

# Who Searches?

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Discussion Paper No 601:2004

ISSN 1459-3696

ISBN 952-10-1531-4

May 5, 2004

## Abstract

This paper answers the question whether buyers seek for sellers or vice versa, in a directed search model with no explicit search costs. We focus on a symmetric mixed-strategy equilibrium where both buyers and sellers can search and wait. The mixed strategy tells the likelihood of searching versus waiting, and it provides insight into the relevance of standard practice of postulating the identity of searchers and waiters. We consider two trading mechanisms, auction and bargaining. The

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main result is that the members of the larger population search and those of the smaller population wait.

JEL Classification: J41, J64, C78, D44, D83

Keywords: search, auction, bargaining

## 1 Introduction

Search theoretic models are widely used with various levels of details as to the actual search and meeting process. In the simplest form agents meet each other with exogenously given rates, while in more detailed models the agents may choose their search efforts and whether to search or wait. The very question of who wait and who search is addressed in a search model of endogenous money by Burdett, Coles, Kiyotaki and Wright (1995). In the model some agents are money holders, or buyers, and others are commodity holders, or sellers. Searching involves an explicit cost which may be different for different kinds of agents, while waiting involves no explicit costs. The authors determine the equilibria of the model when the agents' basic strategic choice is whether to search or wait. Not too surprisingly there are many equilibria depending on the costs. But matters of coordination also generate multiple equilibria. Roughly put, if all sellers wait, then all the buyers must search, and vice versa. To gain some insight into the likelihood that it is the buyers (or sellers) who search, the authors compare the sizes of the parameter space supporting various equilibria. Their result is that the larger the number of different commodities, or the more difficult the double coincidence problem of barter, the more likely it is that the money holders, or buyers, search.

The link between the size of the parameter space supporting an equilibrium and the likelihood of that equilibrium is tenuous at best. In this article we try to offer a more straightforward link between the likelihood of the search decision and the fundamentals of the economy using a model where there are only two kinds of agents called buyers and sellers. We do not introduce any costs of searching, but the search decision turns out to depend on the trading mechanism and the ratio of buyers to sellers. We use auction and bargaining as alternative trading rules. Herreiner (1999) studies also the question about

who searches, but she ignores price formation.

Given a trading mechanism we are interested in how the ratio of buyers to sellers affects the likelihood of searching for both types. We focus on symmetric (mixed strategy) equilibria and ignore the two co-ordination game like equilibria where all buyers search and all sellers wait or vice versa. The mixed strategy then has a straightforward interpretation as the likelihood that a particular type of agent searches rather than waits. It turns out that mixed strategy equilibria do not exist when the numbers of buyers and sellers differ a lot. In this case we postulate that the reasonable equilibrium is the one where the mixed strategy equilibrium converges to when the numbers of buyers and sellers change appropriately. Kultti and Takalo (1999) also investigate this kind of model but their focus is on the evolutionary stability of markets with different trading rules and search patterns.

The main result of the present article is that the members of the larger population search and those of the smaller population wait. We check that this equilibrium is immune against coalitional deviations. If the populations are equal enough, there are two markets: buyers search in one market and sellers in the other. In this case we show that auction is more efficient trading rule than bargaining, because the agents are split into the markets in more advantageous proportions. The rest of the article is organised as follows: Section 2 presents a matching model with decision to search or wait, and in Sections 3 and 4 we incorporate the trading rules into the model. In Section 5 we derive the main result of the paper: who search and who wait. The relative efficiency of auction and bargaining is studied in Section 6. In Section 7 we extend the model to a one where agents do not decide whether to search or wait but they choose which of the many marketplaces to go, and we compare the efficiency of that market structure to that of the basic model. Section 8 concludes. The derivations of the results are in the Appendix.

## 2 The Model

There are  $B$  buyers each with a unit demand and  $S$  sellers each with one indivisible object for sale. The buyers get utility normalised to one from consuming the object, and the sellers get utility normalised to zero from consuming it. The economy extends to infinity, and time proceeds in discrete periods. The agents discount future with factor  $\delta \in (0, 1)$ . When the agents trade they exit the economy and are replaced by identical but unmatched agents. This means that the ratio of buyers to sellers remains the same in every period.

