps most importantly, it affects the occurrence and the timing of investment in the follow-on innovation. This suggests that this doctrine should be taken into consideration from a patent policy perspective.

Appendix

Appendix A: Investment timing when $\alpha \geq \frac{1}{2}$.

In the main text, many analytical steps have been omitted in order to simplify the progression towards the economic results gathered in sections 4 and 5. In Appendix A, I report these omitted steps.
Appendix A.1: The infringer’s decision in the ”no laches regime”.

In Appendix A.1, I report the omitted analytical steps for scenarios 1, 2 and 3.

**Scenario 1**: \( \delta \in [\delta_L, 1] \).

The analysis of this scenario is conducted in detail in section 3.3. Here, I analyze the respective positions of the three boundaries \( \overline{K}_{N,1}(\delta) \), \( \underline{K}_{N,1}(\delta) \) and \( K_{N,1}(\delta) \) defined by (12), (15) and (16).

It can be shown that \( \overline{K}_{N,1}(\delta) \geq \underline{K}_{N,1}(\delta) \). Indeed, this inequality holds if and only if \( (2\alpha - 1)\pi(1 - \delta) + (1 - \alpha)c \geq 0 \) which holds for all \( \alpha \geq \frac{1}{2} \) and \( \delta \in [\delta_L, 1] \). Also, it can be shown that \( \underline{K}_{N,1}(\delta) \geq K_{N,1}(\delta) \). This inequality holds if and only if \( \frac{2\alpha - 1}{1-c} \pi(1 - \delta) + c \geq 0 \) which holds for all \( \alpha \geq \frac{1}{2} \) and \( \delta \in [\delta_L, 1] \). Hence:

\[
\begin{cases}
\overline{K}_{N,1}(\delta) \geq \underline{K}_{N,1}(\delta) \\
\underline{K}_{N,1}(\delta) \geq K_{N,1}(\delta)
\end{cases}
\]  

(26)

It follows that for all \( K \geq \overline{K}_{N,1}(\delta) \) the infringer invests in period 1 if \( K \leq \underline{K}_{N,1}(\delta) \) but does not invest otherwise. And for all \( K \leq \underline{K}_{N,1}(\delta) \) he invests in period 1. So, for all \( K \leq \overline{K}_{N,1}(\delta) \), the infringer invests in period 1. This is stated in lemma 5.

**Analysis of** \( \overline{K}_{N,1}(\delta) = 2\alpha \pi(1 - \delta) - \alpha c \). This function is obviously downward sloping with \( \overline{K}_{N,1}(0) = 2\alpha \pi - \alpha c \) and \( \overline{K}_{N,1}(\delta) = 0 \iff \delta = 1 - \frac{c}{\pi} = \hat{\delta} > \frac{\pi}{\pi} = \delta_L \) since \( \pi \geq 3c \). In addition, \( \overline{K}_{N,1}(\frac{\pi}{\pi}) = 2\alpha \pi - 3ac \) and \( \overline{K}_{N,1}(\frac{\pi}{2\pi}) = 2\alpha \pi - 2ac \).

**Scenario 2**: \( \delta \in [\delta_N, \delta_L] \).

Suppose the infringer delayed investment. His payoff if he invests is \( U_{I,2}^N(i) = \pi - K \). Indeed, the patentholder does not litigate. Hence:

\[
U_{I,2}^N(i) = \pi - K \begin{cases}
< 0 & \text{if } K > K_{N,2} \triangleq \pi \\
\geq 0 & \text{if } K \leq K_{N,2} \triangleq \pi.
\end{cases}
\]  

(27)
In period 1, the infringer’s expected payoff if he does not invest is:

\[ U_{I,1}^N(n) = \begin{cases} 
0 & \text{if } K > \overline{K}_{N,2} \triangleq \pi \\
\alpha(\pi - K) & \text{if } K \leq \overline{K}_{N,2} \triangleq \pi.
\end{cases} \tag{28} \]

By contrast, his payoff if he invests in period 1 is:

\[ U_{I,1}^N(i) = -K + 2\pi\alpha(1 - \delta) - \alpha c, \tag{29} \]

since the patentholder will litigate in period 2 and obtain a share of both period 1 and period 2 profits (the doctrine of laches does not apply).

