Herding with costly information*

Klaus Kultti†
Department of Economics
University of Helsinki

Paavo Miettinen‡
Department of Economics
University of Helsinki

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Abstract

We consider a standard sequential decision to adopt/buy a good in a herding environment. The setup is same as in Sgroi (2002). Contrary to the basic herding case we introduce a cost that the agents have to pay for the information about their predecessors' actions. All agents receive informative signals as in the standard herding models but do not view the actions taken by their predecessors unless they pay the observation costs. In this set up the first and the second agents rely on their own signals when they make the decision to buy/adopt the good. Only the third agent is willing to buy the information on all of the preceding agents' actions. All agents following the third agent buy information on only one agent's action and decide to adopt/buy the good after updating their beliefs. What follows is that the two first agents' actions determine whether the rest of the agents will buy/adopt the good or not when information about the predecessors' actions is cheap enough. If the cost of the information about the predecessors' actions is very expensive then all the agents will act according to their own signals. If observing is free one gets the standard results. Finally we compare welfare between the cases where information about the preceding agents' actions has costs and the case where information about the actions is freely available.

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†Department of Economics, University of Helsinki, P.O. Box 54, Unioninkatu 37, FIN-00014 University of Helsinki, Finland. E-mail: kkultti@valt.helsinki.fi

‡Finnish Post Graduate Program in Economics and Department of Economics , University of Helsinki, P.O. Box 54, Unioninkatu 37. FIN-00014, University of Helsinki, Finland. E-mail: paavo.a.miettinen@helsinki.fi

1 Introduction

The basic set-up of herding or informational cascades has n agents who make sequential decision. Each agent observes the choices of the preceding agents, but not their information, and based on these choices and the agent's own information makes a choice. The outcome of the situation typically features herding, i.e. agents making the same choice as the preceding agent and ignoring his own information. There is now an extensive literature with applications on the subject the seminal articles being Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992). We consider perhaps the simplest set-up used by Sgroi (2002) where there are two choices and two states of the world. If the state of the world is good the agents should make an investment which returns twice the cost, and if the state of the world is bad the agents should not invest because the returns are zero. The cost of investment is independent of the state of the world, and both states are equally possible. Each agent receives an independently and identically distributed informative signal about the state of the world.

This is a totally symmetric situation where the expected return of investment before a signal (or any other information) is received is zero. The complication that we introduce is a cost of observing the previous agents. We think that it is not unreasonable to assume that observing any agent's action or choice is costly, and we simply assume constant observation cost $\omega > 0$ per agent so that observing k previous agents costs $k\omega$. This yields some interesting results: If the cost is very high no-one observes anyone, and each agent acts according to his signal only. If the costs are zero one gets the standard results of herding literature (eg. Bikhchandani, Hirshleifer and Welch (1992), Sgroi (2002)). If the costs are such that one could observe just one agent no-one observes, and one again gets the standard results. If the costs are strictly positive but such that an agent could observe two or more agents' actions only the third agent observes the actions of the two previous agents. All the agents from the third agent on observe only the action of the previous agent. It turns out that in a welfare sense the least welfare is generated when observing is very costly. More is generated when the observation costs are moderate, and the most welfare comes about when there are no costs of observation, and the agents observe all actions taken by their preceding agents. The welfare is measured by the number of agents that choose the right action with respect to the state of world. At least 93% of the expected number of agents who choose the right action when there are no costs of observation choose the right action as the observation has moderate costs.

2 The model

Suppose that N agents have to make a decision to adopt the new technology or product. The agents make their decisions sequentially and the order is exogenous but common knowledge among agents. The new technology/product can be either good or bad. There are two possible states of the world $\Sigma_i = \{\sigma, \overline{\sigma}\}$ where σ denotes the good and $\overline{\sigma}$ the bad state of the world. If the state of the world is good then the adopting agent will receive a benefit of b. If the state of the world is bad then the adopting agent will receive a benefit of b^L that is normalized to zero. The cost of adoption $c = \frac{b}{2}$ is same across the states and agents. The prior probability that the state of the world is good (or bad) is $\Pr[\sigma] = \frac{1}{2} = \Pr[\overline{\sigma}]$. Each agent receives a signal δ_i about the state of the world from the set $\Theta = \{S, \overline{S}\}, \theta_i \in \Theta$, where S denotes a signal that the state is good and $\overline{S}$ is a signal that the state is bad. Agent i's signal $\theta_i = S$ is denoted by S_i and $\theta_i = \overline{S}$ by $\overline{S_i}$. The conditional probabilities of the signals are $\Pr[S_i \mid \sigma] = p = \Pr[\overline{S_i} \mid \overline{\sigma}] > \frac{1}{2}$ and $\Pr[S_i \mid \overline{\sigma}] = (1-p) = \Pr[\overline{S_i} \mid \sigma] < \frac{1}{2}$. Thus the signals are informative about the state of the world. The agents do not observe the actions taken by their predecessors. The agents may get the information about what the previous agents did by paying a fee of ω for observing what some other agent did previously. Thus if an agent wants to know what all the previous agents did it will cost the agent ω times the number of agents that preceded the agent. In the following A_i denotes the agent's decision to buy the good and $\overline{A_i}$ denotes the decision not to buy. Similarly K_j denotes agent's decision to acquire agent j's ($j < i$) information and $\overline{K_j}$ denotes that the agent did not acquire agent j's info. It is assumed that the each agent in turn first decide how many preceding agents' actions to observe. The agent must decide which predecessors to observe simultaneously i.e. decision to view some preceding agents' action is not sequential.[1] The costs from these observations are then realized. Only after the agent has incurred the costs of observation the agent starts to get the information about her predecessors actions.