To model the meeting process we use an urn-ball model (see for example Lu and McAfee, 1996). The agents who decide to wait are in fixed positions, one agent in each location, and the agents who decide to search are randomly and independently allocated on the waiting agents. This meeting technology is well defined and tractable. Further, since multiple meetings are possible one can meaningfully study a variety of trading mechanisms. If  $w$  agents wait and  $m$  agents move, the number of agents a waiting agent meets is a binomial random variable with parameters  $m$  and  $1/w$ . Tractability is achieved by assuming that  $w$  and  $m$  are infinite, since in this case one can approximate the binomial with a Poisson distribution with parameter  $m/w$ . Then the probability that a waiting agent meets exactly  $k$  moving agents is  $e^{-m/w} (m/w)^k / k!$ .

We are going to investigate two different trading mechanisms: auction and bargaining. Auction is modelled as a second price sealed bid procedure in which everyone in a meeting participates. For concreteness, consider a seller who waits and meets exactly one buyer. Then the seller bids his reservation value, which is the same as ignoring the buyer and waiting for new trading opportunities in the next period; and the buyer bids one minus the seller's reservation value. The buyer wins the object and pays the seller's reservation value. If the seller meets two or more buyers, both buyers bid the same and one of them gets the object. It does not matter which buyer gets the object since the buyers are indifferent between getting the object and searching for new trading opportunities. Another way to look at the auction is that in the first case the buyer makes a take-it-or-leave-it offer, and in the second case the buyers engage in a Bertrand-type bidding.

Bargaining is always pairwise, and if a seller meets several buyers he just picks one of them at random for his trading partner. To make things simple we assume that the buyer and seller just split the available surplus in half. One would probably also want to consider the case of posted prices. Kultti (1999) shows that this is equivalent to auction.

Since both buyers and sellers can decide to wait or search, we consider two markets. In one market proportion  $x$  of buyers search and proportion  $y$  of sellers wait. In the other market proportion  $1 - x$  of buyers wait and proportion  $1 - y$  of sellers search. In equilibrium the buyers have to do equally well regardless of whether they wait or search. The same must hold for the sellers. Of course, it is not necessarily the case that this condition is consistent with there being two markets, and in these cases there will be only one market in equilibrium.

Let us denote the ratio of total number of buyers to that of sellers by  $B/S = \theta$ . In the market where the sellers wait the ratio of searchers to waiters is  $xB/yS = x\theta/y \equiv \gamma$ , and in the market where the buyers wait the relevant ratio is  $(1 - y)S/(1 - x)B = (1 - y)/(1 - x)\theta \equiv \varphi$ . In other words, the number of agents that arrive in a seller's or buyer's location is governed by a Poisson process with parameter  $\gamma$  or  $\varphi$ , respectively. Let us next determine the agents' expected utilities in each market with the two trading mechanisms.

### 3 Auction

Consider the market where sellers wait. Let us determine a seller's expected life time utility  $V_s$  and a buyer's expected life time utility  $V_b$ , evaluated in the very end of a period, as

$$V_s = \delta [e^{-\gamma}V_s + \gamma e^{-\gamma}V_s + (1 - e^{-\gamma} - \gamma e^{-\gamma})(1 - V_b)], \quad (1)$$

$$V_b = \delta [e^{-\gamma}(1 - V_s) + (1 - e^{-\gamma})V_b]. \quad (2)$$

In (1) the first term in the square brackets is the probability of the seller meeting no-one in which case he gets his expected life time utility from waiting,  $V_s$ . The second term is the probability of meeting exactly one buyer. In this case the seller also gets  $V_s$  since the buyer makes a take-it-or-leave-it offer to the seller. The take-it-or-leave-it offer

assumption gives the movers a positive probability to get the whole surplus of the trade (like the stayers have, too), treating the stayers and movers as equally as possible. The third term is the probability of meeting two or more buyers in which case the buyers get their reservation utility and the seller gets the rest of the surplus,  $1 - V_b$ . It should be noted that a buyer always meets a seller. Thus, the probability that no other buyers meet the same seller is  $e^{-\gamma}$ , and in this case the buyer gets all the surplus from the meeting,  $1 - V_s$ . This is the first term in the square brackets in (2). If other buyers appear, each of them gets his expected utility from searching,  $V_b$ .

