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Quality Competition and Social Welfare in Markets with Partial Coverage: New Results

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Abstract

We use a vertical product differentiation model under partial market coverage to study the social welfare optimum and duopoly equilibrium when convex costs of quality provision are either fixed or variable in terms of production. We show that, under fixed costs, at the social welfare optimum only one quality variant of the good is provided, while both variants are optimal under variable costs. In the duopoly equilibrium the quality spread is too wide under variable costs, but too narrow under fixed costs, relative to the social optimum. Finally, in both the fixed and variable cost cases, average quality provided by the duopoly equilibrium is too low from the perspective of a social welfare maximizer.

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1. Introduction

A well-established result in vertical product differentiation models is that a duopoly consisting of high and low quality firms leads to product quality dispersion that is too high and average levels of quality that are too low, relative to the socially optimal outcome (see Crampes and Hollander 1995). This result has been established under the assumption that markets are fully covered, i.e., that all consumers purchase positive quantities of the good in question. A consequence of full market coverage is that, even though duopoly qualities differ from the socially optimal ones, the quantities produced by the firms are always equivalent.

The case of a partially covered duopoly is more appealing, in that it allows for some consumers who do not purchase from either firm but could potentially enter the market. In this case, if the duopoly and socially optimal outcomes differ, then not only the qualities but also the quantities differ. In the voluminous literature on partial market coverage, the social welfare outcome has mainly remained an open issue.

Our work fills an important gap in this literature. We characterize the properties of socially optimal qualities and solve for the divergence between duopoly and social outcomes when a market is partially covered. Unlike other work, we analyze and compare both cases of fixed and variable costs of production.[1] For variable costs we characterize the social optimum in the same way that Crampes and Hollander (1995) do for the fully covered market case. Ecchia et al. (2002) have argued that, under fixed costs, it is optimal to provide just one quality level. However, they do not study the problem of socially optimal price setting, nor do they show how the fixed cost case may differ from the variable cost case. Motta (1993) allows for both variable and fixed costs with partial market coverage but does not explicitly solve for the socially optimal outcome. Instead he uses numerical illustrations to compare consumer surpluses in the different equilibria.

Like most existing product differentiation models, we assume consumers derive utility from observed quality of products. Indeed, we retain this assumption because we seek to provide closure on the modeling of one class of product differentiation models under the assumption of partial market coverage. Also like other work, our duopoly outcome is solved as the subgame perfect equilibrium of a two-stage game. Firms maximize profits by first competing in qualities and then competing in prices.

Our characterization of the socially optimal outcome provides several new findings. Unlike in the duopoly equilibrium, it is socially optimal in the fixed cost case to provide only the high quality variant of a good, while the profit maximizing duopoly provides two variants. Moreover, provision of the socially optimal quality level is higher than the high quality variant provided by the market and, therefore, average quality provided by the market is lower than the socially optimal average quality. The social planner in this case is free to charge either a zero or positive price, and to service either some or all of the consumers in the market.

Under the assumption of variable costs the high quality firm in a duopoly will have higher profits but lower market share than the low quality firm. As a new result we show that the spread of product quality observed under a profit maximizing duopoly is too high relative to the socially optimal outcome. At the socially optimal outcome, both firms produce the same amounts, and total output is greater than with the duopoly outcome.

At the social optimum the major difference between fixed and variable cost cases is that, under fixed costs, only one product variant is provided, while both variants are optimal under variable costs. In duopoly equilibrium the quality spread is too wide under variable costs, while it is too narrow under fixed costs. Average product quality in both cases is too low compared to the socially optimal equilibrium.

We proceed as follows. Section 2 presents a basic duopoly model and the profit maximizing solution with partial market coverage for both fixed and variable costs of production. In Section 3 we compare the socially optimal and profit-maximizing qualities. Finally, we provide a brief conclusion.

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[1] Under the assumption of fixed costs, Ronnen (1991) considers minimum quality standards without analyzing the socially optimal quality provision. Lambertini (1996), in turn, considers the variable cost case but does not examine the socially optimal outcome.

