Credibility in Optimal Factor Taxation

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Discussion Paper No. 583:2003
ISBN 952-10-1233-1
ISSN 1459-3696

November 20, 2003

Abstract

This paper examines optimal factor taxation when output is produced from labour and capital and some (or all) households save capital. It is shown that there is a reputational equilibrium in which the government has no incentive to change its announced tax policies. In this equilibrium, the Judd-Chamley assertion that the tax on capital income tends to zero in the limit holds. This is independent of the capital owners' proportion of wages as well as their weight in the social welfare function. The optimal wage tax is determined by a specific elasticity rule.

Journal of Economic Literature: H21, C73, E62

Keywords: optimal taxation, capital accumulation, credibility

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1 Introduction

We consider the *second best policy* where a benevolent government chooses a sequence of tax rates over time to maximize social welfare. According to Benhabib and Rustichini (1997), this policy is inconsistent. It is first optimal for the government to promote capital accumulation by promising low taxes on capital in the future. Once capital has been accumulated, however, it becomes convenient for the government to tax capital (with no distorting effect) rather than to impose distorting taxes on labour. The result of this inconsistency is that promises on tax policy are not credible. We show that if the government can tax consumption, labour and capital, then there is a reputational equilibrium with consistent public policy. Consequently, it is not optimal for the government to renege on its commitments.

The classical Judd-Chamley assertion says that the optimal tax rate on capital income is zero in the limit.[1] Chamley (2001) shows that this depends critically on the existence of a perfect bond market in which private agents take the interest rate as given, households save in bonds, and firms can finance any amount of investment by issuing bonds. In this study, we assume that households own firms directly. We can then aggregate firms and their owners together into capitalists, who decide on capital accumulation. We show that the Judd-Chamley assertion applies to this case.

Lansing (1999) challenges the Judd-Chamley assertion. When the first-order conditions of agents are imposed as constraints on the government's allocation problem, one implicitly presumes that there are anticipation effects', whereby the announced future tax rates can affect current allocations. With a logarithmic utility function, the income and substitution effects of future interest rate movements exactly cancel each other, so that the household needs to observe only the current (after-tax) rate of return to decide how much to consume and save. In such a case, the steady-state optimal tax

[1]Cf. Judd (1985), Chamley (1986) and Correia (1996).

rate on capital income is generally non-zero. In this study, we show that this argument holds only in a very specific case, while the optimal tax on capital income is generally zero in the limit, even with a logarithmic utility function.

In this study, we put the second best public policy in the form of a Stackelberg differential game with the government and the (representative) capitalist as players. Xie (1997) argues that in such a game, one commonly uses a boundary condition which is not a necessary condition of optimal public policy, but which is responsible for the time inconsistency in the models. In this study, this boundary condition stems from the necessary conditions. It does not however violate consistency, because there is a reputational equilibrium in which the government has no incentive to cheat the public.

The remainder of this paper is organized as follows. Sections 2 presents the household and public sectors and section 3 the production sectors of the economy. In section 4, we construct the case where the capitalist tries to anticipate the government's best response, and in sections 5, 6 and 7 the case where the capitalist takes the taxes as given. Section 8 considers the limit case of logarithmic utility.

2 Households and the public sector

The *formal sector* can be taxed and produces goods from capital and labour. The *informal sector*, in which the workers produce goods for themselves without capital, cannot be taxed. We aggregate all goods into one, the price of which is normalized at unity. The informal-sector output N is then a decreasing but concave function of labour supply in the formal sector, L^S:

$$N(L^S), \quad N' < 0, \quad N'' < 0. \tag{1}$$

Hence, more and more labour must be transferred from the formal to the informal sector to produce one more unit in the latter. This two-sector framework is the simplest way of introducing distortion in labour taxation.

There are two groups of households. The *capitalists* save and earn profits and a fixed share α of the total wages W. The *non-capitalists* earn the rest $(1-\alpha)$ of the total wages and consume all their income. We assume that each capitalist owns only a small share of each firm, so that she ignores the effect of her investment policy on her labour income. In this study, we use parameter α as a measure of income distribution. The model is an extension of two special cases. For $\alpha = 0$, we obtain Judd's (1985) case where the capitalists earn profits and do not work, while the workers earn wages and do not save. For $\alpha = 1$, we obtain Chamley's (1986) model of a representative agent who saves and earns both wages and profits.

