Cumulative Innovations: Intellectual Property Regimes and Incentives to Innovate*

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Abstract

When innovations are both sequential and complementary as in the software or the semi-conductor industries, James Bessen and Eric Maskin (2002) argue that patents are likely to reduce firms' incentives to innovate as compared to a regime with no protection. We develop a model close to that of Bessen and Maskin except that we endogenize the probability of success of each innovation (firms choose their R&D investments, reflecting the incentives to innovate and determining the success probability of an R&D program) and introduce an explicit model of a copyright. Our main results contradict Bessen and Maskin: individual and aggregate R&D

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investments are higher under a patent regime or a copyright regime than with no protection. And the socially optimal R&D investment is always higher than the aggregate investment provided in a regime with no protection.

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1 Introduction

During the last twenty years, there has been a trend in the United States toward a strengthening of the patent system. Recent papers by Jaffe (1999), Gallini (2002) or Lerner (2003) acknowledge this evolution. Amongst the different pieces of evidence reflecting this reinforcement, the "expansion of the realm of patentability"[1] has been emphasized by many: there are now patents for gene sequences, financial formulas and computer sofwares, for example. Economists warned against the possible side-effects of this development. James Bessen and Eric Maskin (2002) (denoted BM in the remaining of this paper) contribute to the criticisms by arguing that in industries where innovations are cumulative and complementary, patents might be an impediment to innovation rather than an "engine" as they are traditionnally perceived. Innovations are cumulative when each innovation builds on the previous one. They are complementary in the sense of BM[2] because each firm takes a possibly different research path, which increases the overall probability that at least one firm will come up with the innovation. A patent on one innovation confers its holder a "hold-up" right over subsequent innovations, if performing the latters requires the right to use the former. Assuming no licensing or

[1] Jaffe (1999).

[2] "complementarity" deserves two types of definition. For BM: "by complementary, we mean that each potential innovator takes a different research line and thereby enhances the overall probability that a particular goal is reached within a given time". More commonly, innovations x and y are complementary when the production of x requires y and vice-versa. For example, if y is patented by another firm, the later can hold-up the manufacturer of x.Note that this definition also fits, to some extent, the problem pointed out by BM. Standard references (both theoretical and empirical) for this issue are Heller and Eisenberg (1998), Hall and Ham (1999), Shapiro (2001)or Lerner and Tirole (2002).

imperfect licensing as BM, only the patentholder will have the possibility (the right) to engage in R&D for further innovations. This can restrict the number of firms performing research and the aggregate R&D investment is constrained. Were a patent absent, each innovation could be imitated legally and used for next researches. The (static) disincentives associated with the loss of the patent for a successful firm could be more than compensated by the (dynamic) gains associated with the prospects of being always in the race, being allowed to imitate a winner, and being able to become an innovator. BM´s analysis falls into two parts: they first propose a theoretical model that supports this view, in which they stress the merit of an IP regime with no legal protection as compared to a regime with patents. Then, they conduct an empirical investigation of the transition from an IP regime with copyrights towards a regime with more patents in the US software industry during the late 1980's. They show, in particular, that this trend[3] has generated a decrease of R&D investment at the firm level.

Though BM have had an important influence in terms of policy making, we argue that their theoretical analysis has at least two flaws that lead to a bias in their conclusions. First, they take the absence of intellectual property (IP) protection as a "proxy" for copyright protection, since a copyright protects less against imitation than a patent. We argue that "no protection" and "copyright" can be analyzed as two different regimes of

IP protection.Thus, we explicitely introduce a stylized model of IP rights where patents, copyright and the absence of protection can be distinguished. Second, BM's theoretical model focus on a firm as a decision maker subject to a binary decision problem: given that another firm already performs R&D, given an exogenous R&D cost and an exogenous probability of success, a second firm has to decide whether or not to also perform R&D. We endogenize the probability of success of an R&D program: firms choose non-cooperatively their R&D investment in each of the three IP regimes considered and that determine their probability of success.

When innovations are cumulative, we reach in particular three important conclusions:

[3]This transition came from a series of Court decisions that extended patent protection to software ideas. In the case *Diamond v. Diehr* (1981), the US Supreme Court recognized the patentability of softwares in their function to control an industrial process.

per-period individual R&D investments (at the firm level) are always strictly higher under a patent regime than in a regime without protection. The expected aggregate R&D investment is always strictly higher under a patent regime than in a regime without protection if the firms are symmetric in R&D efficiency. The socially optimal expected R&D investment is always strictly higher than the expected aggregate investment in a regime with no protection. These three results contradict BM. Besides, we show that if the firms are asymmetric, and if the most efficient firm obtains a patent on the first innovation, then the expected aggregate R&D investment in subsequent periods is higher in this patent regime than in a regime with no protection.

The literature on cumulative innovations has grown extensively in the last decade (O'Donoghue (1998) and O´Donoghue, Scotchmer and Thisse (1998) are two examples). Yet, we depart from these models and deliberately remain as close as possible to the model of BM, except for the two modifications previously noted: this allows us to test more accurately the robustness of their analysis. We nevertheless contribute to the literature on IP protection and cumulative innovations by introducing the copyright as a distinct form of protection.

Section 2 presents the main assumptions of our model. Section 3 develops a static model of R&D competition under three IP regimes (patent, copyright andno protection). Section 4 develops the dynamic situation and our main conclusions. Section 5 concludes.

2 The Assumptions of the Model

We consider two firms (players) performing R&D. They differ from each other with respect to their cost of R&D, so that firm 1 is more efficient (has a lower marginal cost) than firm 2. Formally, the cost function is given by $c(x_i) = \alpha_i x_i$ for $i = 1, 2$ and $\alpha_1 = 1$, $\alpha_2 = \alpha > 1$ where x_i is the level of R&D chosen by firm i. This specification allows us to deal with the symmetric situation when necessary ($\alpha = 1$).The probability of success of an R&D program depends on the level of investment and is given by $p(x_i) = 1 - e^{-x_i}$.This functional form[4] is such that $\frac{\partial p(x_i)}{\partial x_i} > 0$ and $\frac{\partial^2 p(x_i)}{\partial x_i^2} < 0$.

For each player, there is a payoff associated with the outcomes of the game (one winner and one looser, two winners or two loosers) and depending on the IP regime that prevails. In the game played in one period, we assume that the value of the innovation for the society is v. This is also the maximal payoff that a successful player can obtain. Our definition of each IP regimes (with patents, with copyrights and without protection) is mainly based on the appropriability of the value v by each player.

In an IP regime with **patents**, we assume that the patent is sufficiently broad to preclude from *any* imitation in a given period. In other words, the scope[5] of the patent is always maximal. In the static case, if only one firm makes the invention, it gets v with probability one. If both succeed, each firm gets the patent with probability $\frac{1}{2}$ (coin flipping assumption).

Copyright is modeled as a less stringent form of protection. This is consistent with the respective features of a patent and a copyright: A copyright protects the expression, not the idea, whereas a patent protects both. To make that distinction clear, a copyright protects a product (or a process) *per se* but allows for a competitive product based on the same idea provided the expression of the idea differs even slightly from the original product. For example, one could think of two varieties of the same good[6]. In order to emphasize the difference between a copyright and a patent, we assume that the patent has a maximal scope (imitation is never legal and never occurs) and that the copyright is only a limited protection against imitation. We formalize copyright protection in a

[4] See Kultti (2003) for a justification of this form in the case of R&D.

[5] A patent confers its owner protection (through the form of a temporary monopoly power) against imitators (backward protection) and against future innovators (forward protection). See O'Donoghue (1998), Denicolo (2000) as well as Merges and Nelson (1990) and Jaffe (1999). In this paper, by "scope" we refer to "backward protection". Section 4 addresses *only partially* the issue of forward protection. As will be seen, following BM, a forward innovation is incremental and does not threatens the profit from a previous innovation.

[6] This is suggested by Waterson (1990). A more structured model of copyright protection should include several varieties of the same good in a general equilibrium framework.

stylized way: if two players succeed they get a duopoly profit $\frac{v}{2}$[7]. However, if only one player succeeds (the winner), his payoff is given by

$$\pi^w(\theta) = \frac{1}{2}(1+\theta)v \tag{1}$$

with $0 < \theta \leq 1$, whereas the payoff of the looser is

$$\pi^l(\theta) = \frac{1}{2}(1-\theta)v \tag{2}$$

These forms come from the fact that a copyright allows for imitation provided the imitation is not an *exact* copy of this invention (hence the condition $0 < \theta$). From the comparison of (1)and (2), given that $0 < \theta \leq 1$,it is clear that the innovator has always a larger payoff than the imitator:

$$\pi^w(\theta) > \pi^l(\theta) \tag{3}$$

This is a rather strong assumption since an imitator could come up with a version (or a variety) that consumers prefer and reap more profit than the original inventor. Yet, it is also sensible to assume that the original inventor posseses an advantage over the imitator: one could think of the original product as a higher-quality variety than the imitation which could justify the R&D race. In this paper, we stick to this assumption[8]. Notice also that $\pi^w(\theta) > \frac{v}{2} > \pi^l(\theta)$: in a regime with copyright, the winner is better off than in a regime with no protection (that is when $\theta = 0$), but the looser is worse-off. Finally, $\frac{\partial \pi^w(\theta)}{\partial \theta} > 0$ and $\frac{\partial \pi^l(\theta)}{\partial \theta} < 0$: the winner is better-off when the imitation under copyright becomes less profitable, that is when θ increases, whereas the looser becomes worse-off. Parameter θ is clearly taken as an exogenous measure of the "scope"of the copyright

[7]This is also assumed as the distinctive feature of copyright protection, for example, in the general equilibrium framework of Kultti and Takalo (2000). Yet, the way we introduce the copyright regime differs from this model: They assume that the main difference between a copyright and a patent regime comes from the number n of the independent identical innovations allowed to receive protection ($n = 1$ in a patent regime and $n > 1$ in a copyright regime), for the *same* scope of protection. Here, we claim that the regimes differ first and foremost in terms of the scope of protection and that, besides, a copyright regime allows for simultaneous identical independent innovations.

[8]This is also a feature of the patent model from Klemperer (1990): imitators come up with product varieties with a lower quality than the variety of the innovator.

protection. Note that the "scope" of a copyright has not exactly the same definition as the "scope" of a patent. The "patent scope" is usually defined by the characteristics of a product that cannot be infringed upon. This "protected area" is controled both by the decisions of the Patent Office and, in case of litigation, by the decisions and the doctrines of the Court of Appeals for the Federal Circuit (CAFC). Thus, the scope of a patent is submitted to a large extent to discretion. In the case of copyright, the only prohibited action is the pure copy which is easily identified and punished[9]. An imitation is legally accepted whenever it is not the reproduction of the original expression and thus the "scope", from a legal view point, is almost zero[10]. Nevertheless, it makes sense to speak of θ as a scope parameter provided it is perceived as the exogenously given, industry-specific, cost of imitation. The higher this cost (the higher θ), the higher the payoff of the winner. It is sensible to assume that θ varies accross industries protected by copyrights: it is perhaps easier to come up with another version of an original book than to produce a new version of a software program designed to compete with the original one.

Finally, the crucial distinctive feature of the copyright, in relation to having a lower scope than a patent, is that it prevents from any "hold-up" by its holder. This is supported by the recent "jurisprudence" in the United States. In the famous case between Apple Computer versus Microsoft and Hewlett-Packard, a ruling was made in favor of the latters against Apple who argued that Microsoft Window´s program and HP´s New Wave software had "copied the "look and feel" of Macintosh's graphic-based operating system"[11]. The Court clearly favored a strict interpretation of the copyright whereby the "idea" of a particular expression can be used for developing a different expression (another software in this case). Note however that this example contrasts with the trend observed

[9]Of course, this assumes that the copyright holder can identify and can engage in costly litigation. See Crampes and Langinier (2002) for a nice model of *patent* monitoring.

[10]See below (case *Lotus Development Corp. v. Borland International Inc.*) for a justification of this idea.

[11]This illustration is taken straight from the example provided, in another context, by Jay Pil Choi (1998), "Patent Litigation as an Information-Transmission Mechanism", *American Economic Review,* vol. 5, pp 1249-1263.

in the 80's that strengthened copyright protection for softwares[12]. A similar judgment by the US Supreme Court was pronounced in the case *Lotus Development Corp. v. Borland International Inc.*, 96 Daily Journal D.A.R. 495 (Jan.16, 1996): The Court let stand a judgment by the CAFC that denied infringement by Borland's spreadsheet program of the Lotus 1-2-3 program. Lotus claimed that the (acknowledged) introduction of Lotus´ menus command hierarchy in Borland's program was illegal. However, the CAFC stated against Lotus by refering to the rule governing copyright protection under title17 of the U.S.Code, Section 102(b):"[I]n no case does copyright protection for an original work of authorship extend to any idea, procedure, process, system, method of operation, concept, principle or discovery, regardless of the form in which it is described, explained, illustrated or embodied in such work" This is remarkably explained by Bunker (2002), who concludes: "(...) it seems clear that broad protection for software under the laws of copyright is dead. (...) copyright protects against copying (...) [but] provides no protection against independent creation. (...)[on the contrary], since a patent can protect an idea or a concept, the patent claims can be written in broad terms to cover the novel combination of elements". This evolution in the US jurisprudence enables us to state that, under a copyright regime, the idea of an innovation can be used to develop further innovations so that there is no hold-up problem (this feature being essential in our dynamic model).

