Gregory S. Amacher* - Erkki Koskela - Markku Ollikainen*****

Forest Rotations and Stand Interdependency: Ownership Structure and Timing of Decisions****

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* Department of Forestry, College of Natural Resources, 307 Cheatham Hall, Virginia Polytechnic Institute and State University, Blacksburg, VA 24060, USA. Email: gamacher@vt.edu.

** Department of Economics, P.O. Box 17 (Arkadiankatu 7), FIN-00014 University of Helsinki, Finland, and the Research Department of the Bank of Finland, P.O. Box 160, 00101 Helsinki, Finland. Email: erkki.koskela@helsinki.fi.

*** Department of Economics and Management, P.O. Box 27, FIN-00014 University of Helsinki, Finland, Email: markku.ollikainen@helsinki.fi

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Forest Rotation under Interdependent Stands: Ownership Structure and Timing of Decisions

Abstract:

This paper extends the Hartman model to study the optimal rotation age of two interdependent stands at the steady state, when the rotation ages of the two stands are equal and the stream of amenities produced from the two stands may be complements or substitutes, both in space and over time. In the presence of stand interdependence both the ownership structure and the sequence of decision making matters. Rotation age choices are examined and compared under a variety of equilibria, including Nash, Stackelberg, and sole owner cases. We show that the sole owner's rotation age is longer than the rotation age under both Nash and Stackelberg assumptions if the stands are spatial complements, but shorter if they are substitutes. The precise relationship between the Nash and Stackelberg rotation ages, and the qualitative properties of rotation ages in terms of timber prices, regeneration costs, and interest rates, also depend on how spatial substitutability and complementarity between stands evolves through time.

Key words: forest rotation, amenity public goods, stand interdependence, nonindustrial landowners
JEL classification: Q23, H21.

1. Introduction

Forest ecosystems comprise complex site-specific interactions between plant and animal species. One aspect of forest ecosystems rarely acknowledged in economics models is the notion of stand interdependence. The majority of models focus on management of only a single stand in isolation, but in practice the management of each stand in a given region should not be undertaken independently of other stands. Biologists have long known this, arguing that trees of many age classes and species mixes are necessary for conservation of biodiversity or contiguous habitat for certain animal species. Stand interdependence may also be anthropogenic in nature. For instance, the recreational opportunities of larger forest areas may be dependent on the interaction of several stands.

It is well known that landowners, especially non-industrial ones, manage forests with both timber and non-timber services in mind (Boyd and Hyde 1989). Managing an interdependent multiple-stand forest is a challenging task. Harvesting even one stand may sometimes pose a threat to the maintenance of an entire ecosystem. While the task is difficult enough for one manager, it becomes even harder under the reality of non-industrial private landownership. Land property rights usually do not follow forest cover types. Although recent survey work reveals that forest landowners may be willing to jointly manage land with other landowners, under various ecosystem management strategies such as preservation of wildlife corridors, the degree of cooperation and coordination differs and is likely to be far from complete (e.g., see Jacobson 2002). In a related survey-based study, Klosowski et al. (2001) shows that landowners in the Northeastern U.S. may be willing to cooperate and make decisions jointly for wildlife, but only if economic incentives exist for coordinated management. In fact, landowners owning one piece of an ecosystem may neglect, knowingly or unknowingly, the impact of their private harvesting on the whole ecosystem or on other nearby landowners.[1]

The presence of landowners who may not be able (or willing) to coordinate actions will be socially costly. The impact of one landowner's decisions on the forest ecosystem used by another landowner represents a type of economic externality associated with private forest management. Only a social planner who manages an entire forest ecosystem

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[1] According to Sample (1996) cooperation may be difficult among landowners, especially when there is heterogeneity of the landscape or landowners are diverse in their preferences for non-timber services.

as a whole has incentives to solve for the rotation age of each stand, conditional on its impacts to all other stands.

In this paper we examine several issues not addressed in the economic management of interdependent forest stands. We assume landowners receive timber and non-timber amenity benefits that depend on rotation ages of an adjacent stand. By assuming a unique steady state equilibrium exists, we examine various assumptions for the timing and implementation of adjacent landowner decisions, when rotation ages are equal. A brief characterization of our main findings are as follows. If landowners behave without regard to their effects on another landowner, then the effectively make forest management decisions simultaneously, i.e., landowners play a Nash game. The Nash game reflects private ownership in practice, where landowners typically border a small number of neighbouring landowners and make decisions without regard to the other landowners. An alternative setting is also examined where one landowner moves first, but makes decisions with the reaction of another landowner in mind. Finally, we examine the rotation age decision for a sole owner who makes forest management decisions taking into account the interdependence between all stands. This is similar to a situation where all landowners perfectly coordinate, or cooperate in, their actions. Comparison of this outcome with the simultaneous and first-mover outcomes will show the importance of coordination, and thereby hint at the social cost of not coordinating forest management actions.

There are very few analytical treatments of the economics problem behind stand interdependence. Stand interdependence was originally addressed in Bowes and Krutilla (1985, 1989). They discuss rents associated with multiple stands under a single (government) owner. Swallow and Wear (1993) and Swallow et al. (1997) were the first to formulate explicit spatial interactions for non-timber amenity benefits between two adjacent stands, but they rely mainly on numerical approximations for the case of an exogenous adjacent stand. Koskela and Ollikainen (2001b) examined analytically the rotation age decision for a single landowner making decisions for a single stand, also under the assumption of a purely exogenous adjacent stand. Moreover, their work does not focus on the different landowner commitment assumptions that we examine, nor do they examine the important sole owner outcome. All of these issues are critical to understanding private landowner behavior under various circumstances.

There is a large literature on stand interdependence in other settings, such as species conservation. This work, when taken in the context of forested areas, promotes the idea that multiple stands are needed to sustain certain species (see e.g. Csuti et al. 1997, Ando et al. 1998, Polasky et al. 2001). An increasing number of empirical studies now exist on conservation, ecosystem management, and forest management. These are, however, typically undertaken from the viewpoint of a benevolent social planner (see e.g. Beavers et al. 1995, Albers 1996, Beavers and Hof 1999, Haight and Travis 1997, Montgomery 1995). Unlike our paper, this literature either considers only the case of the sole owner, or it is based on site-specific empirical data. Hence it abstracts from the interesting practical problems that follow from private landowners who may not coordinate the management of their forests. Yet coordination may be impractical in the U.S., as many landowners do not live close to their properties, thus making the sole owner outcome via cooperation less likely.[2]

The rest of our paper is organized as follows. In section 2 we introduce an extended Hartman model of forest management and make specific the definition of spatial dependence between stands and its evolution over time, i.e. temporal dependence. We then analyse rotation age decisions under simultaneous move, first-mover and sole owner timing assumptions and compare their relationships. Section 3 characterizes the qualitative dependence of rotation ages on important parameters. Finally, in section 4 we provide some concluding remarks.

2. A model of interdependent stands

We first describe a basic framework for the determination of rotation ages for two adjacent stands, denoted by stand 'a' and stand 'b'. It is assumed that landowners value net harvest revenue and the non-timber amenity services produced from the stand, just as in the conventional Hartman model of forest management (Hartman 1976).[3] Following Swallow and Wear (1993) and Koskela and Ollikainen (2001b), we assume that stands are interdependent in terms of amenities but independent with regard to timber production.

In these models, the growth of stands a and b is an S-shaped function of rotation age. Timber volume at harvest is denoted by $f(T)$ and $g(\tau)$, where T refers to the

[2] In North America, there has been much written about absentee landowners who may not be able to coordinate actions on their land. Others have shown recently that landowners are less interested in pursuing joint management of forest land if they are not privy to information regarding the benefits of coordination (Jacobson 2002).

[3] Papers including e.g. Binkley (1981) and Kuuluvainen et al. (1996) provide some indirect empirical evidence in favor of the hypothesis that private landowners value non-timber amenity services.

rotation age for stand *a* and τ refers to the rotation age of the stand *b*. Timber prices p and q and regeneration costs c_T and c_τ for stand *a* and *b,* respectively, are allowed to differ between the stands. These assumptions reflect the typical situation in which stands differ inherently due to site characteristics (such as slope, tree species, aspect, or access). Prices, costs, and the real interest rate r are assumed to be constant over time, as with the basic Hartman model. The present values of timber production over an infinite cycle of rotations for each stand are, respectively,

$$V^a = (1-e^{-rT})^{-1}\left[pf(T)e^{-rT} - c_T\right] \tag{1a}$$

$$V^b = (1-e^{-r\tau})^{-1}\left[qg(\tau)e^{-r\tau} - c_\tau\right]. \tag{1b}$$

We now introduce amenity values in a manner that reflects stand interdependence. Let $F^a(s,\tau)$ describe valuation of amenity benefits provided by stand *a* at *time* s and as a function of the adjacent stand *b*'s *rotation age* of τ. Likewise, $F^b(T,x)$ denotes valuation of amenity benefits of stand *b* at *time* x when stand *a* has a *rotation age* of T. Using this notation, the present value of amenities over an infinite series of rotations of length T and τ for both stands are written,

$$E^a = (1-e^{-rT})^{-1}\int_0^T F^a(s,\tau)e^{-rs}ds \tag{2a}$$

$$E^b = (1-e^{-r\tau})^{-1}\int_0^\tau F^b(T,x)e^{-rx}dx. \tag{2b}$$

Equations (2a) and (2b) reflect the steady-state amenity valuation under certain limiting assumptions. As pointed out in Swallow and Wear (1993), the presence of interdependency might imply that the amenity value of each stand depends on the current calendar age of the adjacent stand. In particular, if the rotation ages of the two stands differ, the dynamics of any steady-state will depend not only on the rotation ages of the stands, but also on the calendar age of the stands. Under these circumstances, one cannot approximate the present value of amenity benefits using equations (2a) and (2b). In Appendix 1 we illustrate an extension of the model by introducing calendar age into (2a) – (2b). Using ex ante given and different rotation ages in a numerical example, Appendix 1 describes how rotation age and actual age of stands interact as the number of rotations is increased.