2. A Duopoly Model of Vertical Product Differentiation with Partial Market Coverage

Under an assumption of partial market coverage, each consumer is typically assumed to purchase either one unit of the good or nothing. Let a consumer have a utility function u (see Tirole 1988, pp 96-97, 296-298),

$$u = q\ s_k - p_k, \tag{1}$$

where s_k and p_k are the quality and price of the kth good.[2] In (1), q represents the consumer's taste parameter, so that the consumer derives a surplus equal to $q\ s_k - p_k$ from a good of quality s_k and price p_k. Assume there are two possible qualities of goods produced by two types of firms, k = H (high quality) and k = L (low quality). A standard assumption is that the consumers' taste parameters are uniformally distributed over qualities on a definite interval, $q \in [\underline{q}, \overline{q}]$ (see e.g. Motta 1993, or Cremer and Thisse 1999).

We assume that the high and low quality firms have quadratic and convex cost functions for providing quality,

$$c_k(s_k) = \frac{1}{2} b s_k^2 \quad \text{for k} = H, L. \tag{2}$$

Because consumers can purchase either one unit or nothing, the consumer who is indifferent between high and low quality goods has a threshold taste parameter defined by $\hat{q} = \dfrac{p_H - p_L}{s_H - s_L}$. Under partial market coverage, some consumers do not enter the market. More specifically, the lowest marginal willingness to pay value can be defined for the consumer who is indifferent between buying and *not buying* the good, i.e.,

[2] Throughout the paper, derivatives of functions with one argument will be denoted by primes, while partial derivatives will be denoted by subscripts of functions with many arguments.

$q^c = \frac{p_L}{s_L}$. Recalling the uniform distribution of consumer types, the demands for high and low quality products then become $q_H = \bar{q} - \hat{q}$ and $q_L = \hat{q} - q^c$, where q_H and q_L are the number of consumers purchasing from the low and high quality firm, respectively.

Based on the above assumptions, we will focus on cases where the costs of providing quality are either fixed or variable with respect to output. The assumption of fixed costs has been widely applied in the literature. Kuhn (2000) recently argued that the variable cost case might be more appealing than the fixed cost case, because it avoids an implausible feature of fixed costs. This is that the high quality firm has both higher profits and a larger market share in equilibrium.[3] In conformity with observations from practice, our variable cost case results in an equilibrium where the profits of the high quality firm are higher that those of the low quality firm. However, the market share of the high quality firm is lower than the low quality firm.

2.1 Price and Quality Games: Fixed costs

The analysis of duopoly competition under fixed costs of production was originally provided by Ronnen (1991). In what follows we develop the features of his model very briefly. When the cost of quality provision is fixed in terms of quantity produced, then given the demands q_k and the cost function in (2), the profit functions of the high and low quality firm are:

$$p_k = p_k q_k - c_k(s_k), \text{for } k = H, L. \qquad (3)$$

There are then two stages of the duopoly game: quality provision (stage 1), and price competition conditional on quality provided (stage 2). Firms move simultaneously in each stage.[4] We can solve for the subgame perfect equilibrium of this game. This equilibrium relies, as usual, on commitment by firms in terms of quality. In the second

[3] This result was originally discovered by Lehmann-Grube (1997). He also showed that it holds irrespective of whether the firms choose their qualities simultaneously or sequentially.

stage, firms choose prices given the costs of quality production. From the first-order conditions, $\partial p^{H} / \partial p_{H} = 0$ and $\partial p^{L} / \partial p_{L} = 0$, we can solve for the optimal prices and their difference as follows,

$$p_H^* = \frac{2s_H(s_H - s_L)\bar{q}}{4s_H - s_L}\ ;\ \ p_L^* = \frac{s_L(s_H - s_L)\bar{q}}{4s_H - s_L}\ ;\ \ p_H^* - p_L^* = \frac{(2s_H - s_L)(s_H - s_L)\bar{q}}{4s_H - s_L} \qquad (4)$$

Thus, duopoly prices depend on the quality differences and the upper bound of the consumer taste distribution. The lower bound of the taste distribution does not matter here, because in partially covered markets the lowest critical value of marginal willingness to pay is endogenous.

Inserting the above prices into the respective profit functions yields the indirect profit functions for each firm's choice of quality,

$$p_H^* = \frac{4s_H^2(s_H - s_L)\bar{q}^2}{(-4s_H + s_L)^2} - \frac{1}{2}bs_H^2; \qquad p_L^* = \frac{s_H(s_H - s_L)s_L\bar{q}^2}{(-4s_H + s_L)^2} - \frac{1}{2}bs_L^2. \qquad (5)$$