Again, I can distinguish between two cases.

\[ \diamond \text{ If } K > \overline{K}_{N,2}, \text{ delaying investment is never profitable. Is it profitable to invest today? The condition for profitability is } U_{I,1}^N(i) \geq 0 \text{ which is equivalent to:} \]

\[ K \leq 2\alpha\pi(1 - \delta) - \alpha c \triangleq \overline{K}_{N,2}. \tag{30} \]

\[ \diamond \text{ If } K \leq \overline{K}_{N,2}, \text{ delaying investment yields a non-negative profit. The infringer invests today if and only if } U_{I,1}^N(i) \geq U_{I,1}^N(n) \text{ or:} \]

\[ K \leq \frac{\alpha}{1 - \alpha} [\pi(1 - 2\delta) - c] \triangleq \overline{K}_{N,2}. \tag{31} \]

As for scenario 1, it remains to analyze in the \((K, \delta)\) space the respective position of \(\overline{K}_{N,2}, \overline{K}_{N,2}, \overline{K}_{N,2}\) defined by (27), (30) and (31). The difficulty in that case is that these positions depend on the value of \(\alpha\). As a result, I must distinguish again distinguish between cases depending on the value of \(\alpha\) on the interval \(\alpha \in \left[\frac{1}{2}, 1\right]\) (remember that I assumed \(\alpha \geq \frac{1}{2}\)).

Notice first that \(\overline{K}_{N,2}(\delta) = \overline{K}_{N,1}(\delta)\). Hence, from the above analysis I know that \(\overline{K}_{N,2}(\delta)\) is downward sloping with \(\overline{K}_{N,2}(\delta) = 0\) at \(\delta = \hat{\delta}\). I can compute \(\overline{K}_{N,2}(\delta_N) = 2\alpha(\pi - c)\). Then, \(\overline{K}_{N,2}(\delta) = \pi\) is a constant. Finally, \(\overline{K}_{N,2}(\delta)\) is linear and decreasing in \(\delta\) and \(\overline{K}_{N,2}(\delta_N) = \frac{\alpha(\pi - 2\alpha c)}{1 - \alpha}\). Notice that the line representing \(\overline{K}_{N,2}(\delta)\) is steeper than that of \(\overline{K}_{N,2}(\delta_N)\). Indeed, \(\frac{2\alpha \pi}{1 - \alpha} \geq 2\alpha\pi\) always holds. Finally, \(\overline{K}_{N,2}(\frac{e}{\pi}) = \frac{\alpha}{1 - \alpha}(\pi - 3c) \geq 0\) since \(\pi \geq 3c\).
In order to analyze the respective positions of $\mathcal{K}_{N,2}(\delta)$, $\overline{\mathcal{K}}_{N,2}(\delta)$ and $\overline{\mathcal{K}}_{N,2}(\delta)$, I define the following values: $\hat{\alpha} = \frac{\pi}{2(\pi - c)}$ and $\hat{\alpha} = \frac{\pi}{2\pi - 3c}$. First, notice that $\hat{\alpha} \geq \frac{1}{2}$ if and only if $c \geq 0$ which holds. Also, notice that $\hat{\alpha} \geq \hat{\alpha}$ if and only if $2\pi(\pi - c) \geq \pi(2\pi - 3c)$ which holds. In addition, $\hat{\alpha} \leq 1$ if and only if $\pi \geq 3c$ which holds by assumption.

\(\diamond\) Consider first the interval $\alpha \in [\frac{1}{2}, \hat{\alpha}]$.

For these values of $\alpha$, we have $\overline{\mathcal{K}}_{N,2}(\delta_N) \leq \mathcal{K}_{N,2}(\delta_N)$ if and only if $\alpha \leq \frac{\pi}{2(\pi - c)} = \hat{\alpha}$ which holds by assumption. And since $\mathcal{K}_{N,2}(\cdot)$ is constant while $\overline{\mathcal{K}}_{N,2}(\cdot)$ is downward sloping, it follows that $\mathcal{K}_{N,2}(\delta) \geq \overline{\mathcal{K}}_{N,2}(\delta)$.

Also, I have $\overline{\mathcal{K}}_{N,2}(\delta_N) \leq \overline{\mathcal{K}}_{N,2}(\delta_N)$ if and only if $\alpha \leq \hat{\alpha}$ which holds by assumption. Since $\overline{\mathcal{K}}_{N,2}(\delta_N)$ is steeper than $\overline{\mathcal{K}}_{N,2}(\delta_N)$ it follows that $\overline{\mathcal{K}}_{N,2}(\delta) \geq \overline{\mathcal{K}}_{N,2}(\delta)$. Hence,

$$\mathcal{K}_{N,2}(\delta) \geq \overline{\mathcal{K}}_{N,2}(\delta) \geq \overline{\mathcal{K}}_{N,2}(\delta).$$

(32)

Hence, for all $K > \overline{\mathcal{K}}_{N,2}(\delta)$ the infringer does not invest. For $K \leq \overline{\mathcal{K}}_{N,2}(\delta)$ he invests in period 1 if $K \leq \overline{\mathcal{K}}_{N,2}(\delta)$ and he delays investment if $K > \overline{\mathcal{K}}_{N,2}(\delta)$. This is stated in lemma 6.