The agents use Bayes' rule to update their beliefs about the state of the world after observing other agents actions. If the agent receives an equal amount of good and bad signals then she will adopt with probability $\frac{1}{2}$.[2] This is the tie-braking rule which implies that her expected utility must be zero: $E[U] = \frac{1}{2}b - c = 0$.

3 First agent

The first agent does not have any agents before her and so she has no access to any information but her own. Thus she acts solely on the basis of her own signal. Thus the first agent buys only when she receives a good signal. If the first agent has a good signal then her expected utility from buying the good is

[1] Assuming that the agents decide sequentially about the predecessors that they observe does not affect the results. The properties of sequential observation are discussed more as we analyze the third agent's choice.

[2] That is $P[V_1|S\overline{S}S\overline{S}...S\overline{S}] = P[V_0|S\overline{S}S\overline{S}...S\overline{S}] = \frac{1}{2}$, where the amount of good and bad signals is equal.

$$E\left[U_1\left(A_1\right)\right] = \Pr\left[\sigma|S_1\right]b - c = \frac{2p-1}{2}b > 0. \quad (1)$$

Where $\Pr\left[\sigma|S_1\right] = \frac{\Pr[S_1|\sigma]\Pr[\sigma]}{\Pr[S_1|\sigma]\Pr[\sigma]+\Pr[S_1|\overline{\sigma}]\Pr[\overline{\sigma}]}$. Thus when her signal is good the expected utility is positive and it is worth buying the good. Suppose instead that the signal for the first agent is bad. Then her expected utility when buying is

$$E\left[U_1\left(A_1\right)\right] = \left(\frac{1}{2} - p\right)b < 0. \quad (2)$$

Thus she will not buy when she receives a bad signal and her utility is thus zero:

$U_1\left(\overline{A_1}\right) = 0.$

4 Second agent

The second agent is the first to consider the purchase of the first agent's info. The second agent can pay ω in order to see what the first agent did. Suppose that the second agent has a good signal. If the second agent refuses to buy the information on the first agents action then her expected utility from buying the good is the same as was the first agents utility above. The second agent will then buy only if her signal is good.

Suppose that the second agent decides to observe the action of the first agent. Given that the second agent has a good or a bad signal she will observe that the first agent either bought the good or not. Since the first agent was acting upon her own information only observing the choice of the first agent fully reveals the signal of the first agent to the observer. Observe that we suppress the observation costs from the calculations of the expected utilities. They are added to the analysis when comparing the expected utilities between acting upon an agent's own signal only or acting upon the agent's own signal plus the additional information gathered from the purchased information on the preceding agents' actions.

Thus observing that the first agent decided to buy the good implies that the first agent's signal is good. When an agent can deduce a preceding agent's signal from the action that the predecessor took we denote this by $A_1 \rightarrowtail S_1$. Then if the second agent observes $S_2; A_1$ the expected utility from buying the good to the second agent is

$E\left[U_2\left(A_2\right)|S_2;A_1\right] = \Pr\left[\sigma|A_1,S_2\right]b - c = \frac{2p-1}{2\left(p^2+(1-p)^2\right)}b$, where

$\Pr\left[\sigma|A_1,S_2\right] = \frac{\Pr[S_1,S_2|\sigma]\Pr[\sigma]}{\Pr[S_1,S_2|\sigma]\Pr[\sigma]+\Pr[S_1,S_2|\overline{\sigma}]\Pr[\overline{\sigma}]}$ and $c = \frac{b}{2}$.

Now if the second agent observes $S_2;\overline{A_1}$ then the expected utility from buying is

$$E\left[U_2\left(A_2\right)|S_2;\overline{A_1}\right]=\tfrac{1}{2}b-\tfrac{1}{2}b=0.$$

The case where the second agent observes $\overline{S_2};A_1$ is analogous to the case where she observes $S_2;\overline{A_1}$ and thus the expected utility is zero.

Finally if the second agent observes $\overline{S_2};\overline{A_1}$ the expected utility from buying is

$$E\left[U_2\left(A_2\right)|\overline{S_2};\overline{A_1}\right]=\tfrac{1-2p}{2\left[(1-p)^2+p^2\right]}b<0.$$

Thus observing $\overline{S_2};\overline{A_1}$ causes the second agent not to buy the good and receive a sure utility of zero.