Differentiating equations in (5) with respect to qualities gives,

$$\frac{\partial p_H}{\partial s_H} = \frac{32\bar{q}^2 s_H^2(s_H - s_L)}{(-4s_H + s_L)^3} + \frac{4\bar{q}^2 s_H^2 + 8\bar{q}^2 s_H(s_H - s_L)}{(-4s_H + s_L)^2} - bs_H = 0 \qquad (6a)$$

$$\frac{\partial p_L}{\partial s_L} = \frac{2\bar{q}^2 s_H s_L(s_H - s_L)}{(-4s_H + s_L)^3} + \frac{\bar{q}^2 s_H(s_H - s_L) - \bar{q}^2 s_H s_L}{(-4s_H + s_L)^2} - bs_L = 0 \qquad (6b)$$

Solving these first-order conditions for high and low quality and their difference with *Mathematica* yields,

$$s_H^* = \frac{0.2533111\bar{q}^2}{b}; s_L^* = \frac{0.0482383\bar{q}^2}{b}; s_H^* - s_L^* = \frac{0.2050727\bar{q}^2}{b}. \qquad (7)$$

[4] In fact, Lambertini (1996) has shown that the simultaneous move game is the only pure strategy equilibrium possible for a partial market coverage model with variable costs of producing quality.

Thus, the equilibrium duopoly qualities and the quality difference between firms depend positively on the square of the upper bound of taste distribution, $\bar{q}^2$, and negatively on the marginal cost parameter of quality provision b.

Using these optimal qualities, we can now solve for the prices and demands of both quality variants as a function of exogenous parameters: $p_H^* = \frac{0.107662\bar{q}^3}{b}$; $p_L^* = \frac{0.010251\bar{q}^3}{b}$; $q_H^* = 0.524994\bar{q}$; $q_L^* = 0.262497\bar{q}$. The overall demand, which indicates the resulting coverage in the market, is therefore given by $q_H^* + q_L^* = 0.787491\bar{q}$. If we now normalize $\bar{q} = 1$ (and $\underline{q} = 0$), then we can conclude that about 79% of consumers enter the market and buy one of the two quality variants. Because the high quality firm charges a higher price and faces a larger demand, it has higher profits and greater market share than the low quality firm. This can also be seen from the profit solutions for high and low quality firms, $\pi_H = \frac{0.0244386\bar{q}^4}{b}$ and $\pi_L = \frac{0.00152741\bar{q}^4}{b}$. As we shall see, this result must be modified for the case of variable costs of production.

2.2 Price and Quality Games: Variable costs

Next we assume the costs of providing quality are variable in terms of output. Under this assumption, and given the demands q_k and the cost function in (2), the profit functions for each firm are written,

$$\pi_k = [p_k - c_k(s_k)]q_k \text{, for } k = H, L. \tag{8}$$

As before, in the second stage firms choose prices given the costs of quality production. From the first-order conditions, $\partial\pi^H/\partial p_H = 0$ and $\partial\pi^L/\partial p_L = 0$, we can solve for optimal prices,

$$p_H^* = \frac{\left[2\bar{q}s_H(s_H - s_L) + bs_H(s_H^2 + (1/2)s_L^2)\right]}{4s_H - s_L} \tag{9a}$$

$$p_L^* = \frac{\left[\bar{q}s_L(s_H - s_L) + bs_H((1/2)s_H^2 + s_L^2)\right]}{4s_H - s_L}. \tag{9b}$$

$$p_H^* - p_L^* = \frac{\bar{q}(2s_H - s_L)(s_H - s_L) + (1/2)bs_H(s_H^2 - s_L^2)}{4s_H - s_L} \tag{9c}$$

Again, duopoly prices and their difference depend on quality differences and on the upper bound of the consumer taste distribution.

Substituting these optimal prices into the profit functions, we can express indirect profits in terms of quality as,

$$p_H^* = \frac{s_H^2(s_H - s_L)\left[-4\bar{q} + b(2s_H^2 + s_L^2)\right]}{4(-4s_H^2 + s_L^2)^2} \tag{10a}$$

$$p_L^* = \frac{s_L s_H(s_H - s_L)\left[2\bar{q} + b(s_H - s_L)^2\right]}{4(-4s_H^2 + s_L^2)^2}. \tag{10b}$$

Optimal second stage qualities then follow from the first-order conditions,

$$\frac{\partial p_H^*}{\partial s_H} = 0 \Leftrightarrow \frac{\Omega\left[-4\bar{q}^2(4s_H^2 - 3s_H s_L + 2s_L^2) - b(24s_H^3 - 22s_H^2 s_L + 5s_H s_L^2 - 2s_L^3)\right]}{4(4s_H - s_L)^3} = 0 \tag{11a}$$

$$\frac{\partial p_L^*}{\partial s_L} = 0 \Leftrightarrow \frac{\Lambda\left[2\bar{q}^2 s_H(4s_H - 7s_L) + b(s_H - s_L)(4s_H^2 - 15s_H s_L + 2s_L^2)\right]}{4(4s_H - s_L)^3} = 0, \tag{11b}$$

where $\Omega = s_H\left[-4\bar{q} + b(2s_H + s_L)\right]$ and $\Lambda = s_H\left[2\bar{q}^2 + b(s_H + s_L)\right]$.