Solving (1) and (2) yields

$$V_s = \frac{\delta(1 - e^{-\gamma} - \gamma e^{-\gamma})}{1 - \delta\gamma e^{-\gamma}}, \quad (3)$$

$$V_b = \frac{\delta e^{-\gamma}}{1 - \delta\gamma e^{-\gamma}}. \quad (4)$$

In an analogous fashion one can calculate the buyers' and sellers' expected life time utilities in the market where the buyers wait. In this case, if only one seller arrives, he makes a take-it-or-leave-it offer to the buyer. The life-time values are

$$W_s = \frac{\delta e^{-\varphi}}{1 - \delta\varphi e^{-\varphi}}, \quad (5)$$

$$W_b = \frac{\delta(1 - e^{-\varphi} - \varphi e^{-\varphi})}{1 - \delta\varphi e^{-\varphi}}. \quad (6)$$

## 4 Bargaining

There is basically only one point that may not be immediate in deriving the expected utilities. When there are two or more buyers in a meeting with a seller, one of the buyers is selected at random, and the probability that a buyer gets to trade with a seller is

$$e^{-\gamma} \sum_{k=0}^{\infty} \frac{\gamma^k}{k!} \frac{1}{k+1} = \frac{1 - e^{-\gamma}}{\gamma}. \quad (7)$$

Consider first the market where sellers wait and use the same notation for the expected utilities as in the auction setting. The life time utilities are determined by

$$V_s = \delta \left[ e^{-\gamma} V_s + (1 - e^{-\gamma}) \left( V_s + \frac{1}{2}(1 - V_b - V_s) \right) \right], \quad (8)$$

$$V_b = \delta \left[ \frac{1 - e^{-\gamma}}{\gamma} \left( V_b + \frac{1}{2}(1 - V_b - V_s) \right) + \frac{\gamma - 1 + e^{-\gamma}}{\gamma} V_b \right], \quad (9)$$

and solving these gives

$$V_s = \frac{\delta \gamma (1 - e^{-\gamma})}{(2 - \delta - \delta e^{-\gamma}) \gamma + \delta (1 - e^{-\gamma})}, \quad (10)$$

$$V_b = \frac{\delta (1 - e^{-\gamma})}{(2 - \delta - \delta e^{-\gamma}) \gamma + \delta (1 - e^{-\gamma})}. \quad (11)$$

The corresponding utilities in the market where sellers search are

$$W_s = \frac{\delta (1 - e^{-\varphi})}{(2 - \delta - \delta e^{-\varphi}) \varphi + \delta (1 - e^{-\varphi})}, \quad (12)$$

$$W_b = \frac{\delta \varphi (1 - e^{-\varphi})}{(2 - \delta - \delta e^{-\varphi}) \varphi + \delta (1 - e^{-\varphi})}. \quad (13)$$

## 5 The Equilibrium Market Structure

So far we have determined the expected utilities of the agents under auction and bargaining, but we have not said anything about the equilibrium of the economy. Since there is an infinite number of agents, Nash-equilibrium does not provide sufficient restrictions as any deviating agent is of measure zero, and his behaviour does not affect anything but his own utility. We require that the equilibrium should be immune against a deviation by a coalition of agents. In more detail, it should be impossible for a coalition of buyers and sellers to put up another market where both of them do better than in equilibrium. It is immediate that this is impossible if there are two markets in equilibrium. The criterion, however, selects one of the two symmetric pure-strategy equilibria: under auction it selects the very equilibrium to which the mixed strategy equilibrium converges when the ratio of buyers to sellers increases or decreases without bound. Under bargaining the criterion gives a sharp and not obvious result as almost surely there are no mixed-strategy equilibria, and it selects from the two pure strategy equilibria.

As an example to motivate coalitional deviations, consider the following situation: sellers advertise that they have an object for sale, buyers observe the advertisements and choose which seller to contact. Then it should not be possible that someone, say, puts

up a magazine and manages to induce some buyers to put ads there and some sellers to buy the magazine (and then contact the buyers).