The whole population has the same constant rate of time preference, $\rho > 0$. The (representative) capitalist's and non-capitalist's instantaneous utilities are given by

$$U(C) \doteq \frac{C^{1-\sigma}-1}{1-\sigma}, \ \sigma > 0, \ \sigma \neq 1, \quad V\big((1-\alpha)W\big), \ V' > 0, \ V'' < 0, \quad (2)$$

where C is the capitalist's consumption, $C_W \doteq (1-\alpha)W$ the non-capitalist's income, which is completely consumed, and constant $1/\sigma$ the intertemporal elasticity of substitution for the capitalist. The capitalist's and non-capitalist's utilities from a flow of consumption starting at time t are

$$\int_t^\infty U(C)e^{-\rho(s-t)}ds, \quad \int_t^\infty V\big((1-\alpha)W\big)e^{-\rho(s-t)}ds, \quad (3)$$

where t is time. The social welfare function is a weighted average of the utilities of the non-capitalist and capitalist:

$$\Psi \doteq \int_t^\infty [V\big((1-\alpha)W\big) + \vartheta U(C)]e^{-\rho(s-t)}ds, \quad (4)$$

where constant $\vartheta > 0$ is the social weight of the capitalist. The marginal rate of substitution between the capitalist's and non-capitalist's consumption when social welfare Ψ is held constant is given by

$$\psi \doteq -\big[dC/dC_W\big]_{\Psi \text{ constant}} = V'/(\vartheta U'). \quad (5)$$

Because the government has fixed expenditures E which must be financed at each moment of time, its budget constraint is given by

$$E = \tau_C C + \tau_K \Pi + \tau_W wL, \tag{6}$$

where $\tau_C > -1$, $\tau_W > -1$ and $\tau_K \leq 1$ are taxes on the capitalist's consumption C, wages in the formal sector, wL, and (gross) capital income Π, respectively.[2] We assume, for technical reasons, that there is a fixed upper limit $\upsilon \in [0, \infty)$ for the capital subsidy $-\tau_K$, so that[3]

$$-\upsilon \leq \tau_K \leq 1. \tag{7}$$

3 Production and investment

Capital K is the only asset in which the capitalist can save. In the formal sector, output is produced from capital K and labour L through the function

$$F(K, L), \;\; F_K > 0, \, F_L > 0, \, F_{KK} < 0, \, F_{LL} < 0, \tag{8}$$

where subscripts K and L denote the partial derivatives of F with respect to K and L.

Because the labour markets are competitive, in the formal sector the labour supply L^S is equal to the demand for labour, L, and the marginal product of labour in the informal sector, N', is equal to the wage w:

$$L^S = L, \quad w = -N'(L^S) = -N'(L). \tag{9}$$

Given (1) and (9), we obtain the elasticity of the labour supply in the formal sector with respect to the wage w as follows:

$$\varepsilon \doteq \left| \frac{w}{L^S} \frac{dL^S}{dw} \right| = \frac{N'}{N'' L}. \tag{10}$$

[2]Because the non-capitalists do not save but both wages and capital income are taxed, it makes no difference whether the government taxes total consumption $(1-\alpha)W + C$ or just the capitalist's consumption C. The cases $\tau_C = -1$ and $\tau_W = -1$ are not feasible, because the effective prices for consumption and labour cannot be zero for a capitalist.

[3]Otherwise, it is possible that $-\tau_K$ would have infinite value in the government's optimal program in section 5.

A labour-surplus economy, for which the wage w is exogenous, is the limit case $\varepsilon \to \infty$. Labour income W is equal to income from the informal sector, N, plus wages in the formal sector, wL. Given (9) and (10), we define

$$W(L) \doteq N + wL = N(L) - N'(L)L, \quad W' = -N''L = -N'/\varepsilon. \tag{11}$$

Firms in the formal sector maximize profit Π by labour L taking the wage w, capital K and the wage tax τ_W as given. This and (9) yield

$$\begin{aligned}&\Pi(K, w, \tau_W) \doteq \max_L [F(K, L) - (1 + \tau_W)wL - \mu K], \;\; L(K, \tau_W) \doteq \arg\max_L \Pi,\\ &\Pi_K \doteq \partial\Pi/\partial K = F_K - \mu, \;\; F_L(K, L) = (1 + \tau_W)w = -(1 + \tau_W)N'(L),\\ &\Pi_{KK} \equiv 0 \;\;\Leftrightarrow\;\; \Pi = \Pi_K K.\end{aligned} \tag{12}$$

The capitalist's budget constraint is given by

$$\dot{K} \doteq dK/dt = \alpha W + (1 - \tau_K)\Pi(K, w, \tau_W) - (1 + \tau_C)C \geq -\mu K, \tag{13}$$

where $\dot{K}$ is capital accumulation (= saving). Inequality $\dot{K} \geq -\mu K$ means that investment is irreversible. In the whole economy, capital accumulation is equal to production in the two sectors, $N(L)+F(K, L)$, minus the capitalist's consumption C, the non-capitalist's consumption $(1 - \alpha)W$, government expenditure E and depreciation μK. Given (11), we then obtain

$$\dot{K} = N(L) + F(K, L) - C - (1 - \alpha)W(L) - E - \mu K \geq -\mu K. \tag{14}$$

Because the labour and goods markets are balanced by (9) and (14), then, by the Walras law, the government budget (6) is balanced as well. Hence the government budget is not a constraint in the problem of public policy.