Given the complexity of the first-order conditions, solving for the actual equilibrium qualities is a bit laborious. Without loss of generality we define $s_H = ds_L$ for some $d > 1$, where d indicates the degree of product differentiation between firms expressed in terms of the quality spread between high and low quality firms. Note that this assumption does not predetermine the results presented later concerning differences between socially optimal and duopoly outcomes. It simply implies that the high quality firm produces higher quality than the low quality firm, which is always the case in these models.

Using $s_H = ds_L$ and solving (11a) - (11b) with *Mathematica*, we obtain the following equilibrium qualities and their difference,

$$s_H^* = \frac{0.8195\bar{q}^2}{b}; \qquad s_L^* = \frac{0.3987\bar{q}^2}{b}, \; s_H^* - s_L^* = \frac{0.4208\bar{q}^2}{b}. \tag{12}$$

Equilibrium duopoly qualities and the degree of quality differentiation are positive functions of the upper bound of the square of the taste distribution, $\bar{q}^2$, and a negative function of the marginal cost parameter of quality provision, b. This result is qualitatively similar to those found in full market coverage models. Note also that the quality difference is higher with variable costs compared to the fixed cost case.

This last finding can be interpreted as follows. Under fixed costs, the costs of producing both quality variants of the good in the second stage are zero (even though the costs of providing quality differ), but they are strictly positive under variable costs. Thus under variable costs of production, quality competition between the firms is tighter because the firms obtain greater rents from differentiating compared to the fixed cost case.

Finally, using the optimal qualities above, we can solve the previous first-order conditions for equilibrium prices and demands: $p_H^* = \frac{0.453313\bar{q}^2}{b}$; $p_L^* = \frac{0.15002\bar{q}^2}{b}$; $q_H^* = 0.279245\bar{q}$; $q_L^* = 0.344503\bar{q}$. Interestingly, for our case of variable costs of production, we find that the high quality firm has higher profits but lower market share than the low quality firm. The overall demand (i.e., coverage) in the market is given by $q_H^* + q_L^* = 0.623748\bar{q}$. Thus, under variable costs, overall market coverage is *smaller* than in the case of fixed costs. This is a natural result since production costs are now positive and the quality spread is wider, which serves to relax price competition between firms and allows the firms to charge higher prices. The firms' indirect profit functions can now be solved to obtain $\tilde{p}^H = \frac{0.0328129\bar{q}^3}{b}$; $\tilde{p}^L = \frac{0.024298\bar{q}^3}{b}$.

3. Socially Optimal versus Profit-Maximizing Quality Decisions

Now we turn to the main part of our paper, i.e., the determination of the socially optimal qualities and their relationship with the equilibrium duopoly qualities under both assumptions of fixed and variable cost of production. The socially optimal levels of quality are those that maximize a social welfare function, which is the sum of surplus to consumers net of costs to produce high and low quality goods,

$$SW = \int_{\hat{q}}^{\bar{q}} (qs_H - \frac{1}{2}bs_H^2)dq \quad + \quad \int_{q^c}^{\hat{q}} (qs_L - \frac{1}{2}bs_L^2)dq \, . \tag{13}$$

3.1 Fixed Costs and the Socially Optimal Qualities

We start by analyzing the properties of the first-best solution under fixed costs. The social planner simultaneously chooses prices and qualities to maximize (13). The planner accounts for the critical taste parameter separating consumers of high and low quality variants, while keeping it open whether it is socially optimal to serve the whole market or not. Thus, the planner uses the following critical values of the taste parameter

$$\hat{q} = \frac{p_H - p_L}{s_H - s_L}; \quad q^c = \underline{q} \, . \tag{14}$$

Differentiating first the social welfare function (13) with respect to high and low quality prices gives $p_H = p_L$.[5] Using this in the social welfare function and differentiating it with respect to high and low qualities yields,

$$SW_{s_H} = \frac{1}{2}(\bar{q}^2 - 2bs_H) = 0, \quad SW_{s_L} = \frac{1}{2}(-\underline{q}^2 - 2bs_L) = 0. \tag{15}$$

Solving for optimal high quality yields $s_H^w = \frac{\bar{q}^2}{2b}$. Note however that $SW_{s_L} < 0$, implying that production of low quality variant is zero. Thus, it is socially optimal to provide just one quality variant (high quality), $s^w = \frac{\bar{q}^2}{2b}$, as pointed out by Ecchia et al. (2002).