Now in order to calculate the expected utilities from observing the first agent's signal we have to calculate the conditional probabilities of observing a specific action of the first action conditional on the second agent's signal. Then the probability of observing a good first agent's signal when the second agent's signal is good is

$$\Pr\left[S_1|S_2\right]=\tfrac{\Pr[S_1,S_2|\sigma]\Pr[\sigma]}{\Pr[S_2]}+\tfrac{\Pr[S_1,S_2|\overline{\sigma}]\Pr[\overline{\sigma}]}{\Pr[S_2]}=2p^2-2p+1.$$

We do not need to calculate the probabilities of observing bad signals since the second agents expected utility from buying after observing one or more bad signals is exactly zero at the optimal decision. Thus the expected utility from viewing the first agent's info is

$$E\left[U_2\left(A_2,K_1\right)|S_2\right]=\Pr\left[S_1|S_2\right]E\left[U_2\left(A_2\right)|S_2;A_1\right]=\frac{2p-1}{2}b. \qquad (3)$$

Thus her expected utility is exactly the same when she uses the first agent's info as when she does not. Since acquiring the first persons info costs ω we conclude that the second agent acts on her own signal alone.

5 Third agent

Now moving on to the third agent we know from the two previous agents' decisions that the third agent's expected utility from deciding on her own information alone is $\frac{2p-1}{2}b$. Since the first two agents have made their actions based solely on their signals then the third agent perceives agents 1 and 2 as identical from informational point of view. Since the second agent did not want to buy the information about the first agent's action the third agent does not want to buy the information about only the first or the second agent's information. It remains to analyze whether the third agent would want to buy the information on both the first and the second agents' actions. Again since the first and the second agents took their actions independently from any other information but their own signal the third agent can directly infer the signals of the second and the first agent by looking at their actions.

The possible observations and the corresponding expected utilities for the third agent are

Observe $S_3; A_1, A_2 \rightarrowtail S_3; S_1, S_2$

$E\left[U_3\left(A_3\right)|S_3; A_1, A_2\right] = \Pr\left[\sigma|A_1, A_2, S_3\right]b - c = \frac{p^3-(1-p)^3}{2\left(p^3+(1-p)^3\right)}b > 0$, since $p > \frac{1}{2}$.

Observing $S_3; \overline{A_1}, A_2 \rightarrowtail S_3; \overline{S_1}, S_2$ is equivalent to observing $S_3; A_1, \overline{A_2} \rightarrowtail S_3; S_1, \overline{S_2}$. The expected utility from buying is

$E\left[U_3\left(A_3\right)|S_3; \overline{A_1}, A_2\right] = \frac{p(1-p)[2p-1]}{2\left[(1-p)p^2+p(1-p)^2\right]} > 0.$

Observing $S_3; \overline{A_1}, \overline{A_2} \rightarrowtail S_3; \overline{S_1}, \overline{S_2}$ yields the expected utility from buying:

$E\left[U_3\left(A_3\right)|S_3; \overline{A_1}, \overline{A_2}\right] = \frac{(1-p)^2p-p^2(1-p)}{2\left[(1-p)^2p+p^2(1-p)\right]}b < 0$. Thus observing $\overline{A_1}, \overline{A_2}$ will induce the agent not to buy.

If the third agent's signal is $\overline{S_3}$ then only if she observes $\overline{S_3}; A_1, A_2 \rightarrowtail \overline{S_3}; S_1, S_2$ will she change her action from the one dictated by her signal and she will then buy. The expected utility of this is

$E\left[U_3\left(A_3\right)|\overline{S_3}; A_1, A_2\right] = \frac{p(1-p)[2p-1]}{2\left[p^2(1-p)+(1-p)^2p\right]}b > 0$, since $p > \frac{1}{2}$.

Observing any other feasible combination of the first and the second agents' signals would lead to a negative expected utility and to a decision of the third agent to refrain from buying when the third agent's signal is $\overline{S_3}$.

Given that the third agent's signal is S_3 the expected utility from observing the first and the second agents' signals is:

$$\begin{gathered} E\left[U_3\left(A_3, K_1, K_2\right)|S_3\right] = \\ \Pr\left[A_1, A_2|S_3\right]E\left[U_3\left(A_3\right)|S_3; A_1, A_2\right] \\ +2\Pr\left[\overline{A_1}, A_2|S_3\right]E\left[U_3\left(A_3\right)|S_3; \overline{A_1}, A_2\right] = \\ \left[p^3\left(3-2p\right)-\left(1+2p\right)\left(1-p\right)^3\right]\frac{b}{2}, \end{gathered} \tag{4}$$

where $\Pr\left[A_1, A_2|S_3\right] = \frac{\Pr[A_1,A_2,S_3|\sigma]\Pr[\sigma]}{\Pr[S_3]} + \frac{\Pr[A_1,A_2,S_3|\overline{\sigma}]\Pr[\overline{\sigma}]}{\Pr[S_3]}$ and

$$\Pr\left[\overline{A_1}, A_2|S_3\right] = \frac{\Pr\left[\overline{A_1}, A_2, S_3|\sigma\right]\Pr[\sigma]}{\Pr[S_3]} + \frac{\Pr\left[\overline{A_1}, A_2, S_3|\overline{\sigma}\right]\Pr[\overline{\sigma}]}{\Pr[S_3]}.$$

Now we can compare this to the expected utility when the agent does not observe any actions by the previous agents:

$\frac{2p-1}{2}b < \left[-2p^3 + 3p^2 + p - 1\right]\frac{b}{2} \Leftrightarrow 1 < p(3-2p)$, which holds for $p \in \left(\frac{1}{2}, 1\right)$.