4 Non-credible public policy

In this section, we assume that the government can renege on an announced sequence of taxes. This means that the game must be solved backwards as follows. At each moment t, both the capitalist and the government make their choices on the assumption that the opponent will make her choices optimally for the whole period (t, ∞). The crucial property of dynamic programming is that the strategies of both parties are independent of the initial time t.[4]

The representative capitalist takes the wage w and total labour income in the economy, W, as given. She maximizes her utility $\int_t^\infty U(C)e^{-\rho(s-t)}ds$ by her consumption C subject to capital accumulation (13), given w, W and taxes (τ_W, τ_K, τ_C). When the strategies of the government and the capitalist are in Stackelberg equilibrium at each point of time from any moment t onwards, the capitalist's utility at that moment t from the flow of consumption starting at time t, given capital accumulation (13), is defined as follows:

$$B(K,t) \doteq \max_{C \text{ s.t. } (13)} \int_t^\infty U(C)e^{-\rho(s-t)}ds.$$

We denote the partial derivatives of B as $B_K \doteq \partial B/\partial K$, $B_t \doteq \partial B/\partial t$, $B_{KK} \doteq \partial^2 B/\partial K^2$, $B_{tt} \doteq \partial^2 B/\partial t^2$ and $B_{Kt} \doteq \partial^2 B/(\partial K \partial t)$, for convenience.

The Hamilton-Jacobi-Bellman equation for the capitalist is given by

$$\rho B - B_t = \max_C \left\{ U(C) + B_K \dot{K} \,\middle|\, \dot{K} \geq -\mu K, \ (13) \right\}, \tag{15}$$

where labour income W, the wage w and taxes (τ_W, τ_K, τ_C) are given. Noting (13), the maximization in (15) yields the first-order condition

$$\frac{C^{-\sigma}}{1+\tau_C} - B_K = \frac{U'}{1+\tau_C} + B_K \frac{\partial \dot{K}}{\partial C} = \nu \begin{cases} = 0 & \text{for } \dot{K} > -\mu K, \\ > 0 & \text{for } \dot{K} = -\mu K, \end{cases} \tag{16}$$

where ν is the Kuhn-Tucker multiplier corresponding to the inequality $\dot{K} \geq -\mu K$. Differentiating equation (15) with respect to K, we obtain

$$\rho B_K - B_{Kt} = B_{KK}[\alpha W + (1-\tau_K)\Pi - (1+\tau_C)C] + (1-\tau_K)B_K \Pi_K.$$

Trying solution $\lambda(t) \doteq B_K(K,t)$ for this equation, we obtain

$$B_{KK} \equiv 0, \quad \dot{\lambda} = B_{Kt} = [\rho - (1-\tau_K)\Pi_K]B_K = [\rho - (1-\tau_K)\Pi_K]\lambda.$$

[4]The formal solution of dynamic programming is given in Kamien and Schwarz (1985), section 21, for example.

Hence, in equilibrium, $B_K = \lambda \equiv 0$. Given (14), (16) and $B_K \equiv 0$, we obtain

$$\dot{K} = -\mu K < 0, \quad C = N(L) + F(K,L) - (1-\alpha)W(L) - E. \tag{17}$$

The government chooses taxes (τ_K, τ_W, τ_C) to maximize social welfare (4), given the reaction of the capitalist (17), the dependence of labour income on employment, (11), the determination of the wage $w = -N'(L)$ by (9), and the constraints (7). Because there is a one-to-one correspondence from τ_W to L through (11), τ_W can be replaced by L as a control variable. The social welfare from the flow of consumption starting at time t is given by

$$G(K,t) \doteq \max_{L,\tau_K,\tau_C \in [-\upsilon,1]} \int_t^\infty \big[V\big((1-\alpha)W(L)\big) + \vartheta U(C)\big] e^{-\rho(s-t)} ds.$$

We denote the partial derivatives of G as $G_K \doteq \partial G/\partial K$, $G_t \doteq \partial G/\partial t$, $G_{KK} \doteq \partial^2 G/\partial K^2$, $G_{tt} \doteq \partial^2 G/\partial t^2$ and $G_{Kt} \doteq \partial^2 G/(\partial K \partial t)$, for convenience.