Consider now the relationships between socially optimal qualities and duopoly qualities (which has been characterized in equation 7),

$$s_H^* - s_H^w = -\frac{0.246689\bar{q}^2}{b} < 0 \tag{16a}$$

$$s_L^* - s_L^w = \frac{0.048238\bar{q}^2}{b} > 0 \tag{16b}$$

$$s_a^* - s_a^w = -\frac{0.24669\bar{q}^2}{b} < 0, \tag{16c}$$

where the subscript 'a' refers to average quality. Clearly, a duopoly provides too little high quality and too much low quality goods. This implies that profit maximization results in a quality dispersion that is socially sub-optimal. Moreover, the average quality provided by the market is too low from the social planner's perspective. We summarize these findings in:

[5] The first-order conditions for the prices of the high and low variants are $SW_{p_H} = -\frac{p_H}{s_H - s_L} + \frac{p_L}{s_H - s_L} = 0$, $SW_{p_L} = \frac{p_H}{s_H - s_L} - \frac{p_L}{s_H - s_L} = 0$.

Proposition 1: Under fixed costs of production, the socially optimal outcome involves production of only the high quality variant. Compared to the socially optimal outcome, the profit maximizing duopoly provides too little high quality and too much low quality.

Using the socially optimal quality, we can also solve for the socially optimal price. Inserting optimal qualities into the first-order conditions for prices would imply that the optimal price is zero, because, when quality is given, the cost of commodity production is zero. There are several possible ways to solve for the optimal prices. First, the social planner could offer the high quality commodity to consumers at a zero price, given that investment in quality is independent of the commodity's price level, and investment as such represents a sunk cost. The second way is to assume that society charges a positive price such that either some subset or all of consumers purchase the commodity. The latter price can be determined from the indifference relation between buying and not buying for the consumer having the lowest preference for quality. By inserting the socially optimal quality into this indifference relationship, we have $\underline{q}s^{sw} - p = 0$. Recall that we solved for the socially optimal quality such that $s^w = \frac{\bar{q}^2}{2b}$. Using this yields the following socially optimal price, $p^w = \frac{\underline{q}\bar{q}^2}{2b}$. Under this price, demand for the commodity is simply given by $q^{swf} = \bar{q} - \underline{q}$, so that relative to the duopoly equilibrium, the socially optimal solution with this pricing strategy yields 21% higher demand for the good (see our earlier analysis of duopoly in Section 2.1).

3.2 Variable Costs and the Socially Optimal Qualities

Next we compare the equilibrium duopoly solution with the socially optimal one in the case of variable costs of production. Unlike with fixed costs of production, the assumption of variable production costs allows the social welfare maximizer to offer products at a nonzero marginal cost. Therefore, replacing duopoly prices by the marginal costs of quality provision in the critical taste parameters $\hat{q}$ and q^c, we can define new threshold critical taste parameters for the upper and lower bounds of the taste distribution,

$$\hat{q} = \frac{1}{2} b(s_H + s_L); \qquad q^C = \frac{1}{2} b s_L. \tag{20}$$

Using equation (20) and differentiating the social welfare function (13) with respect to the qualities s_H and s_L then gives the following first-order conditions,

$$SW_{s_H} = 0 \Leftrightarrow \left[\frac{\bar{q}^2}{2} - \bar{q} b s_H \right] = \left[\frac{\hat{q}^2}{2} - \hat{q} b s_H \right], \tag{21a}$$

$$SW_{s_L} = 0 \Leftrightarrow \left[\frac{\hat{q}^2}{2} - \hat{q} b s_L \right] = \left[\frac{q^{C^2}}{2} - q^C b s_L \right]. \tag{21b}$$

The socially optimal qualities can then be solved from (21a) and (21b) to obtain,

$$s_H^w = \frac{4\bar{q}^2}{5b}, \qquad s_L^w = \frac{2\bar{q}^2}{5b} \qquad \text{and} \quad s_H^w - s_L^w = \frac{2\bar{q}^2}{5b}. \tag{22}$$

Like the quality difference in the profit maximizing duopoly case, the socially optimal quality difference depends positively on the square upper bound of the taste distribution, $\bar{q}$ and negatively on the marginal cost parameter of quality provision b.