Definition 1 *An IP regime with copyright is characterized by payoffs* $\frac{1}{2}v < \pi^w(\theta) \leq v$ *for the winner and* $0 \leq \pi^l(\theta) < \frac{1}{2}v$ *for the looser with* $\theta \in (0,1]$ *capturing the scope of the copyright in the case where only one firm innovates, and a payoff* $\frac{v}{2}$ *when both firms innovate.*

[12] It is important to distinguish two phenomena: first, the extent of the *patent* protection to softwares (initiated by *Diamond v. Diehr (1981)*, it has not been extented to all types of softwares) and the evolution of the *copyright* protection for softwares which shows two trends: In the 1980's, an extension of this protection (strong protection against imitation) and in the1990's a comeback to a strict application of the copyright law (weak protection against imitation). We base our definition of the copyright regime on this last trend.

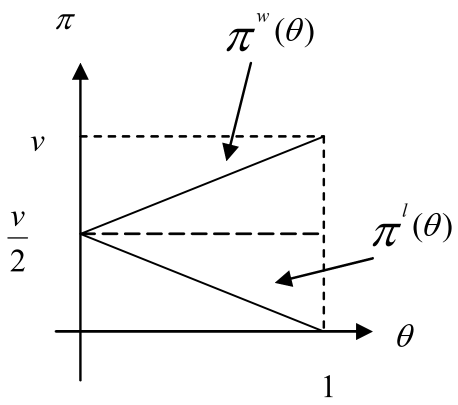

Figure 1: Payoffs of the winner and the looser of the R&D game under a copyright regime

Finally, a regime **without intellectual property protection** allows for legal copy of the innovation. To be consistent with our assumptions, we formally model this regime by assuming $\theta = 0$ in the functions $\pi^l(\theta)$ and $\pi^w(\theta)$. Thus, in this regime, provided at least one player succeeds, the payoff for each of them is $\frac{v}{2}$[13].

3 The Static Model

We call "static model" the situation where, in only one period, firms non-cooperatively choose their R&D investments x_i^l , $i = 1, 2$ and $l = c, p, n$ where c denotes the copyright regime, p denotes the patent regime and n the regime with no protection. The set of feasible actions x_i^l is thus $[0, +\infty)$. Our static model possesses an "equivalence property" between the IP regimes that does not hold anymore in the dynamic model. Indeed, it will be shown that an IP regime with copyright and $\theta = 1$ is formally equivalent to a patent regime whereas a regime with copyright and $\theta = 0$ is formally equivalent to a regime with no protection[14]. In the dynamic model, this property does not hold because of the "hold-up" problem that potentially restricts the number of firms allowed to perform R&D after

[13] Like in BM. This is a drastic assumption yet.

[14] We stress that this equivalence is only *formal*: a copyright with $\theta = 0$ would mean that pure copying is allowed. But that would contradict the very definition of copyright protection.

the first period. Because of the equivalence property, we start with the analysis of the copyright regime and we derive the main results under a patent regime and a regime with no protection as limit cases of the copyright regime. In order to emphasize the difference between the regimes, we assume in this section that the copyright regime is defined for $0 < \theta < 1$.To guarantee the existence of interior solutions, we assume throughout this section that $v > 2\alpha$.

3.1 IP regime with copyrights

Denoting V_i^c the objective function of player i in a regime with *c*opyrights:

$$V_i^c = -\alpha_i x_i^c + p(x_i^c)\left\{p(x_j^c)\frac{v}{2} + (1 - p(x_j^c))\pi^w(\theta)\right\} + (1 - p(x_i^c))p(x_j^c)\pi^l(\theta) \qquad (4)$$

where x_i^cdenotes the R&D investment of player i in the static model and $\alpha_1 = 1$, $\alpha_2 > 1$. $\pi^l(\theta)$ and $\pi^w(\theta)$ are given by (1) and (2).

The first term on the right-hand side is the level of R&D expenditures (the cost of the program). The second term captures the expected payoff of player i if she succeeds: either player j also succeeds in which case each earns the duopoly profit, or player j fails but can imitate and, according to (1), player i will get $\pi^w(\theta)$. The last term captures the expected payoff of player i if she fails: if player j has made the innovation, she has a possibility to imitate and according to (2), to earn $\pi^l(\theta)$.

The objective of player i is defined as:

$$\underset{x_i^c}{Max}\ V_i^c \qquad (5)$$

Lemma 1 *In an IP regime with copyright, in the static model, the equilibrium R&D investments are implicitely defined by the best-response functions:*

$$x_1^c = \ln v - \ln 2 + \ln(\theta + e^{-x_2^c}) \qquad (6)$$

$$x_2^c = \ln v - \ln 2\alpha + \ln(\theta + e^{-x_1^c}) \qquad (7)$$

and the efficient firm (firm 1) invests strictly more than the inefficient one.

Proof. Substituting for $p(x_1)$ and $p(x_2)$ and using $\alpha_1 = 1$, $\alpha_2 = \alpha$,the first-order conditions[15] associated with the program (5) are given by:

$$e^{-x_1^c}(\theta + e^{-x_2^c}) = \frac{2}{v} \tag{8}$$
$$e^{-x_2^c}(\theta + e^{-x_1^c}) = \frac{2\alpha}{v} \tag{9}$$

developing and using the logarithm yields (6)and (7).

To prove the second part of the lemma, substract the two FOC above. It yields:

$$e^{-x_1^c} - e^{-x_2^c} = \frac{2}{\theta v}(1-\alpha) \tag{10}$$

The right-hand side is strictly negative since $\alpha > 1$.It follows that $x_1^c > x_2^c$. ■

We look for a Nash equilibrium in R&D investments. It appears that we cannot derive closed-form solutions for an equilibrium. Yet, we can state the following lemma:

Lemma 2 *In an IP regime with copyright, the static non-cooperative R&D game has a unique Nash equilibrium in R&D investments* $(x_1^{c,NE}; x_2^{c,NE})$.

Proof. See the Appendix. ■

3.2 IP regime with patents

Denoting V_i^p the objective function of player i when a *p*atent is available:

$$V_i^p = -\alpha_i x_i^p + p(x_i^p)\left[p(x_j^p)\frac{1}{2}v + (1-p(x_j^p))v\right] \tag{11}$$

[15] The second order condition is given by $\frac{\partial^2 V_i^c}{\partial x_i^{c2}} = -e^{-x_i^c}\frac{v}{2}(1+\theta e^{-x_j^c}) + e^{-x_i^c}\frac{v}{2}(1-\theta+e^{-x_j^c}(\theta-1)) \leq 0$

The sign comes from the fact that $1-\theta+e^{-x_j^c}(\theta-1)$ is negative. Assume the contrary, then it must be that $e^{-x_j^c} > 1$ which is impossible for all $x_j^c \in [0, +\infty)$.

where x_i^p denotes the level of R&D investment of player i in the static model and $\alpha_1 = 1$, $\alpha_2 = \alpha > 1$.

The first term on the right-hand side is the cost of R&D. There are several possible outcomes: if player i does not succeed, because of the patent protection, she cannot imitate and her payoff is 0 . But if she succeeds (second term on the right-hand side), either player j has also succeeded in which case player i gets the patent with probability $\frac{1}{2}$, or player j has failed and player i gets the monopoly payoff, prize v.

It is clear that $V_i^p = V_i^c(\theta)$ with $\theta = 1$. To see that, substitute $\theta = 1$ in the expression of $\pi^w(\theta)$ and $\pi^l(\theta)$ in the objective function V_i^c.The intuition of this formal equivalence is straightforward: in the static model, the only difference between the three regimes considered comes from their relative strengency in terms of the scope of protection. We have extensively justified this assumption in section 2. We assumed that in a patent regime, the patent holder always get a patent with maximal scope which prevents the looser from any type of imitation. Hence the payoff of the looser must be zero whereas the payoff of the winner must be maximum: v. But this is precisely what is obtained from $\pi^w(\theta)$ and $\pi^l(\theta)$ when $\theta = 1$. Now, consider what happens if the two firms are successful. In the patent regime, we assumed that each firm gets v with the probability $\frac{1}{2}$ so that the expected payoff associated with this outcome is $\frac{v}{2}$. But in the copyright regime, we assumed a right for independent innovators so that in the case of two successful firms, each receive $\frac{v}{2}$.Clearly, the expected payoff associated with this outcome is the same in both regimes.

We can use this equivalence property to derive the best-response functions of the players under a patent regime from the ones obtained in the copyright regime[16].

Lemma 3 *In an IP regime with patent, in the static model, the equilibrium R&D invest-*

[16] Alternatively, one could have solved the program of player i in this regime: $\underset{x_i^p}{Max} V_i^p$ and obtained conditions (12) and (13).

ments are implicitely defined by the best-response functions:

$$x_1^p = \ln v - \ln 2 + \ln(1 + e^{-x_2^p}) \tag{12}$$

$$x_2^p = \ln v - \ln 2\alpha + \ln(1 + e^{-x_1^p}) \tag{13}$$

and the efficient firm invests strictly more than the inefficient one.

Proof. Substitute $\theta = 1$ in the best-response functions (6) and (7) to obtain (12) and (13).

To prove the second part of the lemma, note that the two best-response functions can be re-expressed as:

$$\begin{cases} e^{-x_1^p}(1 + e^{-x_2^p}) = \frac{2}{v} \\ e^{-x_2^p}(1 + e^{-x_1^p}) = \frac{2\alpha}{v} \end{cases} \tag{14}$$

Substracting these two equalities yields:

$$e^{-x_1^p} - e^{-x_2^p} = \frac{2(1-\alpha)}{v} \tag{15}$$

Then, since $\alpha > 1$ by assumption, it follows that

$$e^{-x_1^p} < e^{-x_2^p} \tag{16}$$

from which we can conclude

$$x_1^p > x_2^p$$

■

Again, looking for a Nash equilibrium in R&D investments, we cannot derive closed-form solutions. However, we can state the following lemma:

Lemma 4 *In an IP regime with patents, the static non-cooperative R&D game has a unique Nash equilibrium in R&D investments* $(x_1^{p,NE}; x_2^{p,NE})$.

Proof. See the Appendix. ■

3.3 IP regime without protection

Denoting V_i^n the objective function of player i in a regime with no protection,

$$V_i^n = -\alpha_i x_i^n + \left[1 - (1 - p(x_i^n))(1 - p(x_j^n))\right] \frac{v}{2} \tag{17}$$

where x_i^n denotes the R&D investment of player i and $\alpha_1 = 1$, $\alpha_2 = \alpha > 1$.

The first term on the right-hand side is the cost of R&D for player i. The second term represents the expected payoff of player i if she succeeds: in that case, the outcome concerning player j's R&D is irrelevant: if player j succeeds, then there is *de facto* a duopoly and each earn $\frac{v}{2}$ and if player j does not succeed, she can still imitate, so that the payoff of each player is also $\frac{v}{2}$.However, and for the same reason, if player i fails and player j succeeds, player i can imitate and the payoffs are $\frac{v}{2}$ for both players, hence the last term.

It is clear that $V_i^n = V_i^c(\theta)$ with $\theta = 0$. To see that, substitute $\theta = 0$ in the expressions $\pi^w(\theta)$ and $\pi^l(\theta)$ in the objective function V_i^c. As for the patent case, the intuition for this formal equivalence is straightforward. If there is no protection at all, a looser of the R&D race can imitate freely the innovation so that the two players have the same product and earn $\frac{v}{2}$ acccording to our assumptions. But these are precisely the payoffs given by $\pi^w(0)$ and $\pi^l(0)$.

Here also, we can use this equivalence property to derive the best-response functions of each player in the regime with no protection from the functions obtained in the copyright regime[17].

Lemma 5 *In a regime without protection, in the static model, the equilibrium R&D investments are implicitely defined by the best-response functions:*

$$x_1^n = \ln v - \ln 2 - x_2^n \tag{18}$$

$$x_2^n = \ln v - \ln 2\alpha - x_1^n \tag{19}$$

[17]Alternatively, one could have solved the program of player i in this regime: $\underset{x_i^n}{Max} V_i^n$ and obtained the best-response functions (18) and (19).

Proof. To obtain $x_1^n(x_2^n)$ and $x_2^n(x_1^n)$, simply substitute for $\theta = 0$ in (6) and (7). ■

Lemma 6 *In a regime with no IP right, the non-cooperative R&D game has a unique Nash equilibrium in R&D investments where* $x_1^{n,NE} = \ln v - \ln 2$ *and* $x_2^{n,NE} = 0$.