Equations (2a) and (2b) reflect a case where the interdependent steady-state rotation ages are identical. While this appears to be restrictive, it allows us to develop a qualitative theory of stand interdependency in its simplest form. Such an approach is similar to one proposed in an important contribution of Tobin and Houthakker (1950-1951). They analysed the effects of credit rationing by restricting attention to a margin where rationed and free market solutions were assumed identical. Then, they proceeded with an assessment of the marginal effects of rationing. We follow their approach in spirit by assuming rotation ages are identical, and then examining the behavior of the steady-state within this margin. Most previous forest economics work assumes the existence of a steady state, like we do. Very few studies elaborate on possible transitional dynamics that might arise outside of the steady state. An example is Swallow and Wear (1993), who provide numerical simulation results for stand interdependency in the case of an exogenous adjacent stand. That said, a first and natural step towards developing the analytics of stand interdependency that does not require numerical simulation is to detect the marginal effects of exogenous parameters and agents' behaviour for any form of stand interdependency.[4]

Proceeding under the assumptions of a unique steady state and equal rotation ages, equations (2a) and (2b) define a valuation function of amenity benefits for each stand in terms of its own and the adjacent stand rotation ages. The choice of these rotation ages is made for an infinite series of rotations begun at time $t = 0$, following the Hartman and Faustmann models. In these models, given that the choice of rotation age is made under certainty and exogenous variables remain constant over time, all future rotations will be similar in terms of the path of timber and amenity flows forever. Thus, in this case, the rotation age choice is the appropriate indicator of the path of amenity services consumed by the landowner.

For subsequent analysis we must also characterize how the amenity values in equations (2a) and (2b) behave in terms of changes in their own rotation age and changes in the adjacent stand's rotation age. In describing these effects, we will use the label *own-*

[4] Providing sufficient conditions for the existence of a steady state or specifying the transitional dynamics governing the possible equilibria, which may include steady state, cyclical, chaotic or other solutions, goes beyond the scope of this paper but is certainly an important future research topic.

stand to refer to the stand in question, and *adjacent stand* to refer to the other stand. Neglecting for a moment the present value terms in (2a) and (2b), and differentiating the integrals for each stand with respect to the own rotation age, we can obtain a marginal amenity valuation function defined at harvest times for both stand *a* and *b*: Denote these as $F^a(T,\tau)$ and $F^b(T,\tau)$ respectively.

We now present two definitions from the literature, which characterize *spatial dependence* of stands, as well as the evolution of this over time, i.e., *temporal dependence*. Differentiating each stand's marginal amenity valuation function with respect to the rotation age of the *adjacent* stand indicates how the marginal amenity valuation changes with respect to changes in the rotation age of the other stand. These derivatives define spatial dependence in the literature and are summarized in:[5]

Definition 1 *(Koskela and Ollikainen 2001b)*. S*patial Dependence*

$$F_\tau^a(T,\tau) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \text{ and } F_T^b(T,\tau) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \text{ if stands are } \begin{cases} \text{substitutes wrt amenities} \\ \text{independents wrt amenities} \\ \text{complements wrt amenities} \end{cases}$$

Definition 1 is consistent with ALEP complementarity/substitutability first formalized by Samuelson (1974) and others in a different context.[6] If the stands are spatial substitutes, then the marginal amenity valuation of each stand decreases with the rotation age of the adjacent stand. If the stands are spatial complements, then the opposite is true, i.e., marginal amenities of each stand increase with the rotation age of the adjacent stand.

It is also important to know how spatial dependence between stands is affected by rotation age choices. This is obtained by differentiating the functions in Definition 1 with respect to own-stand rotation ages. The resulting second derivatives define how spatial dependence between stands evolves with own rotation age. This is called 'temporal dependence' in the literature. That is,

[5] In what follows, derivatives of functions will be denoted by subscripts unless otherwise noted.

[6] For the concept of the Auspitz-Liebig-Edgeworth-Pareto (ALEP) complementarity/ substitutability, see Samuelson (1974) and further discussions in Chipman (1977), Kannai (1980) and Weber (2000) regarding its implications for the properties of demand functions.

Definition 2 *(Koskela and Ollikainen 2001b). Temporal Dependence*

$$F^{a}_{\tau T}(T,\tau)\begin{Bmatrix}<\\=\\>\end{Bmatrix}0\,;\; F^{b}_{T\tau}(T,\tau)\begin{Bmatrix}<\\=\\>\end{Bmatrix}0 \text{ if } \begin{cases}\text{stand dependence decreases with stand age}\\ \text{stand dependence unchanged with stand age}\\ \text{stand dependence increases with stand age}\end{cases}$$

From Definition 2, temporal interdependence between two stands may be constant, increasing or decreasing depending on how spatial dependence between the stands changes with increases in the rotation age of each stand. *Temporal independence* results when $F^{a}_{\tau T}(T,\tau)=F^{b}_{T\tau}(T,\tau)=0$. This is the case if spatial substitutability or complementarity (from Definition 1) is merely associated with site-specific properties that remain the same regardless of own-stand rotation age. *Increasing temporal dependence* between the stands means that, for spatial complements, the complementarity between stands increases with own-stand rotation age. But for spatial substitutes, the substitutability between stands decreases with own-stand rotation age. *Decreasing temporal dependence* implies just the opposite: complementarity weakens while substitutability becomes stronger for increases in own-stand rotation age.

Ecologists have shown that amenity production depends on inter-stand relationships like the ones above in Definitions 1 and 2 (see, for example Franklin and Forman (1987) and Giles (1978). As a forestry example of spatial substitutes and complements consider the case of a landowner who values timber and forage production consistent with big game production, where big game requires both forage and cover (see e.g. Swallow and Wear 1993). The own stand and the adjacent stand function in this case exist as substitutes in their production of this amenity, as long as they simultaneously provide both forage and cover. The stands are complements if instead, for example, the own stand provides only forage while the adjacent stand provides only cover. This type of example has special relevance to “ecosystem management” in the U.S. This embodies the holistic management of land in corridors advocated for wildlife habitat protection (Grumbine 1994, Moote and Cortner 1994). Here, corridors function as complementary stands between the stands they are joining. Another example relates to production of water and forest goods discussed in Bowes, Krutilla and Sherman (1984) and Bowes and Krutilla (1989). In their examples, the species composition of trees in a forest (i.e., stands comprising either coniferous or leaf trees) as well as their location on the landscape, are important both to water quality and the likelihood of floods and other hydrological properties. Thus, stands may be complements or substitutes in the production of water.

The case of increasing or decreasing temporal dependence is more complex. Consider the first example. Suppose that the stands are originally spatial substitutes and the adjacent stand rotation age increases. If this decreases (increases) forage and cover for the own stand, due to changes in understory vegetation, then we have the case of decreasing (increasing) temporal dependence between stands. One therefore expects that temporal dependence is important in the case of biodiversity conservation. Typical old-growth species, such as red-cockaded woodpecker, spotted owl or carpagalle, require significant stands of old timber for their propagation. While some old growth species are highly specialized to old growth forests, others are to a lesser extent.[7] Therefore, increasing the age of old growth stands makes the forest more (less) suitable to highly (less) specialized species thus inducing stronger (weaker) interdependence between stands. In our model these situations would be labelled as increasing (decreasing) temporal dependence respectively.

2.1 Three models for rotation age

We now depart from the existing literature and consider rotation age solutions for different ownership structures and timing of decisions. The first rotation age solution corresponds to a Nash game, where there are two different landowners who own stands *a* and *b*, and these landowners make their harvesting decisions simultaneously taking the other's action as given. This mimics the private market solution where landowners take the behavior of others as given. The second rotation age solution follows when there are two landowners, but one is a first-mover, i.e., landowners play a traditional two-stage Stackelberg game. The landowner moving first is assumed able to credibly commit to a harvesting decision before the other landowner moves, so that a leader-follower relationship is established. Finally, the third rotation age solution is derived under the assumption of a sole owner of both stands *a* and *b*. This can be interpreted as the case where landowners coordinate their decisions.

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[7] As metapopulation theory has shown, these species do not occupy all stands, but still a sufficiently high amount of old growth stands are needed (see e.g. Hanski 1998).

In all cases, landowners are assumed to be price takers and, as such, do not account for price-induced demand changes when making rotation age choices. Each landowner does, however, make use of amenities produced by the other stand. Therefore, the sole owner model, by yielding the efficient solution, provides a hint at the social costs associated with uncoordinated harvesting if there are no externalities outside the landowners' control.

2.2. Rotation ages in the Nash game

Here each landowner chooses rotation age taking the other landowner's rotation choice as given by maximizing $\Omega^{aN} = V^a + E^a$ and $\Omega^{bN} = V^b + E^b$. The solution to the Nash game can be obtained for each landowner by solving the following problems simultaneously**:**

$$\underset{\{T\}}{Max}\ \Omega^{aN} = V^a + E^a \tag{3a}$$

$$\underset{\{\tau\}}{Max}\ \Omega^{bN} = V^b + E^b \ , \tag{3b}$$

where the terms in the objective functions are defined in (1) and (2). The labels a and b again refer both to the stand and landowner, and N denotes the Nash game.

The following first-order conditions characterize the optimal rotations T and τ:

$$\Omega_T^{aN} = 0: \qquad pf'(T) + F^a(T,\tau) = rpf(T) + rV^a + rE^a \tag{4a}$$

$$\Omega_\tau^{bN} = 0: \qquad qg'(\tau) + F^b(T,\tau) = rqg(\tau) + rV^b + rE^b \ . \tag{4b}$$

These suggest that both landowners equate their private marginal benefit of delaying harvest (LHS) to the marginal opportunity cost of delaying harvest (RHS). Notice there is an externality evident in the first-order conditions (affecting the last terms on the LHS and RHS of (4a) and (4b)). This arises because landowners do not account for the effect of their rotation age choice on the other landowner's utility and behavior.

The second-order conditions for both landowners are given by

$$\Omega_{TT}^{aN} = pf''(T) - rpf'(T) + F_T^a(T,\tau) < 0 \tag{5a}$$

$$\Omega^{bN}_{\tau\tau} = qg''(\tau) - rqg'(\tau) + F^{b}_{\tau}(T,\tau) < 0 \qquad (5b)$$

We assume that the second-order conditions (5a) and (5b) hold. They hold automatically when the landowner's marginal amenity valuation decreases or remains constant with increasing own-stand rotation age, i.e., for $F^a_T \leq 0$ and $F^b_\tau \leq 0$. They may not always hold when the landowner's amenity valuation increases with own-stand rotation ages, i.e., for $F^a_T > 0$ and $F^b_\tau > 0$ (see Strang 1983).

As for the dynamics of the Nash equilibrium, we assume that landowners adjust their rotation ages in order to increase the sum of net present value of harvest revenue and amenities, taking the behavior of the other landowner as given. These dynamics are represented by the following derivatives,

$$\dot{\Omega}^{aN} = \lambda^{T}\left[\frac{\partial\Omega^{aT}}{\partial T}\right] \text{ and } \dot{\Omega}^{bN} = \lambda^{\tau}\left[\frac{\partial\Omega^{bT}}{\partial \tau}\right] \qquad (5c)$$

where the parameters $\lambda^T, \lambda^\tau > 0$ indicate the speed of adjustment, and the dots refer to "artificial" time, reflecting dynamic adjustments to rotation ages over time (see e.g. Takayama 1988, pp. 302-313).