The Hamilton-Jacobi-Bellman equation for the government is given by

$$\begin{aligned}\rho G - G_t &= \max_{L,\tau_K} \big\{ V\big((1-\alpha)W(L)\big) + \vartheta U(C) + G_K \dot{K} \big|\, (17) \big\} \\ &= \max_{L,\tau_K} \big\{ V\big((1-\alpha)W(L)\big) + \vartheta U(C) - \mu K G_K \big\}. \end{aligned} \tag{18}$$

Noting (5), (11), (12) and (17), the maximization by L in (18) yields

$$\begin{aligned} 0 &= (1-\alpha)V'W' + U'\vartheta\, \partial C/\partial L = (1-\alpha)W'[V' - \vartheta U'] + \vartheta U'[N' + F_L] \\ &= (1-\alpha)W'[V' - \vartheta U'] - \vartheta U' N' \tau_W = \vartheta U' N'[(1-\alpha)(1-\psi)/\varepsilon - \tau_W], \end{aligned}$$

which is equivalent to

$$\tau_W = (1-\alpha)(1-\psi)/\varepsilon. \tag{19}$$

We summarize the result of this section as follows:

Proposition 1 *With non-credible public policy, the optimal wage tax is given by* (19)*, and capital stock K is exhausted at the rate $\mu = -\dot{K}/K$.*

The tax rule (30) can be explained as follows. The lower the elasticity ε, the less distorting labour taxation is and the more labour should be taxed. If all households are capitalists, $\alpha = 1$, then public expenditures should be financed by the non-distorting consumption tax τ_C and wages should not be taxed at all, $\tau_W = 0$. If the workers do not save, $\alpha = 0$, and the capitalists have no political power, $\psi = \vartheta = 0$, then the optimal wage tax τ_W is equal to the inverse of the elasticity ε. Otherwise, labour taxation is between these two extremes as follows. The higher the relative social value of the capitalists' consumption (i.e., the closer ψ to one), or the larger proportion of wages the capitalists earn (i.e. the closer α to one), the closer labour taxation should be to the case where all consumers are capitalists, $\alpha = 1$.

5 The capitalist

Assume that the government is, for some unspecific reason, prevented from reneging on the announced sequence of taxes. The formal structure of the interaction between the government and the capitalist then corresponds to an open-loop Stackelberg equilibrium outcome for a non-cooperative, infinite differential game in which the government is the leader and the capitalist the follower.[5] Technically, the solution of this game is follows. Determine first the unique optimal response of the capitalist to every strategy of the government. The capitalist's choices can be made on the basis of the initial capital stock without making any difference to the solution. Second, find the government's optimal strategy given the capitalist's optimal response. Since the government cannot depart from its announced strategy, then, at each moment, the government makes its choices by the initial capital stock.

Let $t = 0$ be the initial moment. Given (2) and (3), the capitalist maximizes her utility $\int_0^\infty U(C)e^{-\rho t}dt$ by her consumption C subject to capital

[5] Cf. Basar and Olsder (1982), section 7.2, for the formal solution of the Stackelberg leadership in a differential game.

accumulation (13), taking the wage w, labour income W and taxes (τ_C, τ_K) as given. This yields the Hamiltonian H^C and the Langrangean $\mathcal{L}^C$ as:

$$\begin{aligned} H^C &= U(C) + \lambda[\alpha W + (1-\tau_K)\Pi(K, w, \tau_W) - (1+\tau_C)C], \\ \mathcal{L}^C &= H^C + \delta[\alpha W + (1-\tau_K)\Pi(K, w, \tau_W) - (1+\tau_C)C + \mu K], \end{aligned} \tag{20}$$

where the co-state variable λ evolves according to

$$\dot{\lambda} = \rho\lambda - \frac{\partial \mathcal{L}^C}{\partial K} = \rho\lambda - (1-\tau_K)\Pi_K(K, w, \tau_W)(\lambda+\delta), \ \lim_{t\to\infty} \lambda K e^{-\rho t} = 0, \tag{21}$$

and the multiplier δ is subject to the Kuhn-Tucker conditions

$$\delta[\dot{K} + \mu K] = \delta[\alpha W + (1-\tau_K)\Pi(K, w, \tau_W) - (1+\tau_C)C + \mu K] = 0, \ \ \delta \geq 0. \tag{22}$$

The first-order condition for the capitalist's optimization is given by

$$C^{-\sigma} = U'(C) = (1+\tau_C)(\lambda+\delta). \tag{23}$$

Assume first that $\tau_K = 1$. Equation (21) then takes the form $\dot{\lambda} = \rho\lambda$ and, by choosing the initial value $\lambda(0)$ subject to $\lim_{t\to\infty} \lambda K e^{-\rho t} = 0$, we obtain $\lambda \equiv \lambda(0) = 0$. From (22), (23) and $\lambda \equiv 0$ it follows that $\delta = C^{-\sigma}/(1+\tau_C) > 0$,

$$\dot{K} = -\mu K < 0 \ \text{ and } \ C = N + F - (1-\alpha)W - E \ \text{ for } \tau_K = 1. \tag{24}$$