**Definition 1** *An equilibrium is a pair  $(x, y) \in [0, 1]^2$ , where  $x$  is the proportion of buyers who search and  $y$  is the proportion of sellers who wait, such that no coalition of buyers and sellers can put up a new market such that all deviators fare better than in the equilibrium.*

To determine who search and who wait is quite straightforward even though the detailed analysis involves a little computation in some cases. The aim is to determine an equilibrium in which both markets are active, i.e. an equilibrium where some buyers as well as sellers both wait and search. This means that waiting buyers have to be equally well-off as searching buyers. The same condition must hold for sellers, too. This requirement produces two conditions

$$V_s = W_s, \tag{14}$$

$$V_b = W_b, \tag{15}$$

for the two unknowns  $x$  and  $y$ . Of course, we have to solve these for both trading mechanisms. It is not always the case that two markets exist simultaneously. Then there are two equilibria: one in which all buyers search and all sellers wait, and another one where all buyers wait and all sellers search. We return to this point later, and then we evaluate which equilibrium is more likely. One should notice that these two equilibria always exist but since they look like equilibria in a pure co-ordination game, we focus on the other equilibria when they exist.

**Lemma 1** *Under auction two markets exist only if  $1/\theta_3 < \theta < \theta_3$ , where  $\theta_3 \approx 1.146$ . In equilibrium  $x = (\theta_3(\theta\theta_3 - 1)) / (\theta(\theta_3^2 - 1))$  and  $y = (\theta\theta_3 - 1) / (\theta_3^2 - 1)$ .*

**Proof.** In the Appendix. ■

This is already derived in Kultti and Takalo (1999), and it is noticeable that when two markets exist, the ratio of searchers to waiters in both markets is constant, namely  $\theta_3$ .

**Lemma 2** *Under bargaining two markets exist only if  $\theta = 1$ . All configurations  $x = y \in (0, 1)$  are equilibria.*

**Proof.** In the Appendix. ■

This is also derived in Kultti and Takalo (1999). Outside the above regions for  $\theta$ , only buyers or sellers search. Propositions 1 and 2 state the main result of this article:

**Proposition 1** *If trades are consummated by auction, i) all the buyers search and all the sellers wait if  $\theta > \theta_3$ , ii) all the buyers wait and all the sellers search if  $\theta < 1/\theta_3$ .*

**Proof.** In the Appendix. ■

**Proposition 2** *When trades are consummated by bargaining, i) all the buyers search and all the sellers wait if  $\theta > 1$ , ii) all the buyers wait and all the sellers search if  $\theta < 1$ .*

**Proof.** In the Appendix. ■

The utilities from waiting and moving depend on the Poisson parameter that governs the movers' arrival to waiters' locations. With both trading rules, if all buyers search and all sellers wait, a profitable deviation by buyers requires that in the new market the Poisson parameter is large enough, whereas a deviation by sellers requires that it is small enough. These ranges for the value of the parameter do not overlap if  $\theta$  is large enough. On the other hand, if all buyers wait and all sellers search and  $\theta$  is small enough, we cannot find a value for the parameter that induces both types of agents to deviate. There are, however, values of  $\theta$  that enable profitable deviation, indicating that the original market market structure is not an equilibrium.

## 6 The Relative Efficiency of Trading Rules

Here we conduct a rather straightforward comparison of efficiency under bargaining and auction. The efficiency measure is the number of trades per period (which in steady state is the same each period). This is interesting only when two markets exist since if there is only one market under either trading rule, the same type of agents wait, and the number

of trades is the same. Let us denote the number of trades per period under auction or bargaining by  $M_{au}$  and  $M_{ba}$ , and let  $E = M_{au}/M_{ba}$ .

There are several cases to be studied, and we list them below. Note that under bargaining the only case when there are two markets in equilibrium is when the number of buyers equals the number of sellers, i.e.  $\theta = 1$ . But then any configuration  $x = y \in [0, 1]$  is an equilibrium, and in all such equilibria the utility of a given type of agent is the same. The agents' utilities are the same. Thus, the cases are distinguished by the parameter intervals relevant in auction.

Case a)  $\theta < 1/\theta_3$ . All the buyers wait (whether auction or bargaining is used), and the number of matches is  $M_{au} = M_{ba} = (1 - e^{-1/\theta}) B$ .

