# Physical Search*

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**Abstract**

One-sided search is an evolutionarily stable outcome in an economy where each buyer and seller can either search or wait, and where the trading mechanism is auction or bargaining. If the relative number of buyers to sellers increases, the likelihood of all sellers wait and all buyers search –equilibrium increases relative to the likelihood of all buyers wait and all sellers search –equilibrium. In two-sided search, bargaining is more efficient than auction. One-sided search is more efficient

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than two-sided search. In one-sided search, it is more efficient if the larger pool searches and the smaller pool waits, than vice versa.

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# 1 Introduction

Search theoretic models are widely used in labour markets, and recently so called urn-ball models have been successfully applied to study the coordination problems inherent in those markets. This approach has two fruitful features. First, it yields a well-defined equilibrium matching function, and secondly, it is possible to study different ways of determining wages. The economy consists of employers and workers. Production requires one employer and one worker. The workers contact the employers, and if an employer meets at least one worker, a pair is formed. The model has plenty of equilibria: In pure strategy equilibria a worker goes to a certain employer with probability one, and in equilibrium everyone has correct expectation about each other's choice. Especially in large markets, like labour markets, pure strategy equilibria are not regarded as plausible since they seem to involve a lot of coordination. For this reason, the focus is on a symmetric mixed strategy equilibrium in which each worker randomises over which employer to visit. This is exactly the coordination problem which, coupled with the employers' capacity constraint, leads to inefficiency as some employers may not meet any workers and some employers may meet several workers. We call this the one-sided coordination problem.

There is no difference between employers and workers in a sense that we could as well think that employers contact workers. This choice is made in Julien, Kennes and King (2000). It is clear that postulating this kind of market structure already involves plenty of coordination. If employers and workers are treated symmetrically, both of them must be allowed to contact the other side, and we are faced with a two-sided coordination problem. To study this, it is helpful to think that each agent is assigned a location. This assignment is public knowledge. In a two-sided coordination problem the agents have two decisions to make. First, they decide whether to contact an agent of the opposite type or not. In case they contact, they choose a mixed strategy over which location to contact, like in a one-sided coordination problem. If, say, a worker contacts employers, he must leave his own location and go to some location assigned to an employer (this could be called as “physical search”). But now it is possible that there is no one in the location; this happens if the employer has decided to contact workers. In the sequel we say that those who try to contact the opposite type search, or move, and the others wait, or stay. We call the agents as buyers and sellers, and each buyer and seller can search or wait.

The main aim of this study is to explain one-sided search as an evolutionary stable equilibrium in case the trading mechanism is auction or bargaining, and to solve how the probability of ending up to either equilibrium (buyers search for sellers or vice versa) depends on the relative size of the buyer and seller pools. We also study the relative efficiency of the two trading mechanisms in two-sided search equilibrium. Further, the relative efficiency of one-sided search and two-sided search is compared for both trading mechanisms. In addition, we solve which one of the one-sided search equilibria is more efficient (produces more matches): The equilibrium where all sellers wait and all buyers search, or the equilibrium where all buyers wait and all sellers search.

There is quite a lot of related labour market literature, the seminal contribution being Montgomery (1991) where one-sided coordination problem is studied with employers posting wages. Acemoglu and Shimer (1999a, 1999b) also use price posting, while Julien, Kennes, and King (2000, 2001, 2002) have models with both price posting and auctions. Burdett, Coles, Kiyotaki and Wright (1995) emphasise the question who searches and who waits, but they do not stress the coordination problems as such. Herreiner (1999) studies also the question about who searches, but she ignores price formation. Kultti, Miettunen, Takalo, and Virrankoski (2002) study the same question paying explicit attention to different ways of determining prices. As opposed to the present study, they assume that search is one-sided at the outset, and they study if a given search pattern is an equilibrium.

The most important results of the present paper are: 1) Independently of the trading mechanism (auction or bargaining) i) the market where both buyers and sellers use mixed strategies over the wait and move decisions (that is, two-sided search) is not evolutionarily stable, and ii) the market with one-sided coordination problem is evolutionarily stable. 2) For both auction and bargaining, the basin of attraction for the market where sellers wait grows when the ratio of buyers to sellers grows. 3) In the two-sided coordination problem, bargaining is more efficient than auction. 4) One-sided search - whether all buyers search and all sellers wait, or vice versa - is more efficient than two-sided search. This holds for auction as well as for bargaining. 5) A one-sided search equilibrium where all the members of the larger pool search and all the members of the smaller pool wait is more efficient than the equilibrium where all the members of the larger pool wait and all the members of the smaller pool search.

A two-sided coordination equilibrium is evolutionarily unstable and quite inefficient. Solving for the two-sided equilibrium is still important, because we want to find out how the relative likelihood of the two stable (one sided) equilibria changes when the ratio of buyers to sellers changes. The instability accompanied with inefficiency perhaps explains why it is difficult to find real-life examples that fit well into the two-sided framework. With a little stretching one can think that taxi markets and markets where one has to phone to the other party to establish a contact have some features of two-sided search. One can also think of marriage markets from this point of view. But there the idea would be to explain institutions like intermediaries arising from the problems of two-sided coordination.

The rest of the paper is organised as follows. In section 2 we present the basic features of the model: the matching process and the trading mechanisms. In section 3 we solve the search/wait decisions when trades are consummated by auction, and section 4 deals with the same problem when the trading mechanism is bargaining. Section 5 analyses the relative efficiency of different equilibria. Section 6 studies the evolutionary stability of two-sided search. Section 7 concludes.

## 2 The Model

There are $B$ buyers each with a unit demand and $S$ sellers each with one indivisible unit of a good for sale. Let $\theta \equiv B/S$. The buyers get utility normalised to one from consuming the good, and the sellers get utility normalised to zero from consuming it. The economy extends to infinity, and time proceeds in discrete periods. The agents discount future with factor $\delta \in (0,1)$. When the agents trade they exit the economy and are replaced by identical agents who are not yet matched. This means that the ratio of buyers to sellers remains the same in every period.

To model the meeting process we use a familiar urn-ball model. In a basic urn-ball model, the agents who decide to wait are in fixed positions, and the agents who decide to search are randomly allocated on the waiting agents of the opposite pool. This meeting technology is well defined and tractable. Further, since multiple meetings are possible, one can meaningfully study a variety of trading mechanisms. If $w$ agents wait and $m$ agents move, the number of agents a waiting agents meets is a binomial random variable with parameters $m$ and $1/w$. Tractability is achieved by assuming that $w$ and $m$ are large since in this case one can approximate the binomial with a Poisson distribution with parameter $m/w$. Then the probability that a waiting agent meets exactly $k$ moving agents is $\frac{(m/w)^k}{k!}e^{-m/w}$. Assuming that $m$ and $w$ aproach infinity, $e^{-m/w}$ is the probability that a waiting agent does not meet any moving agent, $1-e^{-m/w}$ is the probability that a waiting agent meets at least one any moving agent, and with probability $1-e^{-m/w}-\frac{m}{w}e^{-m/w}$ a waiting agent meets at least two moving agents.