\(\diamond\) Consider then the interval $\alpha \in (\hat{\alpha}, \hat{\alpha}]$.

I know from the preceding analysis that $\overline{\mathcal{K}}_{N,2}(\delta) < \overline{\mathcal{K}}_{N,2}(\delta)$ and $\overline{\mathcal{K}}_{N,2}(\delta) < \overline{\mathcal{K}}_{N,2}(\delta)$. I investigate the condition for the lines representing $\overline{\mathcal{K}}_{N,2}(\cdot)$ and $\overline{\mathcal{K}}_{N,2}(\cdot)$ to intersect $\overline{\mathcal{K}}_{N,2}(\delta) = \pi$ at a point $\delta \in [\delta_N, \delta_L]$. To that end, I solve $\overline{\mathcal{K}}_{N,2}(\delta) = \pi$ for $\delta$. This yields $\delta = \frac{2\alpha\pi - \pi - c}{2\alpha\pi} = \delta$. Then I solve $\overline{\mathcal{K}}_{N,2}(\delta) = \pi$ for $\delta$. This yields $\delta = \delta$ as well. The conditions for $\delta \in [\delta_N, \delta_L]$ are $\delta \geq \delta_N$ and $\delta \leq \delta_L$. The first condition amounts at showing that $\alpha \geq \frac{\pi}{2(\pi - c)} = \hat{\alpha}$, which holds. The second condition implies that $\alpha \leq \frac{\pi}{2\pi - 3c} = \hat{\alpha}$, which holds. These two conditions are clearly satisfied on the interval $(\hat{\alpha}, \hat{\alpha}]$.

Hence we have:

$$\begin{cases} \overline{\mathcal{K}}_{N,2}(\delta) \geq \overline{\mathcal{K}}_{N,2}(\delta) \geq \mathcal{K}_{N,2}(\delta) & \text{if } \delta \in [\delta_N, \delta] \\ \mathcal{K}_{N,2}(\delta) \leq \mathcal{K}_{N,2}(\delta) \leq \overline{\mathcal{K}}_{N,2}(\delta) & \text{if } \delta \in [\delta, \delta_L] \end{cases}$$

(33)
Define the kinked curved $\hat{K}(\delta)$ as:

$$
\hat{K}(\delta) = \begin{cases} 
\overbrace{K_{N,2}(\delta)}^{\text{if } \delta \in [\delta_N, \delta]} \\
\underbrace{K_{N,2}(\delta)}_{\text{if } \delta \in [\delta, \delta_L]} 
\end{cases}
$$

(34)

It follows that for all $K < \hat{K}(\delta)$ the infringer invests in period 1. For $K \geq \hat{K}(\delta)$ but $K \leq K_{N,2}(\delta)$, he delays investment until period 2. Finally, for all $K \geq \hat{K}(\delta)$ and $K > K_{N,2}(\delta)$, he does not invest. This is stated in lemma 6.

$\diamondsuit$ Finally, consider $\alpha \in [\hat{\alpha}, 1]$.

From the preceding analysis, we know that $\delta \geq \delta_L$ for every $\alpha$ in this interval. Hence, for all $\delta \in [\delta_N, \delta_L]$:

$$
\overbrace{K_{N,2}(\delta)}^{\text{if } \delta \in [\delta_N, \delta]} \geq \underbrace{K_{N,2}(\delta)}_{\text{if } \delta \in [\delta, \delta_L]} \geq K_{N,2}(\delta).
$$

(35)

Consequently, for all $K \geq K_{N,2}(\delta)$ and $K \leq K_{N,2}(\delta)$, the infringer invests in period 1. For all $K > K_{N,2}(\delta)$, he does not invest. And for all $K \leq K_{N,2}(\delta)$, he invests in period 1. It follows that for all $K \leq K_{N,2}(\delta) = K_{N,1}(\delta)$ the infringer invests in period 1 and does not invest for larger values of $K$. This is stated in lemma 6.