Case b)  $\theta > \theta_3$ . All the sellers wait (whether auction or bargaining is used), and the number of matches mirrors that of case a:  $M_{au} = M_{ba} = (1 - e^{-\theta}) S$ .

Case c)  $1/\theta_3 < \theta < 1$ . There are two markets if auction is used. In one market there are  $yS$  sellers who wait and  $xB$  buyers who move, in the other market  $(1 - x)B$  buyers wait for the  $(1 - y)S$  sellers to visit. The Poisson parameter in both markets is equal to  $\theta_3$ . The number of matches is the sum of the matches in two markets:  $M_{au} = yS(1 - e^{-\theta_3}) + (1 - x)B(1 - e^{-\theta_3})$ , and using Lemma 1 we have

$$M_{au} = \frac{1 - e^{-\theta_3}}{\theta_3^2 - 1} \left[ (\theta\theta_3 - 1)S + \frac{\theta_3 - \theta}{\theta}B \right]. \quad (16)$$

If bargaining is used, all buyers wait and all sellers search, thus  $M_{ba} = (1 - e^{-1/\theta}) B$ .

The relative efficiency of auction to bargaining is then

$$E = \frac{(1 - e^{-\theta_3})(1 + \theta)}{(1 + \theta_3)(1 - e^{-1/\theta})\theta} > 1, \quad (17)$$

and it can be shown that  $\partial E/\partial\theta > 0$ : The efficiency of auction relative to bargaining increases as the ratio  $B/S$  increases.

Case d)  $1 < \theta < \theta_3$ . In auction, two markets exist, and the sum of matches is  $yS(1 - e^{-\theta_3}) + (1 - x)B(1 - e^{-\theta_3})$ , the same as in case c. In case of bargaining, all sellers wait, and  $M_{ba} = (1 - e^{-\theta}) S$ , and we have

$$E = \frac{(1 - e^{-\theta_3})(1 + \theta)}{(1 + \theta_3)(1 - e^{-\theta})} > 1, \quad (18)$$

and  $\partial E/\partial\theta > 0$ . The smaller the value of  $\theta$ , the more efficient auction is relative to bargaining.

We can summarise the results in

**Proposition 3** *Auction is more efficient than bargaining if  $1/\theta_3 < \theta < \theta_3$ . Outside this region, auction and bargaining are equally efficient.*

This result is based on the comparison of equilibria that satisfy Definition 1. The matching functions above have constant returns to scale, yet two ‘small’ markets can perform better than one large market. The explanation for this is that in one small market buyers search and in the other they wait, and the ratio of movers to stayers in both markets is more advantageous than in the one large market.

## 7 Extension: Common Locations

In the model presented above, either the buyers go to the sellers’ locations or the sellers visit the buyers. This is, however, only one of many conceivable meeting technologies. For example, one could think that in the beginning of each period, each buyer is in his location and each seller is in his location. Then some buyers leave their locations and go to sellers’ locations to visit them. The rest of the buyers stay put in their locations and wait for visiting sellers. Sellers, too, have a choice to stay or to go and visit the buyers. This model is analysed in Kultti, Miettunen and Virrankoski (2003), where the equilibrium choices of staying and visiting are derived. In the sequel we consider an environment where neither buyers nor sellers have their own locations. Instead, there are several marketplaces, or common locations. Each agent goes randomly to one location in every period until she trades and leaves the economy. Trading within a location is frictionless: the number of matches in location  $i$  is equal to  $\min\{B_i, S_i\}$  where  $B_i$  and  $S_i$  are the numbers of buyers and sellers who happen to choose location  $i$ . Frictions arise because the agents are randomly distributed on the locations. We compare the number of matches and agents’ life-time values in this model to those in the basic model analysed above.