In the present model, each buyer and each seller has his own location. Then, a fraction $\beta \in [0,1]$ of buyers remain in their locations, waiting for sellers to visit them, and the rest of the buyers, fraction $1-\beta$, go to sellers' locations, leaving their own locations empty. Each seller, too, makes a decision whether to stay or go. A fraction $\sigma \in [0,1]$ of the sellers remain to wait for moving buyers, and fraction $1-\sigma$ go to buyers' locations. The equilibrium values of these fractions will be solved. Each agent knows where each location is, but a moving agent does not know whether the particular location he is going to will have an occupant or not. This is what we mean by misdirected search. However, the agents are assumed to know the equilibrium fractions of empty locations. As in the basic model, waiters have a risk that nobody comes to their location, and movers may end up with competing with other movers who have chosen to go to the same location. In addition, a mover has the risk of entering an empty location, whose occupant has chosen to move. The Poisson parameter that governs the arrival of buyers to a seller's location is thus

$$\gamma \equiv (1-\beta)\,\theta, \tag{1}$$

and the Poisson parameter that governs the arrival of sellers to a buyer's location is

$$\varphi \equiv \frac{(1-\sigma)}{\theta}. \tag{2}$$

We investigate two trading mechanisms: auction and bargaining. In auction, if a stayer meets at least two movers, the movers are assumed to engage in a Bertrand-type bidding. All the movers bid the same, and one of them trades. Each mover - whether he trades or not - gets his own reservation value, since the movers are indifferent between trading and waiting for an opportunity to trade in the next period. The stayer gets his own reservation value plus the whole surplus of the match, in other words, the stayer gets one minus a mover's reservation value. If a stayer meets just one mover, we assume that the mover makes a take-it-or-leave-it offer. As the result, the stayer gets his own reservation value, and the mover gets his own reservation value plus the whole surplus of the match, that is, the mover gets one minus the stayer's reservation value. Assuming the take-it-or-leave-it offer gives the movers, too, a positive probability to get the whole surplus of a trade, treating the movers and stayers as equally as possible. Bargaining is always pairwise, and if a stayer meets several movers he just picks one of them randomly for his trading partner. To make things simple we assume that the stayer and mover just split the available surplus in half.

# 3 Auction

Denote the life-time values of waiting buyers and sellers as $V_b^w$ and $V_s^w$, the respective values for moving agents are $V_b^m$ and $V_s^m$. The Poisson parameters determining the arrival rates faced by a buyer's and seller's location are $\varphi$ and $\gamma$, respectively. The value functions are

$$V_b^w = \delta \left[ e^{-\varphi} (1+\varphi) V_b^w + \left(1 - e^{-\varphi} - \varphi e^{-\varphi}\right) (1 - V_s^m) \right], \tag{3}$$

$$V_b^m = \delta \left[ (1-\sigma) V_b^m + \sigma e^{-\gamma} (1 - V_s^w) + \sigma \left(1 - e^{-\gamma}\right) V_b^m \right], \tag{4}$$

$$V_s^w = \delta \left[ e^{-\gamma} (1+\gamma) V_s^w + \left(1 - e^{-\gamma} - \gamma e^{-\gamma}\right) (1 - V_b^m) \right], \tag{5}$$

$$V_s^m = \delta \left[ (1-\beta) V_s^m + \beta e^{-\varphi} (1 - V_b^w) + \beta \left(1 - e^{-\varphi}\right) V_s^m \right]. \tag{6}$$

For example, in equation (5), a waiting seller has a probability $e^{-\gamma}(1+\gamma)$ that no buyer or just one buyer arrives, and in both cases the seller gets his reservation value. With probability $1 - e^{-\gamma} - \gamma e^{-\gamma}$ at least two buyers arrive, and the seller gets one minus a buyer's reservation value. In equation (6), a moving seller arrives at an empty location with probability $1-\beta$ and gets his reservation value. With probability $\beta e^{-\varphi}$ the seller arrives at a non-empty location, no other sellers arrive, and the seller makes a take-it-or-leave-it offer to the buyer and gets one minus the buyers reservation value. With probability $\beta(1 - e^{-\varphi})$ the seller arrives at a non-empty location but other sellers arrive, too, and the seller receives his reservation value.

Solving from (3) and (6) gives

$$V_b^w = \frac{\delta \left(1 - e^{-\varphi} - \varphi e^{-\varphi}\right)}{1 - \delta e^{-\varphi} (1 + \varphi - \beta)}, \tag{7}$$

$$V_s^m = \frac{\delta \beta e^{-\varphi}}{1 - \delta e^{-\varphi} (1 + \varphi - \beta)}, \tag{8}$$

and solving from (4) and (5) gives

$$V_b^m = \frac{\delta \sigma e^{-\gamma}}{1 - \delta e^{-\gamma} (1 + \gamma - \sigma)}, \tag{9}$$

$$V_s^w = \frac{\delta \left(1 - e^{-\gamma} - \gamma e^{-\gamma}\right)}{1 - \delta e^{-\gamma} (1 + \gamma - \sigma)}. \tag{10}$$

In equilibrium, staying and moving give equal life-time utilities: $V_b^w = V_b^m$, and $V_s^w = V_s^m$. If both of these equations hold, then

$$\sigma e^{-\gamma} = 1 - e^{-\varphi} - \varphi e^{-\varphi}, \tag{11}$$

$$\beta e^{-\varphi} = 1 - e^{-\gamma} - \gamma e^{-\gamma}. \tag{12}$$

Equations (11) and (12) together determine the equilibrium proportions of waiting buyers and sellers. Using $\theta e^{\varphi} = e^{\gamma}$ implied by (11) and (12), gives the equilibrium fractions of staying buyers and sellers:

$$\beta = 1 - \frac{1}{\theta}\ln(1+\theta), \tag{13}$$

$$\sigma = 1 - \theta \ln\left(1 + \frac{1}{\theta}\right). \tag{14}$$

**Proposition 1** *If trades are consummated by auction, the proportion of moving sellers increases and the proportion of moving buyers decreases if $B/S$ increases.*

**Proof.** Equation (13) gives $\frac{d\beta}{d\theta} = \frac{1}{\theta}\left[\frac{1}{\theta}\ln(1+\theta) - \frac{1}{1+\theta}\right] \geq 0$ because $\ln(1+\theta) = \frac{\theta}{1+\theta}$ if $\theta = 0$, and $\ln(1+\theta) > \frac{\theta}{1+\theta}$ if $\theta > 0$. The latter holds because $\frac{\partial\left[\ln(1+\theta) - \frac{\theta}{1+\theta}\right]}{\partial\theta} = \frac{1}{1+\theta} - \frac{1}{(1+\theta)^2} > 0 \forall\theta > 0$. Equation (14) gives $\frac{d\sigma}{d\theta} = \frac{1/\theta}{1+1/\theta} - \ln(1+1/\theta) < 0 \forall\theta > 0$,because $\frac{x}{1+x} - \ln(1+x) < 0 \forall x > 0$. ■

That is, the larger the relative size of the population, the larger is the fraction of waiting agents and the smaller is the fraction of moving agents.

One might ask whether making the present model static will change the results in any way. The answer is no. In a one-period model, the reservation values in the right-hand sides of value functions (3) - (6) are zeroes, and the equilibrium conditions that result from solving the new value functions are just (13) and (14). This holds in the case of bargaining, too.

We solve the matching function, that is, the aggregate number of trades, $M_a$, that will emerge per period:

**Remark 1** *The matching function implied by the matching process with empty locations, when the trading mechanism is auction, is* $M_a = \frac{\theta S}{1+\theta}\left(2 - \theta\ln\left(1+\frac{1}{\theta}\right) - \frac{1}{\theta}\ln(1+\theta)\right)$.