Assume next that $\tau_K < 1$. In such a case, $\dot{K} > -\mu K$ holds. Noting (21), (22) and (23), we obtain $\delta = 0$, $C^{-\sigma} = \lambda$,

$$\dot{C}/C = -(1/\sigma)\dot{\lambda}/\lambda = [(1-\tau_K)\Pi_K(K, w, \tau_W) - \rho]/\sigma \ \text{ for } \tau_K < 1. \tag{25}$$

Variables K and C are governed by the system (13) and (25). With decreasing returns to scale, $\Pi_{KK} < 0$, the dynamics is as follows. Because

$$\frac{\partial \dot{K}}{\partial K} = (1-\tau_K)\Pi_K > 0, \ \frac{\partial \dot{K}}{\partial C} < 0, \ \frac{\partial \dot{C}}{\partial K} = (1-\tau_K)\Pi_{KK}C < 0, \ \left.\frac{\partial \dot{C}}{\partial C}\right|_{\dot{C}=0} = 0,$$

we obtain

$$\frac{\partial \dot{K}}{\partial K} + \frac{\partial \dot{C}}{\partial C}\bigg|_{\dot{C}=0} > 0, \quad \frac{\partial \dot{K}}{\partial K}\frac{\partial \dot{C}}{\partial C}\bigg|_{\dot{C}=0} < \frac{\partial \dot{K}}{\partial C}\frac{\partial \dot{C}}{\partial K},$$

and a saddle-point solution for the system exists. Hence, the co-state variable C (which represents λ) jumps onto the saddle path which leads to the steady state in which K, C and λ are constants, and $\lim_{t\to\infty} \lambda K e^{-\rho t} = 0$ holds. With constant returns to scale, $\Pi_{KK} \equiv 0$, there must be $\Pi = \Pi_K K$ by (12). Given $\Pi = \Pi_K K$, (13), (21) and $\delta = 0$, we obtain

$$\left[\frac{\dot{K}}{K} + \frac{\dot{\lambda}}{\lambda} - \rho\right]_{\dot{K}=0} = \left[\alpha\frac{W}{K} - (1+\tau_C)\frac{C}{K}\right]_{\dot{K}=0} = (\tau_K - 1)\Pi_K < 0.$$

This implies the transversality condition $\lim_{t\to\infty} K\lambda e^{-\rho t} dt = 0$.

6 The government

Given (2) and (4), the government sets taxes (τ_W, τ_K, τ_C) to maximize social welfare $\int_0^\infty [V((1-\alpha)W) + \vartheta U(C)]e^{-\rho t}dt$ subject to the response of the private sector, (14), (24) and (25), the function $L(K, \tau_W)$ in (12), the determination of the wage $w = -N'(L)$ by (9), and constraints (7). Because there is one-to-one correspondence from τ_W to L through $L(K, \tau_W)$ in (12), τ_W can be replaced by L as a control variable.

Assume $\tau_K \equiv 1$ for $t \in [0, \infty)$. Then, given (24), $\dot{K} = -\mu K$ holds and the Hamiltonian for the government's maximization is given by

$$H^I = V\big((1-\alpha)W(L)\big) + \vartheta U\big(N(L) + F(K,L) - (1-\alpha)W(L) - E\big) - \gamma\mu K, \tag{26}$$

where the co-state variable γ evolves according to $\dot{\gamma} = \rho\gamma - \partial H^I/\partial K$ and $\lim_{t\to\infty} K\gamma e^{-\rho t} = 0$. The first-order condition for L is given by (19). These results can be rephrased as:

Proposition 2 *If $\tau_K \equiv 1$ for $t \in [0, \infty)$, the dynamics of the economy is the same as with non-credible public policy in proposition 1.*

Next, assume that the government chooses $\tau_K \in [-\mu, 1]$ freely. The Hamiltonian and the Lagrangean for the government's maximization are then

$$\begin{aligned} H^{II} &= V\big((1-\alpha)W(L)\big) + \vartheta U(C) + \eta[(1-\tau_K)\Pi_K(K,w,\tau_W) - \rho]C/\sigma \\ &\quad + \gamma\big[N(L) + F(K,L) - (1-\alpha)W(L) - C - E - \mu K\big], \\ \mathcal{L} &= H^{II} + \chi_1(1-\tau_K) + \chi_2(\tau_K + \upsilon), \end{aligned} \tag{27}$$

where the co-state variables γ and η evolve according to

$$\dot{\gamma} = \rho\gamma - \frac{\partial\mathcal{L}}{\partial K}, \quad \lim_{t\to\infty} K\gamma e^{-\rho t} = 0, \quad \dot{\eta} = \rho\eta - \frac{\partial\mathcal{L}}{\partial C}, \quad \lim_{t\to\infty} C\eta e^{-\rho t} = 0, \tag{28}$$

and variables χ_1 and χ_2 satisfy the Kuhn-Tucker conditions

$$\chi_1(1-\tau_K) = 0, \;\; \chi_1 \geq 0, \quad \chi_2(\tau_K + \upsilon) = 0, \;\; \chi_2 \geq 0. \tag{29}$$