Proof. Note that the two conditions (18) and (19) cannot hold together. There are two possible equilibria: $x_1^n > 0$ and $x_2^n = 0$ or $x_1^n = 0$ and $x_2^n > 0$.Consider the first possibility. $x_1^n(0) = \ln v - \ln 2$. And $x_2^n(\ln v - \ln 2) = -\ln \alpha < 0$. But a negative investment is not feasible, so it must be that $x_2^n = 0$. So $\{x_1^n = \ln v - \ln 2; x_2^n = 0\}$ is an equilibrium. Consider the second possibility. $x_2^n(0) = \ln v - \ln 2\alpha$ and $x_1^n(\ln v - \ln 2\alpha) = \ln \alpha > 0$. So $\{x_1^n = 0; x_2^n > 0\}$ is not an equilibrium. ■

3.4 The Socially optimal R&D investment

From the society´s point of view[18], what matters is that at least one firm makes the invention so that society gets v.

Denoting V_s the objective function of the social planner:

$$V_s = -\sum_{i=1}^{2} \alpha_i x_i^s + \left[1 - (1 - p(x_1^s))(1 - p(x_2^s))\right] v \tag{20}$$

The first term on the right-hand side is the total (industry-wide) cost of R&D. The second term is the probability that at least one player succeeds times the prize v.

The objective of the social planner is defined as:

$$\underset{x_1^s, x_2^s}{Max}\ V_s \tag{21}$$

[18]It is interesting to note that, for small open economies with a consumers'demand mainly *abroad*, the sum of the utilities of each firm can be a relevant proxy for the measure of the social (national) welfare: the number of national consumers can be neglected and the total (national) surplus is thus approximated by the industry surplus.

Lemma 7 *Social surplus is maximized when only the most efficient firm invests a positive amount in R&D:*

$$x^{s,*} = x_1^s = \ln v \tag{22}$$

Proof. Solving the program (21) yields the two first-order conditions:

$$x_1^s = \ln v + \ln e^{-x_2^s} \tag{23}$$

$$x_2^s = \ln v - \ln \alpha + \ln e^{-x_1^s} \tag{24}$$

These two conditions cannot hold together. Thus in equilibrium it must be that: $x_i^s > 0$ and $x_j^s = 0$ for $i \neq j$.

If $x_1^s = 0$, then $x_2^s = \ln v - \ln \alpha$. Substituting for these values in V_s yields:

$$V_s^2 = \alpha \ln \alpha - \alpha \ln v + v - \alpha \tag{25}$$

If $x_2^s = 0$, then $x_1^s = \ln v$. Substituting fot these values in V_s yields:

$$V_s^1 = -\ln v + v - 1 \tag{26}$$

Showing that $V_s^1 > V_s^2$ is equivalent to show that, for $\alpha > 1$:

$$\alpha \ln v + \alpha - \ln v - 1 - \alpha \ln \alpha = \Psi(\alpha) > 0 \tag{27}$$

But $\lim_{\alpha \mapsto 1^+} \Psi(\alpha) = 0$ and, remembering the condition for interior solutions, $v > 2\alpha$, we have:

$$\frac{\partial \Psi(\alpha)}{\partial \alpha} = \ln v + 1 - \ln \alpha - 1 = \ln(\frac{v}{\alpha}) > 0. \tag{28}$$

It follows that for $\alpha > 1$, $V_s^1 > V_s^2$. ■

3.5 Comparison

We have proved the existence of a unique Nash equilibrium in R&D investments in each of the three IP regimes considered and we have derived the socially optimal investment. Even if we do not have a closed-form solution for the Nash equilibria,we can still compare the R&D investments (both at the firm and at the aggregate levels) between each regime and with the social optimum, from our results regarding the best-response functions. Lemma 8 states important properties of the upper and lower bounds of the best-response functions that will help to conduct the comparative analysis. Figure 2 illustrates these properties.

Lemma 8 *The upper bounds of the best-response functions of player i in each IP regime are such that*

$$\overline{x}_i^p \geq \overline{x}_i^c \geq x_i^{n,NE}, \tag{29}$$

for $i = 1, 2$,

And the lower bounds are such that

$$\begin{aligned} \underline{x}_1^p &= x_1^{n,NE} \geq \underline{x}_1^c, & (30) \\ x_2^{n,NE} &= 0 < \underline{x}_2^c \leq \underline{x}_2^p & (31) \end{aligned}$$

Besides,

$$x_2^{n,NE} = 0 < \underline{x}_2^c < \underline{x}_2^p < \overline{x}_2^c < \overline{x}_2^p \tag{32}$$

Finally,

$$x^{s,*} = \overline{x}_1^p = x_1^p(0) \tag{33}$$

Proof. The values of $\overline{x}_i^l$ for $i = 1, 2$ and $l = c, p, n$ have been computed in the proof of lemmas 2 and 4 in the appendix and are given by:

$$\begin{cases} \overline{x}_i^p = \ln v - \ln 2\alpha_i + \ln 2 \\ \overline{x}_i^c = \ln v - \ln 2\alpha_i + \ln(\theta + 1) \\ x_1^{n,NE} = \ln v - \ln 2 \end{cases} \tag{34}$$

Clearly, with $0 < \theta \leq 1$, we have:

$$\ln v - \ln 2\alpha_i + \ln 2 > \ln v - \ln 2\alpha_i + \ln(\theta + 1) > \ln v - \ln 2 > 0$$

hence (29).

We also have:

$$\left\{\begin{array}{c} \underline{x}_i^p = \ln v - \ln 2\alpha_i \\ \underline{x}_i^c = \ln v - \ln 2\alpha_i + \ln\theta \\ x_1^{n,NE} = \ln v - \ln 2 \end{array}\right. \tag{35}$$

and because $0 < \theta \leq 1$:

$$\left\{\begin{array}{c} \ln v - \ln 2 > \ln v - \ln 2 + \ln\theta \\ \ln v - \ln 2\alpha > \ln v - \ln 2\alpha + \ln\theta \end{array}\right. \tag{36}$$

hence (30) and (31).

Besides, it is clear that:

$$\ln v - \ln 2\alpha + \ln(\theta + 1) > \ln v - \ln 2\alpha \tag{37}$$

Combining with (36) yields (32).

Finally,

$$\overline{x}_1^p = \ln v - \ln 2 + \ln 2 = \ln v = x^{s,*}$$

■

Assumption 1: $v > \frac{2\alpha}{\theta}$ or equivalently: $\theta > \frac{2\alpha}{v}$.

This assumption enables us to draw the following illustrative graphic, with $\widehat{\underline{x}}_2^c > 0$. It is not fundamental for our analytical results.

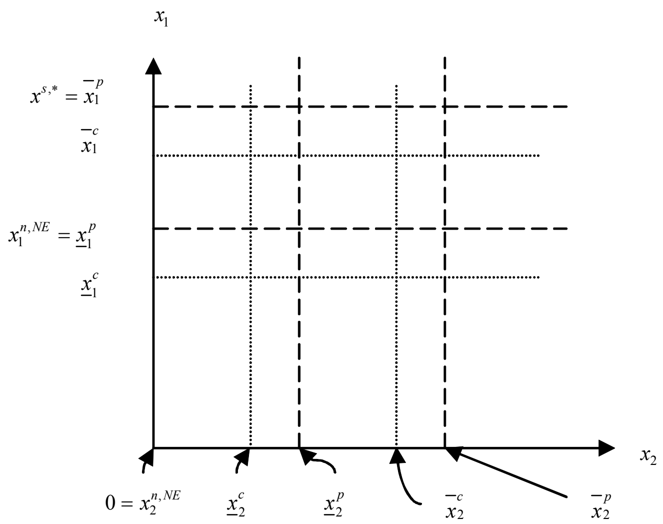

Figure 2: Upper and lower bounds of the best-response functions in each IP regime for the static model

Proposition 1 *The individual and aggregate R&D investments are always higher under a patent regime than under a copyright regime or a regime with no protection.*

Proof. We show that for all $x_j^p = x_j^c = x_j$, $x_i^p(x_j) > x_i^c(x_j)$ for $i \neq j$. But this is obvious from the expression of the best-response functions given by (6), (7), (12) and (13):

$$\begin{aligned} x_i^p(x_j) &> x_i^c(x_j) \\ &\Leftrightarrow \ln v - \ln 2\alpha_i + \ln(1 + e^{-x_j}) > \ln v - \ln 2\alpha_i + \ln(\theta + e^{-x_j}) \\ &\Leftrightarrow 1 > \theta \end{aligned}$$

which holds. And clearly $x_i^p(x_j) > x_i^c(x_j)$ for $i \neq j$ implies that $x_i^{p,NE} > x_i^{c,NE}$ for $i = 1, 2$.

Also,

$$x_1^p(x_2^p) > x_1^{n,NE}$$

always holds since the lower bounds of $x_1^p(.)$ is equal to $x_1^{n,NE}$ from lemma 8. ■

Proposition 2 *The socially optimal R&D investment is always larger than the aggregate investment provided by the firms in a regime without protection.*

Proof. In the remaining of the paper,we denote X^l the aggregate R&D investment in IP regime l with $l = c, p, n$.

$x^{s,*} = \ln v$ and $X^n = x_1^{n,NE} = \ln v - \ln 2$.

Clearly $\ln v > \ln v - \ln 2$ ■

Proposition 3 *The socially optimal R&D investment is always smaller than the aggregate investment provided by the firms in a regime with patents.*

Proof. The proof is by contradiction. Firts, note that $e^{-x^{s,*}} = \frac{1}{v}$.

1) Assume that the social optimum coincides with the aggregate equilibrium investment: $x_1^p + x_2^p = x^{s,*}$.

Then, one can rewrite (12) as $e^{-x_1^p} + e^{-x_1^p} e^{-x_2^p} = \frac{2}{v}$.

This implies that $e^{-x_1^p} + e^{-x^{s,*}} = \frac{2}{v}$ since by assumption $x_1^p + x_2^p = x^{s,*}$.

Or, $e^{-x_1^p} = \frac{2}{v} - \frac{1}{v} = \frac{1}{v} = e^{-x^{s,*}}$.

But $e^{-x_1^p} = e^{-x^{s,*}}$ would imply $x_1^p = x^{s,*}$. Again, by assumption $x^{s,*} = x_1^p + x_2^p$

Combining would yield: $x_1^p = x_1^p + x_2^p$

But we have shown that $x_2^p > 0$,which contradicts the last equality.

Thus, $x_1^p + x_2^p \neq x^{s,*}$

2) Assume that $x_1^p + x_2^p < x^{s,*}$.

This would imply: $e^{-x_1^p - x_2^p} > e^{-x^{s,*}} = \frac{1}{v}$.

But as long as $x_2^p > 0$,one must have: $x_1^p < x_1^p + x_2^p$,

which implies: $e^{-x_1^p} > e^{-x_1^p - x_2^p} > \frac{1}{v}$.

We end up with two conditions:

$$\begin{cases} e^{-x_1^p - x_2^p} > \frac{1}{v} \\ e^{-x_1^p} > \frac{1}{v} \end{cases} \tag{38}$$

Adding these two conditions yields: $e^{-x_1^p} + e^{-x_1^p} e^{-x_2^p} > \frac{2}{v}$.

But this contradicts the first-order condition (12).

Hence, $x_1^p + x_2^p < x^{s,*}$ does not hold.

We can now conclude that $x^{s,*} < x_1^p + x_2^p$. ■

4 The Dynamic Model

Now, we consider the case where there is an infinite sequence of innovations, and each innovation "builds" on the previous one: we assume that the development of the innovation at date $t+1$ requires the right to use the innovation obtained at date t.

In the IP regime with **patents**, like in BM, a patent on the first innovation constrains the set of firms able to perform R&D for the subsequent innovations: supposing that there is no licensing, only the patentholder will have the right to perform R&D and eventually realize the next innovations[19].

As in the static model, we assume that a copyright regime allows for simultaneous independent similar innovations: if both firms succeed, each earns the duopoly payoff $\frac{v}{2}$. Again, if there is a winner and a looser, we assume the same payoff functions as in the static model: $\pi^w(\theta) = \frac{1}{2}(1+\theta)v$ and $\pi^l(\theta) = \frac{1}{2}(1-\theta)v$ with $0 < \theta \leq 1$.

Finally, in a regime without protection, whenever a firm is successful, it earns the duopoly payoff $\frac{v}{2}$ as well as its rival (the latter can imitate) and of course there are always two firms involved in R&D at each period.

We assume that, at each period, v represents an *incremental value.* As stated in BM (1999), "an invention is simply an improvement that enhances the value of the first innovation". This allows us to avoid the complication associated with the replacement effect. Our model assumes a memoryless R&D process. Although simplifying, this assumption is very common in the literature, for example in Poisson-process models of R&D[20].

In the dynamic model, the equivalence property from the static analysis between a copyright regime with $\theta = 1$ and a patent regime does not hold anymore as will be seen.

[19] We assume licensing (and infringement) do not happen, to deal with the most extreme patent regime.

[20] See for example the seminal papers by Loury (1979), Lee and Wilde (1980), Reinganum (1983) as well as Reinganum (1989).

Yet, this formal equivalence still holds between the copyright with $\theta = 0$ and the regime with no protection. Again, to guarantee interior solutions, we assume throughout that $v > 2\alpha$.