**Scenario 3:** $\delta \in [0, \delta_N]$.

Suppose the infringer delayed investment. In period 2, his net payoff from investing is $U_{I,2}^N(i) = \pi - K$ since the patentholder does not litigate. Hence:

$$
U_{I,2}^N(i) = \pi - K \begin{cases} 
< 0 & \text{if } K > K_{N,3} \triangleq \pi \\
\geq 0 & \text{if } K \leq K_{N,3} \triangleq \pi.
\end{cases}
$$

(36)

In period 1, the infringer’s expected payoff if he does not invest is:

$$
U_{I,1}^F(n) = \begin{cases} 
0 & \text{if } K > K_{N,3} \\
\alpha(\pi - K) & \text{if } K \leq K_{N,3}.
\end{cases}
$$

(37)
If the infringer invests in period 1, he obtains:

\[ U^N_{I,1}(i) = -K + \alpha 2\pi, \]  

(38)

as the patentholder will never litigate.

\[ \diamond \text{ If } K \geq \overline{K}_{N,3}, \text{ delaying investment is never profitable. Investing today is profitable if and only if } U^N_{I,1}(i) \geq 0 \text{ or:} \]

\[ K \leq 2\alpha \pi \triangleq \overline{K}_{N,3}. \]  

(39)

\[ \diamond \text{ If } K < \overline{K}_{N,3}, \text{ delaying yields a non-negative profit. The infringer invests today if and only if } U^N_{I,1}(i) \geq U^N_{I,1}(n) \text{ or:} \]

\[ K \leq \frac{\alpha}{1 - \alpha} \pi \triangleq \overline{K}_{N,3}. \]  

(40)

Again, I have to analyze in the \((K, \delta)\) space the respective position of \(\overline{K}_{N,3}(\delta)\), \(\overline{K}_{N,3}(\delta)\) and \(\overline{K}_{N,3}(\delta)\), respectively defined by (36), (39) and (40).

Obviously, \(2\alpha \pi \geq \pi\) if and only if \(\alpha \geq \frac{1}{2}\) which holds, so that \(\overline{K}_{N,3}(\delta) \geq \overline{K}_{N,3}(\delta)\). And \(\frac{\alpha}{1 - \alpha} \pi \geq \pi\) if and only if \(\alpha \geq \frac{1}{2}\) which holds as well, so that \(\overline{K}_{N,3}(\delta) \geq \overline{K}_{N,3}(\delta)\). Hence,

\[ \left\{ \begin{array}{l} \overline{K}_{N,3}(\delta) \geq \overline{K}_{N,3}(\delta) \\ \overline{K}_{N,3}(\delta) \geq \overline{K}_{N,3}(\delta). \end{array} \right. \]  

(41)

It follows that for all \(K \geq \overline{K}_{N,3}(\delta)\) the infringer invests in period 1 provided that \(K \leq \overline{K}_{N,3}(\delta)\), otherwise he does not invest. And for all \(K \leq \overline{K}_{N,3}(\delta)\) the infringer invests in period 1. So, for all \(K \leq \overline{K}_{N,3}(\delta)\) the infringer invests in period 1. This is stated in lemma 7

Analysis of \(\overline{K}_{N,3}(\delta) = 2\alpha \pi\). Notice only that \(\overline{K}_{N,3}(\delta) = 2\alpha \pi\) is a constant.
The following lemma combines the above analysis with the analysis of litigation conducted in section 3. It gives the exact values of the parameters for which a specific equilibrium outcome occurs. Figures $\mathbf{N1}$, $\mathbf{N2}$ and $\mathbf{N3}$ capture these features in a livelier manner.

**Lemma 19 (Equilibrium outcomes when the doctrine of laches does not apply).** When the probability of commercial success is high ($\alpha \geq \frac{1}{2}$),

- If patent protection is strong ($\delta \in [\delta_L, 1]$), the infringer invests in period 1 if the sunk cost is low enough ($K \leq K_{N,1}$) and does not invest otherwise. If he invests, the patentholder delays litigation.

- If patent protection is weak ($\delta \in [0, \delta_N)$), the infringer invests in period 1 if the sunk cost is low enough ($K \leq K_{N,3}$) and does not invest otherwise. If he invests, the patentholder does not litigate.