Given that the third agent's signal is $\overline{S_3}$ the expected utility from observing the first and the second agents' signals is:

$$E\left[U_3(A_3, K_1, K_2,)\,|\overline{S_3}\right] = \frac{p(1-p)[2p-1]}{2}b. \tag{5}$$

Now since the agent's signal was bad, the agent would receive a sure benefit of zero if she will not observe. Thus since $\frac{p(1-p)[2p-1]}{2}b > 0$ it is worth for the third agent to observe both the first and the second agents' signals given that the cost of observation is moderate.

Definition 1 *Observation costs are moderate if they are strictly positive and if it is worthwhile for the third agent to observe both her predecessors' actions.*

6 Fourth agent

Now the fourth agent has multiple choices to make. First we note that the fourth agent may always copy the third (and thus the second or the first) agent since the same options are available to the fourth as were to the third agent. Additionally the fourth agent may wish to extract some extra information by observing some combination of the first, second and third agents' actions. Since the first two agents make their actions based only on their signals the fourth agent can deduce some things using the information contained in the first or second and third agents' actions. More specifically: the fourth agent can deduce in some cases what was the second agent's signal from the knowledge of the first and the third agents' actions or vice versa the fourth agent may deduce the first agent's actions by observing the second and the third agents' actions.

6.1 Fourth agent observes all preceding agents' actions

If the fourth agent decides to observe all actions and pay 3ω then the possible things that she may see are:

1. $A_1, \underline{A_2}, A_3$
2. $\underline{A_1}, A_2, A_3$
3. $A_1, \underline{A_2}, \underline{A_3}$
4. $\underline{A_1}, A_2, \underline{A_3}$
5. A_1, A_2, A_3

6. $\overline{A_1}, \overline{A_2}, \overline{A_3}$
$(A_1, A_2, \overline{A_3})$
$(\overline{A_1}, \overline{A_2}, A_3)$

Now since agents 1 and 2 are symmetric cases 2 and 3 can be treated as symmetric and respectively cases 4 and 5 are symmetric. Thus we calculate expected utilities in 4 cases. The cases in parenthesis do not matter[3]. Suppose that the fourth agent's signal is good:

$$E\left[U_4\left(A_4\right)|S_4; A_1, A_2, A_3\right] = \Pr\left[\sigma|A_1, A_2, A_3, S_4\right] b - c = \frac{p^3 - (1-p)^3}{2\left[p^3 + (1-p)^3\right]} > 0.$$

$$E\left[U_4\left(A_4\right)|S_4; A_1, \overline{A_2}, A_3\right] = \frac{p^3(1-p) - (1-p)^3 p}{p^3(1-p) + (1-p)^3 p} > 0$$

$$E\left[U_4\left(A_4\right)|S_4; A_1, \overline{A_2}, \overline{A_3}\right] = 0.$$

$$E\left[U_4\left(A_4\right)|S_4; \overline{A_1}, \overline{A_2}, \overline{A_3}\right] = \frac{(1-p)p\left[(1-p)^2 - p^2\right]}{2\left[(1-p)^3 p + p^3(1-p)\right]} < 0$$

Now calculating the expected utility of the fourth agent when she observes all the preceding agents' actions is

$$\begin{gathered} E\left[U_4\left(A_4, K_1, K_2, K_3\right)|S_4\right] = \\ E\left[U_4\left(A_4\right)|S_4; A_1, \overline{A_2}, A_3\right] \Pr\left[A_1, \overline{A_2}, A_3|S_4\right] + \\ E\left[U_4\left(A_4\right)|S_4; A_1, A_2, A_3\right] \Pr\left[A_1, A_2, A_3|S_4\right] = \\ \frac{b}{2}\left[p^3\left(3 - 2p\right) - \left(1 - p\right)^3\left[1 + 2p\right]\right]. \end{gathered} \tag{6}$$

Thus observing all three previous agents' actions is strictly preferred to deciding solely on the basis on the agent's own signal, since the expected utility of using only the agent's own signal is $\frac{2p-1}{2}b$. Comparing this to the case where the fourth agent observes only 1 and 2 we see directly that the expected utility form observing all three previous agents' actions coincides with the expected utility of observing the two first agents' actions.

Next suppose that the fourth agent's signal is bad. The only case that we need to consider is when the fourth agent observes A_1, A_2, A_3. Observing anything else would induce the fourth agent not to buy. Then her expected utility from observing all three preceding actions is

$$E\left[U_4\left(A_4, K_1, K_2, K_3\right)|\overline{S_4}\right] = \frac{p\left(1-p\right)\left(2p-1\right)}{2} b > 0. \tag{7}$$

[3] The cases that are in parenthesis are not possible since agent 3 has decided to view both the first and the second agents' actions. If the two first agents would both have an opposing signal to that of the third agent's then the actions of the two first agents would dictate the third agent's action.

Thus buying the information gives the fourth agent a positive expected utility where as refusing to buy would give her zero utility. In addition we note that the fourth agent's utility is equally good when she would observe only agents 1 and 2. This fact comes from the third agent's case where we calculated that the expected utility of the third agent is $\frac{(1-p)p(2p-1)}{2}b$ when her signal is bad and she observed agents 1 and 2. Thus we may already conclude that buying only the first two agents' information is more beneficial than buying all three preceding agents' information simply because $3\omega > 2\omega$.