The uniqueness and stability condition for the Nash game can be expressed as

$$\Delta^N = \Omega^{aN}_{TT}\Omega^{bN}_{\tau\tau} - \Omega^{aN}_{T\tau}\Omega^{bN}_{\tau T} > 0, \qquad (5d)$$

where $\Omega^{aN}_{T\tau} = F^a_\tau(T,\tau) - r(1-e^{-rT})^{-1}\int_0^T F^a_\tau(s,\tau)e^{-rs}ds$ and

$\Omega^{bN}_{\tau T} = F^b_T(T,\tau) - r(1-e^{-r\tau})^{-1}\int_0^\tau F^b_T(T,x)e^{-rx}dx$, so that the determinant of the second-order derivatives matrix in (5d) must be positive.[8] The second order conditions (5a) – (5b) imply that the first part of (5d) is positive. Therefore, the uniqueness and stability of the Nash game depends on the product of cross-derivatives $\Omega^{aN}_{T\tau}\Omega^{bN}_{\tau T}$, which jointly with the second-order conditions define the slopes of the reaction functions for the landowners.

[8] For further details about uniqueness and stability, see Dixit (1986) and Vives (1999, pp. 49-58).

The reaction functions for landowner a and b respectively can be obtained from the first-order conditions by totally differentiating them with respect to the rotation age of adjacent stands,

$$\left.\frac{dT}{d\tau}\right|_a \equiv a(\tau) = -\frac{\Omega_{T\tau}^{aN}}{\Omega_{TT}^{aN}}, \qquad \left.\frac{d\tau}{dT}\right|_b \equiv b(T) = -\frac{\Omega_{\tau T}^{bN}}{\Omega_{\tau\tau}^{bN}}. \tag{6}$$

Lemma 1 characterizes how the cross derivatives of the reaction functions depend on properties of the amenity valuation functions.

Lemma 1. $\Omega_{T\tau}^{aN} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \;\; if \;\; F_{\tau T}^{a} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \;\; and \;\; \Omega_{\tau T}^{bN} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \;\; if \;\; F_{T\tau}^{b} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0.$

Proof. See Appendix 2.

According to Lemma 1 and equation (6), the reaction functions of landowners a and b have different slopes depending on temporal stand interdependence (Definition 2). There are three cases, which we summarize in:

Result 1. Properties of the reaction functions

a) *Under temporally independent stands, the reaction functions are vertical lines in (T, τ) space and the equilibrium is stable.*
b) *Under increasing temporal dependence, the reaction functions are increasing in (T, τ) space. Stability of the equilibrium requires that the reaction function for stand a is steeper than the reaction function for stand b.*
c) *Under decreasing temporal dependence, the reaction functions are decreasing in (T, τ) space. Stability of the equilibrium requires that the reaction function for stand a is steeper than the reaction function for stand b.*

Result 1 is illustrated in Figures 1-4 for both decreasing and increasing temporal dependence. Drawn in the figures are the reaction functions for the landowner of stand a and b, $a(\tau)$ and $b(T)$, each of which is a function of the rotation age choice of the other landowner. In Figures 1–2 the reaction functions are downward-sloping, reflecting decreasing temporal dependence between the stands. The upward sloping reaction functions in Figures 3 and 4 reflect increasing temporal dependence between the stands. As we explain later, Figures 1 and 3 are drawn assuming the stands are spatial substitutes, while Figures 2 and 4 are drawn assuming the stands are spatial complements. The unique and stable Nash equilibrium solution satisfying (4a) and (4b) occurs at the point where the reaction functions for the landowners cross.

2.3. Rotation ages in the Stackelberg game

In the Stackelberg game, the leader moves with knowledge of how the following landowner responds; the follower takes the rotation age of the leader as given. While this model may mimic some private market situations, the leader might also be interpreted as a government formally setting a long run harvest policy, effectively leading.

Assume the leader is the landowner holding stand *a.* This landowner maximizes the following objective function,

$$\underset{\{T\}}{Max}\ \Omega^{aS} = V^a + E^a \tag{7a}$$

$$s.t. \quad \tau^S = \tau(T^S, q, r, c_\tau), \tag{7b}$$

where the superscript S refers to the Stackelberg game, and $\tau^S = \tau(T^S, q, r, c_\tau)$ describes the reaction function of the follower (who holds stand *b*). Utilizing the Nash first-order condition (4b) for the follower's reaction function, the leader and the follower first-order conditions are, respectively,

$$\Omega_T^{aS} = \Omega_T^{aN} + e^{rT}\tau_T(.)\int_0^T F_\tau^a(s,\tau)e^{-rs}ds = 0, \tag{8a}$$

$$\Omega_\tau^{bS} = qg'(\tau) + F^b(T,\tau) - rqg(\tau) - rV^b - rE^b = 0. \tag{8b}$$

We assume that the second-order condition holds for this problem.[9] In reality, whether it holds depends again on the amenity valuation function and on the properties of the follower's reaction function $\tau^S = \tau(T^S, q, r, c_\tau)$.

[9] Thus we assume that $\Omega_{TT}^{aS} = \Omega_{TT}^{aN}e^{-rT} - r(pf'(T) - rpf(T))e^{-rT} + F^a(T,\tau)$

$$+\tau_{TT}\int_0^T F_\tau^a(s,\tau)e^{-rs}ds + \tau_T^2\int_0^T F_{\tau\tau}^a(s,\tau)e^{-rs}ds + 2\tau_T F_\tau^a(T,\tau)e^{-rT} < 0.$$

Consider first the *follower's* behavior given in (8b). This condition is qualitatively the same as the necessary condition for landowner b in the Nash game; that is, given T^S, the follower chooses the rotation age τ. This is not so for the leader (eqn (8a)). Compared to the Nash game, there is an additional term in the Stackelberg first order condition, reflecting the impact of the follower's rotation choice on the leader's marginal amenity benefits. The presence of this additional term implies that the leader partly accounts for the externality that arises from the effect of the follower's rotation age on the amenities of the leader's stand. This difference between Stackelberg and Nash outcomes will become important later when we study comparative statics effects.

Whether the leader has a longer or shorter rotation age compared to the Nash equilibrium rotation age depends on the last term in (8a), i.e., on the slope of the follower's reaction curve and the integral term. To sign this integral term and make the analysis tractable, we assume that the amenity function has a quadratic shape, and we also use a second order approximation for $e^{-rT} = 1/(1 + rT + (1/2)r^2T^2)$.[10] The quadratic amenity function $F^a(T,\tau) = \alpha(T+\tau) - \frac{1}{2}\beta(T+\gamma\tau)^2$ has the following properties: $F_T^a(T,\tau) = \alpha - \beta(T+\gamma\tau)$ and $F_{T\tau}^a(T,\tau) = -\beta\gamma \geq (<) 0$ as $\gamma \leq (>) 0$, so that it exhibits all relevant cases described in Definitions 1 and 2. We now have,

Lemma 2. *Under the quadratic amenity valuation function,*

$F^a(T,\tau) = \alpha(T+\tau) - \frac{1}{2}\beta(T+\gamma\tau)^2$, *we have*

$$\int_0^T F_\tau^a(s,\tau)e^{-rs}ds \begin{Bmatrix} > \\ < \end{Bmatrix} 0 \text{ as } F_\tau^a(0,\tau) \begin{Bmatrix} > \\ < \end{Bmatrix} 0.$$

Proof. See Appendix 3.

According to Lemma 2 the sign of the integral term depends on how the rotation age of the follower's stand affects the marginal amenity valuation of the leader's stand at the margin, when $T = 0$. Returning to Figures 1-4, consider now the iso-net-present-value-of-revenue curves. The iso-net-present-value-of-revenue curves are lines along which the net present value is constant for given interest rates, timber prices and regeneration costs. A family of these therefore exist for each set of constant parameters.

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[10] We can show that higher order approximations for the discount factor will not change the nature of the results in Lemma 2 below.

When the stands are spatial substitutes, the iso-net-present-value-of-revenue curves are decreasing in the rotation age of the other stand, while spatial complements imply the iso-net-present-value-of-revenue curves are increasing in the rotation age of the other stand. This means that, for complements (substitutes) the net present value of profits for the forest landowner is increasing when moving up (down) the reaction functions. In the figures, the Stackelberg rotation age for leader and follower corresponding to (8a) – (8b) above is defined by the point where the leader's highest iso-net-present-value-of-revenue curve is tangent to the follower′s reaction function, $b(T)$.

Using Figures 1-4, Lemma 2, Lemma 1, and Result 1, we can now examine the relationship between the Nash and the Stackelberg rotation ages. Consider first the leader. If the stands are temporally independent, then $F^a_{\tau T}(.) = 0$ and the reaction functions are vertical lines. Here, the sign of $F^a_\tau(T,\tau)$ does not matter, therefore, the Stackelberg rotation age coincides with the Nash rotation age. Under decreasing (increasing) temporal dependence and spatial substitutability between the stands, we have $\Omega^{aN}_T < 0$ ($\Omega^{aN}_T > 0$), because now the last term in (8a) is positive (negative). In this case, the leader's rotation age, T^S, is longer (shorter) than the Nash rotation age, T^N. If the stands are spatial complements, then under decreasing (increasing) temporal dependence we have $\Omega^{aN}_T > 0$ ($\Omega^{aN}_T < 0$); now the leader's rotation age is shorter (longer) than the Nash rotation age. For spatial substitutes (complements), the follower's rotation age is shorter (longer) than the Nash rotation age under decreasing temporal dependence.

The interpretation is as follows. The follower observes the rotation age of the leader prior to moving. When the stands are substitutes, the follower's rotation age must be shorter because, under decreasing temporal dependence, the leader's rotation age will be longer than the Nash rotation age, and the follower's reaction curve will be downward-sloping (Figure 1). Intuitively, the longer rotation age of the leader allows the follower to harvest sooner but still derive foregone amenity benefits from the leader's stand. For spatial complements, the leader's rotation age is shorter than the Nash age, and the best response of the follower is to lengthen the rotation age relative to Nash age, because that decreases stand complementarity. Similar reasoning can be applied to increasing temporal dependence.