The first-order conditions for the capital tax τ_K are given by

$$\partial\mathcal{L}/\partial\tau_K = -(C/\sigma)\Pi_K\eta - \chi_1 + \chi_2 = 0. \tag{30}$$

Examine first the case $-\upsilon < \tau_K < \upsilon$, in which $\chi_1 = \chi_2 = 0$. Because $\partial^2\mathcal{L}/\partial w^2 = \partial^2 H^{II}/\partial w^2 \equiv 0$ then holds, the capital tax τ_K must be solved through the generalized Legendre-Clebsch conditions:[6]

$$\begin{aligned} &\frac{\partial}{\partial\tau_W}\Big(\frac{d^p}{dt^p}\frac{\partial H^{II}}{\partial\tau_W}\Big) = 0 \text{ for any odd integrer } p, \\ &(-1)^q\frac{\partial}{\partial\tau_W}\Big(\frac{d^{2q}}{dt^{2q}}\frac{\partial H^{II}}{\partial\tau_W}\Big) \geq 0 \text{ for any integrer } q, \end{aligned} \tag{31}$$

where t is time. Since $C > 0$ by (23), equation (30) yields $\eta = 0$. Differentiating (30) with respect to time t and noting (27) and (28) produces

$$\begin{aligned} &\frac{d}{dt}\Big(\frac{\partial H^{II}}{\partial\tau_W}\Big) = -\frac{C}{\sigma}\Pi_K\dot{\eta}\Big|_{\eta=0} = \frac{C}{\sigma}\Pi_K\frac{\partial H^{II}}{\partial C} = \frac{C}{\sigma}\Pi_K(\vartheta C^{-\sigma} - \gamma) = 0, \\ &\frac{\partial}{\partial\tau_W}\frac{d}{dt}\Big(\frac{\partial H^{II}}{\partial\tau_W}\Big) = 0. \end{aligned} \tag{32}$$

[6] Cf. Bell and Jacobson (1975), pp. 12-19.

Given these and (25), we furthermore obtain

$$\frac{d^2}{dt^2}\Big(\frac{\partial H^{II}}{\partial \tau_W}\Big) = -\Pi_K \vartheta C^{-\sigma}\dot{C} = 0, \quad \frac{\partial}{\partial \tau_W}\frac{d^2}{dt^2}\Big(\frac{\partial H^{II}}{\partial \tau_W}\Big) = \Pi_K^2 \vartheta C^{-\sigma}\frac{C}{\sigma} > 0. \quad (33)$$

Results (32) and (33) satisfy the Clebsch-Legendre conditions (31).

Xie (1997) argues that in Stackelberg differential games, one frequently uses a boundary condition which is not necessary for the optimality of public policy, but which is largely responsible for the time inconsistency in the models. In this model, that particular condition is equivalent to $\eta = 0$, which makes the Hamiltonian H^{II} in (27) independent of τ_K, so that nothing seems to pin down the capital tax τ_K between $-\nu$ and 1. In contrast to Xie (1997), however, $\eta = 0$ is not imposed as a boundary condition, but results from the necessary conditions (31). In the next section, we show that, despite $\eta = 0$, time consistency can be maintained through a reputational equilibrium in which the government has no incentive to cheat the capitalist.

7 Policy rules

From (32) it follows that

$$\gamma = \vartheta U' = \vartheta C^{-\sigma}. \quad (34)$$

Given this and (33), $\dot{C} = 0$ holds, and C and γ are kept constant. Noting (12), (27), (28), $\chi_1 = \chi_2 = 0$ and $\eta = 0$, we then obtain $0 = \dot{\gamma} = \rho\gamma - \partial H^{II}/\partial K = (\rho + \mu - F_K)\gamma$ and

$$\rho = F_K(K, L) - \mu = \Pi_K. \quad (35)$$

Noting $\eta = 0$, (12) and (34), we obtain the first-order condition for L as:

$$\begin{aligned}\partial H^{II}/\partial L &= (1-\alpha)(V' - \gamma)W' + \gamma(N' + F_L) = (1-\alpha)(V' - \gamma)W' - \gamma N'\tau_W \\ &= \gamma N'\big\{(1-\alpha)[V'/(\vartheta U') - 1](W'/N') - \tau_W\big\} = 0, \quad (36)\end{aligned}$$

which is equivalent to (19). Hence we obtain the following result:

Proposition 3 *The optimal wage tax in the steady state is* (19).