6.2 Fourth agent observes only two preceding agents' actions

Since the first and the second agents have equally good information when making their decisions it is sufficient to consider the case where fourth agent would buy the information on first and the third agents' actions. When the fourth agent has a good signal the possible observations are

1. $\underline{S_1}, A_3 \rightarrowtail S_2$ or $\overline{S_2}$
2. $S_1, \underline{A_3} \rightarrowtail \underline{S_2}$
3. $\underline{S_1}, \underline{A_3} \rightarrowtail S_2$
4. $S_1, A_3 \rightarrowtail S_2$ or $\overline{S_2}$

Now in the first case the expected utility is

$$E\left[U_4\left(A_4\right)|S_4; S_1, A_3\right] = \frac{p^3(2-p)-(1-p)^3(1+p)}{2\left[p^3+p^3(1-p)+(1-p)^3+(1-p)^3p\right]}b.$$

The expected utility in the second case is

$$E\left[U_4\left(A_4\right)|S_4; \overline{A_1}, A_3\right] = b\frac{(1-p)p\left(p^2-(1-p)^2\right)}{2\left[(1-p)p^3+p(1-p)^3\right]}$$

And in the third case

$$E\left[U_4\left(A_4\right)|S_4; A_1, \overline{A_3}\right] = b\frac{p^2(1-p)^2-p^2(1-p)^2}{2\left[p^2(1-p)^2+p^2(1-p)^2\right]} = 0.$$

The fourth case then yields

$$E\left[U_4\left(A_4\right)|S_4; \overline{A_1}, \overline{A_3}\right] = b\frac{(1-p)p(1-2p)}{2\left[p^2(1-p)^2+(1-p)^2p+p^2(1-p)^2+p^2(1-p)\right]} < 0.$$

Thus we can calculate the expected utility of buying the info from agents 1 and 3:

$$
\begin{gathered}
E\left[U_4\left(A_4, K_1, \overline{K_2}, K_3\right)|S_4\right] = \\
\Pr\left[A_1, A_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; S_1, A_3\right] + \Pr\left[\overline{A_1}, A_3|S_4\right] E\left[U_4\left(A_4\right)|S_4; \overline{A_1}, A_3\right] = \\
\frac{b}{2}\left[p^3\left(3-2p\right) - \left(1-p\right)^3\left(1+2p\right)\right].
\end{gathered} \tag{8}
$$

Thus the fourth agent will get the same expected benefit from observing only agents 1 and 3 as observing all preceding agents' actions. Since observations are costly observing only 1 and 3 (or 1 and 2) is better than observing all three preceding agents. Similarly we get that the utility from observing agents' 1 and 3 is equal to the utility of observing all agents' actions when the fourth agent's signal is bad.

6.3 Fourth agent observes only one preceding agent's action

The cases where the fourth agent observes only agents' 1 or 2 action have been analyzed with agent 3. It remains to check whether the fourth agent would want to observe only agent 3's action. The possible observations are:

1. $A_3 \rightarrowtail S_1, S_2$ or $\overline{S_1}, S_2$ or $S_1, \overline{S_2}$
2. $\overline{A_3} \rightarrowtail \overline{S_1}, \overline{S_2}$ or $\overline{S_1}, S_2$ or $S_1, \overline{S_2}$

Suppose again that the fourth agent has a good signal. Then the expected utility from making the first observation and buying is

$$E\left[U_4\left(A_4\right)|S_4; A_3\right] = \frac{b}{2}\frac{p^3(3-2p)-(1-p)^3(1-2p)}{\left[p^3+2p^3(1-p)+(1-p)^3+2(1-p)^3p\right]}.$$

Then the expected utility when observing the second case is

$$E\left[U_4\left(A_4\right)|S_4; \overline{A_3}\right] = \frac{b}{2}\frac{(1-p)p(1-2p)}{\left[(1-p)^2p+2p^2(1-p)^2+p^2(1-p)+2p^2(1-p)^2\right]} < 0.$$

Thus if the fourth agent observes a that the third agent did not buy then the fourth agent will not buy either and her utility will be zero less the observation cost. Now calculating the expected utility from observing only the third agent yields:

$$
\begin{gathered}
E\left[U_4\left(A_4, \overline{K_1}, \overline{K_2}, K_3\right)|S_4\right] = \Pr\left[A_3|S_4\right] E\left[U\left(A_4\right)|S_4; A_3\right] = \\
\frac{b}{2}\left[p^3\left(3-2p\right) - \left(1-p\right)^3\left(1+2p\right)\right]
\end{gathered} \tag{9}
$$

which is equal to the expected utility of observing only two or all preceding agents' actions.