**Proof.** Let us add together the number of matches that waiting agents make. The probability that a waiting seller gets matched is equal to $1 - e^{-(1-\beta)\theta}$. The number of waiting sellers is $\sigma S$, so waiting sellers form $\sigma S(1 - e^{-(1-\beta)\theta})$ matches. Similarly, waiting buyers form $\beta B(1 - e^{-(1-\sigma)/\theta})$ matches. Using the equilibrium values of $\beta$ and $\sigma$ gives

$1 - e^{-(1-\beta)\theta} = \frac{B}{B+S}$, and $1 - e^{-(1-\sigma)/\theta} = \frac{S}{B+S}$. We get $M_a = \beta B \left( \frac{S}{B+S} \right) + \sigma S \left( \frac{B}{B+S} \right) = \frac{BS}{B+S} (\beta + \sigma)$. Using (9) and (10) again we get $M_a$. ■

**Remark 2** *Matching function $M_a$ has constant returns to scale.*

**Proof.** A matching function has constant returns to scale if individual matching probabilities do not depend on the absolute number of agents but on their relative number. Using $M_a = \frac{BS}{B+S} (\beta + \sigma)$, we have $\frac{M_a}{B} = \frac{1}{1+\theta} (\beta + \sigma)$, and $\frac{M_a}{S} = \frac{\theta}{1+\theta} (\beta + \sigma)$ ■

# 4 Bargaining

If at least one mover comes to a stayer, the stayer picks one mover in random, and the two split the available surplus in half. The life-time values of waiting buyers and sellers are $W_b^w$ and $W_s^w$, and the respective values for moving agents are $W_b^m$ and $W_s^m$. The Poisson parameters are $\varphi$ and $\gamma$ for buyers' and sellers' locations, respectively. The value functions are

$$W_b^w = \delta \left\{ e^{-\varphi} W_b^w + \left(1 - e^{-\varphi}\right) \left[ W_b^w + \frac{1}{2} \left(1 - W_b^w - W_s^m\right) \right] \right\}, \tag{15}$$

$$W_s^m = \delta \left\{ \begin{array}{c} (1-\beta) W_s^m + \frac{\beta \left(1 - e^{-\varphi}\right)}{\varphi} \left[ W_s^m + \frac{1}{2} \left(1 - W_b^w - W_s^m\right) \right] \\ + \frac{\beta \left(\varphi - 1 + e^{-\varphi}\right)}{\varphi} W_s^m \end{array} \right\}, \tag{16}$$

$$W_s^w = \delta \left\{ e^{-\gamma} W_s^w + \left(1 - e^{-\gamma}\right) \left[ W_s^w + \frac{1}{2} \left(1 - W_s^w - W_b^m\right) \right] \right\}, \tag{17}$$

$$W_b^m = \delta \left\{ \begin{array}{c} (1-\sigma) W_b^m + \frac{\sigma \left(1 - e^{-\gamma}\right)}{\gamma} \left[ W_b^m + \frac{1}{2} \left(1 - W_s^w - W_b^m\right) \right] \\ + \frac{\sigma \left(\gamma - 1 + e^{-\gamma}\right)}{\gamma} W_b^m \end{array} \right\}. \tag{18}$$

In equation (15), a waiting buyer meets no seller with probability $e^{-\varphi}$ and continues to the next period. With probability $1 - e^{-\varphi}$ he meets at least one seller and gets his reservation value plus one half of the surplus. In equation (16), a moving seller comes to an empty location with probability $1 - \beta$. With probability $\beta$ the seller arrives at an occupied location, trades with probability $(1-e^{-\varphi})/\varphi$, and does not trade with probability $(\varphi-1+e^{-\varphi})/\varphi$.

Solving from (15) and (16) yields

$$W_b^w = \frac{\delta\varphi(1-e^{-\varphi})}{\delta(1-e^{-\varphi})(\beta+\varphi)+2(1-\delta)\varphi}, \tag{19}$$

$$W_s^m = \frac{\delta\beta(1-e^{-\varphi})}{\delta(1-e^{-\varphi})(\beta+\varphi)+2(1-\delta)\varphi}, \tag{20}$$

and solving from (17) and (18) gives

$$W_s^w = \frac{\delta\gamma(1-e^{-\gamma})}{\delta(1-e^{-\gamma})(\sigma+\gamma)+2(1-\delta)\gamma}, \tag{21}$$

$$W_b^m = \frac{\delta\sigma(1-e^{-\gamma})}{\delta(1-e^{-\gamma})(\sigma+\gamma)+2(1-\delta)\gamma}. \tag{22}$$

Using the familiar conditions for equal utilities for moving and waiting ($W_b^w = W_b^m$ and $W_s^w = W_s^m$), gives, when both these conditions hold, that

$$\left(1-e^{-\varphi}\right)\gamma = \sigma\left(1-e^{-\gamma}\right), \tag{23}$$

$$\left(1-e^{-\gamma}\right)\varphi = \beta\left(1-e^{-\varphi}\right). \tag{24}$$

Equations (23) and (24) imply $\beta+\sigma=1$. Plugging this into (23) yields

$$1-\theta-e^{-\sigma\theta}+\theta e^{\frac{\sigma-1}{\theta}}=0 \tag{25}$$

which tells us the equilibrium proportion $\sigma$ of waiting sellers as a function of $B/S$.

**Proposition 2** *If trades are consummated by bargaining, the proportion of moving sellers increases and the proportion of moving buyers decreases if $B/S$ increases.*

**Proof.** See the Appendix. ■

The decisions to stay or move show a similar pattern than in the case of auction: the larger the relative size of a population is, the larger is the fraction of stayers in that population and the larger is the fraction of movers in the opposite population.

**Remark 3** *The matching function implied by the matching process with empty locations, when the trading mechanism is bargaining, is $M_b = S\left(1-e^{-\sigma\theta}\right)$, where $\sigma$ and $\theta$ satisfy equation (25).*

**Proof.** Add together the matches made by waiting agents. We have $M_b = \sigma S(1 - e^{-(1-\beta)\theta}) + \beta B(1 - e^{-(1-\sigma)/\theta})$. Then use $\beta + \sigma = 1$ and condition (25). ■

**Remark 4** *Matching function $M_b$ has constant returns to scale.*

**Proof.** The argument is similar as in the case of $M_a$. We have $M_b/S = \left(1 - e^{-\sigma\theta}\right)$ and $M_b/B = \frac{1}{\theta}\left(1 - e^{-\sigma\theta}\right)$, and because $\sigma$ is a function of $\theta$ only, the individual matching probabilities depend only on $\theta$. ■

# 5 Efficiency

We determine efficiency simply as the number of trades made per period. It is conceivable that different trading mechanisms imply different number of trades made: The number of trades depends on how many agents on each side of the market move and how many agents stay, and the trading mechanism affects the incentives of moving and staying. In this section we first study the relative efficiency of auction and bargaining when search is two-sided. Then we compare the efficiency of one-sided and two-sided search. Finally, we compare the two alternative patterns in one-sided search.

## 5.1 Efficiency of Auction and Bargaining in Two-Sided Coordination Problem

Letting $E = M_a/M_b$, we have

$$E = \frac{\frac{\theta}{1+\theta}\left(2 - \theta \ln\left(1 + \frac{1}{\theta}\right) - \frac{1}{\theta}\ln(1+\theta)\right)}{1 - e^{-\sigma\theta}}, \tag{26}$$

where the denominator satisfies condition (25).