We can summarize the above discussion in:

Proposition 1. *The relationship between Nash and Stackelberg rotation ages depends on the nature of stand interdependence:*

a) *Under temporal independence,* $T^N = T^S$ *and* $\tau^N = \tau^S$.

b) *Under decreasing temporal dependence,* $T^N > T^S$ *and* $\tau^N < \tau^S$ *for spatial complements, while* $T^N < T^S$ *and* $\tau^N > \tau^S$ *for spatial substitutes.*

c) *Under increasing temporal dependence,* $T^N < T^S$ *and* $\tau^N < \tau^S$ *for spatial complements, while* $T^N > T^S$ *and* $\tau^N > \tau^S$ *for spatial substitutes.*

2.4. Rotation ages for the sole owner

The sole owner chooses rotation ages of both stands to maximize,

$$\underset{\{T,\tau\}}{Max}\, W = V^a + V^b + E^a + E^b \quad . \tag{9}$$

The first-order conditions characterizing sole owner rotation age choices can be expressed using the following modification of the Nash conditions (4a) and (4b),

$$W_T = \frac{e^{-rT}}{(1-e^{-rT})}\Omega_T^{aN} + \frac{\int_0^{\tau} F_T^b(T,x)e^{-rx}dx}{(1-e^{-r\tau})} = 0 \tag{10a}$$

$$W_\tau = \frac{e^{-r\tau}}{(1-e^{-r\tau})}\Omega_\tau^{bN} + \frac{\int_0^{T} F_\tau^a(s,\tau)e^{-rs}ds}{(1-e^{-rT})} = 0 \tag{10b}$$

The second-order conditions $W_{TT} < 0$, $W_{\tau\tau} < 0$, and $\Delta^{SO} > 0$ are presented in Appendix 4 and are assumed to hold.

Equations (10a) and (10b) imply that the sole owner chooses rotation ages for both stands taking into account how amenities are affected by rotation age of both stand *a* (as in the Stackelberg game) and now also stand *b* (see the last term in 10a and 10b). Hence, all potential externalities arising from the effects of harvesting one stand on the other stand's amenities are internalized. The sole owner outcome is therefore the efficient solution for our problem.

The last terms in equations (10a) and (10b) determine how the sole-owner rotation age of both stands compares to Nash and Stackelberg rotation ages. From Lemma 2 we know that these last terms are positive when the stands are spatial complements and negative when the stands are spatial substitutes. Thus, relative to the Nash rotation age, the sole owner chooses longer rotation ages for both stands when they are spatial complements, but shorter rotation ages when they are spatial substitutes.

How does the sole owner rotation age compare to the Stackelberg rotation age? The sole owner first order conditions differ from the Stackelberg conditions by the last term (compare (10a) with (8a)). Due to the symmetric sole owner first-order conditions (10a) and (10b), we can graphically distinguish the sole owner optimum in Figures 1-4 as points where the iso-net-present-value-of-revenue-curves from both stands are *tangent* to each other. Referring to the figures, for spatial complements (substitutes) the sole owner rotation age is longer (shorter) than the Stackelberg age for both leader and follower.

The important driving factor in the comparison of the sole owner's rotation age with Nash and Stackelberg rotation ages is the spatial complementarity or substitutability of stands. The sole owner internalizes all externalities associated with amenities. When stands are temporally independent, it is natural that the sole owner rotation age coincides with the other rotation ages because there is no external effect of harvesting one stand on the other stand. However this is not the case when stands are temporally interdependent. Now the comparison of rotation ages depends on how the two stands are related spatially; if they are spatial complements, the sole owner increases rotation ages, while the opposite is true under spatial substitutes. We can express the relationship between Nash, Stackelberg and sole owner rotation ages as follows

Proposition 2.

a) *Under temporal independence, the spatial complementarity or substitutability between the stands does not matter and Nash and Stackelberg rotation ages coincide with the sole owner rotation ages.*
b) *Under temporal dependence, the sole owner rotation age is longer (shorter) than Nash and Stackelberg solutions when stands are spatial complements (substitutes).*

The differences in rotation ages under our various solutions will undoubtedly lead to differences in welfare for forest landowners.[11] Obviously, landowners are by definition better off at the sole owner optimum relative to the other outcomes. An interesting comparison of welfare under the other outcomes can be obtained from Figures 1-4, by noting the position of the equilibria on the iso-net-present-value-of-revenue curves. Figures 3-4 show that when temporal dependence is increasing, the welfare of both landowners is higher under the Stackelberg solution than under the Nash equilibrium. This result occurs because in the Stackelberg game, the leader partially accounts for the effects of his rotation age decision on the follower landowner, and this increases the welfare of both leader and follower. In the case of decreasing temporal dependence, Figures 1-2 show that the welfare comparison between Nash and Stackelberg solutions is ambiguous. Thus, we have an additional Corollary:[12]

Corollary 1. *Under increasing temporal independence, both landowners are better off when rotation ages are chosen according to the Stackelberg game, relative to the case when rotation ages are solved under the Nash equilibrium.*

3. Comparative Static Analysis

We have shown that the Nash, Stackelberg, and sole owner rotation ages differ from each other in the presence of amenity benefits when there is stand interdependence. Now we study the qualitative properties of these rotation ages. An important point to realize is that market parameters for both stands can differ given that site characteristics inherent to both

[11] What happens if only one of the two owners values amenities? Suppose only landowner a values amenity services. In the Nash equilibrium, landowner b's reaction function would then be a vertical line in (τ, T) space. If landowner b is the Stackelberg follower, then the integral term in (8a) would equal zero because of this reaction function. Finally, in the sole owner case stand a is managed like in the conventional Hartman model. However, stand b is not managed like in the Faustmann model, because the effect of its rotation age on amenities provided by stand a are taken into account.

[12] Hamilton and Slutsky (1990) have shown that, in models of endogenous timing regarding the decisions of firms, there are incentives for firms to move sequentially if there is one Stackelberg equilibria that Pareto dominates all other simultaneous move Nash equilibria. With two firms, the necessary condition for the Stackelberg to obtain is that the leader's profits are higher when moving first, compared to its profits in the Nash game, otherwise no firm would choose to move first and both would, effectively, play a Nash game. A sufficient condition for this is that both leader and follower profits in the sequential move game are at least as high as their Nash profits.

stands could differ, and their rotation ages are not generally equivalent, as we showed above. The results of this section are condensed in Table 1.

3.1 Nash game

For the effect of a timber price p of stand a on both rotation ages we obtain, through total differentiation of the first order conditions,

$$T_p^N = -\Delta^{N^{-1}} \left\{ \Omega_{Tp}^{aN} \Omega_{\tau\tau}^{bN} \right\} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \quad \text{as} \quad A \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0, \tag{12a}$$

$$\tau_p^N = \Delta^{N^{-1}} \left\{ \Omega_{Tp}^{aN} \Omega_{T\tau}^{bN} \right\}, \tag{12b}$$

where $\Omega_{Tp}^{aN} = f'(T) - rf(T) - rf(T)e^{-rT}(1-e^{-rT})$, $A = rc_T(1-e^{-rT})^{-1} + F^a(T,\tau) - rE^a$ and

$F^a(T,\tau) - rE^a \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ as $F_T^a(T,\tau) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ (see Koskela-Ollikainen 2001b).

Given that the present value of regeneration costs is always positive, we have the conventional effect of a shorter rotation age due to an increase in the own-stand harvesting price when $F_T^a(T,\tau) \geq 0$ (i.e., the marginal amenity valuation increases or remains constant with the age of stands).

Equation (12b) reveals that the rotation age of stand b may also be affected by a change in stand a's timber price if the stands are not temporally independent, i.e., if $\Omega_{T\tau}^{bN} \neq 0$. Assuming that the marginal amenity valuation does not decrease with the age of the own stand, the other landowner will shorten (lengthen) his rotation age as a result of a rise in p when temporal dependence between stands is increasing (decreasing). Note that the signs of T_q^N and τ_q^N are symmetric given (12a) and (12b).

To assess the impacts of a change in regeneration costs of stand a, c_T, on rotation ages, we obtain

$$T_{cT}^{N} = -\Delta^{N^{-1}} \left\{ \frac{r}{(1-e^{-rT})} \Omega_{TT}^{aN} \right\} > 0 \qquad (13a)$$

$$\tau_{cT}^{N} = \Delta^{N^{-1}} \left\{ \frac{r}{(1-e^{-rT})} \Omega_{\tau T}^{bN} \right\} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \text{ as } \Omega_{\tau T}^{bN} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 . \qquad (13b)$$

Thus, the effect of an increase in the own-stand regeneration cost is qualitatively the same as in the Faustmann and Hartman model-based literature. However, with respect to the adjacent stand, the Nash solution brings a new result, from (13b). The reaction of the adjacent stand's landowner to a change in the regeneration costs of the other landowner depends on the temporal dependence between stands. More specifically, if the dependence between the stands increases (decreases) with a longer rotation age for the own stand, then the owner of the adjacent stand lengthens (shortens) his rotation age. Again, the signs of $T_{c_\tau}^{N}$ and $\tau_{c_\tau}^{N}$ are symmetric, given (13a) and (13b).

Finally, for a change in the real interest rate we obtain,

$$T_{r}^{N} = -\Delta^{N^{-1}} \left\{ \Omega_{Tr}^{aN} \Omega_{\tau\tau}^{bN} - \Omega_{\tau r}^{bN} \Omega_{T\tau}^{aN} \right\} \qquad (14a)$$

$$\tau_{r}^{N} = -\Delta^{N^{-1}} \left\{ \Omega_{\tau r}^{bN} \Omega_{TT}^{aN} - \Omega_{Tr}^{aN} \Omega_{\tau T}^{bN} \right\}, \qquad (14b)$$

where $\Omega_{Tr}^{aN} < 0$ and $\Omega_{\tau r}^{bN} < 0$. Under increasing temporal dependence both rotation ages will unambiguously shorten from (14a) and (14b). However, under decreasing temporal dependence the effect is a priori ambiguous. Naturally, a sufficient condition for a shorter rotation age here is that the own-stand direct effect of the interest rate dominates all other effects. Summarizing we have

Result 2. *In a symmetric Nash equilibrium with interdependent stands,*

a) A higher own-stand price shortens the rotation age under increasing temporal dependence, but may increase rotation age under strong decreasing temporal dependence. The effect of own-stand regeneration cost on rotation age is positive.

b) The effects of higher adjacent-stand timber price and regeneration costs on the own-stand rotation age depend on the nature of temporal dependence between the stands.

c) *A higher interest rate decreases rotation ages under increasing temporal dependence, but is a priori ambiguous under decreasing temporal dependence*

3.2 Stackelberg game

In the Stackelberg game we have only to solve the comparative statics for the leader's rotation age.[13] A change in the timber price for stand a impacts the leader's rotation age as follows

$$T_p^S = -\frac{\Omega_{Tp}^{aS}}{\Omega_{TT}^{aS}} \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \; as \; A \quad + \tau_T(.)\int_0^T F_\tau^a(s,\tau)e^{-rs}ds \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0, \tag{15}$$

where $\Omega_{Tp}^{aS} = f'(T) - rf(T) - rf(T)e^{-rT}(1-e^{-rT})$ and
$A = rc_T(1-e^{-rT})^{-1} + F^a(T,\tau) - rE^a$.