Then if the fourth agent has a bad signal then in an equivalent manner we have that

$$E\left[U_4\left(A_4\right)|\overline{S_4};A_3\right]=\Pr\left[\sigma|A_3,\overline{S_4}\right]b-c=$$

$$\frac{b}{2}\frac{p(1-p)(2p-1)}{\left[p^2(1-p)+2p^2(1-p)^2+(1-p)^2p+2p^2(1-p)^2\right]}>0.$$

Since observing $\overline{A_3}$ when the fourth agent's signal is bad is sure to induce the fourth agent not to buy and yield her a utility of minus information acquisition cost we can calculate the expected utility from the acquisition of the third agent's information:

$$E\left[U_4\left(A_4,\overline{K_1},\overline{K_2},K_3\right)|\overline{S_4}\right]=\frac{b}{2}\left[p\left(1-p\right)\left(2p-1\right)\right]>0. \tag{10}$$

Thus it is optimal for the fourth agent to observe only agent 3. Given that the fourth agent does this she will then buy if her own signal is good and she observes that the third agent has bought. If the fourth agent has a good signal but observes that the third agent did not buy then the expected utility form buying is negative and she will not buy either. If the fourth agent has a bad signal then observing a buying decision from the third agent induces the fourth agent to buy as well and an observation of the third agent not buying will encourage the fourth agent to refrain from buying as well. Thus the fourth agent follows exactly what the third agent does.

Proposition 1 *Information aggregation stops after the third agent and all agents after the third agent observe only one agent i from the sequence such that $i \geq 3$.*

Above we have shown that the fourth agent mimics the third agent. Then since the fourth agent's action tells absolutely nothing about her information to someone who observes the fourth agent then observing the fourth agent is equivalent to observing the third agent in expected utility terms. Since observing only the third agent one obtains more utility than when one observes agents 1 and 2 since the observation is costly one is thus better of observing only the third agent or some agent after the third.

7 Welfare comparison

We define the social welfare as the number of agents that choose the "right" action. I.e. when the state of the world is good then the welfare is measured as the number of agents that choose to buy the good rather than forgo the opportunity of buying. The case where the state of the world is bad is symmetric. We compare the welfare in three different cases. We compare the situation when the cost of information is high to the case when the cost of information is low. Additionally we compare the situation where the cost of information is low to the case when information is freely available (I.e. the benchmark herding case).

The cost of the information may be high in the sense that it is not optimal for any agent to see what the preceding agent did. It suffices to consider the case where the cost of information is such that the third agent's expected utility is rendered negative if she were to buy the information on the actions taken by the first and the third agent. Since in this case it is optimal for the third agent to rely only on her own signal buying information on only one agent's action is not beneficial in expected utility terms as argued above in the second agent's case.

7.1 Expected number of agents that buy the good when the cost of information is high

Given that the cost of acquiring the information about the past actions taken by the preceding agents is high then all agents make their decisions based on their own signal only. It is sufficient that the cost of observing two persons is such that the third agent would have a negative expected utility from observing the actions taken by the first and the second agent.

When the state of the world is good the expected number of agents buying the good in that case is simply the fraction p of N.

$$E[\#\text{ agents buying the good}|\sigma] = pN. \tag{11}$$

7.2 Expected number of agents that buy the good when the cost of information is low

This is basically the case that we have analyzed above. Thus in this case the first and the second agents rely on their own signals when making the decision to purchase the good or not, the third agent buys the information on the first and the second agents' actions and then decides whether to buy or not. The fourth agent will then see what the third did and imitate the third agent. Then the fifth, sixth and all agents from there onwards will all view the action taken by some agent $n \geq 3$ and they will all take the same action as their observed agent since the information contained in any agent's $n \geq 3$ decision is the same as the information that the decision maker would get by viewing the third agent. Thus we have that if the state of the world is good then the correct cascade will arise if one of the three following action sequences are initially taken:

$A_1, A_2 \rightarrowtail$ correct cascade with probability $p^2 \rightarrowtail N$ agents buy the good.

$\overline{A_1}, A_2, A_3 \rightarrowtail$ correct cascade with probability $(1-p)p^2 \rightarrowtail N-1$ agents buy the good.

$A_1, \overline{A_2}, A_3 \rightarrowtail$ correct cascade with probability $(1-p)p^2 \rightarrowtail N-1$ agents buy the good.

$A_1, \overline{A_2}, \overline{A_3} \rightarrowtail$ incorrect cascade with probability $p\,(1-p)^2 \rightarrowtail 1$ agent buys the good.

$\overline{A_1}, A_2, \overline{A_3} \rightarrowtail$ incorrect cascade with probability $p\,(1-p)^2 \rightarrowtail 1$ agent buys the good.

Thus we have that the expected number of agents that make the right choice when the state is good is:

$$E[\#\text{ agents buying the good}|\sigma] = \tag{12}$$
$$p^2\left[N + 2\,(1-p)\,(N-1)\right] + 2p\,(1-p)^2\,.$$

Now we can compare whether the number of agents buying the good is larger in the case when the information is costly compared to the case when it is cheap[4]. We can leave out the term $p\,(1-p)^2$ and still the following inequality holds.

$(12) > (11) \Leftrightarrow p^2\left[N + 2\,(1-p)\,(N-1)\right] > pN \Leftrightarrow p > \frac{N}{2(N-1)}$.

Thus this holds always if p is sufficiently larger than $\frac{1}{2}$ or if N is large.