4.1 IP regime with patents

Denoting $U_i^{k,p}$ the objective function of firm i when there are k firms performing R&D ($k = 1$ or $k = 2$) in a regime with patents, we have

$$U_i^{2,p} = -\alpha_i \widehat{x}_i^p + p(\widehat{x}_i^p) \left\{ p(\widehat{x}_j^p) \left[\frac{1}{2}(v + U_i^{1,p}) \right] + (1 - p(\widehat{x}_j^p) \left[v + U_i^{1,p} \right] \right\} \tag{39}$$

with

$$U_i^{1,p} = -\alpha_i \widetilde{x}_i + p(\widetilde{x}_i) \left[v + U_i^{1,p} \right] \tag{40}$$

where the p superscript denotes the *p*atent regime. $\widehat{x}$ denotes the R&D investment when there are two players, that is only in the first period. Indeed, the winner of the first period ($t = 0$) obtains a patent that cannot be infringed upon and because a right on the first innovation is a requirement for developing the next ones ($t = 1, 2...$), the patent holder is the only one able to perform R&D in the next periods. Therefore, $\widetilde{x}$ denotes the R&D investment of the winner of the first R&D game in periods $t = 1$, $t = 2$ etc...

In equation (39), the first term on the right-hand side is the cost of R&D. If player i does not succeed she gets nothing (she cannot imitate). If she succeeds, they are two possible outcomes captured by the term into brackets: either player j also succeeds in which case, with probability $\frac{1}{2}$, player i gets the patent *and* is the only one able to perform R&D in the next periods hence: $v + U_i^{1,p}$,or player j fails and for sure player i gets the patent and is the only one performing R&D in the next periods, hence $v + U_i^{1,p}$.

We look for the best response functions of each player for the first R&D game, $\widehat{x}_i^p(\widehat{x}_j^p)$, and for an optimal investment in the subsequent R&D program by the winner of the first patent, $\widetilde{x}_i^p$.

Lemma 9 *In an IP regime with patents, when innovations are cumulative, the equilibrium R&D investments in the first patent race are implicitely defined by the best response functions:*

$$\widehat{x}_1^p = v - 1 - \ln 2 + \ln(1 + e^{-\widehat{x}_2^p}) \tag{41}$$

$$\widehat{x}_2^p = \frac{v - \alpha}{\alpha} - \ln 2 + \ln(1 + e^{-\widehat{x}_1^p}) \tag{42}$$

and the efficient firm invests strictly more than the inefficient one. Moreover, the investment of the winner of the first race in the subsequent R&Ds is given by:

$$\widetilde{x}_1 = v - 1 \tag{43}$$

$$\widetilde{x}_2 = \frac{v - \alpha}{\alpha} \tag{44}$$

Proof. see the appendix. ■

The equilibrium R&D investments in all periods but the first one are thus given by (43) and (44). Concerning the R&D investments in the first period, we can state the following lemma:

Lemma 10 *In an IP regime with patents, when innovations are cumulative, the non-cooperative R&D game of the first period has a unique Nash equilibrium* $(\widehat{x}_1^{p,NE}; \widehat{x}_2^{p,NE})$.

Proof. See the Appendix. ■

4.2 IP regime with copyrights

Denoting $U_i^{k,c}$ the objective function of firm i when there are k firms performing R&D in a regime with copyrights, we have

$$\begin{aligned} U_i^{2,c} = & -\alpha_i \widehat{x}_i^c + p(\widehat{x}_i^c) \left\{ p(\widehat{x}_j^c) \left[\frac{v}{2} + U_i^{2,c} \right] + (1 - p(\widehat{x}_j^c) \left[\pi^w(\theta) + U_i^{2,c} \right] \right\} + \\ & (1 - p(\widehat{x}_i^c)) p(\widehat{x}_j^c) \left[\pi^l(\theta) + U_i^{2,c} \right] \end{aligned} \tag{45}$$

where the superscript c stands for $c$opyright, $\widehat{x}$ denotes the R&D investment when there are two players and $\pi^l(\theta)$ and $\pi^w(\theta)$ are given by (1) and (2).

In this objective function, the first term, $-\alpha_i \widehat{x}_i^c$, represents the cost of R&D for player i, the second term represents the expected payoff of player i if she succeeds in her R&D program: with probability $p(\widehat{x}_j^c)$ the other player succeeds and in that case each player earns the duopoly payoff $\frac{v}{2}$. Because both players have made the innovation they can of course compete again in the next period, hence $U_i^{2,c}$; with probability $(1 - p(\widehat{x}_j^c))$ the rival does not succeed and in that case the winner receives the payoff $\pi^w(\theta)$, but because a copyright protects the work and not the underlying idea, the looser can still use the idea not only to imitate *but also to develop the next innovation* and the game in the next period will involve two players as well, hence $U_i^{2,c}$. Finally, the last term represents the expected payoff of player i if she looses the R&D race. For the same reason, because of copyright protection, she is allowed to imitate, hence $\pi^l(\theta)$, and she is still allowed to perform R&D in the next period, provided the other player has found the innovation at the previous period which happens with probability $p(\widehat{x}_j^c)$.

Again, we look for the best-response functions of each player: $\widehat{x}_i^c(\widehat{x}_j^c)$.

Lemma 11 *In an IP regime with copyrights, when innovations are cumulative, the equilibrium R&D investments in each period are implicitely defined by the best-response functions:*

$$\widehat{x}_1^c = \frac{-2 + v(1 + \theta e^{-\widehat{x}_2^c})}{2} \quad (46)$$

$$\widehat{x}_2^c = \frac{-2\alpha + v(1 + \theta e^{-\widehat{x}_1^c})}{2\alpha} \quad (47)$$

Proof. See the appendix. ■

As for the previous cases analyzed, we cannot derive a closed-form solution for a Nash equilibrium in R&D investment. Yet, we can state:

Lemma 12 *In an IP regime with copyrights, when innovations are cumulative, the non-cooperative R&D game has an equilibrium in R&D investments* $(\widehat{x}_1^{c,NE}; \widehat{x}_2^{c,NE})$ *and this equilibrium is unique.*

Proof. See the Appendix. ■

4.3 IP regime without protection

Denoting $U_i^{k,n}$ the objective function of player i when there are k players in such a regime, we have

$$U_i^{2,n} = -\alpha_i \widehat{x}_i^n + \{1 - (1 - p(\widehat{x}_i^n))(1 - p(\widehat{x}_j^n))\} \left[\frac{v}{2} + U_i^{2,n}\right]$$

The first term represents of cost of R&D for player i. The second term represents the expected payoff of player i in a regime without protection. The term $\{1-(1-p(\widehat{x}_i^n))(1-p(\widehat{x}_j^n))\}$ is the probability that *at least* one player makes an innovation. Indeed, if two players make an innovation they share the market and earn $\frac{v}{2}$. If only one player makes the innovation, they also share the market and earn $\frac{v}{2}$ because the looser can freely imitate the innovation (we assume it is done immediately and at no cost). In any case, when at least an innovation is made at period t , both players can use it to perform, R&D in the next period, hence $U_i^{2,n}$.

It is clear that $U_i^{2,n} = U_i^{2,c}(\theta)$ with $\theta = 0$. As for the static model, the intuition for this equivalence is straightforward: If there is no protection, the looser of an R&D race can legally imitate the invention so that each firms gets $\frac{v}{2}$, but this is precisely the payoff obtained from $\pi^w(0)$ and $\pi^l(\theta)$. Moreover, the looser will still be in the race in the subsequent periods.

We can use this equivalence property to derive the best-response functions of each player from the ones obtained in the copyright case[21].

[21] Alternatively, one could have solved the program of player i in this regime: $\underset{\widehat{x}_i^n}{Max}\ U_i^{2,n}$ and obtained the best-response functions (48) and (49).

Lemma 13 *In an IP regime without protection, when innovations are cumulative, the equilibrium R&D investments in each period are*

$$\widehat{x}_1^n = \frac{v-2}{2} \quad (48)$$

$$\widehat{x}_2^n = \frac{v-2\alpha}{2\alpha} \quad (49)$$

Proof. Substitute $\theta = 0$ in (46) and (47). ■

4.4 The Socially optimal R&D investment

From the society's point of view, what matters is that at least one firms makes the innovation so that society gets v. Denotingt U_s the objective function of the social planner in the dynamic case:

$$U_s = -\sum_{i=1}^{2} \alpha_i \widehat{x}_i^s + [1 - (1 - p(\widehat{x}_1^s))(1 - p(\widehat{x}_2^s))][v + U_s] \quad (50)$$

The objective of the social planner is

$$\underset{\widehat{x}_1^s, \widehat{x}_2^s}{Max}\ U_s \quad (51)$$

Lemma 14 *The social surplus is maximized when the most efficient firm invests a positive amount in R&D:*

$$\widehat{x}^{s,*} = \widehat{x}_1^s = v - 1 \quad (52)$$

Proof. Solving the program (51) yields the two first-order conditions:

$$\widehat{x}_1^s = v - 1 - \alpha \widehat{x}_2^s \quad (53)$$

$$\widehat{x}_2^s = \frac{v - \alpha - \widehat{x}_1^s}{\alpha} \quad (54)$$

These two conditions cannot hold together. It must be that in equilibrium, $\widehat{x}_i^s > 0$ and $\widehat{x}_j^s = 0$ for $i \neq j$.If $\widehat{x}_2^s = 0$ then $\widehat{x}_1^s = v - 1$ and substituting for these values in U_s yields:

$$U_s^1 = e^{v-1} - v \quad (55)$$

If $\widehat{x}_1^s = 0$ then $\widehat{x}_2^s = \frac{v-\alpha}{\alpha}$ and:

$$U_s^2 = \alpha e^{\frac{v-\alpha}{\alpha}} - v \tag{56}$$

We must show that $\Phi(\alpha) = U_s^1 - U_s^2 = e^v - \alpha e^{\frac{v}{\alpha}} > 0$ for all $\alpha > 1$

First:

$$\lim_{\alpha \mapsto 1^+} \Phi(\alpha) = 0 \tag{57}$$

Second, remembering that the condition for interior solutions is $v > 2\alpha$,we have:

$$\frac{\partial \Phi(\alpha)}{\partial \alpha} = e^{\frac{v}{\alpha}}(\frac{v}{\alpha} - 1) > 0 \tag{58}$$

Hence, taking (57) into account, we can conclude that $U_s^1 > U_s^2$. ■

4.5 Comparison of the three IP regimes

As for the static case, we cannot derive a comparative analysis from the equilibrium R&D investments. However, we can use the results on the best-response functions. First, we state a useful lemma concerning the upper and lower bounds of these functions.

Lemma 15 *The upper and lower bounds of the best-response functions of player i in each IP regime are such that:*

$$\begin{cases} \overline{\widehat{x}}_i^p \geq \overline{\widehat{x}}_i^c \\ \widehat{x}_i^{n,NE} = \underline{\widehat{x}}_i^c \\ \underline{\widehat{x}}_i^p > \underline{\widehat{x}}_i^c \end{cases} \tag{59}$$

for $i = 1, 2$.

Proof. The values of $\overline{\widehat{x}}_i^l$ and $\underline{\widehat{x}}_i^l$ for $i = 1, 2$ and $l = p, c, n$ have been computed in the proof of lemmas 10 and 12 in the appendix and are given by:

$$\begin{cases} \overline{\widehat{x}}_i^p = \frac{v-\alpha_i}{\alpha_i} \\ \underline{\widehat{x}}_i^p = \frac{v-\alpha_i}{\alpha_i} - \ln 2 \end{cases} \tag{60}$$

$$\begin{cases} \overline{\widehat{x}}_i^c = \frac{-2\alpha_i + v(1+\theta)}{2\alpha_i} \\ \underline{\widehat{x}}_i^c = \frac{-2\alpha_i + v}{2\alpha_i} \end{cases} \tag{61}$$

Finally, $\widehat{x}_i^{n,NE}$ is given by (48) and (49).

Thus, the necessary condition for $\overline{\widehat{x}}_i^p \geq \overline{\widehat{x}}_i^c$ is that $v(1-\theta) \geq 0$ which is satisfied since $\theta \in [0,1]$.

Then, a simple comparison shows that $\widehat{x}_i^{n,NE} = \underline{\widehat{x}}_i^c$.