Recall from Result 1 that the sign of the reaction function $\tau_T(.)$ depends on temporal dependence between the stands, and that from Lemma 2 the sign of the integral term depends on whether the stands are spatial complements or substitutes. We can first see that if the stands are temporally independent ($F_{T\tau}^a = 0$), then the last term in (15) is zero. Thus, the price effect on the Stackelberg rotation age is identical to the price effect on rotation age in the single stand Hartman model. Second, a combination of increasing marginal valuation, spatial complements and increasing temporal dependence, or spatial substitutes with decreasing temporal dependence, implies unambiguously the conventional effect of a shorter rotation age.

Given that the Stackelberg game is not symmetric like the Nash game, we next solve for the effect of the price of stand b on the leader's rotation age. Differentiating the first-order condition (8a) with respect to this price (q) and noting that all effects emerge from the follower's response function, we obtain,

$$T_q^S = -\frac{\Omega_{Tq}^{aS}}{\Omega_{TT}^{aS}}. \tag{16}$$

We show in Appendix 5 that, while this price effect is a priori ambiguous for both decreasing and increasing temporal dependence, it is zero for temporally independent

[13] Comparative statics for the follower is identical to the case of an exogenous adjacent stand; this

stands. The ambiguity under non-constant temporal dependence results from the fact that the direct and indirect effects of timber price q are offsetting (via a shift in the follower's reaction function and its slope).

The effect of higher regeneration cost of the leader's stand on the leader's rotation age is given by,

$$T^S_{c_T} = -\frac{r}{(1-e^{-rT})\Omega^{aS}_{TT}} > 0\,. \tag{17a}$$

$$T^S_{c_\tau} = 0\,, \tag{17b}$$

Hence, the leader's rotation age unambiguously lengthens. The interpretation of this finding is the same as before. If the regeneration cost of the follower's stand increases, it affects the leader's first-order condition only via the reaction function of the follower. Given that $\tau_{Tc_\tau}(.) = 0$, there is no effect on the rotation age of the leader's stand.

Finally, the effect of an interest rate change on the leader's rotation age can be expressed as,

$$T^S_r = -\frac{\Omega^{aS}_{Tr}}{\Omega^{aS}_{TT}}\,, \tag{18}$$

where

$$\Omega^{aS}_{Tr} = -\left(1+\frac{Te^{-rT}}{(1-e^{-rT})}\right)\left[pf(T)+V^a+E^a\right] - \int_0^T sF(s,\tau)e^{-rs}ds$$
$$-\tau_T\int_0^T sF_\tau(s,\tau)e^{-rs}ds + \tau_T\tau_r\int_0^T F_{\tau\tau}(s,\tau)e^{-rs}ds + \tau_{Tr}\int_0^T F_\tau(s,\tau)e^{-rs}ds$$

According to this expression for Ω^{aS}_{Tr}, the interest rate effect arises through three channels: directly i) through the profitability of the leader's stand, and indirectly both ii) via the slope of the reaction function, and iii) through the position of the follower's reaction function. In Appendix 5 we show via Lemma 3 that these effects counter each other, so that the overall impact of the interest rate is ambiguous. We can summarize our findings in:

Result 3. *In a Stackelberg equilibrium with interdependent stands,*

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case has been analyzed in Koskela-Ollikainen (2001b).

a) *Under increasing marginal amenity valuation, an increase in the leader's own-stand price will shorten the leader's rotation age when the stands are spatial complements with increasing temporal dependence, or spatial substitutes with decreasing temporal dependence. A higher own-stand regeneration cost increases the leader's rotation age unambiguously.*

b) *An increase in the follower's own-stand price has an a priori ambiguous effect on rotation age when stands are not temporally independent, while higher own-stand regeneration costs for the follower have no effect on the leaders' rotation age.*

c) *A higher interest rate has an a priori ambiguous effect on the leader's rotation age.*

3.3. Sole owner solution

Finally, we consider comparative statics in the sole owner case. For the effect of higher timber price of stand *a* on both rotation ages we have,

$$T_p^{SO} = -\Delta^{SO^{-1}}\{W_{Tp}W_{\tau\tau}\} \tag{19a}$$

$$\tau_p^{SO} = \Delta^{SO^{-1}}\{W_{Tp}W_{\tau T}\}, \tag{19b}$$

where $W_{Tp} = \frac{e^{-rT}}{(1-e^{-rT})}\left[f'(T) - rf(T) - \frac{rf(T)e^{-rT}}{(1-e^{-rT})}\right]$, and

$$W_{\tau T} = \frac{e^{-r\tau}}{(1-e^{-r\tau})}\Omega_{T\tau}^{aN} + \frac{e^{-rT}}{(1-e^{-rT})}\left[F_\tau^a(T,\tau) - \frac{r\int_0^T F_\tau^a(s,\tau)e^{-rs}ds}{(1-e^{-rT})}\right].$$

By using the first-order conditions (1a) and (10b), we can show that: $\operatorname{sgn} W_{Tp}$ = - $\operatorname{sgn}\left[A + \int_0^\tau F_T^b(T,x)e^{-rx}dx\right]$, where $A = rc_T(1-e^{-rT}) + F^a(T,\tau) - rE^a$. The sufficient conditions for $W_{Tp} < 0$, and the impact of own-stand price on the rotation age to be negative, are that marginal amenity valuations are non-decreasing with the rotation age of the stand $(F_T^a(.) \geq 0)$ and the stands are spatial complements $(F_T^b(.) > 0)$ (see Lemma 2). Under the sufficient condition for $W_{Tp} < 0$, the effect of the adjacent stand price on own-stand rotation age is negative under increasing temporal dependence and positive under decreasing temporal dependence; because according to Lemma 1 $W_{\tau T} > (<) 0$, as the temporal dependence between stands increases (decreases). Thus, the price effects here resemble the classic case of complements and substitutes, except in our model complementarity is specified in a temporal sense.

The effect of a change in the own-stand regeneration costs of stand *a* on the rotation age is given by,

$$T_{c_T}^{SO} = -\Delta^{SO^{-1}} \left\{ \frac{re^{-rT}}{(1-e^{-rT})^2} W_{TT} \right\} > 0 \tag{20a}$$

$$\tau_{c_T}^{SO} = \Delta^{SO^{-1}} \left\{ \frac{re^{-rT}}{(1-e^{-rT})^2} W_{T\tau} \right\} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} > 0 \text{ as } W_{T\tau} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0. \tag{20b}$$

A higher own-stand regeneration cost lengthens own rotation age, while its effect on the other stand depends again on the temporal dependence between stands. The rotation age of the other stand lengthens under increasing temporal dependence and shortens under decreasing temporal dependence.

Finally, for the effects of an increase in the real interest rate we have,

$$T_r^{SO} = -\Delta^{SO^{-1}} \{ W_{Tr} W_{\tau\tau} - W_{\tau r} W_{T\tau} \} \tag{21a}$$

$$\tau_r^{SO} = -\Delta^{SO^{-1}} \{ W_{\tau r} W_{TT} - W_{Tr} W_{\tau T} \}, \tag{21b}$$

where

$$W_{Tr} = \frac{e^{-rT}}{(1-e^{-rT})} \Omega_{Tr}^{aN} + \frac{T}{(1-e^{-rT})^2} \left\{ (1-e^{-rT}) \int_0^{\tau} F_T^b(T,x) e^{-rx} dx - \frac{1}{T} \int_0^{\tau} x F_T^b(T,x) e^{-rx} dx \right\}$$

$$W_{\tau r} = \frac{e^{-r\tau}}{(1-e^{-r\tau})} \Omega_{\tau r}^{bN} + \frac{\tau}{(1-e^{-r\tau})^2} \left\{ (1-e^{-rT}) \int_0^{T} F_\tau^a(s,\tau) e^{-rs} ds - \frac{1}{\tau} \int_0^{T} s F_\tau^a(s,\tau) e^{-rs} ds \right\}$$

The first terms in W_{Tr} and $W_{\tau r}$ are negative. In Appendix 6 we show that, under plausible assumptions concerning the interest rate and rotation ages, the braced terms are negative if the stands are spatial complements. Thus, under increasing temporal dependence and spatial complementarity of stands, both rotation ages will unambiguously shorten; otherwise the effects are a priori ambiguous. We therefore have,

Result 4. *In the case of a sole owner with interdependent stands,*

a) A higher own-stand price shortens rotation age if stands are either spatial complements, and temporal dependence is either unchanged or increasing. A higher own-stand regeneration cost unambiguously lengthens the rotation age

b) Under a certain sufficient condition, the effect of the adjacent stand price on own-stand rotation age depends on the nature of temporal dependence. The effect of the adjacent stand's regeneration cost will increase (decrease) the own-stand rotation age under increasing (decreasing) temporal dependence.

c) If stands are spatial complements and temporal interdependence is increasing, then a higher interest rate will shorten the rotation ages. [14]

4. Discussion and Policy Implications

Sustaining forest ecosystems requires that stands are managed in concert rather than in isolation. Most forest economics models, however, consider only a single isolated stand or a single landowner. This is not necessarily the case for private land ownership, where individual property rights make proper coordination of management decisions across large numbers of landowners very difficult. The lack of coordination among landowners can be detrimental to amenities that depend on the ecosystem as a whole, such as those derived from recreational experiences or the existence of certain wildlife species.

Under the assumption that rotation ages are identical in the steady state, we have examined the possibility that stands can be temporally or spatially interdependent in different ways regarding the production of amenities by allowing for several cases of landowner decision timing and commitment, including landowners making simultaneous decisions, landowners coordinating their actions, or one landowner acting as a first mover. Our results extend the basic Hartmann model of forest management that first introduced

[14] Suppose that in the case of sole owner solution both stand a and stand b have the same timber price or regeneration cost change. Then assuming that $W_{Tp} < 0$ a higher timber price will decrease both rotation ages when marginal amenity valuations are non-decreasing with the rotation age of stands and temporal dependence between stands is non-decreasing. The effect of changes in the uniform regeneration cost is positive (ambiguous) when temporal dependence between stands is increasing (decreasing). Proofs are available upon request.

amenities for the case of an isolated stand, as well as other models of multiple stands based on numerical simulations or the assumption of only sole ownership.

We demonstrate that rotation age decisions depend on how adjacent stands are spatially and temporally dependent with regards to amenity valuation, and on the structure of landowner decision timing. Sole ownership represents the social optimum in our model. Comparing this with the Nash and Stackelberg outcomes gives a qualitative indication of the social costs associated with landowners who do not seek to jointly maximize total revenue and amenity rents from owning forests.