The tradable object is indivisible, and there are  $B$  buyers and  $S$  sellers with the same preferences as in the basic model. In order to trade, they have to go to marketplaces. The number of such locations is  $L$ , and we assume that  $L = B + S$ . In the beginning of each period, each buyer and seller chooses randomly which location to go. Every location then has more buyers than sellers, or more sellers than buyers, or equally many of them. Some locations may remain empty. The trading mechanism is a mixture of auction and bargaining in the following way: If the number of buyers and sellers in a location is unequal, there is an auction. The more numerous type gets his reservation value, and the less numerous type gets one minus the other's reservation value. If there is an equal number of buyers and sellers, they bargain such that everyone gets his reservation value plus one half of the surplus. In the equal-numbered case we assume bargaining because we want to treat buyers and sellers symmetrically. Agents who have traded exit the market, and they are replaced by identical but unmatched agents. Let  $a$  ( $b$ ) be the probability that there are more (less) buyers than sellers, respectively, in a location chosen by a buyer. Let  $c$  ( $d$ ) be the probability that there are more (less) sellers than buyers, respectively, in a location chosen by a seller. The value functions are, for a buyer and for a seller,

$$V_b^{co} = \delta \left[ aV_b + b(1 - V_s) + (1 - a - b) \left( V_b + \frac{1}{2}(1 - V_b - V_s) \right) \right], \quad (19)$$

$$V_s^{co} = \delta \left[ cV_s + d(1 - V_b) + (1 - c - d) \left( V_s + \frac{1}{2}(1 - V_b - V_s) \right) \right]. \quad (20)$$

Solving for  $V_b$  and  $V_s$  gives

$$V_b^{co} = \frac{\delta(1 - a + b)}{2 - \delta(a - b + c - d)}, \quad (21)$$

$$V_s^{co} = \frac{\delta(1 - c + d)}{2 - \delta(a - b + c - d)}. \quad (22)$$

The probabilities  $a$ ,  $b$ ,  $c$  and  $d$  cannot be solved analytically. Instead, they are solved numerically by calculating the cumulative Poisson distribution functions for the number of buyers and sellers in a location. We have

- i)  $a = G_0F_0 + (G_1 - G_0)F_1 + (G_2 - G_1)F_2 + \dots$ ,
- ii)  $b = G_0(1 - F_1) + (G_1 - G_0)(1 - F_2) + (G_2 - G_1)(1 - F_3) + \dots$ ,
- iii)  $c = F_0G_0 + (F_1 - F_0)G_1 + (F_2 - F_1)G_2 + \dots$ , and

$$\text{iv) } d = F_0(1 - G_1) + (F_1 - F_0)(1 - G_2) + (F_2 - F_1)(1 - G_3) + \dots,$$

where  $F_n$  is the probability that at most  $n$  other sellers have come to the location, and  $G_n$  is the probability that at most  $n$  buyers have arrived. The probabilities  $a$ ,  $b$ ,  $c$  and  $d$  are solved for cases  $\theta = 1/3, 1/2, 1, 2$ , and  $3$ . Note that the absolute values of  $B$  and  $S$  do not matter, but  $\theta (= B/S)$  does. We let  $\delta = 0.9$ .

We first compare the efficiency of the common-locations model to that of the basic model (with auction and bargaining). In the common-locations model, the number of matches is

$$M_{co} = bB + dS + (1 - a - b)B, \quad (23)$$

where  $bB$  is the number of matches that form in locations where there are less buyers than sellers,  $dS$  is the number of matches in locations where there are less sellers than buyers, and  $(1 - a - b)B$  matches form in locations where there are equally many buyers and sellers. We can write  $M_{co} = dS + (1 - a)B$ , and as a measure of efficiency we use the number of matches per seller,  $M_{co}/S = d + (1 - a)\theta$ . The efficiency measures in the basic model are  $M_{au}/S$  and  $M_{ba}/S$ , where we take into account that there are two markets for certain values of  $\theta$ . Table 1 summarises the efficiency in each model:

Table 1: Efficiency in the common-locations model and in the basic model

	$M_{co}/S$	$M_{au}/S$	$M_{ba}/S$
$\theta = 1/3$	0.162	0.317	0.317
$\theta = 1/2$	0.217	0.432	0.432
$\theta = 1$	0.326	0.636	0.632
$\theta = 2$	0.435	0.865	0.865
$\theta = 3$	0.485	0.950	0.950

The meeting technology of the basic model is roughly two times as efficient as that of the common locations model.