**Proposition 3** *In two-sided coordination equilibrium, bargaining is more efficient than auction.*

**Proof.** By numerically solving equation (26) we see that $E < 1$ with all positive values of $\theta$. ■

In Figure 1, the horizontal axis is for $\theta$, and the vertical axis is for $E$. We see that $\partial E/\partial\theta < 0$ if $\theta < 1$, and $\partial E/\partial\theta > 0$ if $\theta > 1$. In other words, auction performs worst with respect to bargaining when $B = S$. Plugging $\theta = 1$ into (13) and (14) we get $\beta = \sigma = 1 - \ln 2 \approx 0.3$. That is, if the there are equally many buyers and sellers and the trading mechanism is auction, 30 percent of buyers and sellers stay, and 70 percent of them move. Using $\theta = 1$ in (25) we have $\beta = \sigma = 1/2$: if the number of buyers equals that of the sellers, and bargaining is used, one half of buyers and sellers move and one half of them stays. If the two pools of traders are equally large, there is too much moving and too little staying if auction is used. Auction gives the agents a too strong

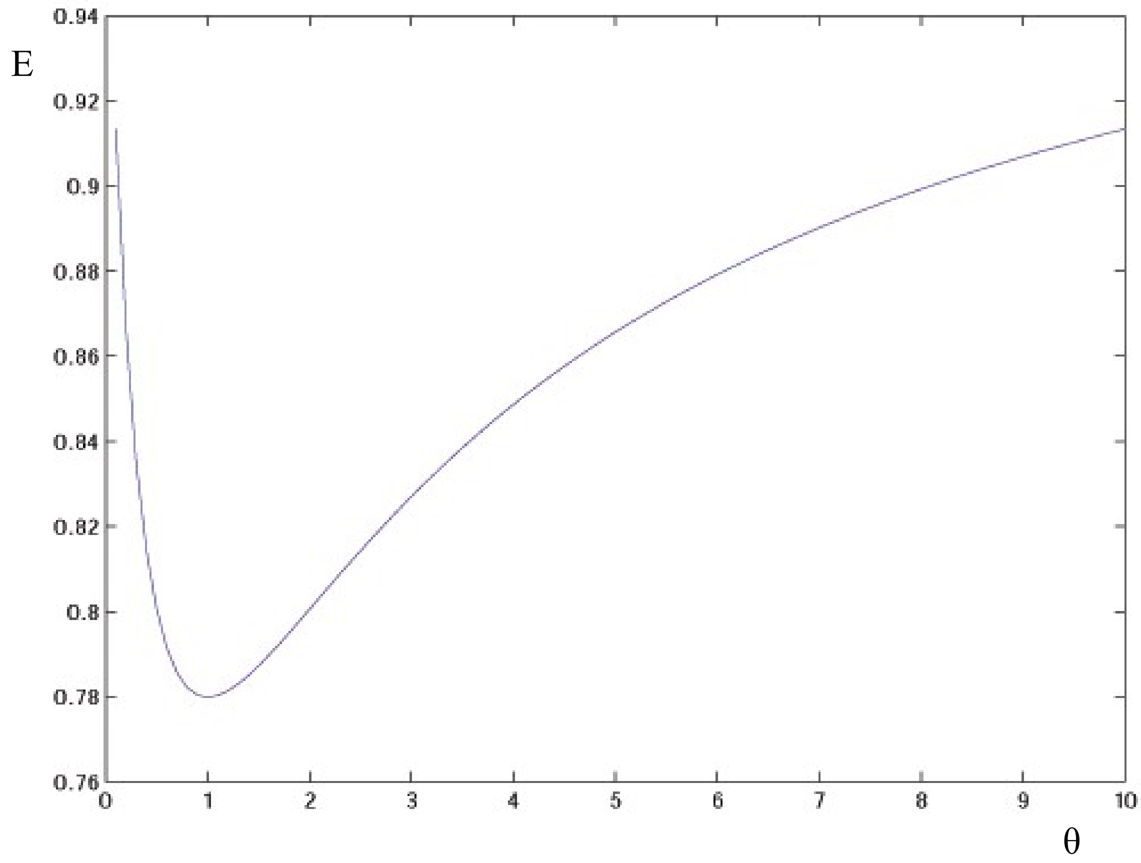


Figure 1: The relative efficiency of auction versus bargaining

incentive to move, which leads to an excessive amount of empty locations. Bargaining treats the agents more evenly when sharing the surplus in a trade, and this in turn makes the populations of movers and stayers more balanced. A simulation (not presented here) showed that for all values of $\theta$, the proportions of staying sellers and staying buyers is larger in case of bargaining than in case of auction.

## 5.2 Efficiency of Two-Sided versus One-Sided Search

Here we compare the number of matches resulting from one-sided search versus two-sided search. If all the sellers wait and all the buyers move, the number of matches is $S\left(1-e^{-\theta}\right)$, whereas if all the buyers wait and all the sellers move, the number of matches is $B\left(1-e^{-1/\theta}\right)$. These hold whether the trading mechanism is auction or bargaining.

**Proposition 4** *One-sided search is more efficient than two-sided search. This holds for auction as well as for bargaining.*

**Proof.** i) Auction: In two-sided search, the number of matches is $\frac{\theta S}{1+\theta}\left(2-\theta\ln\left(1+\frac{1}{\theta}\right)-\frac{1}{\theta}\ln(1+\theta)\right)$(see Remark 1), and in one-sided search it is $S\left(1-e^{-\theta}\right)$ if all the sellers wait and all the buyers search, and $B\left(1-e^{-1/\theta}\right)$ if all the buyers wait and all the sellers search. Using fact $\ln(1+x)\geq\frac{1}{1+x^{-1}}$ gives that

$$S\left(1-e^{-\theta}\right)-\frac{\theta S}{1+\theta}\left(2-\theta\ln\left(1+\frac{1}{\theta}\right)-\frac{1}{\theta}\ln(1+\theta)\right)$$

$$\geq S\left\{1-e^{-\theta}-\frac{\theta}{1+\theta}\left(2-\frac{\theta}{1+\theta}-\frac{1}{1+\theta}\right)\right\}=S\left(1-e^{-\theta}-\frac{\theta}{1+\theta}\right),$$ which has the same sign as $1-e^{-\theta}-\theta e^{-\theta}$, which is positive. In the latter case $B\left(1-e^{-1/\theta}\right)-\frac{\theta S}{1+\theta}\left(2-\theta\ln\left(1+\frac{1}{\theta}\right)-\frac{1}{\theta}\ln(1+\theta)\right)$ has the same sign as

$$\theta\left(1-e^{-1/\theta}\right)-\frac{\theta}{1+\theta}\left(2-\theta\ln\left(1+\frac{1}{\theta}\right)-\frac{1}{\theta}\ln(1+\theta)\right),$$ which is at least as large as $\theta\left(1-e^{-1/\theta}\right)-\frac{\theta}{1+\theta}$, which has the same sign as $1-e^{-1/\theta}-\frac{1}{\theta}e^{-1/\theta}$, which is positive. That is, one-sided search produces more matches than two-sided search.