The collective results from our paper are summarized in Propositions 1, 2, Corollary 1, and Table 1. The sole owner's rotation age is longer than the Nash and Stackelberg rotation ages if the stands are spatial complements, but shorter if they are spatial substitutes with regard to marginal amenity valuation. Additionally, the relationship between the Nash and Stackelberg rotation ages also depends on how this substitutability and complementarity evolves over time. The differences between these solutions largely reflect the ability (or inability) of landowners to either benefit from another landowner's decisions. Interestingly, we show that under increasing temporal interdependence, it can be the case that one landowner moving first may make both landowners better off relative to the case where they do not coordinate at all and play a Nash game.

As a basis for policy analysis, we also characterize in these new models how rotation ages depend on timber prices, interest rates, and regeneration costs. We find that the effects of these parameters, derived in single stand models, does not usually hold. Instead, the results depend on the spatial and temporal dependence between the stands, the ability of landowners to commit to harvesting, and whether the parameters changing are for the stand in question or the adjacent stand. By and large, most of the differences between our models and existing models occur for two reasons. First, the possibility of increasing or decreasing temporal dependence often determines the signs of comparative statics results, for instance, the interest rate effect on rotation age (for instance) is not necessarily negative like in existing models with one forest stand (see Table 1). Second, stand interdependence implies that parameters from one stand can affect the choices made in the other stand, as exemplified by our price and regeneration cost effects.

Our new results and approach to studying landowner behavior suggests that existing models of policy design should be revised. We demonstrate that both Nash and Stackelberg equilibrium rotation ages are longer than sole owner rotation ages when stands are spatial substitutes, but Nash and Stackelberg ages are shorter than sole owner ages when stands are spatial complements. Clearly the scope for using taxes or subsidies to adjust rotation ages toward their efficient levels depends on the nature of stand interdependence regarding amenities, and also on the ability of landowners to commit to rotation age actions. These ideas have not been previously uncovered, even within the spatial forestry literature. In the end, the design of a proper Pigouvian tax system which mimics the efficiency of the sole owner solution will be much more complicated than previously thought. It would be useful to extend the optimal forest taxation analysis in the Hartman framework by Koskela and Ollikainen (2003) for allow for stand interdependencies. What we can learn from empirical work regarding this interdependence will also be crucial to practical policy work.

Our work here suggests many other research topics, most of which are motivated by the need to continue generalizing the modelling approach. As pointed out in the text and illustrated in Appendix 1, it would be useful to understand interdependency when adjacent stand rotation ages differ. Here, the amenity value of each stand might depend on the calendar age of the adjacent stand, as well as the dynamics of the steady-state involving the two stands. Providing an analytical solution for this problem would be a demanding and interesting topic for future research, but if solved it would provide an analysis of transitional dynamics and identification of possible outcomes that might arise as rotation ages adjust between various equilibria. For this reason, the application of our model to uneven-aged managed forests with adjacent landowners would be interesting and probably lead to different results. In models where rotation ages of stands are different, we expect that the asymmetries we identify between landowner interactions will continue to exist and become even more complicated.

Finally, it is clear from our work that gains from cooperation among landowners could be highly relevant to the application of any forest policy, especially if the policy takes stand interdependency as its starting point. Analyzing the policy problem theoretically, and quantifying and comparing the gains from different policy instruments in the different equilibria we find here would be important in assessing the effectiveness and scope for policy.

Figure 1. Decreasing temporal dependence when the stands are spatial substitutes

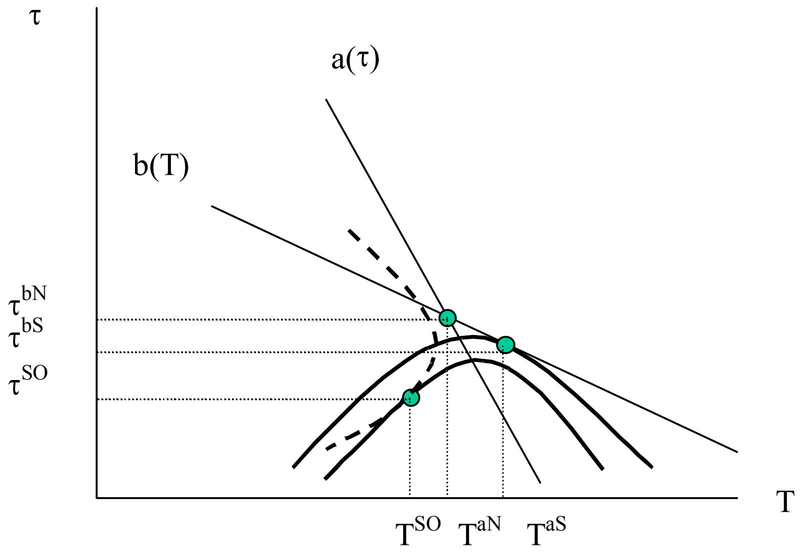

Figure 2. Decreasing temporal dependence when the stands are spatial complements

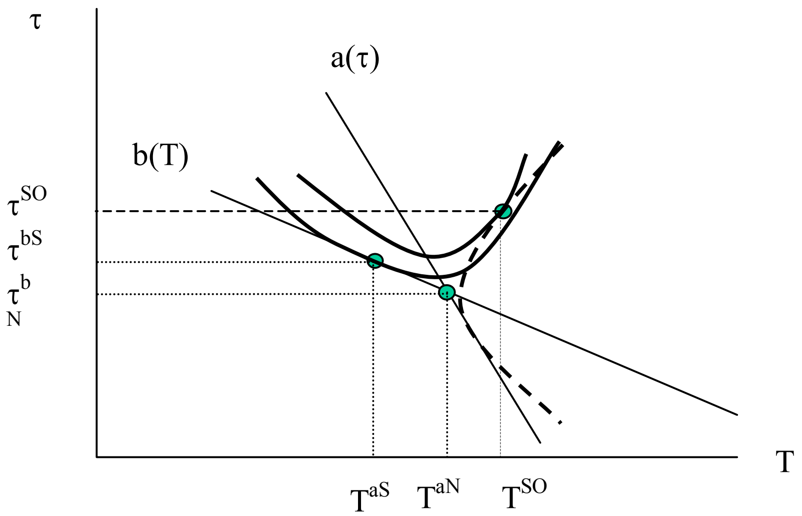

Figure 3. Increasing temporal dependence when the stands are spatial substitutes

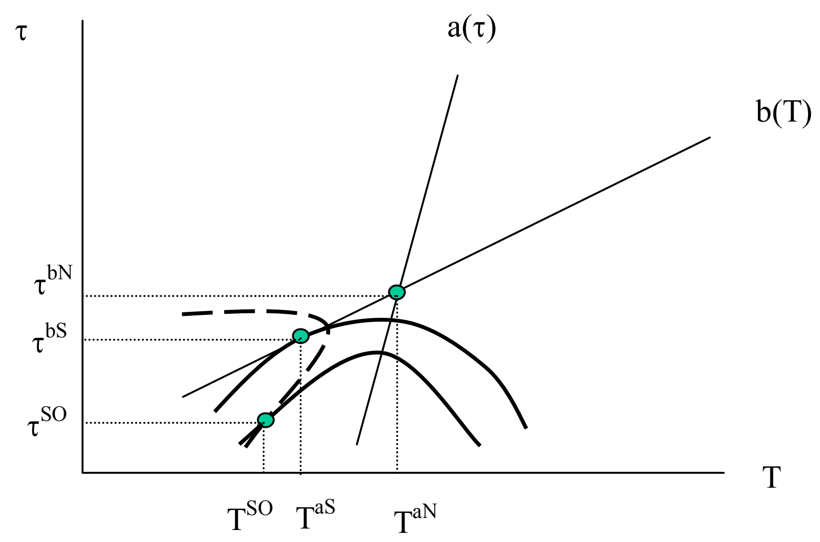

Figure 4. Increasing temporal dependence when the stands are spatial complements

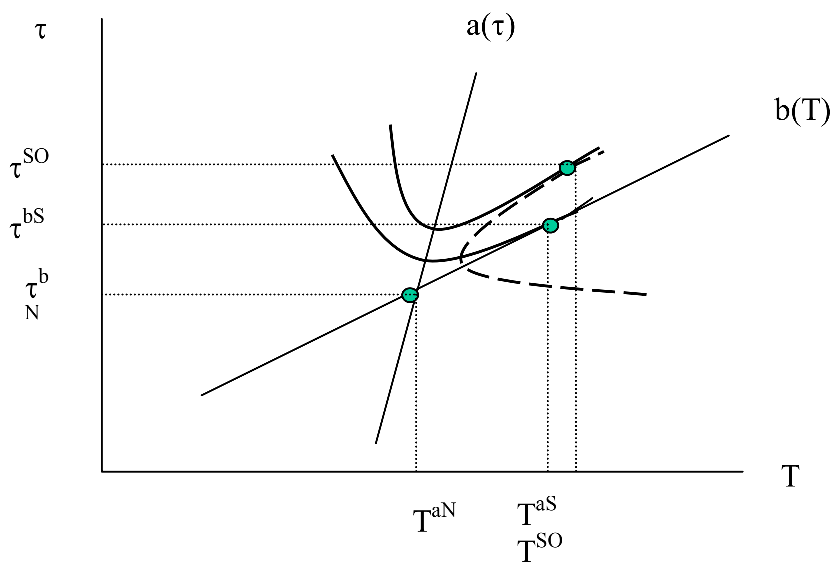

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TABLE 1. Comparative statics of the rotation age in Nash, Stackelberg, and sole owner cases.

Solutions for stand a.* '+' indicates an increase in rotation age, while '-' indicates a decrease in rotation age. Marginal amenity function signs correspond to Definitions 1 and 2 in the text.

Exogenous Parameter	*Nash Rotation Age* (T^N)	*Stackelberg Rotation Age Leader* (T^S)	*Sole Owner Rotation Age* (T^{SO})
Own stand price	- if $F_T^a \geq 0$ +/- otherwise	- if $F_T^a > 0$, $F_{T\tau}^a > 0$, $F_\tau^a > 0$ *or* if $F_T^a > 0$, $F_{T\tau}^a < 0$, $F_\tau^a < 0$	- *If* $F_T^a \geq 0; F_T^b > 0$ +/- *otherwise*
Own-stand regeneration cost	+	+	+
Adjacent stand price as $F_T^a \geq 0$	+ if $F_{\tau T}^a > 0$ - if $F_{\tau T}^a < 0$	+/-	+ if $F_\tau^b \geq 0; F_\tau^a > 0;$ $F_{\tau T}^a < 0$ - If $F_\tau^b \geq 0; F_\tau^a > 0;$ $F_{\tau T}^a > 0$
Interest rate	- if $F_{\tau T}^a > 0$ +/- otherwise	+/-	- if $F_{T\tau}^b > 0$, $F_T^b > 0$ +/- otherwise
Adjacent stand regeneration cost	+ if $F_{\tau T}^a > 0$ - if $F_{\tau T}^a < 0$	0	+ if $F_{\tau T}^a > 0$ - if $F_{\tau T}^b < 0$

*By symmetry, the comparative statics results in the table would also hold for stand b, with the parameters redefined for this stand.