Next we compare the life-time utilities of buyers and sellers in the common-locations model to those in the basic model. In the basic model we consider both auction and bargaining. The identity of movers and stayers is determined as stated in propositions 1 and 2. For example, if trades are consummated by bargaining, all the buyers search and all the sellers wait if  $\theta = 2$  or if  $\theta = 3$ , and so on. Let us use the following notation:

$V_b^{au}$  and  $V_s^{au}$  are the life-time utilities for buyers and sellers in the basic model when the trading mechanism is auction;  $V_b^{ba}$  and  $V_s^{ba}$  are the respective values when bargaining is used, and  $V_b^{co}$  and  $V_s^{co}$  are the utilities in the common-locations model. Table 2 presents the utilities:

Table 2: Welfare of buyers and sellers in the common-locations model and in the basic model

	$V_b^{co}$	$V_b^{au}$	$V_b^{ba}$	$V_s^{co}$	$V_s^{au}$	$V_s^{ba}$
$\theta = 1/3$	0.628	0.833	0.639	0.135	0.052	0.213
$\theta = 1/2$	0.548	0.708	0.570	0.206	0.161	0.285
$\theta = 1$	0.372	0.426	0.425	0.372	0.426	0.425
$\theta = 2$	0.206	0.161	0.285	0.548	0.708	0.570
$\theta = 3$	0.135	0.052	0.213	0.628	0.833	0.639

Table 3 shows the ordering of utilities:

Table 3: Ordering of utilities

	Buyers	Sellers
$\theta = 1/3$ and $\theta = 1/2$	$V_b^{au} > V_b^{ba} > V_b^{co}$	$V_s^{ba} > V_s^{co} > V_s^{au}$
$\theta = 1$	$V_b^{au} > V_b^{ba} > V_b^{co}$	$V_s^{au} > V_s^{ba} > V_s^{co}$
$\theta = 2$ and $\theta = 3$	$V_b^{ba} > V_b^{co} > V_b^{au}$	$V_s^{au} > V_s^{ba} > V_s^{co}$

We see that the configuration with common locations is dominated by the basic configuration with either auction or bargaining. This is due to large number of locations, as it is clear that letting the number of locations go to one yields the maximum number of meetings. We also see that when the number of buyers and sellers in the economy is equal, the buyers and sellers in the basic configuration prefer auction to bargaining. This result holds with all  $\delta \leq 1$ .

## 8 Conclusion

We take a standard urn-ball model with buyers and sellers and study a symmetric mixed strategy equilibrium when the agents' choice set consist of the decision to wait

or search. Focusing on mixed symmetric strategies looks reasonable as in large markets pure-strategy equilibria seem to require plenty of co-ordination. On top of that, symmetric strategies indicate how likely it is for a certain type of agent to search. This way we get a quite clear-cut picture of the search and wait decisions. The main conclusion is that the more numerous party is more likely to search, regardless of how terms of trade are determined. If the difference between the number of buyers and sellers is large, there exists only a pure-strategy equilibrium where the more numerous agents search and the less numerous agents wait.

Our results should be of interest for modelling purposes as it is customary to assume that one type, eg. employers in labour market, or sellers in decentralised goods market, or women in marriage market, wait and the other type searches. These assumptions seem well motivated if the waiting type is not significantly more numerous, but in the opposite case the practice is in doubt. The model also has an obvious empirically testable implication. The results are very clear due to the small number of equilibria, as we do not assume that there are any costs of searching or waiting, or any kind of heterogeneity. One may want to relax these assumptions, but it is likely to lead to a huge number of equilibria while in our simple setting there are practically at most three configurations that may be regarded as equilibria under any reasonable criterion.