And the necessary condition for $\underline{\widehat{x}}_i^p > \underline{\widehat{x}}_i^c$ is that $1 > \ln 2$, which holds. ■

The following graphic illustrates lemma 15:

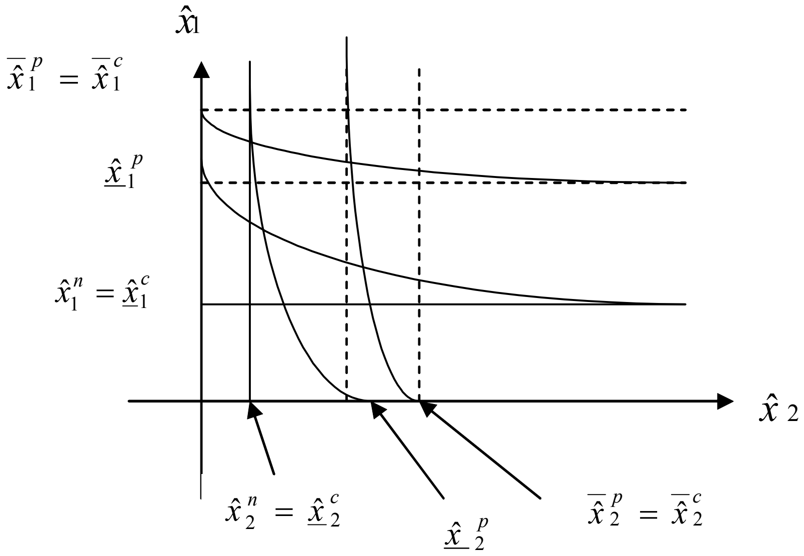

Figure 3: Reaction curves in each IP regimes when innovations are cumulative

Proposition 4 ***(individual per-period investments)***. *The individual per-period R&D investment is always larger under a patent regime than without protection.*

Proof. Under a patent regime, from period 2 on, the individual per-period R&D investments are $\widetilde{x}_1 = v - 1$ if firm 1 obtained the patent and $\widetilde{x}_2 = \frac{v-\alpha}{\alpha}$ if firm 2 obtained the patent. With no protection, $\widehat{x}_1^n = \frac{v-2}{2} < v - 1$ and $\widehat{x}_2^n = \frac{v-2\alpha}{2\alpha} < \frac{v-\alpha}{\alpha}$. Concerning the first period, since $\underline{\widehat{x}}_i^p = \frac{v-\alpha i}{\alpha_i} - \ln 2$,it follows that $\widehat{x}_i^p > \widehat{x}_i^n$ if $\widehat{x}_i^n < \underline{\widehat{x}}_i^p$. This holds provided that:

$$\begin{aligned} \frac{v-\alpha_i}{\alpha_i} - \ln 2 &> \frac{v-2\alpha_i}{2\alpha_i} \\ &\Leftrightarrow \; v > \alpha_i \ln 2 \end{aligned} \tag{62}$$

But by assumption $v > 2\alpha$. So it must be that (62) is satisfied. ■

The comparative analysis between the IP regimes in terms of *aggregate* investments yields a first set of results summarized by proposition 5:

Proposition 5 ***(Aggregate R&D investments)***

i) The aggregate R&D investment is strictly higher in a regime with patents than in a regime with no protection if the efficient firm obtains the patent in the first period.

ii) If the firms are symmetric ($\alpha = 1$), the aggregate R&D investment is always strictly higher in a regime with patents than in a regime with no protection, regardless which firm obtained the patent in the first period.

iii) Moreover, the aggregate R&D investment is strictly higher under a copyright regime than without protection.

Proof. In what follows, by a slight abuse of notation, we denote $\widehat{x}_i^{l,NE}$ (for $l = p, c, n$) by $\widehat{x}_i^l$.

i) If the efficient firm has obtained the patent in the first period, we compute the expected aggregate investment in the subsequent periods:

$$\widetilde{X}_1 = \left\{\widetilde{x}_1 + (1 - e^{-\widetilde{x}_1})\widetilde{x}_1 + (1 - e^{-\widetilde{x}_1})^2\widetilde{x}_1 + ...\right\} \tag{63}$$

which is equivalent to:

$$\widetilde{X}_1 = \widetilde{x}_1 e^{\widetilde{x}_1} \tag{64}$$

Equivalently, in the non-protection case where there are always two firms performing R&D, the expected aggregate investment from period 2 on is:

$$\widehat{X}^n = (\widehat{x}_1^n + \widehat{x}_2^n)e^{(\widehat{x}_1^n + \widehat{x}_2^n)} \tag{65}$$

We can compute, from lemmas 9 and 13:

$$\begin{cases} a = \widetilde{x}_1 = v - 1 \\ b = \widehat{x}_1^{n,NE} + \widehat{x}_2^{n,NE} = \frac{v-2}{2} + \frac{v-2\alpha}{2\alpha} \end{cases} \tag{66}$$

Inequality $a > b$ holds provided that $\frac{v-2}{2} + \frac{v-2\alpha}{2\alpha} < v - 1$.

This is true if:

$$v(1-\alpha) - 2\alpha < 0 \tag{67}$$

which always holds since $\alpha > 1$.

We can write $\widehat{X}^n = be^b$ and $\widetilde{X} = ae^a$ Since $a > b$, and $\frac{\partial \widehat{X}^n}{\partial k} > 0$ for $k = a, b$,it follows that:

$$\widetilde{X} > \widehat{X}^n$$

ii) If firms are symmetric, then, from lemma 13:

$$\widehat{X}^n = \left[2 \times \frac{v-2}{2}\right] e^{2(\frac{v-2}{2})} = (v-2)e^{v-2} \tag{68}$$

And the R&D investment in all periods but the first one is given by lemma 9,

$$\widetilde{X} = \widetilde{x}_i e^{\widetilde{x}_i}(1 - e^{-\widetilde{x}_i}) = (v-1)e^{v-1} \tag{69}$$

for $i = 1, 2$. Since $v - 1 > v - 2$,it follows that $\widetilde{X} > \widehat{X}^n$.

iii) The expected aggregate investment under copyright is given by:

$$\begin{cases} \widehat{X}^c = se^s(1 - e^{-s}) \\ s = \widehat{x}_1^c + \widehat{x}_2^c \end{cases} \tag{70}$$

and we have computed above $\widehat{X}^n = be^b(1 - e^{-b})$ with b given by (66)

We cannot compute s explicitely, but we can still show that $s > b$.Indeed, adding the two best-response functions from lemma 11:

$$s = \frac{-4\alpha + v(1+\alpha) + v\theta(\alpha e^{-\widehat{x}_2^c} + e^{-\widehat{x}_1^c})}{2\alpha} \tag{71}$$

But since $0 < \alpha e^{-\widehat{x}_2^c} \leq \alpha$ and $0 < e^{-\widehat{x}_1^c} \leq 1$, it follows that $0 < v\theta(\alpha e^{-\widehat{x}_2^c} + e^{-\widehat{x}_1^c}) \leq (\alpha + 1)v\theta$. And consequently:

$$\frac{v(1+\alpha) - 4\alpha}{2\alpha} < s \leq \frac{v(1+\alpha) - 4\alpha + (1+\alpha)v\theta}{2\alpha} \tag{72}$$

Call $\underline{s}$ the lower bound of this inequality. We can show that $b \leq \underline{s}$. Indeed:

$$\frac{v(1+\alpha)-4\alpha}{2\alpha} \leq \frac{v(1+\alpha)-4\alpha}{2\alpha} \tag{73}$$

which holds. We thus have $b \leq \underline{s} < s$. From which we can conclude that $\widehat{X}^c > \widehat{X}^n$. ■

Moreover, we can draw two important conclusions concerning the socially optimal expected aggregate investment:

Proposition 6 ***(Social Optimum)***

i) The expected aggregate R&D investment in a regime with no protection is always less than the socially optimal investment.

ii) The socially optimal R&D investment is less than the expected aggregate R&D investment in a patent regime if the firms are symmetric.

Proof. See the Appendix. ■

Proposition 5 i) and iii) and proposition 6 contradict BM's claim that there are circumstances where a regime with no protection leads to more R&D investments than a regime with patents and that this might be socially optimal.

5 Conclusion

The primary objective of this paper was to show that Bessen and Maskin's model of cumulative innovations is not robust when the success probability of an R&D program is endogenized. We reached the conclusion that even if a patent regime can restrict the set of firms authorized to perform R&D, the aggregate R&D investment is still higher in this regime than in a regime with no protection. This suggests that the static disincentives associated with the absence of protection are stronger than the dynamic benefits resulting from this regime. Moreover, a regime with no intellectual property protection always yields an expected aggregate R&D investment less than the socially optimal one.

Our paper also introduced a stylized model of IP rights where three regimes coexist. In particular, we introduced the copyright regime as a regime with three distinctive features: a lower scope than the patent regime, the impossibility to exclude the "looser" of an R&D race from future research (since the "idea" of an invention is not protected under copyright), and a right for independent simultaneous identical inventions.

Yet, since we followed the assumptions of the original model proposed by Bessen and Maskin (the only modification concerning the probability of success), we could not elaborate more on the IP regimes. Our model still suggests several possibilities for further research in the analysis of copyright as a distinct form of protection and in the welfare comparison of different IP regimes.

Appendix.

Proof of Lemma 2: The proof proceeds in two steps: first, we show that the best-response functions of each player have an upper-bound and a lower bound denoted $\overline{x}_i^c$ and $\underline{x}_i^c$; then we show that there exists a unique Nash equilibrium in R&D investments.

1) The best-response function of player i is given by

$$x_i^c(x_j^c) = \ln v - \ln 2\alpha_i + \ln(\theta + e^{-x_j^c}) \tag{A1}$$

for $i \neq j$. Note first that x_i^c and x_j^c are strategic substitutes since $\frac{\partial x_i(x_j^c)}{\partial x_j^c} = \frac{-e^{-x_j^c}}{\theta + e^{-x_j^c}} < 0$ and thus the upper bound of x_i^c is reached for $x_j^c = 0$ whereas the lower bound is given by the constant asymptot defined by $\lim_{x_j^c \mapsto +\infty} x_i^c(x_j^c)$.We have:

$$x_i^c(0) = \overline{x}_i^c = \ln v - \ln 2\alpha_i + \ln(1+\theta) \tag{A2}$$

$$\lim_{x_j^c \mapsto +\infty} x_i^c(x_j^c) = \underline{x}_i^c = \ln v - \ln 2\alpha_i + \ln \theta \tag{A3}$$

Or, substituting $\alpha_1 = 1$ and $\alpha_2 = \alpha$ in (A2) and (A3),

$$\begin{cases} \overline{x}_1^c = \ln v - \ln 2 + \ln(1+\theta) \\ \underline{x}_1^c = \ln v - \ln 2 + \ln \theta \end{cases} \tag{A4}$$

$$\begin{cases} \overline{x}_2^c = \ln v - \ln 2\alpha + \ln(1+\theta) \\ \underline{x}_2^c = \ln v - \ln 2\alpha + \ln \theta \end{cases} \tag{A5}$$

2) Now consider the system of best-response functions defined by (6) and (7):

$$\begin{aligned} x_1^c(x_2^c) &= \ln v - \ln 2 + \ln(\theta + e^{-x_2^c}) \\ x_2^c(x_1^c) &= \ln v - \ln 2\alpha + \ln(\theta + e^{-x_1^c}) \end{aligned}$$

Denote $x_1^c(x_2^c) \overset{def}{=} g(x_2^c)$. From $x_2^c(x_1^c)$ one can derive:

$$\begin{aligned} x_2^c &= \ln v - \ln 2\alpha + \ln(\theta + e^{-x_1^c}) \\ &\Leftrightarrow \; x_1^c = -\ln\left[e^{x_2^c - \ln v + \ln 2\alpha} - \theta\right] \overset{def}{=} f(x_2^c) \end{aligned} \tag{A6}$$

The equilibrium x_2^c,denoted $x_2^{c,NE}$ solves the following equation:

$$f(x_2^c) = g(x_2^c) \tag{A7}$$

We cannot derive a closed-form solution for $x_2^{c,NE}$. However, we can show that an equilibrium value *exists* and is *unique*. To show that, we consider the functions $f(.)$ and $g(.)$ on the interval $(\underline{x}_2^c, \overline{x}_2^c)$ (because $g(.)$ is defined only on this interval). We first show that the following inequalities hold: $g(\underline{x}_2^c) < \lim_{x_2^c \mapsto \underline{x}_2^c} f(x_2^c)$ and $g(\overline{x}_2^c) > f(\overline{x}_2^c)$ (existence). Then we show that both functions are strictly decreasing on this interval with $\left|\frac{\partial f(x_2^c)}{\partial x_2^c}\right|$ larger than $\left|\frac{\partial g(x_2^c)}{\partial x_2^c}\right|$ (uniqueness).

We have:

$$\lim_{x_2^c \mapsto \underline{x}_2^c} f(x_2^c) = -\lim_{x_2^c \mapsto \underline{x}_2^c} \ln\left[e^{x_2^c - \ln v + \ln 2\alpha} - \theta\right] = -\lim_{Z \mapsto 0} \ln Z = +\infty,$$

whereas $g(\underline{x}_2^c) = \ln v - \ln 2 + \ln(\theta + e^{-\ln v + \ln 2\alpha - \ln(1+\theta)}) = \zeta_c$ with ζ_c being a positive finite number. It follows that:

$$g(\underline{x}_2^c) < \lim_{x_2^c \mapsto \underline{x}_2^c} f(x_2^c). \tag{A8}$$

Finally, $f(\overline{x}_2^c) = -\ln\left[e^{\ln v - \ln 2\alpha + \ln(1+\theta) - \ln v + \ln 2\alpha} - \theta\right] = 0$ and $g(\overline{x}_2^c) = x_1^c(\overline{x}_2^c) > \underline{x}_1^c$ since $\underline{x}_1^c$ is the lower bound of $g(.)$. And by assumption, we have $\underline{x}_1^c > 0$, from which we conclude that

$$f(\overline{x}_2^c) < g(\overline{x}_2^c). \tag{A9}$$

Thus an equilibrium exists.