Appendix 1. *Stand interdependence, present value of amenity benefits and different rotation ages for stands.*

Consider the amenity function for stand a in the case of three rotations T when the rotation age of stand b is $\tau \neq T$. Assume that the first rotation for each stand begins with bare land. The amenity function can be written as follows,

$$E^a = \int_{0(i=0)}^{T(i=0)} F^a(t, t - i\tau)e^{-rt}dt + \int_{T(i=0)}^{2T(i=1)} F^a(t - T, t - i\tau)e^{-rt}dt + \int_{2T(i=1)}^{3T(i=2)} F^a(t - 2T, t - i\tau)e^{-rt}dt \qquad \text{A1.1}$$

where $i = 0$ initially and at the end of 1st rotation of stand a, $i = 1$ at the end 2nd rotation of stand a, and $i = 2$ at the end 3rd rotation of stand a. We can describe the amenity function for stand b in an analogous way. Differentiating equation A1.1. with respect to rotation age T yields,

$$E^a_T = \left[F^a(T,T)e^{-rT} - F^a(0,0)\right] + \left[F^a(T,2T - \tau)e^{-rT} - F^a(0,T)\right]e^{-rT} + \left[F^a(T,3T - 2\tau)e^{-rT} - F^a(0,2T - \tau)\right]e^{-2rT} \qquad \text{A1.2}$$

One can see from equation A1.2 that, in the presence of stand interdependence, the dynamics of both stands should be included in the amenity value function for each stand if rotation ages are different from each other. But, if rotation ages are equal, one ends up with the specification presented in the text – this can be seen from A1.2.

In order to examine the amenity function with different rotation ages further, assume that $T = 20$ and $\tau = 30$. Now, A1.1 and A1.2 can be expressed respectively as

$$E^a = \int_{0(i=0)}^{20(i=0)} F^a(t, t - i\tau)e^{-rt}dt + \int_{20(i=0)}^{40(i=1)} F^a(t - 20, t - i30)e^{-rt}dt + \int_{40(i=1)}^{60(i=2)} F^a(t - 40, t - i30)e^{-rt}dt \qquad \text{A1.3}$$

and

$$E^a_T = \left[F^a(20,20)e^{-r20} - F^a(0,0)\right] + \left[F^a(20,10)e^{-r20} - F^a(0,20)\right]e^{-r20} + \left[F^a(20,0)e^{-r20} - F^a(0,10)\right]e^{-r40.} \qquad \text{A1.4}$$

Appendix 2. *Proof of Lemma 1*

The proof is given only for $\Omega^{aN}_{T\tau}$, because the proof for $\Omega^{bN}_{\tau T}$ is analogous. The cross-derivative $\Omega^{aN}_{T\tau}$ can be re-expressed as

$$\Omega^{aN}_{T\tau} = \int_0^T F^a_\tau(s,\tau)e^{-rs}ds \left[\frac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} - \frac{r}{1-e^{-rT}} \right] \qquad \text{A.2.1}$$

- Temporal Independence: $F^a_{\tau T} = 0 \;\Rightarrow\; \frac{dT}{d\tau} = 0$

Proof. If $F^a_{\tau T} = 0$, $\Omega^{aN}_{T\tau}$ reduces to $\Omega^a_{T\tau} = F^a_\tau(T,\tau) - (1-e^{-rT})^{-1}\left[F^a_\tau(0,\tau) - F^a_\tau(T,\tau)e^{-rT}\right]$. There are two possibilities. If $F^a_\tau = 0$, then trivially $\Omega^{aN}_{T\tau} = 0$. Under $F^a_\tau \neq 0$, $F^a_{\tau T} = 0$ implies $\left[F^a_\tau(0,\tau) - F^a_\tau(T,\tau)e^{-rT}\right] = F^a_\tau(1-e^{-rT})$ so that $\Omega_{T\tau} = F_\tau(T,\tau) - (1-e^{-rT})^{-1}F_\tau(T,\tau)(1-e^{-rT}) = 0$. Hence, $\frac{dT}{d\tau} = 0$.

- Increasing Temporal Dependence: $F^a_{\tau T} > 0 \;\Rightarrow\; \frac{dT}{d\tau} > 0$

Proof. i) If $F^a_\tau > 0$ then $\Omega^{aN}_{T\tau} > 0 \Leftrightarrow \frac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} > \frac{r}{1-e^{-rT}}$. Now $F^a_{\tau T} > 0 \Rightarrow$

$$\int_0^T F^a_\tau(T,\tau)e^{-rs}ds > \int_0^T F^a_\tau(s,\tau)e^{-rs}ds \Leftrightarrow \frac{F^a_\tau(T,\tau)(1-e^{-rT})}{r} > \int_0^T F^a_\tau(s,\tau)e^{-rs}ds \Leftrightarrow$$

$\frac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} > \frac{r}{1-e^{-rT}}$. Hence, $\Omega^{aN}_{T\tau} > 0$ so that $\frac{dT}{d\tau} > 0$.

ii) If $F^a_\tau < 0$ then $\Omega^{aN}_{T\tau} < 0 \Leftrightarrow \frac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} < \frac{r}{1-e^{-rT}}$. Now $F^a_{\tau T} > 0 \Rightarrow$

$$\int_0^T F^a_\tau(T,\tau)e^{-rs}ds > \int_0^T F^a_\tau(s,\tau)e^{-rs}ds \Leftrightarrow \frac{F^a_\tau(T,\tau)(1-e^{-rT})}{r} > \int_0^T F^a_\tau(s,\tau)e^{-rs}ds \Leftrightarrow$$

$\frac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} < \frac{r}{1-e^{-rT}}$. Hence, $\Omega^{aN}_{T\tau} > 0$ so that $\frac{dT}{d\tau} > 0$.

- Decreasing Temporal Dependence: $F^a_{\tau T} < 0 \;\Rightarrow\; \dfrac{dT}{d\tau} < 0$

Proof. i) If $F^a_\tau > 0$ then $\Omega^{aN}_{T\tau} > 0 \;\Leftrightarrow\; \dfrac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} < \dfrac{r}{1-e^{-rT}}$. Now $F^a_{\tau T} < 0 \;\Rightarrow$

$$\int_0^T F^a_\tau(T,\tau)e^{-rs}ds \;<\int_0^T F^a_\tau(s,\tau)e^{-rs}ds \Leftrightarrow \frac{F^a_\tau(T,\tau)(1-e^{-rT})}{r} < \int_0^T F^a_\tau(s,\tau)e^{-rs}ds \;\Leftrightarrow$$

$\dfrac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} < \dfrac{r}{1-e^{-rT}}$. Hence, $\Omega^{aN}_{T\tau} < 0$ so that $\dfrac{dT}{d\tau} < 0$.

ii) If $F^a_\tau < 0$ then $\Omega^{aN}_{T\tau} < 0 \;\Leftrightarrow\; \dfrac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} > \dfrac{r}{1-e^{-rT}}$. Now $F^a_{\tau T} < 0 \;\Rightarrow$

$$\int_0^T F^a_\tau(T,\tau)e^{-rs}ds \;<\int_0^T F^a_\tau(s,\tau)e^{-rs}ds \;\Leftrightarrow\; \frac{F^a_\tau(T,\tau)(1-e^{-rT})}{r} < \int_0^T F^a_\tau(s,\tau)e^{-rs}ds \;\Leftrightarrow$$

$\dfrac{F^a_\tau(T,\tau)}{\int_0^T F^a_\tau(s,\tau)e^{-rs}ds} > \dfrac{r}{1-e^{-rT}}$. Hence, $\Omega^{aN}_{T\tau} < 0$ so that $\dfrac{dT}{d\tau} < 0$. Q.E.D.

Appendix 3. *Proof of Lemma 2*

Integrating the term $\Psi = \int_0^T F^a_\tau(s,\tau)e^{-rs}ds$ in (8a) by parts and assuming that the third derivative of the amenity function is zero, i.e. $F^a_{\tau TT} = 0$, we get

$$\Psi = \frac{1}{r}\left\{F^a_\tau(0,\tau) - F^a_\tau(T,\tau)e^{-rT} + \frac{1}{r}(F^a_{\tau T}(0,\tau) - F^a_{\tau T}(T,\tau)e^{-rT})\right\} \qquad \text{A3.1}$$

Under the quadratic amenity valuation function

$$F(T,\tau) = \alpha(T+\tau) - \tfrac{1}{2}\beta(T+\gamma\tau)^2, \qquad \text{A3.2}$$

with $\beta > 0$, $\gamma > (<)\, 0$, we have $F_\tau(T,\tau) = \alpha - \beta\gamma(T+\gamma\tau)$, and $F_{\tau T}(T,\tau) = -\beta\gamma$. Moreover, we define

$$F_\tau(0,\tau) = \alpha - \beta\gamma^2\tau \begin{Bmatrix} > \\ < \end{Bmatrix} 0 \text{ as } \begin{Bmatrix} \text{stands are complements} \\ \text{stands are substitutes} \end{Bmatrix} \qquad \text{A3.3}$$

Hence, we can express A3.1 as

$$\Psi = \frac{1}{r}\left\{(\alpha - \beta\gamma^2\tau)(1-e^{-rT}) + \beta\gamma T e^{-rT} - \frac{\beta\gamma(1-e^{-rT})}{r}\right\} \qquad \text{A3.4}$$

Using second-order approximation $e^{-rT} = \dfrac{1}{1 + rT + (1/2)r^2T^2}$ yields

$$\Psi = \frac{(1-e^{-rT})}{r}\left\{(\alpha - \beta\gamma^2\tau) - \beta\gamma\frac{T}{2+rT}\right\} = \frac{(1-e^{-rT})}{r}\left\{F_\tau^a(0,\tau) - \beta\gamma\frac{T}{2+rT}\right\} \qquad \text{A3.5}$$

Now,

- For complements $\gamma < 0$, and under increasing temporal dependence $F_\tau^a(T,\tau) = \alpha - \beta\gamma^2\tau - \beta\gamma T > 0$, while decreasing temporal dependence implies that $F_\tau^a(T,\tau) = \alpha - \beta\gamma^2\tau - \beta\gamma T < 0$. Thus A3.5 is positive in both cases: the first one automatically, the second one, because $T > \dfrac{T}{2+rT}$.
- For substitutes $\gamma > 0$ condition A3.5 is automatically negative for decreasing temporal dependence; and negative also under increasing temporal dependence, because $T > \dfrac{T}{2+rT}$. QED.