ii) Bargaining: In two-sided search, the number of matches is $S\left(1-e^{-\sigma\theta}\right)$, where $\sigma\leq 1$, and where $\sigma$ and $\theta$ satisfy equation (25) (see Remark 3). It is easily noted that $S\left(1-e^{-\sigma\theta}\right)\leq S\left(1-e^{-\theta}\right)$, that is, one-sided search when all sellers wait produces at least as many matches as two-sided search. If in one-sided search all the buyers wait, the difference in the number of matches is $S\left(1-e^{-\sigma\theta}\right)-B\left(1-e^{-1/\theta}\right)$, which has the same sign as $1-\theta-e^{-\sigma\theta}+\theta e^{-1/\theta}$. In two-sided search, $\sigma$ and $\theta$ satisfy equilibrium condition $1-\theta-e^{-\sigma\theta}+\theta e^{\frac{\sigma-1}{\theta}}=0$, and plugging the latter into $1-\theta-e^{-\sigma\theta}+\theta e^{-1/\theta}$ implies that $S\left(1-e^{-\sigma\theta}\right)-B\left(1-e^{-1/\theta}\right)$ has the same sign as $1-e^{\sigma/\theta}$, which is negative. One-sided search when all buyers wait produces at least as many matches as two-sided search. ■

### 5.3 Efficiency in One-Sided Search

One-sided search has two alternative configurations: i) all sellers wait and all buyers search, so the number of matches is $S\left(1-e^{-\theta}\right)$, or ii) all buyers wait and all sellers search, which produces $B\left(1-e^{-1/\theta}\right)$ matches. Their relative efficiency depends only on $\theta$. Note that the trading rule has no effect on the number of matches if search is one-sided, because all searchers just choose each location with equal probability.

**Proposition 5** *If $B>(<)S$, a market where all buyers search and all sellers wait produces more (less) matches than a market where all sellers search and all buyers wait.*

**Proof.** See the Appendix. ■

## 6 Stability

Let us study replicator dynamics in order to gain insight to the nature of equilibria. There are two variables that determine the equilibria, namely the proportions of buyers and sellers, $\beta$ and $\sigma$, who stay at their locations. This means that the analysis can be conducted graphically in a $(\beta,\sigma)$-plane (see Lu and McAfee, 1996). The idea behind the replicator dynamics presupposes myopically behaving agents. Those who trade are replaced by identical agents and these agents go to the market where their type did best in the previous period. The stable points under replicator dynamics are a subset of Nash-equilibria, and we can think of the dynamics as a kind of criterion to choose between equilibria.

To conduct the analysis we determine two curves called the buyers' equilibrium curve BE, and the sellers' equilibrium curve SE. The BE is got by equating the buyers' expected utility in the market where they wait and in the market where they move. The SE is determined analogously, and the equilibrium we study is determined by the intersection of the BE and the SE.

## 6.1 Stability in Case of Auction

Equating $V_b^w$ and $V_b^m$ from (7) and (9) yields the BE which after some manipulations is given by

$$\left(1 - e^{-\varphi} - \varphi e^{-\varphi}\right)\left(1 - \delta e^{-\gamma} - \delta\gamma e^{-\gamma}\right) - \sigma e^{-\gamma}\left(1 - \delta + \delta\beta e^{-\varphi}\right) = 0. \tag{27}$$

Equating $V_s^w$ and $V_s^m$ from (8) and (10) yields SE:

$$\left(1 - e^{-\gamma} - \gamma e^{-\gamma}\right)\left(1 - \delta e^{-\varphi} - \delta\varphi e^{-\varphi}\right) - \beta e^{-\varphi}\left(1 - \delta + \delta\sigma e^{-\gamma}\right) = 0 \tag{28}$$

On the $\beta, \sigma$-plane the point $(0, 1)$ corresponds to the market where all the buyers move and all the sellers wait, and point $(1, 0)$ to the market where all the sellers move and all the buyers wait. Next we establish the position of curves on the plane. First we show that they slope downwards. Totally differentiating (27) and (28) yield

$$\frac{d\sigma}{d\beta} = \frac{\delta\gamma\theta e^{-\gamma}\left(1 - e^{-\varphi} - \varphi e^{-\varphi}\right) + \sigma\theta e^{-\gamma}\left(1 - \delta + \delta\beta e^{-\varphi}\right) + \delta\sigma e^{-\gamma}e^{-\varphi}}{-\theta^{-1}\varphi e^{-\varphi}\left(1 - \delta e^{-\gamma} - \delta\gamma e^{-\gamma}\right) - e^{-\gamma}\left(1 - \delta + \delta\beta e^{-\varphi}\right) - \delta\theta^{-1}\beta\sigma e^{-\gamma}e^{-\varphi}} < 0, \tag{29}$$

$$\frac{d\sigma}{d\beta} = \frac{\gamma\theta e^{-\gamma}\left(1 - \delta e^{-\varphi} - \delta\varphi e^{-\varphi}\right) + e^{-\varphi}\left(1 - \delta + \delta\sigma e^{-\gamma}\right) + \delta\beta\sigma\theta e^{-\gamma}e^{-\varphi}}{-\delta\theta^{-1}\varphi e^{-\varphi}\left(1 - e^{-\gamma} - \gamma e^{-\gamma}\right) - \beta\theta^{-1}e^{-\varphi}\left(1 - \delta + \delta\sigma e^{-\gamma}\right) - \delta\beta e^{-\gamma}e^{-\varphi}} < 0. \tag{30}$$

That is, BE and SE are downward-sloping. Next we determine how the curves behave close to points $(0, 1)$ and $(1, 0)$. Let us first consider BE. There are four cases.

**1.** $(\beta, \sigma) \to (0, \sigma)$ in which case $\gamma = \theta$ and BE becomes $\left(1 - e^{-\varphi} - \varphi e^{-\varphi}\right) \times \left(1 - \delta e^{-\theta} - \delta\theta e^{-\theta}\right) - \sigma e^{-\theta}(1 - \delta) = 0$, which is equivalent to $\frac{\sigma(1-\delta)e^{\varphi}}{e^{\varphi}-1-\varphi} = e^{\theta} - \delta(1+\theta)$. The RHS of the latter equation is fixed; the LHS is zero when $\sigma = 0$, and it grows without limit when $\sigma$ approaches unity. The LHS is also increasing in $\sigma$. Thus, we know that BE always goes through point $(0, \sigma)$ where $\sigma < 1$.

**2.** $(\beta, \sigma) \to (\beta, 1)$ in which case $\varphi = 0$, and BE becomes $-\sigma e^{-\gamma}(1 - \delta + \delta\beta) = 0$ which is never true. Thus, BE does not go through $(\beta, 1)$ where $\beta > 0$.

**3.** $(\beta, \sigma) \to (\beta, 0)$ in which case $\varphi = \theta^{-1}$, and the BE becomes $\left(1 - e^{-1/\theta} - \dfrac{1}{\theta}e^{-1/\theta}\right) \times$

$(1-\delta e^{-\gamma}-\delta\gamma e^{-\gamma}) = 0$ which never holds. Thus, we know that BE never goes through $(\beta, 0)$ where $\beta < 1$.

**4.** $(\beta, \sigma) \to (1, \sigma)$ in which case $\gamma = 0$, and BE becomes $(1-e^{-\varphi}-\varphi e^{-\varphi})(1-\delta) - \sigma(1-\delta+\delta e^{-\varphi}) = 0$ which is equivalent to $\frac{\sigma(e^{\varphi}-\delta e^{\varphi}+\delta)}{e^{\varphi}-1-\varphi} = 1-\delta$. Here the RHS is fixed, and the LHS is zero when $\sigma = 0$ and it grows without limit when $\sigma$ approaches unity. The LHS is also increasing in $\sigma$. Thus, we know that BE always goes through point $(1, \sigma)$ where $\sigma > 0$.