- If patent protection is intermediate ($\delta \in [\delta_N, \delta_L]$), the timing of investment depends on the probability of commercial success:
  - When success is moderately likely ($\alpha \in \left[\frac{1}{2}, \alpha^\dagger\right]$), the infringer invests in period 1 if $K \leq K_{N,1}$ and the patentholder delays litigation. The infringer delays investment until period 2 if $K \in [K_{N,2}, K_{N,2}]$ and the patentholder does not litigate. And the infringer does not invest if $K \geq K_{N,2}$.
  - When success is likely ($\alpha \in (\alpha^\dagger, \alpha]$), the infringer invests in period 1 if $K \leq K_{N,2}$ and the patentholder delays litigation. The infringer delays investment if $K \in [K, K_{N,2}]$ and the patentholder does not litigate. And the infringer does not invest if $K \geq K_{N,2}$.
  - When success is very likely ($\alpha \in (\alpha^\dagger, 1]$), the infringer invests in period 1 if $K \leq K_{N,1}$ and the patentholder delays litigation. Otherwise, the infringer does not invest.
Appendix A.2. The infringer’s decision in the ”laches regime”

The methodology here is similar to that in Appendix A.1.

**Scenario 1**: $\delta \in [\delta_L, 0]$.

I want to analyze the respective of the three functions $K_{L,1}(\delta)$, $\overline{K}_{L,1}(\delta)$ and $\underline{K}_{L,1}(\delta)$, respectively defined by (20), (23) and (24). I can show that $\overline{K}_{L,1}(\delta) \geq \underline{K}_{L,1}(\delta)$. Indeed, this inequality holds if and only if $\pi(1-\delta)(2\alpha - 1) \geq 0$ which holds for all $\alpha \geq \frac{1}{2}$ and $\delta \in [\overline{\delta}_L, 1]$. Also, we can show that $\overline{K}_{L,1}(\delta) \geq \underline{K}_{L,1}(\delta)$. This inequality holds if and only if $\frac{\alpha \pi (1-\delta) + \alpha c - c}{1-\alpha} \geq \pi(1-\delta) - c$ which holds for all $\alpha \geq \frac{1}{2}$.

From that, we can conclude that if $K \geq \overline{K}_{L,1}(\delta)$, the infringer invests in period 1 if and only if $K \leq \overline{K}_{L,1}(\delta)$ and if $K \leq \underline{K}_{L,1}(\delta)$, the infringer always invest in period 1. So, for all $K \leq \overline{K}_{L,1}(\delta)$, the infringer invests in period 1. Otherwise he does not invest. This is stated in lemma 8.

*Analysis of $\overline{K}_{L,1}(\delta)$. The function $\overline{K}_{L,1}(\delta) = 2\alpha \pi (1-\delta) - c$ is linear and decreasing in $\delta$. In addition, $\overline{K}_{L,1}(\delta_L) = 2\alpha \pi - c(2\alpha + 1)$ and $\overline{K}_{L,1}(\delta) = 0$ for $\delta = 1 - \frac{c}{2\alpha \pi} = \hat{\delta}$.

**Scenario 2**: $\delta \in [\hat{\delta}_L, \delta_L]$.

If the infringer did not invest in period 1, his net payoff from investing in period 2 is $U_{L,2}^I(i) = -K + \pi$. Indeed, he faces no litigation in period 2. It follows that:

$$U_{L,2}^I(i) = -K + \pi \begin{cases} < 0 & \text{if } K > \overline{K}_{L,2} \triangleq \pi \\ \geq 0 & \text{if } K \leq \overline{K}_{L,2} \triangleq \pi. \end{cases}$$

(43)

In period 1, the infringer’s payoff if he does not invest is:

$$U_{L,1}^I(n) = \begin{cases} 0 & \text{if } K > \overline{K}_{L,2} \triangleq \pi \\ \alpha (\pi - K) & \text{if } K \leq \overline{K}_{L,2} \triangleq \pi. \end{cases}$$

(44)
His payoff if he invests in period 1 is:

\[ U^L_{I,1}(i) = -K - c + 2\alpha \pi (1 - \delta), \]  

(45)

since the patentholder litigates in period 1 (before uncertainty is resolved).

\[ \diamond \text{Suppose } K \geq \overline{K}_{N,2}. \text{ Then } U^L_{I,1}(i) \geq 0 \text{ if and only if:} \]

\[ K \leq 2\alpha \pi (1 - \delta) - c \triangleq \overline{K}_{L,2}. \]  

(46)

\[ \diamond \text{Suppose } K < \overline{K}_{N,2}. \text{ Then } U^L_{I,1}(i) \geq U^L_{I,1}(n) \text{ if and only if:} \]

\[ K \leq \frac{\alpha \pi (1 - 2\delta) - c}{1 - \alpha} \triangleq \overline{K}_{L,2}. \]  