Appendix 4: *Second order conditions for the sole owner*

The second order conditions rely on the following derivatives, from (10a) and (10b)

$$W_{TT} = -\frac{e^{-rT}}{(1-e^{-rT})}\left[\frac{r}{(1-e^{-rT})}\Omega_T^{aN} - \Omega_{TT}^{aN}\right] + \frac{\int_0^\tau F_{TT}^b(T,x)e^{-rx}dx}{(1-e^{-r\tau})} < 0$$

$$W_{\tau\tau} = -\frac{e^{-r\tau}}{(1-e^{-r\tau})}\left[\frac{r}{(1-e^{-r\tau})}\Omega_\tau^{bN} - \Omega_{\tau\tau}^{bN}\right] + \frac{\int_0^T F_{\tau\tau}^a(s,\tau)e^{-rs}ds}{(1-e^{-rT})} < 0$$

$$\Delta^{SO} = W_{TT}W_{\tau\tau} - W_{T\tau}W_{\tau T} > 0.$$

Appendix 5. *Comparative statics of adjacent stand's timber price and interest rate in Stackelberg model*

- *Price (q):*

By differentiation for the Stackelberg leader we obtain,

$$\Omega_{Tq}^{aS} = \tau_q(.)\left[F_\tau(T,\tau)e^{-rT} - \frac{re^{-rT}}{(1-e^{-rT})}\int_0^T F_\tau(s,\tau)e^{-rs}ds + \tau_T(.)\int_0^T F_{\tau\tau}(s,\tau)e^{-rs}ds\right] \qquad \text{A5.1}$$

$$+\tau_{Tq}(.)\int_0^T F_\tau(s,\tau)e^{-rs}ds,$$

where the term τ_{Tq} is defined as:

$$\tau_{Tq} = \frac{F_T^b - r(1-e^{-r\tau})^{-1}\int_0^\tau F_T^b e^{-rx}dx)(g''(\tau)-rg'(\tau))}{(\Omega_{\tau\tau}^{bS})^2}\begin{Bmatrix}>\\=\\>\end{Bmatrix}0 \ \ as \ \ F_{T\tau}^b\begin{Bmatrix}<\\=\\>\end{Bmatrix}0. \qquad \text{A5.2}$$

Thus Ω_{Tq}^{aS} consists of a shift in the follower's reaction function as well as a change in its slope in terms of τ. The sign of all terms depends on the nature of temporal dependence between the stands. On the basis of our previous analysis the sum of the first two terms of eqn A5.1 in brackets are negative (positive) for increasing (decreasing) temporal dependence. The third and fourth terms are of opposite sign, being positive (negative) for increasing (decreasing) temporal dependence, indicating that the sign of Ω_{Tq}^{aS} is a priori ambiguous for both decreasing and increasing temporal dependence. For temporally independent stands we have $\Omega_{Tq}^{aS}=0$.

- *Interest rate (r):*

We first develop the cross-derivative of the follower's reaction function, $\tau_{Tr}(.)$. Recalling equation (8b), the derivative with respect to r is,

$$\tau_{Tr} = \frac{1}{\phi^2}\left\{F_T^b - r(1-e^{-r\tau})^{-1}\int_0^T F_T^b e^{-rx}dx + \phi\left[r(1-e^{-r\tau})^{-1}\int_0^T xF_T^b e^{-rx}dx - \omega\int_0^T F_T^b e^{-rx}dx\right]\right\}, \text{ A5.3.}$$

where $\phi = qg''(\tau)-rqg'(\tau)+F_\tau^b<0$ and $\omega = \frac{1}{1-e^{-r\tau}}\left(1-\frac{\tau re^{-r\tau}}{1-e^{-r\tau}}\right)$. The sign of the first two terms in braces depends on the sign of $F_{T\tau}^b$ by Lemma 1. Approximating the ω term using $e^{-r\tau} = \frac{1}{1+r\tau+(1/2)r^2\tau^2}$ implies that $\omega = \frac{1}{1-e^{-r\tau})}\left[\frac{\frac{1}{2}r^2\tau^2}{1+r\tau+(1/2)r^2\tau^2}\right]>0$. We determine the sign of the integral term $\int_0^T xF_T^b e^{-rx}dx$ by using the quadratic amenity valuation function given in A.3.2 in Lemma 3.

Lemma 3. *If the amenity valuation function is quadratic, then*

$$\int_0^\tau xF_T^b e^{-rx}dx\begin{Bmatrix}>\\=\\<\end{Bmatrix}0 \text{ as } F_T^b(T,0\begin{Bmatrix}>\\=\\<\end{Bmatrix}0.$$

Proof. Integrating $\int_0^{\tau} xF_T^b e^{-rx}dx$ by parts and assuming that the third derivative of the amenity function is zero, i.e. that $F_{T\tau\tau}=0$, we get

$$\int_0^{\tau} xF_T^b e^{-rx}dx = \int_0^{\tau} F_T^b e^{-rx}dx + \int_0^{\tau} xF_{T\tau}^b e^{-rx}dx \qquad \text{A5.4}$$

Under the quadratic amenity valuation function

$$F(T,\tau) = \alpha(T+\tau) - \tfrac{1}{2}\beta(T+\gamma\tau)^2, \qquad \text{A5.5}$$

A6.1 yields, so that we can write,

$$\int_0^{\tau} xF_T^b e^{-rx}dx = (\alpha - \beta T)\int_0^{\tau} xe^{-rx}dx - \beta\gamma\int_0^{\tau} x^2 e^{-rx}dx \qquad \text{A5.6}$$

By integrating the first RHS of A5.6 one gets $\int_0^{\tau} xe^{-rx}dx = -\frac{1}{r}\left[\tau e^{-r\tau} - \frac{1}{r}(1-e^{-r\tau})\right]$ and using second-order approximation $e^{-rT} = \frac{1}{1+rT+(1/2)r^2T^2}$ yields

$$\int_0^{\tau} xe^{-rx}dx = \frac{\tau^2}{2r(1+r\tau+\frac{1}{2}r^2\tau^2)} > 0, \qquad \text{A5.7}$$

For the second term we get via integration by parts that

$$-\beta\gamma\int_0^{\tau} x^2 e^{-rx}dx = -\beta\gamma\left\{-\frac{\tau^2}{r(1+r\tau+\frac{1}{2}r^2\tau^2)} + \frac{2}{r}\frac{\tau^2}{2(1+r\tau+\frac{1}{2}r^2\tau^2)}\right\} = 0 \qquad \text{A5.8}$$

Given that A5.7 is positive, the sign of the integral $\int_0^{\tau} xF_T^b e^{-rx}dx$ depends on the sign of $(\alpha - \beta T)$. Hence, we have shown that

$$\int_0^{\tau} xF_T^b e^{-rx}dx \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \text{ as } F_T^b(T,0) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0. \qquad \text{A5.9}$$

QED.

See Appendix6.

Hence, we have established that $\tau_{Tr} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$ as $F_{\tau T}^a \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0$. Now the overall sign of the three latter terms in Ω_{Tr}^{aS} can be revealed. The first term is positive, but the last two terms are negative when the stands exhibit increasing temporal dependence, while the opposite holds for decreasing temporal dependence. Hence, the interest rate effect in the Stackelberg game is genuinely ambiguous.

Appendix 6.

Given (21a) and (21b) in the text, the first terms in W_{Tr} and $W_{\tau r}$ are negative. Next we study sign of the braced terms in the derivative W_{Tr} (by symmetry this holds also for the sign of the braced terms in $W_{\tau r}$). According to Lemma 2 we have,

$$\int_0^\tau F_T^b(T,x)e^{-rx}dx = \frac{(1-e^{-r\tau})}{r}\left\{(\alpha-\beta T)-\beta\gamma\frac{\tau}{2+r\tau}\right\} = \frac{(1-e^{-r\tau})}{r}\left\{F_T^b(T,0)-\beta\gamma\frac{\tau}{2+r\tau}\right\} \quad \text{A6.1}$$

where $1-e^{-r\tau} = (r\tau+(1/2)r^2\tau^2)/(1+r\tau+(1/2)r^2\tau^{2\prime})$. Now we have,

$$(1-e^{-rT})\int_0^\tau F_T^b(T,x)e^{-rx}dx = \frac{\left[rT+(\frac{1}{2})r^2T^2\right]\left[r\tau+(\frac{1}{2})r^2\tau^2\right]}{r\left[1+rT+(\frac{1}{2})r^2T^2\right]\left[1+r\tau+(\frac{1}{2})r^2\tau^2\right]}\left\{F_T^b(T,0)-\beta\gamma\frac{\tau}{2+r\tau}\right\}$$

A6.2

Using Lemma 3 we have,

$$\int_0^\tau xF_T^b e^{-rx}dx = (\alpha-\beta T)\frac{\tau^2}{2r(1+r\tau+(1/2)r^2\tau^2)} = F_T^b(T,0)\frac{\tau^2}{2r(1+r\tau+(1/2)r^2\tau^2)} \quad \text{A6.3}$$

Combining A6.2. and A6.3 gives

$$(1-e^{-rT})\int_0^\tau F_T^b(T,x)e^{-rx}dx - \frac{1}{T}\int_0^\tau xF_T^b(t,x)e^{-rx}dx =$$

$$\frac{1}{r\left[1+r\tau+(\frac{1}{2})r^2\tau^2\right]}\left\{\frac{\left[rT+(\frac{1}{2})r^2T^2\right]\left[r\tau+(\frac{1}{2})r^2\tau^2\right]}{\left[1+rT+(\frac{1}{2})r^2T^2\right]}F_T^b(T,0)-\beta\gamma\frac{\tau}{2+r\tau}-\frac{\tau^2}{2T}F_T^b(T,0)\right\}$$

A6.4

Analogously we can write,

$$(1-e^{-rT})\int_0^\tau F_T^b(T,x)e^{-rx}dx - \frac{1}{T}\int_0^\tau xF_T^b(t,x)e^{-rx}dx =$$

$$\frac{1}{r\left(1+rT+(\frac{1}{2})r^2T^2\right)(}\left\{\frac{\left[rT+(\frac{1}{2})r^2T^2\right]\left[r\tau+(\frac{1}{2})r^2\tau^2\right]}{\left(1+r\tau+(\frac{1}{2})r^2\tau^2\right)}F_\tau^a(0.\tau)-\beta\gamma\frac{\tau}{2+r\tau}-\frac{T^2}{2\tau}F_\tau^a(0.\tau)\right\}$$

A6.5

Under plausible assumptions concerning the interest rate and rotation ages, the last term in equations A6.4 and A6.5 dominates. Q.E.D.