In sum, for all $\theta$, BE goes through points $(0, \sigma)$ where $\sigma < 1$, and $(1, \sigma)$ where $\sigma > 0$. Completely analogous reasoning shows that for all $\theta$, SE goes through $(\beta, 1)$ where $\beta > 0$, and $(\beta, 0)$ where $\beta < 1$. From (13) and (14) we know that BE and SE intersect exactly once. The BE and SE-curves are presented in Figure 2. The arrows depicting the behavior out of steady state are result of evolutionary stability analysis presented below.

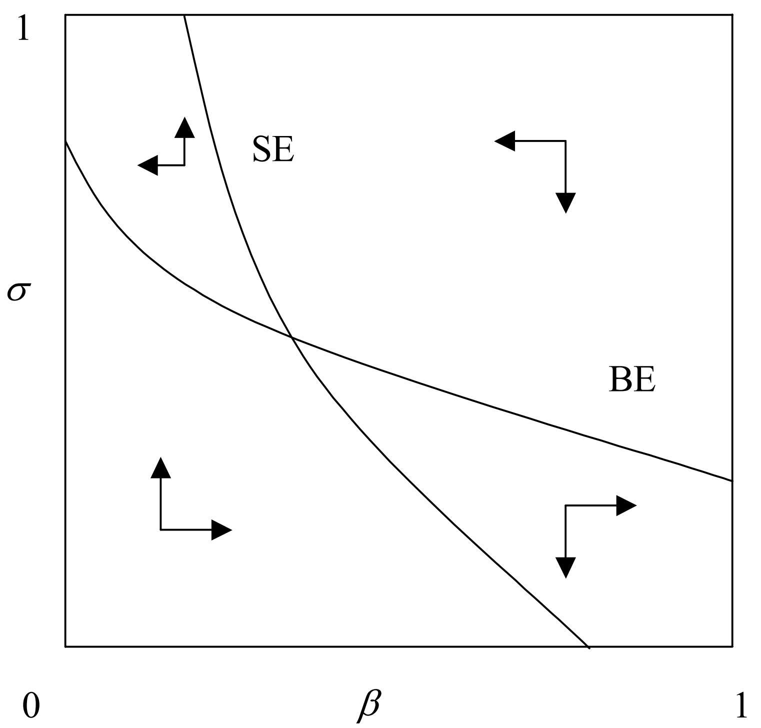


Figure 2: Two-sided search is evolutionarily unstable

The entering agents' action set consists of two actions corresponding to the two markets. Let us denote the buyers' strategy by $\beta$ and the sellers' strategy by $\sigma$. The first is the probability of buyers going to the market where buyers wait and the second is the probability of sellers going to the market where sellers wait. The probabilities of going to the other market are naturally $1 - \beta$ for the buyers and $1 - \sigma$ for the sellers. One can also think of these as the population shares of agents going to the two markets. To define replicator dynamics let us first establish notation for the buyers' and sellers' average expected utilities $A_b$ and $A_s$, given population shares $\beta$ and $\sigma$ in the markets where they wait. Now $A_b = (1 - \beta)V_b^m + \beta V_b^w$ and $A_s = (1 - \sigma)V_s^m + \sigma V_s^w$. In the replicator dynamics the population shares are determined by the following differential equations: $\frac{d\beta}{dt} = \beta\left(V_b^w - A_b\right)$ and $\frac{d\sigma}{dt} = \sigma\left(V_s^w - A_s\right)$.

**Definition 1** *An equilibrium $(\beta, \sigma)$ is evolutionarily stable if there exists a neighbourhood of $(\beta, \sigma)$, where the replicator dynamics converges to the equilibrium.*

Since there are only two possible choices for the agents, the analysis of the replicator dynamics can be easily performed graphically (see eg. Lu and McAfee, 1996). We get one of the the main results of this paper:

**Theorem 1** *If the trading mechanism is auction, then i) the market where both buyers and sellers use mixed strategies over the wait and move decisions is not evolutionarily stable, and ii) the market with one-sided coordination problem is evolutionarily stable.*

**Proof.** It is enough to locate the buyers' and sellers' equilibrium curves and determine the entering agents' behaviour off the equilibrium curves, where the shares adjust according to the replicator dynamics. When the ratios of agents in the two markets are such that $(\beta, \sigma)$ is above the $BE$-curve, the entering buyers go to, or prefer, the market where sellers wait. This is almost self-evident since on the $BE$-curve the buyers are indifferent between the markets. Above the $BE$-curve there are relatively more sellers in the market where they wait, and this makes the market preferable to the buyers. By a similar logic, when the ratios of agents in the two markets are such that $(\beta, \sigma)$ is above the $SE$ curve, the entering sellers go to, or prefer, the market where the buyers wait. Of course, below the curves the agents' preferences are just opposite. With this knowledge we can draw the arrows that depict the direction of the dynamics in Figure 2. ∎

Let us finally see how the intersection of the BE and the SE depends on $\theta$:

**Theorem 2** *In case of auction, the basin of attraction for the market where sellers wait grows when the ratio of buyers to sellers grows.*

**Proof.** We see from the proof of Proposition 1 that $\frac{d\beta}{d\theta} > 0$ and $\frac{d\sigma}{d\theta} < 0$. This means that as $\theta$ grows, the intersection of BE and SE goes towards the point $(1, 0)$, and the basin of attraction for the market where the sellers wait and the buyers move grows. In this sense the larger is $\theta$, the more likely it is that the equilibrium market structure is at point $(0, 1)$. ■

This result is in accordance with the results where buyers and sellers may choose to move or wait but where there is still only one-sided coordination problem in any market, i.e. there are no empty locations but those who move always meet someone. See for example Kultti, Miettunen, Takalo, and Virrankoski (2002).

## 6.2 Stability in Case of Bargaining

The analysis goes analogously to the case of auction. Equating $W_b^w$and $W_b^m$ from (19) and (22) gives the buyers' equilibrium, BE, where

$$\frac{\gamma}{\sigma} - \frac{\beta}{\varphi} - \frac{2(1-\delta)}{\delta(1-e^{-\varphi})} + \frac{2(1-\delta)\gamma}{\delta\sigma(1-e^{-\gamma})} = 0, \tag{31}$$

and setting $W_s^m = W_s^w$ from (20) and (21) gives the sellers' equilibrium SE:

$$\frac{\sigma}{\gamma} - \frac{\varphi}{\beta} - \frac{2(1-\delta)\varphi}{\delta\beta(1-e^{-\varphi})} + \frac{2(1-\delta)}{\delta(1-e^{-\gamma})} = 0. \tag{32}$$

It can be shown that BE and SE are downward sloping, like in the case of auction. The behavior of BE and SE near points $(0, 1)$ and $(1, 0)$ is analyzed next.

**1.** $(\beta, \sigma) \to (0, \sigma)$ in which case $\gamma = \theta$ and BE becomes, after some manipulation, as

$$\frac{\sigma}{1-e^{-\varphi}} = \frac{\delta\theta}{2(1-\delta)} + \frac{\theta}{1-e^{-\theta}}. \tag{33}$$

The RHS is constant, and the LHS approaches infinity when $\sigma$ approaches unity, and LHS approaches zero when $\sigma$ approaches zero. The LHS is increasing in $\sigma$. Then we know that BE goes through point $(0, \sigma)$, where $\sigma \in (0, 1)$.