Simple computations yields:

$$\frac{\partial f(x_2^c)}{\partial x_2^c} = -\frac{e^{x_2^c - \ln v + \ln 2\alpha}}{e^{x_2^c - \ln v + \ln 2\alpha} - \theta} < 0 \tag{A10}$$

$$\frac{\partial g(x_2^c)}{\partial x_2^c} = \frac{-e^{-x_2^c}}{\theta + e^{-x_2^c}} < 0 \tag{A11}$$

The sign of (A10) comes from the fact that $e^{x_2^c - \ln v + \ln 2\alpha} - \theta > 0 \Leftrightarrow x_2^c > \ln v - \ln 2\alpha + \ln \theta = \underline{x}_2^c$ which holds here because we consider the interval $(\underline{x}_2^c, \overline{x}_2^c]$.

To compare the slopes, note that

$$\left|\frac{\partial f(x_2^c)}{\partial x_2^c}\right| > \left|\frac{\partial g(x_2^c)}{\partial x_2^c}\right|$$
$$\Leftrightarrow -e^{-x_2^c} < e^{x_2^c - \ln v + \ln 2\alpha},$$

which always holds.

Thus the equilibrium is unique.

3) There exists a unique Nash equilibrium in R&D investments, ($x_1^{c,NE}, x_2^{c,NE}$) where $x_2^{c,NE}$ solves $f(x_2^c) = g(x_2^c)$ and $x_1^{c,NE} = g(x_2^{c.NE}) = f(x_2^{c,NE})$. Moreover, it is clear that $x_1^{c,NE} \in (\underline{x}_1^c, \overline{x}_2^c]$ and $x_2^{c,NE} \in (\underline{x}_2^c, \overline{x}_2^c]$. *QED.*

Here, we provide a graphical illustration of the unique equilibrium that we proved above.

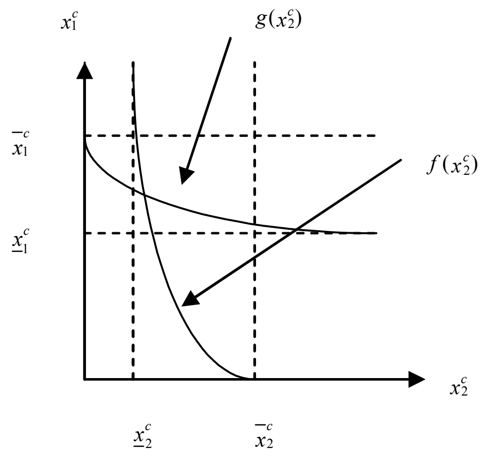

Figure 4: Nash equilibrium of the R&D game under a copyright regime, assuming $\theta > \frac{2\alpha}{v}$.

Proof of Lemma 4: As for the proof of lemma 2, this proof proceeds in two steps: first, we show that the best-response functions of each player have a upper bound and a lower bound denoted $\overline{x}_i^p$ and $\underline{x}_i^p$ for $i = 1, 2$; then we show that there exists a unique Nash equilibrium in R&D investments. Yet, we can use the equivalence property emphasized in subsection 3.2 to shorten the first step of the proof.

1) Substituting $\theta = 1$ in (A2) and (A3) yields the expressions for the upper and the lower bounds $\overline{x}_i^p$ and $\underline{x}_i^p$:

$$\overline{x}_i^p = \ln v - \ln 2\alpha_i + \ln 2 \tag{A12}$$

$$\underline{x}_i^p = \ln v - \ln 2\alpha_i \tag{A13}$$

And substituting $\alpha_1 = 1$ and $\alpha_2 = \alpha > 1$ in (A12) and (A13):

$$\begin{cases} \overline{x}_1^p = \ln v \\ \underline{x}_1^p = \ln v - \ln 2 \end{cases} \tag{A14}$$

$$\begin{cases} \overline{x}_2^p = \ln v - \ln \alpha \\ \underline{x}_2^p = \ln v - \ln 2\alpha \end{cases} \tag{A15}$$

2) Now consider the system of best-response functions as defined by (11) and (12):

$$\begin{aligned} x_1^p(x_2^p) &= \ln v - \ln 2 + \ln(1 + e^{-x_2^p}) \\ x_2^p(x_1^p) &= \ln v - \ln 2\alpha + \ln(1 + e^{-x_1^p}) \end{aligned}$$

Denote $x_1^p(x_2^p) \overset{def}{=} g(x_2^p)$. From $x_2^p(x_1^p)$ one can derive:

$$\begin{aligned} x_2^p &= \ln v - \ln 2\alpha + \ln(1 + e^{-x_1^p}) \\ &\Leftrightarrow \; x_1^p = -\ln\left[e^{x_2^p - \ln v - \ln 2\alpha} - 1\right] \overset{def}{=} f(x_2^p) \end{aligned} \tag{A16}$$

(11) and (A16) imply that the equilibrium value of x_2^p, denoted $x_2^{p,NE}$, solve the following equation:

$$f(x_2^p) = g(x_2^p) \tag{A17}$$

As in the proof of lemma 2 for the copyright regime, we cannot derive a closed-form solution for $x_2^{p,NE}$. However, we can show that an equilibrium value *exists* and is *unique*. To show that, we consider the functions $f(.)$ and $g(.)$ on the interval $(\underline{x}_2^p, \overline{x}_2^p]$ (because $g(.)$ is only defined on this interval). We first show that both functions are strictly decreasing on this interval with $\left|\frac{\partial f(x_2^p)}{\partial x_2^p}\right| > \left|\frac{\partial g(x_2^p)}{\partial x_2^p}\right|$,and then that the following inequalities hold: $g(\underline{x}_2^p) < \lim_{x_2^p \mapsto \underline{x}_2^p} f(x_2^p)$ and $g(\overline{x}_2^p) > f(\overline{x}_2^p)$.

Simple computations yields:

$$\frac{\partial f(x_2^p)}{\partial x_2^p} = -\frac{e^{x_2^p - \ln v + \ln 2\alpha}}{e^{x_2^p - \ln v + \ln 2\alpha} - 1} < 0 \tag{A18}$$

$$\frac{\partial g(x_2^p)}{\partial x_2^p} = \frac{-e^{-x_2^p}}{1 + e^{-x_2^p}} < 0 \tag{A19}$$

The negative sign in (A18) comes from the fact that $e^{x_2^p - \ln v + \ln 2\alpha} - 1 > 0 \Leftrightarrow x_2^p > \ln v - \ln 2\alpha = \underline{x}_2^p$ which is assumed here since we consider the interval $(\underline{x}_2^p, \overline{x}_2^p]$.

It is clear that

$$\begin{aligned} \left|\frac{\partial f(x_2^p)}{\partial x_2^p}\right| &> \left|\frac{\partial g(x_2^p)}{\partial x_2^p}\right| \\ &\Leftrightarrow \; -e^{-x_2^p} < e^{x_2^p - \ln v + \ln 2\alpha} \end{aligned}$$

which always holds.

Then, $\lim_{x_2^p \mapsto \underline{x}_2^p} f(x_2^p) = -\lim_{x_2^p \mapsto \underline{x}_2^p} \ln\left[e^{x_2^p - \ln v + \ln 2\alpha} - 1\right] = -\lim_{Z \mapsto 0} \ln Z = +\infty,$

whereas $g(\underline{x}_2^p) = \ln v - \ln 2 + \ln(1 + e^{-\ln v + \ln 2\alpha}) = \zeta_p$ with ζ_p being a positive finite number. It follows that:

$$g(\underline{x}_2^p) < \lim_{x_2^p \mapsto \underline{x}_2^p} f(x_2^p) \tag{A20}$$

Finally, $f(\overline{x}_2^p) = -\ln\left[e^{\ln v - \ln \alpha - \ln v - \ln 2\alpha} - 1\right] = 0$ whereas $g(\overline{x}_2^p) = x_1^p(\overline{x}_2^p) > \underline{x}_1^p$ since $\underline{x}_1^p$ is the lower bound of $g(.)$.By assumption, we have $\underline{x}_1^p > 0$ so we can conclude that

$$f(\overline{x}_2^p) < g(\overline{x}_2^P). \tag{A21}$$

3) We have shown that $f(.)$ and $g(.)$ are strictly decreasing on $(\underline{x}_2^p, \overline{x}_2^p]$ with $\lim_{x_2^p \mapsto \underline{x}_2^p} f(x_2^p) > g(\underline{x}_2^p)$ and $f(\overline{x}_2^p) > g(\overline{x}_2^p)$.From that, it follows that there exists a unique intersection between $f(.)$ and $g(.)$ on $(\underline{x}_2^p, \overline{x}_2^p]$ such that $f(x_2^p) = g(x_2^p)$.Consequently, there exists a unique Nash equilibrium in R&D investments, $(x_1^{p,NE}; x_2^{p;NE})$ where $x_2^{p,NE}$solves $f(x_2^p) = g(x_2^p)$ and $x_1^{p,NE} = g(x_2^{p,NE}) = f(x_2^{p,NE})$.Moreover, it is clear that $x_1^{p,NE} \in (\underline{x}_1^p, \overline{x}_1^p]$ and $x_2^{p,NE} \in (\underline{x}_2^p, \overline{x}_2^p)$. *QED.*

Proof of lemma 9. Consider equation (32) first. Solving for $U_i^{1,p}$, we obtain:

$$U_i^{1,p} = \frac{-\alpha_i \widetilde{x}_i + (1 - e^{-\widetilde{x}i})v}{e^{-\widetilde{x}i}} \tag{A22}$$

from this expression we obtain

$$v + U_i^{1,p} = \frac{v - \alpha_i \widetilde{x}_i}{e^{-\widetilde{x}_i}} \tag{A23}$$

and substituting for $v + U_i^{1,p}$ into equation (31) yields the following objective function for player i

$$U_i^{2,p} = -\alpha_i \widehat{x}_i^p + (1 - e^{-\widehat{x}_i^p}) \left\{ (1 - e^{-\widehat{x}_j^p}) \frac{v - \alpha_i \widetilde{x}_i}{2e^{-\widetilde{x}_i}} + e^{-\widehat{x}_j^p} \frac{v - \alpha_i \widetilde{x}_i}{e^{-\widetilde{x}_i}} \right\} \tag{A24}$$

which simplifies to

$$U_i^{2,p} = -\alpha_i \widehat{x}_i^p + \frac{1+e^{-\widehat{x}_j^p}}{2e^{-\widetilde{x}_i}}(1-e^{-\widehat{x}_i^p})(v-\alpha_i \widetilde{x}_i) \tag{A25}$$

We now solve for a Nash equilibrium in $\widehat{x}_i^p$, $i=1,2$, given $\widetilde{x}_i$. To determine $\widetilde{x}_i$ we compute the first-order condition associated with the optimization program of the winner of the first patent with respect to the R&D investment levels for the next researches. That is

$$\underset{\widetilde{x}_i}{Max} U_i^{1,p}$$

where $U_i^{1,p}$ is given by (A22).

The first-order condition is

$$\widetilde{x}_i = \frac{v-\alpha_i}{\alpha_i} \tag{A26}$$

Given our assumptions about the asymmetry between the two firms (firm 1 being more efficient than firm 2 in performing R&D), we can replace α_i by 1 for firm 1 and by α for firm 2 (with $\alpha > 1$).We thus obtain the optimal investment of the winner of the first race in the subsequent R&Ds

$$\widetilde{x}_1 = v-1 \tag{A27}$$

$$\widetilde{x}_2 = \frac{v-\alpha}{\alpha} \tag{A28}$$

Now, given $\widetilde{x}_i$ we consider the optimization program of the two players before the race for the first patent starts. That is

$$\underset{\widehat{x}_i}{Max} U_i^{2,p}$$

where $U_i^{2,p}$ is given by (A25).

The first order condition associated with this program defines the best response functions of each firm

$$e^{-\widehat{x}_i^p} = \frac{\alpha_i 2e^{-\widetilde{x}_i}}{(1+e^{-\widehat{x}_j^p})(v-\alpha_i \widetilde{x}_i)} \tag{A29}$$

Again, we can replace α_i by 1 for firm 1 and by α for firm 2. Also, we susbstitute $\widetilde{x}_i$ for its optimal value obtained in (A27) and (A28).

$$e^{-\widehat{x}_1^p} = \frac{2e^{1-v}}{1+e^{-\widehat{x}_2^p}} \tag{A30}$$
$$e^{-\widehat{x}_2^p} = \frac{2e^{-\frac{v-\alpha}{\alpha}}}{1+e^{-\widehat{x}_1^p}} \tag{A31}$$

which can be re-expressed as

$$\widehat{x}_1^p = v-1-\ln 2+\ln(1+e^{-\widehat{x}_2^p}) \tag{A32}$$
$$\widehat{x}_2^p = \frac{v-\alpha}{\alpha}-\ln 2+\ln(1+e^{-\widehat{x}_1^p}) \tag{A33}$$

(A30) and (A31) can be written:

$$e^{-\widehat{x}_1^p}(1+e^{-\widehat{x}_2^p}) = 2e^{1-v}$$
$$e^{-\widehat{x}_2^p}(1+e^{-\widehat{x}_1^p}) = 2e^{-\frac{v-\alpha}{\alpha}}$$

Substracting these two equalities yields $e^{-\widehat{x}_1^p}-e^{-\widehat{x}_2^p}=2(e^{1-v}-e^{\frac{\alpha-v}{\alpha}})$ which is negative since $v-1>\frac{v-\alpha}{\alpha}\Longrightarrow e^{1-v}<e^{\frac{\alpha-v}{\alpha}}$ for all $\alpha>1$.