(47)

\[ \square \text{ On this interval, I analyze the functions } \overline{K}_{L,2}(\delta), \overline{K}_{L,2}(\delta) \text{ and } \overline{K}_{L,2}(\delta) \text{ respectively defined by } (43), (46) \text{ and } (47). \text{ Notice first that } \overline{K}_{L,2}(\delta) = \overline{K}_{L,1}(\delta). \text{ Hence, from the above analysis, I know that } \overline{K}_{L,2}(\delta) \text{ is downward sloping with } \overline{K}_{L,2}(\delta) = 0 \text{ at } \delta = \hat{\delta}. \text{ I can compute } \overline{K}_{L,2}(\delta_L) = 2\alpha \pi - 2c. \text{ Then, } \overline{K}_{L,2}(\delta) = \pi \text{ is a constant. Finally, } \overline{K}_{L,2}(\delta_L) = \frac{\alpha \pi - 2c}{1 - \alpha}. \text{ Notice that the line representing } \overline{K}_{L,2}(\delta) \text{ is steeper than that of } \overline{K}_{L,2}(\delta). \text{ Indeed, } \frac{2\alpha \pi}{1 - \alpha} \geq 2\alpha \pi \text{ always holds.} \]

\[ \square \text{ In order to analyze the respective positions of } \overline{K}_{L,2}(\delta), \overline{K}_{L,2}(\delta) \text{ and } \overline{K}_{L,2}(\delta), \text{ I define the following values: } \tilde{\alpha} = \frac{\pi + 2c}{2\pi} \text{ and } \tilde{\alpha} = \frac{\pi + c}{2(\pi - c)}. \text{ Notice first that } \tilde{\alpha} \geq \frac{1}{2} \text{ if and only if } 2c \geq 0 \text{ which holds. Also, notice that } \tilde{\alpha} \geq \tilde{\alpha} \text{ if and only if } (\pi + 2c)(\pi - c) \leq (\pi + c)\pi \text{ or } -2c^2 \leq 0 \text{ which holds. In addition, } \tilde{\alpha} \leq 1 \text{ if and only if } \pi - 3c \geq 0 \text{ which holds by assumption.} \]

\[ \diamond \text{ Consider first the interval } \alpha \in \left[ \frac{1}{2}, \tilde{\alpha} \right]. \]

For these values of } \alpha, \overline{K}_{L,2}(\delta) \leq \overline{K}_{L,2}(\delta) = \pi. \text{ To establish this result, it is sufficient to show that } \overline{K}_{L,2}(\delta_L) \leq \overline{K}_{L,2}(\delta_L) = \pi \text{ because } \overline{K}_{L,2}(\delta) \text{ is decreasing in } \delta \text{ while } \overline{K}_{L,2}(\delta) = \pi \text{ is constant. And } \overline{K}_{L,2}(\delta_L) \leq \overline{K}_{L,2}(\delta_L) \text{ if and only if } 2\alpha \pi - 2c \leq \pi \text{ or } \alpha \leq \frac{\pi + 2c}{2\pi} = \tilde{\alpha} \text{ which holds.} \]
Then, I can show that $K_{L,2}(\delta) \leq K_{L,2}(\delta)$. Again, to establish this result, it is sufficient to show that $K_{L,2}(\delta) \leq K_{L,2}(\delta)$ since $K_{L,2}(\delta)$ is steeper than $K_{L,2}(\delta)$. But this inequality amounts at $\frac{2\pi - 2c}{2\alpha} \leq 2\pi - 2c$ which holds for all $\alpha \leq \tilde{\alpha}$.

Hence, I have:

$$K_{L,2}(\delta) \leq K_{L,2}(\delta) \leq K_{L,2}(\delta). \quad (48)$$

It follows that for all $K > K_{L,2}(\delta)$, the infringer does not invest. For $K \leq K_{L,2}(\delta)$, he invests in period 1 provided $K \leq K_{L,2}(\delta)$ and he delays if $K \in (K_{L,2}(\delta), K_{L,2}(\delta)]$. This is stated in lemma 9.

\[ \diamond \text{ Consider then } \alpha \in \left[\frac{\pi}{2}, \tilde{\alpha}\right]. \]

I know from the preceding analysis that $K_{L,2}(\delta) \geq K_{L,2}(\delta) = \pi$ and in addition $K_{L,2}(\delta) \geq K_{L,2}(\delta)$. I investigate the condition for both $K_{L,2}(\delta)$ and $K_{L,2}(\delta)$ to intersect $K_{L,2}(\delta) = \pi$ at a point $\delta \in \left[\delta_L, \tilde{\delta}_L\right]$. To that end, I solve $K_{L,2}(\delta) = \pi$ for $\delta$. This gives $\delta = \frac{2\alpha \pi - c - \pi}{2\alpha} = \delta$ and I solve $K_{L,2}(\delta) = \pi$ for $\delta$. This also yields $\delta = \frac{2\alpha \pi - c - \pi}{2\alpha} = \tilde{\delta}$. The conditions for $\delta \in \left[\delta_L, \tilde{\delta}_L\right]$ are:

\[ \delta \geq \delta_L \text{ and } \delta \leq \tilde{\delta}_L. \]