Equations $\dot{C} = 0$, (25), (35) and τ_K imply $0 = (1 - \tau_K)\Pi_K - \rho = -\tau_K \rho$, $\tau_K = 0$ and the Judd-Chamley result:

Proposition 4 *The capital tax τ_K should be zero in the steady state.*

This result is a dynamic version of aggregate production efficiency. Because capital is an intermediate good, appearing only in the production function but not in the utility function, it should not be taxed if there are enough instruments to separate consumption and production decisions.

In the steady-state, $F_K(K^*, L(K^*)) = \rho + \mu$ holds. Given (29) and (30), we obtain the following. If $\eta > 0$, then capital is heavily subsidized, $-\tau_K = \upsilon$, and the capitalist accumulates wealth, $\dot{K} > 0$. If $\eta < 0$, then capital income is taxed away, $\tau_K = 1$, and the capitalist exhausts its wealth, $\dot{K} < 0$. In equilibrium $\eta = 0$, capital is K held constant K^*. We conclude as follows:

Proposition 5 *The steady-state level for capital, K^*, is determined by $\rho + \mu = F_K(K^*, L(K^*))$ and, outside the steady state, the capital tax evolves according to $\tau_K = 1$ for $K < K^*$ and $\tau_K = -\upsilon < 0$ for $K > K^*$.*

Because the system produces a steady state in which K, C and γ are constants and $\eta = 0$, $\lim_{t\to\infty} K\gamma e^{-\rho t} = 0$ and $\lim_{t\to\infty} C\eta e^{-\rho t} = 0$ hold.

Because the government's choice set is more restrictive with the rule $\tau_K \equiv 1$ for $t \in [0, \infty)$ than with $\tau_K \in [-\nu, 1]$, in the former case the welfare is lower, $H^I < H^{II}$. Hence, proposition 1 yields the following result:

Proposition 6 *The government prefers credible to non-credible public policy. In other words, it has no incentive cheat the public.*

A government with a good reputation can always impose the same outcome as one with a bad reputation, but it will never have an incentive do so. Because the capitalist knows this, it relies on the announced tax policy and invests in capital. In propositions 1-6, parameters α and ϑ do not affect the main results – the zero capital taxation in the limit and the equalization of the marginal utility of income. Hence, despite changes in the capitalist's social weight ϑ or in her share of wages, α, the capitalist can expect that the main principles of the tax policy will remain invariant. Because the consumption tax τ_C does not appear in propositions 1-6, it balances the government budget.

8 Logarithmic utility

Now assume logarithmic preferences for the capitalist, $\sigma \to 1$. Since with $\tau_K = 1$, the analysis is the same as in sections 5-7 and proposition 2 holds, we can focus on the cases with $\tau_K < 1$.

If $\Pi_{KK} < 0$, the system (13) and $\dot{\lambda}/\lambda = \rho - (1 - \tau_K)\Pi_K$ converges to a steady state where K and λ are positive constants and $\lim_{t\to\infty} \lambda K e^{-\rho t} = 0$. Next, consider the case $\Pi_{KK} \equiv 0$, for which $\Pi = \Pi_K K$ holds by (12). Noting (13) and (25), a steady state with $\dot{C}/C = \dot{K}/K$ is then true only if

$$(1 + \tau_C)C/K - \alpha W/K = \rho. \tag{37}$$

Given this, the equation $\dot{C}/C = \dot{K}/K$ is true for $\alpha > 0$ only if K, C and $\lambda = C^{-\sigma}$ are positive constants. We conclude that in the cases where either $\Pi_{KK} < 0$ or $\alpha > 0$ (or both) holds, the analysis is the same as in sections 5-7. Consequently, propositions 3-6 hold.

Finally, examine the remaining case with $\Pi_{KK} = 0$ and $\alpha = 0$. Given (37), we then obtain $C = \rho K/(1 + \tau_C)$. With the initial value $C(0) = \rho K(0)/(1 + \tau_C)$ for the co-state variable C, the system immediately jumps into the steady state with $\dot{K}/K = \dot{C}/C$. This means that for a given wage w and given taxes (τ_K, τ_W, τ_C), the capitalist plans to increase her capital K and consumption C at the same rate $\dot{K}/K = \dot{C}/C$ indefinitely. The common growth rate $\dot{K}/K = \dot{C}/C$ is positive (negative), when the average product of capital, $(1 - \tau_K)\Pi_K$, is greater (lower) than the rate of time preference for the capitalist, ρ.