**2.** $(\beta, \sigma) \to (\beta, 1)$ in which case $\varphi = 0$, and BE becomes

$$-\beta = \frac{2(1-\delta)}{\delta}\left[\frac{\varphi}{(1-e^{-\varphi})}\right] \tag{34}$$

The limit of RHS of (34) when $\sigma$ approaches one is, using L´Hospital's rule, equal to $\frac{2(1-\delta)\theta}{\delta}$, and we conclude that BE does not go through $(\beta, 1)$ where $\beta \geq 0$.

**3.** $(\beta,\sigma) \rightarrow (\beta, 0)$ in which case $\varphi = \theta^{-1}$, and BE becomes

$$\gamma = \frac{-2(1-\delta)\gamma}{\delta(1-e^{-\gamma})} \tag{35}$$

which never holds. Thus, BE does not go through $(\beta, 0)$ where $\beta < 1$.

**4.** $(\beta,\sigma) \rightarrow (1,\sigma)$ in which case $\gamma = 0$, and BE becomes

$$-\frac{1}{\varphi} = \frac{2(1-\delta)}{\delta}\left[\frac{1}{1-e^{-\varphi}} - \frac{\gamma}{\sigma(1-e^{-\gamma})}\right], \tag{36}$$

where $\lim_{\beta\rightarrow 1}\frac{\gamma}{1-e^{-\gamma}} = 1$. When $\beta$ approaches one, BE becomes

$$1 = \frac{2(1-\delta)\varphi}{\delta}\left[\frac{1}{\sigma} - \frac{1}{1-e^{-\varphi}}\right]. \tag{37}$$

The term in brackets approaches infinity if $\sigma$ approaches zero. The term in brackets is decreasing in $\sigma$.We can conclude that BE goes through point $(1,\sigma)$ where $\sigma > 0$. The analysis for SE-curve goes analogously and it is not presented here. In sum, BE and SE are situated like in the case of auction, presented in Figure 2. Also, the arrows depicting out of steady state behavior are similar to the case of auction. We can state the following (the proofs are similar to proofs of Theorems 1 and 2, and they are omitted):

**Theorem 3** *If the trading mechanism is bargaining, then i) the market where both buyers and sellers use mixed strategies over the wait and move decisions is not evolutionarily stable, and ii) the market with one-sided coordination problem is evolutionarily stable.*

**Theorem 4** *In case of bargaining, the basin of attraction for the market where sellers wait grows when the ratio of buyers to sellers grows.*

Like in case of auction, the intersection of BE and SE moves toward point $(1, 0)$ when $\theta$ grows. We see that from Proposition 2 and its proof.

# 7 Conclusion

Traditionally, the directed search literature has simply *assumed* that either all sellers wait and all buyers search, or vice versa. This paper makes a step towards *explaining* who search and who wait. We show that one-sided search as an evolutionarily stable equilibrium in a model where both buyers and sellers have the opportunity to search or wait. We begin with two-sided search equilibrium which is shown to be unstable, and show that the market converges to either of the two stable one-sided search equilibria: i) all sellers wait and all buyers search, or ii) all buyers wait and all sellers search. The relative likelihoood of these one-sided equilibria depends on the relative number of buyers and sellers: Increasing (decreasing) the number of buyers relative to sellers increases the probability that the market converges to an equilibrium where all sellers wait (search) and all buyers search (wait). This result holds whether we assume that the trading mechanisms is auction or bargaining.

In addition to the above main result, we derive several results about efficiency: i) In two-sided search equilibrium, bargaining is more efficient than auction. The result originates from the assumed physical nature of search: if an agent leaves his location in order to search, the location becomes empty, and an agent of the opposite side who happens to choose this location will not trade. Bargaining internalises this externality better than auction. ii) One-sided search is more efficient than two-sided search. This a very intuitive result, because in two-sided search, searching agents can end up in an empty location. iii) In one-sided search it is more efficient if the members of the larger pool search and the members of the smaller pool wait, than vice versa.

Of course, auction and equal-split bargaining are not the only trading rules one can apply in this kind of model. One could use a mixture of auction and bargaining in the following way: If the stayer meets just one mover, they split the resulting surplus in half. If more than one mover comes to the stayer, there is an auction, and each of the movers get their reservation values, and the stayer gets one minus a mover's reservation value. However, the results regarding stability are not likely to change, and the efficiency of mixture of auction and equal-split bargaining is likely to lie somewhere between the two.

It seems that in principle one could find a surplus-sharing rule that maximises the number of trades, given the matching process. Such rules are presented by Hosios (1990) and Mortensen (1982). The “Hosios rule” states that efficiency is obtainend if and only if the surplus going to an agent equals his marginal contribution to matches. The Hosios rule is applicable only if the matching function has constant returns to scale. Mortensen’s rule is applicable also in case of non-constant returns, but it is required that the initiator of a match (which, in the present model, would be a mover rather than the stayer) can be identified. The rule - in case of constant returns to scale matching function - says that efficiency is achieved if the initiator is allocated the whole surplus of the match. However, Hosios’s and Mortensen’s rules are designed for pairwise meetings, and applying them in the present model is not straightforward since a stayer may meet many movers in one period. Recently, Julien, Kennes, and King (2002) have presented a similar kind of rule in a search model where there may be multiple meetings. However, the coordination problem in their model is one-sided, and they assume that it is the seller side (sellers of labour) that waits.

One could give up the assumption that search is physical in nature. Then agents could take and receive contacts at the same time. This seems to lead to a very difficult analysis, since one would have to deal with a vast net of contacts. While maintaining the physical search, one could introduce heterogeneity into the model: sellers could have different kind of goods, or different amounts of them, or different buyers could value the same good differently. It may then well happen that a two-sided search equilibrium is stable.

# 8 Appendix

### Proof of Proposition 2

For the proof, we use items 1 - 4 in section 6.2. The buyers’ equilibrium condition $W_b^w = W_b^m$, called BE, can be written as

$$\frac{\gamma}{\sigma} - \frac{\beta}{\varphi} - \frac{2(1-\delta)}{\delta(1-e^{-\varphi})} + \frac{2(1-\delta)\gamma}{\delta\sigma(1-e^{-\gamma})} = 0, \tag{A1}$$

and using $\gamma \equiv (1-\beta)\theta$ and $\varphi \equiv \dfrac{1-\sigma}{\theta}$, it can be written as

$$\frac{(1-\beta)\theta}{\sigma} - \frac{\beta\theta}{1-\sigma} - \frac{2(1-\delta)}{\delta\left(1-e^{\frac{\sigma-1}{\theta}}\right)} + \frac{2(1-\delta)(1-\beta)\theta}{\delta\sigma\left(1-e^{(\beta-1)\theta}\right)} = 0. \tag{A2}$$

Differentiating (A2) with respect to $\beta$ and $\theta$ gives

$$\frac{d\beta}{d\theta} = \frac{\dfrac{1-\beta}{\sigma} - \dfrac{\beta}{1-\sigma} - \dfrac{2(1-\delta)\varphi e^{-\varphi}}{\delta\left(1-e^{-\varphi}\right)^2\theta}}{\dfrac{\theta}{\sigma} + \dfrac{\theta}{1-\sigma} + \dfrac{2(1-\delta)\theta\left(1-e^{-\gamma}-\gamma e^{-\gamma}\right)}{\delta\sigma\left(1-e^{-\gamma}\right)^2}}. \tag{A3}$$

The denominator of the RHS of (A3) is positive. In the numerator, term $\dfrac{1-\beta}{\sigma} - \dfrac{\beta}{1-\sigma}$ has the same sign as term $1-\beta-\sigma$, which is equal to zero in equilibrium, implied by equations (23) and (24). We conclude that if $\theta$ increases, $\beta$ decreases for any given $\sigma$.