The first condition amounts at $\frac{2\alpha \pi - c - \pi}{2\alpha} \leq \frac{c}{2\alpha}$ or $\alpha \geq \bar{\alpha}$ while the second condition amounts at $\frac{2\alpha \pi - c - \pi}{2\alpha} \leq \frac{\pi}{2}$ or $\alpha \leq \frac{\pi}{2}(\pi - c) = \bar{\alpha}$. These two conditions are clearly satisfied.

Hence, I have:

$$\left\{ \begin{array}{ll} K_{L,2}(\delta) \geq K_{L,2}(\delta) \geq K_{L,2}(\delta) & \text{if } \delta \in [\delta_L, \tilde{\delta}_L] \\ K_{L,2}(\delta) \leq K_{L,2}(\delta) \leq K_{L,2}(\delta) & \text{if } \delta \in [\delta, \tilde{\delta}_L]. \end{array} \right. \quad (49)$$

Defines the kinked curved $K(\delta)$ by:

$$K(\delta) = \left\{ \begin{array}{ll} K_{L,2}(\delta) & \text{if } \delta \leq \delta \\ K_{L,2}(\delta) & \text{if } \delta > \tilde{\delta} \end{array} \right. \quad (50)$$
It follows that for all $K \leq \bar{K}(\delta)$ the infringer invests in period 1, and for all $K$ such that $K \leq \bar{K}_L,2(\delta)$ and $K \geq \bar{K}(\delta)$ he delays investment until period 2. For $K > \bar{K}(\delta)$ and $K > \bar{K}_L,2(\delta)$ he does not invest. This is stated in lemma 9.

\[ \diamond \text{Consider finally } \alpha \in [\bar{\alpha}, 1]. \]

From the preceding analysis, I know that $\delta \geq \bar{\delta}_L$. Hence, for all $\delta \in [\bar{\delta}_L, \delta_L]$, I have:

\[ \bar{K}_L,2(\delta) \geq \bar{K}_L,2(\delta) \geq \bar{K}_L,2(\delta). \] (51)

It follows that for all $K \leq \bar{K}_L,2(\delta)$ the infringer invests in period 1, otherwise he does not invest. This is again stated in lemma 9.

\[ \text{Scenario 3: } \delta \in [0, \bar{\delta}_L]. \]

The analysis is identical to the ”no laches regime”. See Appendix A.1.

On this interval, three functions must be considered: $\bar{K}_{L,3}(\delta)$, $\bar{K}_{L,3}(\delta)$ and $\bar{K}_{L,3}(\delta)$. The analysis is equivalent to the analysis of the ”no laches” regime since $\bar{K}_{L,3}(\delta) = \bar{K}_{N,3}(\delta)$, $\bar{K}_{L,3}(\delta) = \bar{K}_{N,3}(\delta)$ and $\bar{K}_{L,3}(\delta) = \bar{K}_{N,3}(\delta)$. Therefore, it is not necessary to detail the analysis and, following the result for the no laches regime, I can state that for all $K \leq \bar{K}_{L,3} = 2\alpha\pi$, the infringer invests in period 1. Otherwise, he does not invest. This is stated in lemma 7.

The following lemma combines the above analysis with the analysis of litigation conducted in section 3. It gives the exact values of the parameters for which a specific equilibrium outcome occurs. Figures L1, L2 and L3 capture these features in a livelier manner.

**Lemma 20** *Equilibrium outcomes under the doctrine of laches.*

*When the probability of commercial success is high ($\alpha \geq \frac{1}{2}$),*
• If patent protection is strong ($\delta \in [\delta_L, 1]$), the infringer invests in period 1 if the sunk cost is low enough ($K \leq K_{L,1}$) and the patentholder litigates in period 1. If $K > K_{L,1}$ he does not invest.

• If patent protection is weak ($\delta \in [0, \delta_L]$), the infringer invests in period 1 if the sunk cost is low enough ($K \leq K_{L,3}$) and the patentholder does not litigate. He does not invest if $K > K_{L,3}$.