7.3 Expected number of agents that buy the good when there is no cost of observation

Suppose again that the state of the world is good. Then if there is no cost of observation and the agents will see the decisions taken by their predecessors the situation is equivalent to the standard herding model as in Bikhchandani et al.(1992), Banerjee (1992) or Sgroi (2002). Here it is possible that a cascade leading every subsequent agent to buy or to refrain from buying will not get started. If two subsequent agents take different actions their actions will effectively cancel out in the probabilistic sense. Thus if the first and the second agent make different decisions about buying the good the third agent is actually left with her own signal alone since the actions taken by the first and the second agent cancel each other out in informative sense. The posterior probability that the state of the world is good after observing two conflicting actions is $\frac{1}{2}$. Thus the third agent makes her decision according to her own signal alone. Then if the fourth agent again makes an action that is different from the third agent's action the the fifth agent is again in the same situation as the third agent and the process begins anew. One might thing about the sequence of agents as

[4] We could also compare the expected utility to the agents when costs are high or low. In this case we note that the first two agents make the same independent decision in both settings. The third agent could also make her decision independently when costs are moderate but she gets a higher expected utility from viewing both her predecessors. Since all agents that follow the third get more utility than the third agent since they pay less observation costs, utility to all agents from the third agent onwards is larger in the case when the observation costs are moderate.

a sequence of pairs. The cascade will then get started if there exists a pair where both agents make the same decision. Only if two agents in a pair make the same decision will all the subsequent agents follow. Otherwise the agents may be choosing actions that cancel each other out and the cascade never gets started or the cascade may get started but goes to another direction than when information is costly. We now need to calculate the expected number of buyers that will buy the good in this setting in order to be able to make comparisons between this case and the case where information is only moderately costly.

Thus if the first two agents buy the good the "yes-cascade" gets started. The probability of this is p^2. In this case all N agents would adopt the good. If both the first and the second agent decide that they will not buy the good a cascade where no agent after the first two agents buys the good gets started. The probability of this "no-cascade" is $(1-p)^2$. Neither of these cascades will start after the two first agents if they take different actions. The probability of this is of course $2(1-p)p$. After two conflicting actions this process starts anew. Thus the probability that the third and the fourth agent will buy the good after the first and the second made different choices is then $2(1-p)p\left(p^2\right)$. In this case the number of agents adopting the good would be $N-1$. If the third and the fourth agents have bad signals they will not buy the good and a no-cascade gets started. The probability of this is then $2(1-p)p(1-p)^2$. In a similar way the probability that the third and the fourth agent will not make same purchasing decisions when first and the second agents did not make the same decisions is again $2(1-p)p(2(1-p)p)$. Thus we can write out the expected number of agents that will buy the good when the state of the world is good as

$$E[\# \text{ agents buying the good}|\sigma] =$$
$$p^2N + 2(1-p)p\left(\begin{array}{c} p^2[N-1] + (1-p)^2 + \\ 2(1-p)p\left([N-2]p^2 + 2(1-p)^2\right) + \\ (2(1-p)p)^2\left([N-3]p^2 + 3(1-p)^2\right) + \ldots + \\ (2(1-p)p)^{\frac{N}{2}-2}\left(\left[N-\left(\frac{N}{2}-1\right)\right]p^2 + \left(\frac{N}{2}-1\right)(1-p)^2\right) + \\ (2(1-p)p)^{\frac{N}{2}-1}\frac{N}{2} \end{array}\right)$$

We are calculating the pairs of decisions that are made and thus we have assumed that the number of agents N (here even) is finite then there are $\frac{N}{2}$ terms to the sum. Defining $y = 2(1-p)p$ this can be re-written after some manipulations as

$$E[\# \text{ agents buying the good}|\sigma] =$$
$$p^2N\frac{1-y^{\frac{N}{2}}}{1-y} + (1-2p)\left(\frac{y - \frac{N}{2}y^{\frac{N}{2}}(1-y) - y^{\frac{N}{2}+1}}{(1-y)^2}\right) + y^{\frac{N}{2}}\frac{N}{2}. \qquad (13)$$

Now lets compare the expected number of agents in cases when the observation is free and in the case when observing predecessors actions is only moderately costly.

Suppose that the number of agents is largest when information of the preceding agents actions is not free but moderately costly. Then this implies that (12) > (13). This holds if

$$(1-y)^2 \left(p^2 \left[N + 2(1-p)(N-1)\right] + 2p(1-p)^2\right) >$$
$$p^2 N \left(1 - y^{\frac{N}{2}}\right)(1-y) - (1-2p)\left(y - \tfrac{N}{2} y^{\frac{N}{2}} (1-y) - y^{\frac{N}{2}+1}\right) + (1-y)^2 y^{\frac{N}{2}} \tfrac{N}{2}.$$

After some manipulations this reduces to

$$\tfrac{N}{2}(1-y) < 2 - y - y^{\frac{N}{2}-1}.$$

If $N = 4$ then the above holds with equality. If $N = 6$ or 8 we have a contradiction. Thus it can be shown by induction that

$$\frac{N}{2}(1-y) > 2 - y - y^{\frac{N}{2}-1}. \tag{14}$$

holds. Thus when observing other agents actions is costly the number of agents that make the right choice is always less than when there is no cost of observation. Additionally we want to know what is the share of the agents that make the right choice when the observation is costly in proportion to the case when the observation is free as N tends to infinity. Dividing(12) with (13) and additionally dividing by N we get that as N grows with out bounds then

$$\lim_{N \longrightarrow \infty} \frac{p^2 + 2(1-p)p^2 - \frac{2(1-p)p^2}{N} + \frac{2p(1-p)^2}{N}}{p^2 \frac{1-y^{\frac{N}{2}}}{1-y} + \left(\frac{\frac{(1-2p)y}{N} - (1-2p)\frac{1}{2} y^{\frac{N}{2}} (1-y) - \frac{(1-2p) y^{\frac{N}{2}+1}}{N}}{(1-y)^2}\right) + y^{\frac{N}{2}} \frac{1}{2}} = 1 + 2(1-p)^2 (1-2p).$$