Next we show that there is a unique equilibrium $\sigma$ for any given $\theta$. Write (25) as

$$1-\theta-e^{-\sigma\theta} = -\theta e^{\frac{\sigma-1}{\theta}}. \tag{A4}$$

Fix $\theta$ and differentiate both sides of (A4) with respect to $\sigma$ : $\dfrac{\partial\left(1-\theta-e^{-\sigma\theta}\right)}{\partial\sigma} = \theta e^{-\sigma\theta} > 0$, and $\dfrac{\partial\left(-\theta e^{\frac{\sigma-1}{\theta}}\right)}{\partial\sigma} = -e^{\frac{\sigma-1}{\theta}} < 0$ . That is, $\sigma$ is a function of $\theta$.

In order to solve how the equilibrium $\sigma$ changes when $\theta$ changes, we must find how the buyers' and sellers' equilibrium curves $BE$ and $SE$ are situated in $(\beta,\sigma)$ plane. For this, look at items 1-4 in section 6.2., and Figure 2 which applies for both auction and bargaining. At an intersection of $BE$ and $SE$, the $BE$ -curve is steeper than the $SE$ -curve. The equilibria lie on the downward-sloping diagonal line (because $\beta+\sigma=1$ in equilibrium). If $\theta$ increases, the $BE$ -curve moves to the left, and the new equilibrium value of $\sigma$ must situate to the right and down from the old equilibrium $\sigma$. ■

**Proof of Proposition 5**

Expression $S\left(1-e^{-\theta}\right) - B\left(1-e^{-1/\theta}\right)$ has the same sign as expression $1-e^{-\theta}-\theta+\theta e^{-1/\theta}$ which equals zero if $\theta=0$ or if $\theta=1$. Differentiating yields $\partial\left(1-e^{-\theta}-\theta+\theta e^{-1/\theta}\right)/\partial\theta =$

$e^{-\theta} + (1 + 1/\theta)\, e^{-1/\theta} - 1$ which is positive if $\theta = 1$. Next we show that equation

$$1 - e^{-\theta} - \theta + \theta e^{-1/\theta} = 0 \tag{A5}$$

has exactly one strictly positive solution, $\theta = 1$. Let us study function $g(h) = e^h - 1 - he^h + he^{h-1/h}$, which has the same zeroes as the LHS of (A5).

i) Here it is shown that equation $1 - e^{-\theta} - \theta + \theta e^{-1/\theta} = 0$ has no solution in interval $\theta \in (0, 1)$. We immediately see that $g(0) = g(1) = 0$. The derivative of $g$ is

$$g'(h) = -he^h + e^{h-1/h}\left(h + 1 + \frac{1}{h}\right). \tag{A6}$$

We can see that $g'(1) = 3 - e > 0$. In the RHS of (A6) we have $\lim_{h\to 0}\left(\frac{e^{h-1/h}}{h}\right) = \lim_{h\to 0}\left(\frac{e^{\frac{h^2-1}{h}}}{h}\right)$, and using L´Hôspital's rule, it is equal to $\lim_{h\to 0}\left(e^{\frac{h^2-1}{h}}\left(\frac{h^2+1}{h^2}\right)\right) = \lim_{h\to 0}\left(e^{h-\frac{1}{h}}\right) = 0$. We see that $\lim_{h\to 0} g'(h) = 0$. The second derivative of $g$ is

$$g''(h) = -e^h - he^h + he^{h-1/h}\left(2 + h + \frac{2}{h} + \frac{1}{h^3}\right). \tag{A7}$$

Using L´Hôspital's rule we get $\lim_{h\to 0} g''(h) = -1$. Thus, at first $g(h)$ is decreasing. Next we show that $g(h) \neq 0$ in interval $h \in (0, 1)$. If $g(h) = 0$ in interval $h \in (0, 1)$ and if $g$ attained strictly positive values, there should be at least two zeroes. Before the last zero $g$ would reach a maximum and its derivative would be zero. Let us denote the last maximum of $g$ (where it is positive) by $k$. Thus we know that $g'(k) = -ke^k + e^{k-1/k}\left(k + 1 + \frac{1}{k}\right) = 0$ and $g(k) = e^k - 1 - ke^k + ke^{k-1/k} > 0$. From these conditions we get

$$g(k) = e^k - 1 - ke^k + \frac{k^3 e^k}{1 + k + k^2} > 0, \tag{A8}$$

which holds if $e^k - 1 - k - k^2 > 0$. We have $\partial\left(e^k - 1 - k - k^2\right)/\partial k = e^k - 2k - 1$ which is equal to zero at $k = 0$ and negative at $k = 1$. We also have $\partial\left(e^k - 2k - 1\right)/\partial\theta = e^k - 2$ which is equal to $-1$ at $k = 0$, and $\partial^2\left(e^k - 2k - 1\right)/\partial\theta^2 = e^k > 0$. We conclude that $e^k - 1 - k - k^2$ is negative in interval $k \in (0, 1]$. Thus, the assumption that $g$ is positive at $k \in (0,1)$ leads to a contradiction. The result is that equation $1-e^{-\theta}-\theta+\theta e^{-1/\theta}=0$ has no solution in interval $\theta \in (0,1)$.

ii) To show that equation $1-e^{-\theta}-\theta+\theta e^{-1/\theta}=0$ has no solution at $\theta>1$ it is enough to show that $g'(h)=-he^{h}+e^{h-1/h}\left(h+1+\frac{1}{h}\right)>0$ when $h>1$, because $g(1)=0$ and $g'(h)>0$ at $h=1$. The sign of $g'(h)$ is positive if and only if $v(h)\equiv 1-e^{-1/h}-\frac{1}{h}e^{-1/h}-\frac{1}{h^2}e^{-1/h}<0$. We see that $v(1)<0$ and $\lim_{h\to\infty}v(h)=0$. Further, $v'(h)=1+e^{-1/h}\frac{1}{x^3}\left(1-\frac{1}{x}\right)$, which is positive if $h>1$. That is, $v(h)<0$ if $h>1$, thus $g'(h)=-he^{h}+e^{h-1/h}\left(h+1+\frac{1}{h}\right)>0$ when $h>1$.

We have shown that equation $1-e^{-\theta}-\theta+\theta e^{-1/\theta}=0$ has exactly two solutions, $\theta=0$ and $\theta=1$. If $\theta\in(0,1)$, then $1-e^{-\theta}-\theta+\theta e^{-1/\theta}<0$, and if $\theta>1$, then $1-e^{-\theta}-\theta+\theta e^{-1/\theta}>0$, resulting in $S\left(1-e^{-\theta}\right)<B\left(1-e^{-1/\theta}\right)$ if $\theta\in(0,1)$, and $S\left(1-e^{-\theta}\right)>B\left(1-e^{-1/\theta}\right)$ if $\theta>1$.■

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