• If patent protection is intermediate ($\delta \in [\delta_L, \delta_L]$), the timing of investment depends on the probability that the innovation is profitable:
  - When $\alpha \in \left[\frac{1}{2}, \tilde{\alpha}\right]$, the infringer invests in period 1 if $K \leq K_{L,2}$ and the patentholder litigates in period 1. He delays investment until period 2 for $K \in [K_{L,2}, K_{L,2}]$ and the patentholder does not litigate. He does not invest if $K \geq K_{L,2}$.
  - When $\alpha \in \left[\tilde{\alpha}, \tilde{\alpha}\right]$, the infringer invests in period 1 if $K \leq K$ and the patentholder litigates in period 1. If $K \in [K_{L,2}, K_{L,2}]$, the infringer delays investment and the patentholder does not litigate. If $K > K$ and $K > K_{L,2}$, the infringer does not invest.
  - When $\alpha \in \left[\tilde{\alpha}, 1\right]$, the infringer invests in period 1 if $K \leq K_{L,1}$ and the patentholder litigates in period 1. He does not invest if $K > K_{L,1}$.

Appendix B

Appendix B.1: I show that $\frac{1}{2} \leq \hat{\alpha} \leq \tilde{\alpha} \leq \tilde{\alpha} \leq \tilde{\alpha}$.

I have: $\hat{\alpha} = \frac{\pi}{2(\pi-c)}$, $\tilde{\alpha} = \frac{\pi}{2\pi-3c}$, $\tilde{\alpha} = \frac{\pi+3c}{2\pi}$ and $\tilde{\alpha} = \frac{\pi+c}{2(\pi-c)}$.

Notice that $\frac{1}{2} \leq \hat{\alpha}$ if and only if $\pi \geq \pi - c$ which holds. Then, $\hat{\alpha} \leq \tilde{\alpha}$ if and only if $2(\pi - c) \geq 2\pi - 3c$ which holds as well. And $\tilde{\alpha} \leq \tilde{\alpha}$ if and
only if $2\pi^2 \leq (\pi + 2c)(2\pi - 3c)$ which is equivalent to $\pi \geq 6c$ which holds by assumption. Finally, $\tilde{\alpha} \leq \tilde{\alpha}$ if and only if $(\pi + 2c)2(\pi - c) \leq (\pi + c)2\pi$ which is equivalent to $-2c^2 \leq 0$. This clearly holds.

I can conclude that $\frac{1}{2} \leq \tilde{\alpha} \leq \tilde{\alpha} \leq \tilde{\alpha} \leq \tilde{\alpha}$. QED.

Appendix B.2. I conduct the comparative analysis between the laches and the no laches regime, for each of the following intervals: $\alpha \in \left[\tilde{\alpha}, \tilde{\alpha}\right]$, $\alpha \in \left[\tilde{\alpha}, \tilde{\alpha}\right]$, $\alpha \in \left[\tilde{\alpha}, 1\right]$ and $\alpha \in \left[1, \tilde{\alpha}\right]$. I proceed with a graphical comparison, as in section 5.1.

When $\alpha \in \left[\tilde{\alpha}, \tilde{\alpha}\right]$, the relevant graphics to compare are N2 (for the "no laches regime) and L1 (for the laches regime). I superpose these two graphics to obtain the following figure. As in Figure 4, the solid lines represent boundaries between the different equilibria in the no laches regime. The dotted lines represent the boundaries in the (new) laches regime. The capital letters represent parameters configurations that are affected by a switch to the laches regime. Notice that all these configurations $(I, I', M, P, J)$ have been encountered when analyzing the case $\alpha \in \left[\frac{1}{2}, \tilde{\alpha}\right]$. Hence, there is no additional insight when $\alpha \in \left[\tilde{\alpha}, \tilde{\alpha}\right]$. 

![Figure 4](image-url)
I repeat this analysis for the case where $\alpha \in [\hat{\alpha}, \tilde{\alpha}]$. The relevant graphics to compare are now N3 and L1. As shown in the following figure, the parameters configurations affected by the change of regime $(I, I', M, P, J)$ have been encountered in the case $\alpha \in \left[\frac{1}{2}, \tilde{\alpha}\right]$

![Diagram](image)

When $\alpha \in \left[\hat{\alpha}, \tilde{\alpha}\right]$ I compare N3 and L2. All the parameters configurations affected by the change of regime $(I, I', M, P, J)$ have been analyzed before:
When $\alpha \in [\tilde{\alpha}, 1]$ I compare N3 and L3. Again, all parameters configurations $(I, I', M, P)$ have been encountered before:

Hence, focusing only on $\alpha \in \left[\frac{1}{2}, \tilde{\alpha}\right]$ entails no loss of generality.
References