This ratio is at the minimum as $p = \frac{2}{3}$ then the ratio is $25/27 \approx 0,93$. Were $p = \frac{1}{2}$ or $p = 1$ the ratio would be equal to one. That is when the signals carry no information at all or when they are fully informative the expected number of agents that make the right choice is the same between the cases when observation is moderately costly and when it is free. When $p \in (0,1)$ then at least 93% of the number of agents that make the right choice when the observation is free make the right choice as observation is moderately costly.

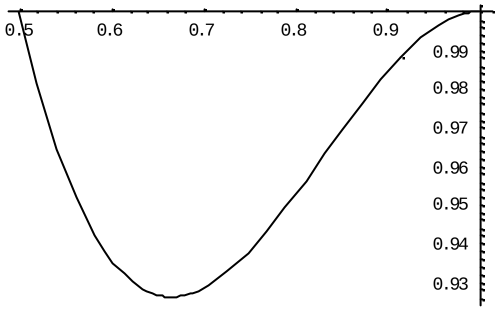

Figure 1: The ratio of expected number of agents that make the right choice when observation has moderate costs/observation is costless.

In the similar way we may calculate the ratio between the cases when the observation costs are high and when they are moderate. Thus dividing (11) with (12) and additionally dividing by N we get that as N grows with out bounds then

$$\lim_{N \longrightarrow \infty} \frac{p}{p^2+2(1-p)p^2-\frac{2(1-p)p^2}{N}+\frac{2p(1-p)^2}{N}} = \frac{1}{p(3-2p)}.$$

This ratio reaches its minimum as $p = \frac{3}{4}$ the ratio then being $\frac{8}{9} \approx 0,89$. Thus when $p \in (0,1)$ then at least 89% of the number of agents that make the right choice when the observation moderately costly make the right choice when relying on their own signals alone.

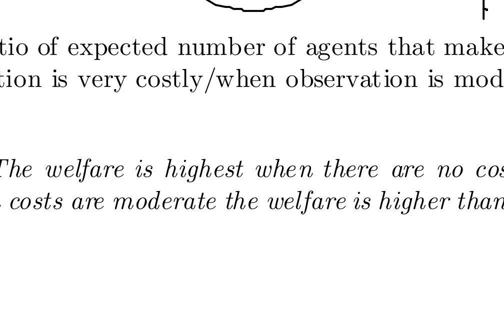

Figure2: The ratio of expected number of agents that make the right choice when observation is very costly/when observation is moderately costly.

Proposition 2 *The welfare is highest when there are no costs of observation. When observation costs are moderate the welfare is higher than when observation costs are high.*

8 Conclusion

Let's elaborate the case when observations are made sequentially. It is clear that the first agent to make sequential decisions about observing her predecessors is the third agent. It is also clear that in the case that she would observe first one agent and her observation would indicate that the agent that she observed had the same signal as the third agent then the third agent would not want to observe any additional actions since they could not change her desired action from the expected utility point of view. In the case that the third agent were first to observe an action that implies that the signal of her predecessor is different from that of her own then she would have an equal posterior probability about both states of the world. In this case she would want to observe another predecessors since the expected utility from buying with two contradicting signals is zero. The expected utility from observing another predecessor given that the first observation was different from her own signal is equal to

$$\begin{gathered} E\left[U\left(observe\left\{a_2\right\}\right)|\overline{A_1}S_3\right] = \\ \Pr\left[A_2|\overline{A_1}S_3\right] E\left[U\left(A_3\right)|\overline{A_1}A_2S_3\right] + \Pr\left[\overline{A_2}|A_1S_3\right]\left(0-2\omega\right) = \\ \left(2p-1\right)\frac{b}{4} - 2\omega. \end{gathered} \tag{15}$$

, where $E\left[U\left(A_3\right)|\overline{A_1}A_2S_3\right] =$

$\Pr\left(\sigma|\overline{A_1}A_2S_3\right) b - c = \frac{p(1-p)\left[2p-1\right]}{2\left[(1-p)p^2+p(1-p)^2\right]} b.$

Then given that the observation costs are moderate in this sequential observation case, i.e. that (15) is more or equal to zero, then it will be worth for the third agent to observe the second predecessor in the case that she has observed one predecessor to have a signal that is different from her own. Again in this case it will be optimal for the fourth agent to observe only the third agent's action and to mimic the third agent rather that replicate the sequential decision making of the third agent[5].

This paper shows that the simplest herding model is not robust to small changes in the cost of observing other agents' actions. If observation costs are moderate enough to provide the third agent with sufficient incentives to observe both her predecessors actions, then a herd gets started after the third agent with probability one. If no such incentives for the third agent exist then all agents will rather make their decisions independently of all other agents.

[5] The authors have exact calculations on this that may be sent upon request.

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