We can conclude that $\widehat{x}_1^p>\widehat{x}_2^p$.

QED.

Proof of lemma 10: Again, the proof proceeds in two steps: we start by computing the upper and lower bounds of the best-response functions given by (33) and (34). Then, we show that there exists a unique Nash equilibrium in R&D investments.

1) The best-response function of player i is given by:

$$\widehat{x}_i^p(\widehat{x}_j^p)=\frac{v-\alpha_i}{\alpha_i}-\ln 2+\ln(1+e^{-\widehat{x}_j^p}) \tag{A34}$$

for $i\neq j$. Note that $\widehat{x}_i^p$ and $\widehat{x}_j^p$ are strategic substitutes since $\frac{\partial\widehat{x}_i^p(\widehat{x}_j^p)}{\partial\widehat{x}_j^p}=\frac{-e^{\widehat{x}_j^p}}{1+e^{-\widehat{x}_j^p}}<0$ and the upper bound of $\widehat{x}_i^p(.)$ is reached for $\widehat{x}_j^p=0$ whereas the lower bound is given by the constant asymptot defined by $\lim\limits_{\widehat{x}_j^p \mapsto +\infty} \widehat{x}_i^p(\widehat{x}_j^p)$. We have:

$$\overline{\widehat{x}_i^p} = \widehat{x}_i^p(0) = \frac{v-\alpha_i}{\alpha_i} \tag{A35}$$

$$\underline{\widehat{x}_i^p} = \lim_{\widehat{x}_j^p \mapsto +\infty} \widehat{x}_i^p(\widehat{x}_j^p) = \frac{v-\alpha_i}{\alpha_i} - \ln 2 \tag{A36}$$

Or, substituting $\alpha_1 = 1$ and $\alpha_2 = \alpha > 1$ in (A35) and (A36),

$$\begin{cases} \overline{\widehat{x}_1^p} = v - 1 \\ \underline{\widehat{x}_1^p} = v - 1 - \ln 2 \end{cases} \tag{A37}$$

$$\begin{cases} \overline{\widehat{x}_2^p} = \frac{v-\alpha}{\alpha} \\ \underline{\widehat{x}_2^p} = \frac{v-\alpha}{\alpha} - \ln 2 \end{cases} \tag{A38}$$

2) Now consider the system of best-response functions formed by (33) and (34):

$$\widehat{x}_1^p(\widehat{x}_2^p) = v - 1 - \ln 2 + \ln(1 + e^{-\widehat{x}_2^p}) \tag{A39}$$

$$\widehat{x}_2^p(\widehat{x}_1^p) = \frac{v-\alpha}{\alpha} - \ln 2 + \ln(1 + e^{-\widehat{x}_1^p}) \tag{A40}$$

Denote $\widehat{x}_1^p(\widehat{x}_2^p) \overset{def}{=} g(\widehat{x}_2^p)$ and from $\widehat{x}_2^p(\widehat{x}_1^p)$ we can derive:

$$\widehat{x}_2^c = \frac{v-\alpha}{\alpha} - \ln 2 + \ln(1 + e^{-\widehat{x}_1^p})$$

$$\Leftrightarrow \ \widehat{x}_1^p = -\ln\left[e^{\widehat{x}_2^p + 1 + \ln 2 - \frac{v}{\alpha}} - 1\right] \overset{def}{=} f(\widehat{x}_2^p) \tag{A41}$$

(33) and (A41) imply that the equilibrium value of $\widehat{x}_2^p$, denoted $\widehat{x}_2^{p,NE}$ solve the following equation:

$$f(\widehat{x}_2^p) = g(\widehat{x}_2^p) \tag{A42}$$

We can show that an equilibrium value *exists* and is *unique*. To show that, we consider the functions $f(.)$ and $g(.)$ on the interval $(\underline{\widehat{x}_2^p}, \overline{\widehat{x}_2^p}]$ (because $g(.)$ is defined only on this interval). We first show that both functions are strictly decreasing on this interval with $\left|\frac{\partial f(\widehat{x}_2^p)}{\partial \widehat{x}_2^p}\right| > \left|\frac{\partial g(\widehat{x}_2^p)}{\partial \widehat{x}_2^p}\right|$ and then that the following inequalities hold: $g(\underline{\widehat{x}_2^p}) < \lim\limits_{\widehat{x}_2^p \mapsto \underline{\widehat{x}_2^p}} f(\widehat{x}_2^p)$ and $g(\overline{\widehat{x}_2^p}) > f(\overline{\widehat{x}_2^p})$.

simple computations yield:

$$\frac{\partial f(\widehat{x}_2^p)}{\partial \widehat{x}_2^p} = -\frac{e^{\widehat{x}_2^p + 1 + \ln 2 - \frac{v}{2}}}{e^{\widehat{x}_2^p + 1 + \ln 2 - \frac{v}{2}} - 1} < 0 \tag{A43}$$

$$\frac{\partial g(\widehat{x}_2^p)}{\partial \widehat{x}_2^p} = \frac{-e^{-\widehat{x}_2^p}}{1 + e^{-\widehat{x}_2^p}} < 0 \tag{A44}$$

The sign of (A43) comes from the fact that $e^{\widehat{x}_2^p+1+\ln 2-\frac{v}{\alpha}}-1>0 \Leftrightarrow \widehat{x}_2^p > \frac{v-\alpha}{\alpha}-\ln 2=\underline{\widehat{x}}_2^p$ which holds here because we consider the interval $(\underline{\widehat{x}}_2^p,\overline{\widehat{x}}_2^p]$.

It is clear that:

$$\left|\frac{\partial f(\widehat{x}_2^p)}{\partial \widehat{x}_2^p}\right| > \left|\frac{\partial g(\widehat{x}_2^p)}{\partial \widehat{x}_2^p}\right|$$
$$\Longleftrightarrow \quad -e^{-\widehat{x}_2^p} < e^{\widehat{x}_2^p+1+\ln 2-\frac{v}{\alpha}}$$

which always holds.

Then, $\lim_{\widehat{x}_2^p\mapsto\underline{\widehat{x}}_2^p} f(\widehat{x}_2^p) = -\lim_{\widehat{x}_2^p\mapsto\underline{\widehat{x}}_2^p} \ln\left[e^{\widehat{x}_2^p+1+\ln v-\frac{v}{\alpha}}-1\right] = -\lim_{Z\mapsto 0}\ln Z=+\infty$

whereas $g(\underline{\widehat{x}}_2^p)=v-1-\ln 2+\ln\left[1+e^{-\frac{v-\alpha}{\alpha}+\ln 2}\right]=\widehat{\zeta}_p$ with $\widehat{\zeta}_p$ being a positive finite number. It follows that:

$$g(\underline{\widehat{x}}_2^p) < \lim_{\widehat{x}_2^p\mapsto\underline{\widehat{x}}_2^p} f(\widehat{x}_2^p). \tag{A45}$$

Finally, $f(\overline{\widehat{x}}_2^p)=-\ln\left[e^{\frac{v-\alpha}{\alpha}+1+\ln 2-\frac{v}{\alpha}}-1\right]=0$ and $g(\overline{\widehat{x}}_2^p)=\widehat{x}_1^p(\overline{\widehat{x}}_2^p)>\underline{\widehat{x}}_1^p$ since $\underline{\widehat{x}}_1^p$ is the lower bound of $g(.)$. And by assumption we have $\underline{\widehat{x}}_1^p>0$. we can thus conclude that

$$f(\overline{\widehat{x}}_2^p) < g(\overline{\widehat{x}}_2^p). \tag{A46}$$

3) We have shown that $f(.)$ and $g(.)$ are strictly decreasing on $(\underline{\widehat{x}}_2^p,\overline{\widehat{x}}_2^p]$ with $\lim_{\widehat{x}_2^p\mapsto\underline{\widehat{x}}_2^p} f(\widehat{x}_2^p) > g(\underline{\widehat{x}}_2^p)$ and $f(\overline{\widehat{x}}_2^p) < g(\overline{\widehat{x}}_2^p)$. From that it follows that there exists a unique intersection between $f(.)$ and $g(.)$ on $(\underline{\widehat{x}}_2^p,\overline{\widehat{x}}_2^p]$ such that $f(\widehat{x}_2^p = g(\widehat{x}_2^p)$.Consequently, there exists a unique Nash equilibrium in R&D investments, $(\widehat{x}_1^{p,NE},\widehat{x}_2^{p,NE})$ where $\widehat{x}_2^{p,NE}$ solves $f(\widehat{x}_2^p)=g(\widehat{x}_2^p)$ and $\widehat{x}_1^{p,NE}=f(\widehat{x}_2^{p,NE})=g(\widehat{x}_2^{p,NE})$. Moreover, it is clear that $\widehat{x}_1^{p,NE}\in(\underline{\widehat{x}}_1^p,\overline{\widehat{x}}_1^p]$ and $\widehat{x}_2^{p,NE}\in(\underline{\widehat{x}}_2^p,\overline{\widehat{x}}_2^p)$. *QED.*

Proof of lemma 11: Consider the objective function of player i given by (37). Substituting for $p(\widehat{x}_i)$ and $p(\widehat{x}_j)$ and solving for $U_i^{2,c}$, it can be rewritten as:

$$U_i^{2,c}=\frac{1}{e^{-\widehat{x}_i^c}e^{-\widehat{x}_j^c}}\left[-\alpha_i\widehat{x}_i+\frac{v}{2}+\frac{v}{2}\theta e^{-\widehat{x}_j^c}-\frac{v}{2}\theta e^{-\widehat{x}_i^c}-\frac{v}{2}e^{-\widehat{x}_i^c}e^{-\widehat{x}_j^c}\right] \tag{A47}$$

The objective of player i is defined as:

$$\underset{\widehat{x}_i}{Max} U_i^{2,c}$$

The first-order condition associated with this problem is:

$$\widehat{x}_i^c = \frac{-2\alpha_i + v(1+\theta e^{-\widehat{x}_j^c})}{2\alpha_i} \tag{A48}$$

Again, plugging in $\alpha_1 = 1$ and $\alpha_2 = \alpha > 1$ into (A48), we obtain the best-response functions $\widehat{x}_1^c(\widehat{x}_2^c)$ and $\widehat{x}_2^c(\widehat{x}_1^c)$. *QED.*

Proof of lemma 12: We proceed as for the proof of lemmas 2,4 and 10 in deriving the upper and lower bounds. The proof of the existence and uniqueness of the Nash equilibrium requires nevertheless another approach.

1) The best response function of player i is given by:

$$\widehat{x}_i^c = \frac{-2\alpha_i + v(1+\theta e^{-\widehat{x}_j^c})}{2\alpha_i} \tag{A49}$$

for $i \neq j$. $\widehat{x}_i^c$ and $\widehat{x}_j^c$ are strategic substitutes since $\frac{\partial \widehat{x}_i^c(\widehat{x}_j^c)}{\partial \widehat{x}_j^c} = -\frac{\theta v}{2\alpha_i} e^{-\widehat{x}_j^c} < 0$ and thus the upper bound of $\widehat{x}_i^c$ is reached for $\widehat{x}_j^c = 0$ whereas the lower bound is given $\lim_{\widehat{x}_j^c \mapsto +\infty} \widehat{x}_i^c(\widehat{x}_j^c)$. We have:

$$\widehat{x}_i^c(0) = \overline{\widehat{x}}_i^c = \frac{-2\alpha_i + v(1+\theta)}{2\alpha_i} \tag{A50}$$

$$\lim_{\widehat{x}_j^c \mapsto +\infty} \widehat{x}_i^c(\widehat{x}_j^c) = \underline{\widehat{x}}_i^c = \frac{-2\alpha_i + v}{2\alpha_i} \tag{A51}$$

Substituting for $\alpha_1 = 1$ and $\alpha_2 = \alpha > 1$ in (A50) and (A51):

$$\begin{cases} \overline{\widehat{x}}_1^c = \frac{-2+v(1+\theta)}{2} \\ \underline{\widehat{x}}_1^c = \frac{-2+v}{2} \end{cases} \tag{A52}$$

$$\begin{cases} \overline{\widehat{x}}_2^c = \frac{-2\alpha+v(1+\theta)}{2\alpha} \\ \underline{\widehat{x}}_2^c = \frac{-2\alpha+v}{2\alpha} \end{cases} \tag{A53}$$

2) Now, considering the system of best-response functions defined by (38) and (39). Let us denote $f(\widehat{x}_2^c) \overset{def}{=} \widehat{x}_1^c$ and $g(\widehat{x}_1^c) \overset{def}{=} \widehat{x}_2^c$:

$$\widehat{x}_1^c(\widehat{x}_2^c) = \frac{-2 + v(1+\theta e^{-\widehat{x}_2^c})}{2} \overset{def}{=} f(\widehat{x}_2^c)$$

$$\widehat{x}_2^c(\widehat{x}_1^c) = \frac{-2\alpha + v(1+\theta e^{-\widehat{x}_1^c})}{2\alpha} \overset{def}{=} g(\widehat{x}_1^c)$$

Clearly,

$$\begin{aligned} f(\widehat{x}_2^c) + 1 &= \frac{v(1+\theta e^{-\widehat{x}_2^c})}{2} \\ g(\widehat{x}_1^c) + 1 &= \frac{v(1+\theta e^{-\widehat{x}_1^c})}{2\alpha} \end{aligned}$$

Denoting $y_1 = \widehat{x}_1^c + 1 = f(\widehat{x}_2^c) + 1$ and $y_2 = \widehat{x}_2^c + 1 = g(\widehat{x}_1^c) + 1$,

$$y_1 = \frac{v(1+\theta e^{-(y_2-1)})}{2} \stackrel{def}{=} \phi(y_2) \tag{A54}$$

$$y_2 = \frac{v(1+\theta e^{-(y_1-1)})}{2\alpha} \tag{A55}$$

Inverting (A55) yields:

$$y_1 = 1 - \ln[\frac{2\alpha y_2 - v}{\theta}] \stackrel{def}{=} \mu(y_2) \tag{A56}$$

$\mu(.)$ is defined only on $[\frac{v}{2\alpha}, +\infty)$. To show the existence of an equilibrium, we compute:

$$\begin{cases} \phi(\frac{v}{2\alpha}) = \frac{v(1+\theta e^{-(\frac{v}{2\alpha}-1)})}{2} \\ \lim_{y_2 \mapsto \frac{v}{2\alpha}} \mu(y_2) = +\infty \end{cases} \tag{A57}$$

And

$$\begin{cases} \lim_{y_2 \mapsto +\infty} \phi(y_2) = \frac{v}{2} \\ \lim_{y_2 \mapsto +\infty} \mu(y_2) = -\infty \end{cases} \tag{A58}$$

Thus an equilibrium exists.