The government maximizes welfare $\int_0^\infty [V((1-\alpha)W) + \vartheta U(C)]e^{-\rho t}dt$ by taxes (τ_W, τ_C) subject to (14), $C = \rho K/(1+\tau_C)$ and the function $L(K, \tau_W)$ in (12). Given $C = \rho K/(1+\tau_C)$ and $L(K, \tau_W)$, there is a one-to-one correspondence from (τ_C, τ_W) to (C, L), so that (τ_C, τ_W) can be replaced by (C, L) control variables. The Hamiltonian for the government is then given by

$$H^{III} = V\big((1-\alpha)W(L)\big) + \vartheta \log C \\ + \zeta\big[N(L) + F(K,L) - (1-\alpha)W(L) - C - E - \mu K\big], \qquad (38)$$

where the co-state variable ζ evolves according to

$$\dot{\zeta} = \rho\zeta - \partial H^{III}/\partial K = (\rho + \mu - F_K)\zeta = (\rho - \Pi_K)\zeta, \quad \lim_{t\to\infty} K\zeta e^{-\rho t} = 0.$$

Given (38), the first-order conditions for C and L are $\partial H^{III}/\partial C = 0$ and $\partial H^{III}/\partial C = 0$, which are equivalent to $U' = \vartheta/C = \zeta$ and

$$0 = (1-\alpha)(V' - \zeta)W' + \zeta(N' + F_L).$$

Noting (5), (11) and (12), this equation is equivalent to (36) and (19). Given $\vartheta/C = \zeta$, the capitalist's consumption is determined by $\dot{C}/C = -\dot{\zeta}/\zeta = \Pi_K - \rho$. Because the system $\dot{C}/C = \Pi_K - \rho$ and (14) produces a steady state in which K, C and $\zeta = \vartheta/C$ are constants, the transversality condition $\lim_{t\to\infty} K\zeta e^{-\rho t} = 0$ holds. Hence the government sets its tax parameters so that the average product of capital, $(1-\tau_K)\Pi_K$, is equal to the rate of time preference for the capitalist, ρ, and the capitalist's common growth rate $\dot{K}/K = \dot{C}/C$ will be zero. We summarize the results of this section as:

Proposition 7 *If* (*i*) *the capitalist has logarithmic utility* $\sigma \to 1$, (*ii*) *she earns no wages,* $\alpha = 0$, *and* (*iii*) *the formal sector is subject to constant returns to scale,* $F_{KK} \equiv 0$, *then capital income should be taxed at a non-zero rate and the optimal wage tax is given by* (19). *Otherwise, zero taxation on capital income holds.*

The result that with logarithmic utility capital should be taxed at a non-zero rate is the same as in Lansing (1999), but our interpretation is different. If the capitalist does not earn wages from elsewhere in the economy and her production is subject to constant returns to scale, her private economy will grow in fixed proportion to her capital stock K. Furthermore, if the capitalist has a logarithmic utility function, the income and substitution effect of future interest rate movements exactly cancel each other, and she needs to observe only the current (after-tax) rate of return to decide how much to consume and save. From all of this it follows that the capitalist chooses a common growth rate for her capital, income and consumption on the basis of the current tax rate. In this situation, the government needs the capital tax as a means of controlling the growth rate of the economy.

9 Conclusions

This paper examines optimal taxation in an economy with the following properties: (*i*) There is a formal sector which can be taxed, and a informal sector which cannot be taxed. (*ii*) The numeraire good, which can be consumed, invested and used in public spending, is in the formal sector produced from capital and labour, but in the informal sector from labour only. (*iii*) Some households called capitalists save capital and earn a fixed proportion of wages. (*iv*) The government taxes wages, capital income and consumption to finance fixed public expenditures. The main results are as follows.

If the government cheated the public by promising low taxes in the earlier periods and charging high taxes in the later periods, the capitalists would stop investing and social welfare would fall. For this reason, the government never has an incentive to cheat. Because the capitalists know this, there is a reputational equilibrium in which the government follows its announced strategy and the capitalists accumulate capital. In the advent of exogenous changes, the consumption tax adjusts to balance the government budget, but the tax rules for capital and wages will be unchanged.

There is a socially optimal level of capital at which the marginal product of capital equals the rate of time preference plus the depreciation rate and the marginal product of labour is uniform in the economy. When capital stock is above this optimal level, the government should discourage investment by raising the capital tax as high as possible, and when capital stock is below this optimal level, it should encourage investment by subsidizing capital as much as possible. When capital is at its optimal level, it should generally taxed at a zero rate. Only in the very special case where (a) a capitalist has logarithmic utility and (b) earns no wages, and (c) the formal sector is subject to constant returns to scale can the optimal capital tax differ from zero in the steady state. In such a case, the capitalists' consumption changes in proportion to capital and the government needs the capital tax to bring capital to the socially optimal level.

The optimal wage tax is determined by a specific elasticity rule. If all households save capital, then public expenditures should be financed by the non-distorting consumption tax and wages should be taxed at a zero rate. The higher the proportion of wages the capitalists earn or the higher the relative social value of their consumption, the higher the optimal wage tax. The more inelastic the labour supply in the formal sector, the less the wage tax distorts the economy and the more wages should be taxed to finance government spending.

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