As for the relationship between socially optimal qualities in the duopoly and social welfare maximization cases, we obtain (using equation 12),

$$s_H^* - s_H^w = \frac{0.0195\bar{q}^2}{b} > 0, \tag{23a}$$

$$s_L^* - s_L^w = -\frac{0.087\bar{q}^2}{b} < 0. \tag{23b}$$

The magnitude of these expressions depends on the size of the squared upper bound of the taste distribution, which indicates how many consumers can potentially be captured by differentiating product qualities. Unlike in the case of fixed costs, the profit-maximizing duopoly produces too much high quality and too little low quality than would the social planner. Further, if we compare quality differences across the outcomes, we see that profit maximization gives a quality dispersion that is too wide, i.e.,

$$\left(s_H^w - s_L^w\right) \; - \; (s_H^* - s_L^*) = -\frac{0.140\bar{q}^2}{5b} < 0 \quad . \qquad (23c)$$

This implies that, in order to relax price competition, firms will behave in a manner that increases the spread of quality dispersion too much by maximizing profits. Such behavior decreases social welfare. We can summarize these findings in:

Proposition 2: Under variable costs of production, the socially optimal outcome involves provision of both high and low quality variants. Compared to the socially optimal outcome, the profit maximizing duopoly provides too much high quality and too little low quality.

Next, we solve the demands for qualities in the socially optimal outcome. Using (12) in (9a) and (9b) and accounting for the definition of demands yields $q_H^w = q_L^w = 0.4\bar{q}$. Hence, the high and low quality firms will have demand of equal size at the social welfare optimum. The difference between total demand in the socially optimal and duopoly outcomes are given by: $(q_H^w + q_L^w) - (q_H^* + q_L^*) = 0.176252\bar{q} > 0$, implying that production of each variety and market coverage are both too small under the duopoly. Intuitively, the duopoly restricts production in order to charge prices higher than marginal production costs. This is strikingly different from the well-known result derived in fully covered markets, which states that the size of the economy's production of quality is equal under duopoly and socially optimal outcomes. Our new finding may have important policy implications for achieving efficient levels of quality in markets.

Finally, it is interesting to compare the features of socially optimal quality provision under fixed and variables costs. The major difference between these two cases is as follows: under fixed costs we showed that only one quality variant of the good is provided. Both variants are optimal under variable costs, and we can show that provision of the high quality variant is greater than in the case of fixed production costs. In the duopoly equilibrium, we also showed that the quality spread is too wide under variable costs, but it is too narrow under fixed costs of production. In both the fixed and variable cost cases, average quality is too low from the perspective of the social welfare maximizer.

4. Conclusion

We have used a vertical product differentiation model under the assumption of partial market coverage to characterize the social welfare and profit maximizing duopoly outcomes, in terms of quality provision and quantities produced. We consider these under both variable and fixed convex costs of production.

Under an assumption of fixed costs, the high quality firm has higher profits and greater market share than the low quality firm. Unlike in the duopoly equilibrium, however, we demonstrated, as has been pointed out in Ecchia et al. (2002), that it is socially optimal to provide the high quality commodity and set low quality production to zero at the social optimum. We also demonstrated that production of high quality at the social optimum is higher than that provided by the market, and therefore the average quality provided by the market is lower than at the social optimum. As production is costless and investment in quality is lump sum, the planner may be free to set a zero or positive price, and to serve either all or part of the market.

Under an assumption of variable costs, we also find a new result that the spread of product quality in the profit maximization duopoly outcome is too high relative to the social welfare maximizing outcome. At the social welfare optimum, overall output is greater than the output produced under the duopoly.

At the social optimum there are two major differences between fixed cost and variable cost assumptions. First, under fixed costs only one variant of quality is provided, while both variants are optimal under variable costs. Second, provision of high quality is greater under variable costs of production. Comparing social welfare maximizing and duopoly outcomes, we also show that the product differentiation spread is too wide under a variable cost assumption, but it is too narrow under fixed costs. Average quality in both variable and fixed cost duopoly cases is too low from a social planner's perspective.

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