To show the uniqueness, we prove that:

$$\left|\frac{\partial \mu(y_2)}{\partial y_2}\right| > \left|\frac{\partial \phi(y_2)}{\partial y_2}\right| \tag{A59}$$

at the equilibrium point.

We have:

$$\begin{cases} \left|\frac{\partial \mu(y_2)}{\partial y_2}\right| = \frac{2\alpha}{2\alpha y_2 - v} \\ \left|\frac{\partial \phi(y_2)}{\partial y_2}\right| = \frac{v\theta}{2} e^{-(y_2-1)} \end{cases} \tag{A60}$$

The equilibrium condition is given by (A54) and (A56):

$$\frac{v\theta e^{-(y_2-1)})}{2} = 1 - \frac{v}{2} - \ln(\frac{2\alpha y_2 - v}{\theta}) \tag{A61}$$

Inserting into (A59) yields:

$$\frac{2\alpha}{2\alpha y_2 - v} > 1 - \frac{v}{2} - \ln[\frac{2\alpha y_2 - v}{\theta}] \tag{A62}$$

Since we have the general result: $\ln x \geq 1 - x^{-1}$, the last inequality holds for certain if:

$$\begin{aligned} \frac{2\alpha}{2\alpha y_2 - v} &> 1 - \frac{v}{2} - (1 - \frac{\theta}{2\alpha y_2 - v}) \\ &\iff \frac{2\alpha}{2\alpha y_2 - v} > -\frac{v}{2} + \frac{\theta}{2\alpha y_2 - v} \end{aligned} \tag{A63}$$

This holds if:

$$\frac{2\alpha - \theta}{2\alpha y_2 - v} > -\frac{v}{2} \tag{A64}$$

But, given that $\alpha > 1$,

$$\begin{cases} \theta \in [0,1] \Longrightarrow 2\alpha - \theta > 0 \\ y_2 > \frac{v}{2\alpha} \Longrightarrow 2\alpha y_2 - v > 0 \end{cases}$$

Hence (A64) holds.

We can conclude that there exists a unique equilibrium value for y_2 and thus a unique equilibrium value for y_1. Then, since $\widehat{x}_1^c = y_1 - 1$ and $\widehat{x}_2^c = y_2 - 1$, there exists as well a unique equilibrium value for $\widehat{x}_1^c$and $\widehat{x}_2^c$ denoted $\widehat{x}_1^{c,NE}$ and $\widehat{x}_2^{c,NE}$. *QED.*

Proof of Proposition 6:

i) The expected aggregate investment in a regime with no protection has been computed in the proof of proposition 4 i):

$$\begin{cases} \widehat{X}^n = be^b(1 - e^{-b}) \\ b = \frac{v(\alpha+1) - 4\alpha}{2\alpha} \end{cases} \tag{A65}$$

and the socially optimal expected investment comes from lemma 14. Since $\widehat{x}^{s,*} = v - 1$, we have:

$$\begin{cases} \widehat{X}^{s,*} = re^r(1 - e^{-r}) \\ r = \widehat{x}^{s,*} = v - 1 \end{cases} \tag{A66}$$

But $r > b$ implies that $\widehat{X}^n < \widehat{X}^{s,*}$.

ii) The expected aggregate investment in a patent regime *from period 1 on* is:

$$\begin{cases} \widehat{I}^p = \widehat{x}_1^p + \widehat{x}_2^p + (1 - e^{-\widehat{x}_1^p})e^{-\widehat{x}_2^p}\Omega_1 + (1 - e^{-\widehat{x}_2^p})e^{-\widehat{x}_1^p}\Omega_2 \\ \qquad +(1 - e^{-\widehat{x}_1^p})(1 - e^{-\widehat{x}_2^p}) \left[\frac{1}{2}\Omega_1 + \frac{1}{2}\Omega_2\right]. \\ \Omega_1 = \widetilde{x}_1 e^{\widetilde{x}_1} \\ \Omega_2 = \widetilde{x}_2^p e^{\widetilde{x}_2^p} \end{cases} \tag{A67}$$

whereas the socially optimal expected investment is given by $\widehat{X}^{s,*}$.

We can rewrite (A67) as follows:

$$\widehat{I}^p = \widehat{x}_1^p + \widehat{x}_2^p + \frac{1}{2}(1 - e^{-\widehat{x}_1^p} + e^{-\widehat{x}_2^p} - e^{-\widehat{x}_1^p - \widehat{x}_2^p})\widetilde{x}_1 e^{\widetilde{x}_1} + \frac{1}{2}(1 - e^{-\widehat{x}_2^p} + e^{-\widehat{x}_1^p} - e^{-\widehat{x}_1^p - \widehat{x}_2^p})\widetilde{x}_2 e^{\widetilde{x}2} \tag{A68}$$

We now evaluate how this expected aggregate investment evolves when the degree of asymmetry α between the firms increases, from the situation where the firms are symmetric $(\alpha = 1)$: $\frac{d\widehat{I}^p}{d\alpha}|_{\alpha=1}$. . We use the implicit function theorem to express $\frac{d\widehat{x}_i^p}{d\alpha}$ for $i = 1, 2$.

Given the best-response functions in lemma 9, we have:

$$\begin{cases} F(\widehat{x}_1^p, \widehat{x}_2^p; \alpha) = \widehat{x}_1^p - v + 1 + \ln 2 - \ln(1 + e^{-\widehat{x}_2^p}) = 0 \\ G(\widehat{x}_1^p, \widehat{x}_2^p; \alpha) = \widehat{x}_2^p - \frac{v-\alpha}{\alpha} + \ln 2 - \ln(1 + e^{-\widehat{x}_1^p}) = 0 \end{cases} \tag{A69}$$

We compute the following derivatives:

$$\begin{cases} F_1 = \frac{\partial F(.)}{\partial \widehat{x}_1^p} = 1; \; F_2 = \frac{\partial F(.)}{\partial \widehat{x}_2^p} = \frac{1}{1+e^{-\widehat{x}_2^p}} e^{-\widehat{x}_2^p}; \; F_3 = \frac{\partial F(.)}{\partial \alpha} = 0 \\ G_1 = \frac{\partial G(.)}{\partial \widehat{x}_1^p} = \frac{1}{1+e^{-\widehat{x}_1^p}} e^{-\widehat{x}_1^p}; \; G_2 = 1; \; G_3 = \frac{v}{\alpha^2} \end{cases} \tag{A70}$$

Now, we determine the following Jacobians:

$$\begin{cases} J = \begin{vmatrix} F_1 & F_2 \\ G_1 & G_2 \end{vmatrix} = 1 - \frac{e^{-\widehat{x}_1^p - \widehat{x}_2^p}}{(1+e^{-\widehat{x}_1^p})(1+e^{-\widehat{x}_2^p})} \neq 0 \\ J_1 = \begin{vmatrix} -F_3 & F_2 \\ -G_3 & G_2 \end{vmatrix} = \frac{v}{\alpha^2} \frac{e^{-\widehat{x}_2^p}}{1+e^{-\widehat{x}_2^p}} \\ J_2 = \begin{vmatrix} F_1 & -F_3 \\ G_1 & -G_3 \end{vmatrix} = -\frac{v}{\alpha^2} \end{cases} \tag{A71}$$

From which it follows that:

$$\begin{cases} \frac{d\widehat{x}_1^p}{d\alpha} = \frac{\frac{v}{\alpha^2} e^{-\widehat{x}_2^p}(1+e^{-\widehat{x}_1^p})}{1+e^{-\widehat{x}_2^p}+e^{-\widehat{x}_1^p}} > 0 \\ \frac{d\widehat{x}_2^p}{d\alpha} = -\frac{\frac{v}{\alpha^2}(1+e^{-\widehat{x}_1^p})(1+e^{-\widehat{x}_2^p})}{1+e^{-\widehat{x}_1^p}+e^{-\widehat{x}_2^p}} < 0 \end{cases} \tag{A72}$$

Moreover, it is easy to see from lemma 9 that $\frac{d\widetilde{x}_1}{d\alpha} = 0$ and $\frac{d\widetilde{x}_2}{d\alpha} = -\frac{v}{\alpha^2}$

Given (A68):

$$\begin{aligned}\frac{d\widehat{I}^p}{d\alpha} &= \frac{d\widehat{x}_1^p}{d\alpha}\left[1+\frac{1}{2}e^{-\widehat{x}_1^p}\widetilde{x}_1 e^{\widetilde{x}_1}+\frac{1}{2}e^{-\widetilde{x}_1-\widetilde{x}_2}\widetilde{x}_1 e^{\widetilde{x}_1}-\frac{1}{2}e^{-\widetilde{x}_1}\widetilde{x}_2 e^{\widetilde{x}_2}+\frac{1}{2}e^{-\widehat{x}_1^p-\widehat{x}_2^p}\widetilde{x}_2 e^{\widetilde{x}_2}\right]+ \quad \text{(A73)}\\ &\frac{d\widehat{x}_2}{d\alpha}\left[1-\frac{1}{2}e^{-\widehat{x}_2^p}\widetilde{x}_1 e^{\widetilde{x}_1}+\frac{1}{2}e^{-\widehat{x}_1^p-\widehat{x}_2^p}\widetilde{x}_1 e^{\widetilde{x}_1}+\frac{1}{2}e^{-\widehat{x}_2^p}\widetilde{x}_2 e^{\widetilde{x}_2}+\frac{1}{2}e^{-\widehat{x}_1^p-\widehat{x}_2^p}\widetilde{x}_2 e^{\widetilde{x}_2}\right]-\\ &\frac{v}{\alpha^2}(e^{\widetilde{x}_2}+\widetilde{x}_2 e^{\widetilde{x}_2})\frac{1}{2}(1-e^{-\widehat{x}_2^p}+e^{-\widehat{x}_1^p}-e^{-\widehat{x}_1^p-\widehat{x}_2^p})\end{aligned}$$

Noting that, at $\alpha = 1$, we have $\widetilde{x}_1 = \widetilde{x}_2 = \widetilde{x} = v - 1$, and $\widehat{x}_1^p = \widehat{x}_2^p = \widehat{x}$:

$$\begin{aligned}\frac{d\widehat{I}^p}{d\alpha}|_{\alpha=1} &= \frac{d\widehat{x}_1^p}{d\alpha}(1+e^{-2\widehat{x}}\widetilde{x}e^{\widetilde{x}})+\frac{d\widehat{x}_2^p}{d\alpha}(1+e^{-2\widehat{x}}\widetilde{x}e^{\widetilde{x}})\\ &-v(e^{\widetilde{x}}+\widetilde{x}e^{\widetilde{x}})\frac{1}{2}(1-e^{-2\widehat{x}}) \qquad \text{(A74)}\end{aligned}$$

Hence:

$$\frac{d\widehat{I}^p}{d\alpha} < 0$$

Thus, from the point where the firms are symmetric ($\alpha = 1$), when the degree of asymmetry increases, the expected aggregate investment decreases. Since the investment of the efficient firm is maximal *and equal to the social optimum* $v-1$ when the investment of the inefficient one is 0,and since the investment of the inefficient firm is 0 for a high enough α (v fixed), it follows that for the smallest α ($\alpha = 1$), the expected aggregate investment is higher than the socially optimal investment.

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