## 9 Appendix

### Proof of Lemma 1

Equilibrium conditions  $W_b = V_b$  and  $W_s = V_s$  yield by (3)-(6) that

$$e^\gamma - 1 - \gamma = \frac{1}{e^\varphi - 1 - \varphi} \tag{A1}$$

which is equivalent to

$$e^\gamma = \frac{e^\varphi - \varphi}{e^\varphi - 1 - \varphi} + \gamma. \tag{A2}$$

The buyers' equilibrium condition  $W_b = V_b$  yields

$$\frac{e^\gamma - 1 - \gamma}{e^\gamma - \delta\gamma} = \frac{1}{e^\varphi - \delta\varphi} \tag{A3}$$

which is equivalent to

$$e^\gamma = \frac{e^\varphi - \delta\varphi}{e^\varphi - 1 - \varphi} + \delta\gamma. \quad (\text{A4})$$

Next we show that there is a unique solution  $\varphi = \gamma = \theta_3$  for (A1) and (A3) to hold where  $e^{\theta_3} - 2 - \theta_3 = 0$ . Equations (A2) and (A4) yield

$$\gamma = \frac{\varphi}{e^\varphi - 1 - \varphi}. \quad (\text{A5})$$

Equations (A2) and (A5) yield  $e^\gamma/\gamma = e^\varphi/\varphi$  which is equivalent to  $e^{f(\varphi)}/f(\varphi) = e^\varphi/\varphi$  where  $f(\varphi) = \varphi/(e^\varphi - 1 - \varphi)$ . Still, a more convenient form is

$$f(\varphi) - \varphi = \ln \frac{f(\varphi)}{\varphi}. \quad (\text{A6})$$

The left-hand side and the right-hand side of (A6) are plus infinity at  $\varphi = 0$ . The derivative of the left-hand side is

$$f'(\varphi) - 1 = \frac{e^\varphi - 1 - \varphi e^\varphi}{(e^\varphi - 1 - \varphi)^2} - 1 \quad (\text{A7})$$

which is negative for  $\varphi > 0$ . The derivative of the right-hand side is  $(1 - e^\varphi)/(e^\varphi - 1 - \varphi)$  which is also negative for  $\varphi > 0$ .

Let us compare the derivatives; we show that the derivative of the left-hand side is smaller than that of the right-hand side for small values of the argument. Formally,

$$\frac{e^\varphi - 1 - \varphi e^\varphi}{(e^\varphi - 1 - \varphi)^2} - 1 < \frac{1 - e^\varphi}{e^\varphi - 1 - \varphi} \quad (\text{A8})$$

which is equivalent to  $e^\varphi - 1 - \varphi - \varphi^2 < 0$ , which holds as long as  $\varphi$  is less than 1.8 (approximately). Thus, there is at most one zero. One immediately sees that  $\theta_3 > 1$  is a root of equation (A6) where  $\theta_3$  satisfies  $2 + \theta_3 - e^{\theta_3} = 0$ , yielding  $\theta_3 = \varphi = \gamma \approx 1.146$ . Two auction markets exist only if

$$\gamma = \frac{x\theta}{y} = \varphi = \frac{1 - y}{(1 - x)\theta} = \theta_3. \quad (\text{A9})$$

Solving this yields  $y = (x/\theta_3)\theta$ , which inserted into the latter equation gives

$$x = \frac{\theta_3(\theta\theta_3 - 1)}{\theta(\theta_3^2 - 1)}, \quad (\text{A10})$$

from which we see that equilibrium with two auction markets exists if and only if  $\theta_3 > \theta > 1/\theta_3$ .  $\square$

### Proof of Lemma 2

The equilibrium conditions  $W_b = V_b$  and  $W_s = V_s$  yield by (10)-(13) that  $\gamma = 1/\varphi$ . Inserting this back to one of the equilibrium conditions gives

$$1 - e^{-\gamma} - \gamma(1 - e^{-1/\gamma}) = 0. \quad (\text{A11})$$

Next we show that (A11) has exactly one positive solution,  $\gamma = 1$ . Let us study function  $g(h) = e^h - 1 - he^h + he^{h-1/h}$  that has the same zeroes as the left-hand side in (A11). We immediately see that  $g(0) = 0$ . The derivative of  $g$  is

$$g'(h) = -he^h + e^{h-1/h} \left( h + 1 + \frac{1}{h} \right), \quad (\text{A12})$$

and since  $(e^{-1/h}/h) = 0$  at  $h = 0$  we see that  $g'(0) = 0$ . The second derivative of  $g$  is

$$g''(h) = -e^h - he^h + he^{h-1/h} \left( 2 + h + \frac{2}{h} + \frac{1}{h^3} \right), \quad (\text{A13})$$

and  $g''(0) = -1$ . Thus, at first  $g$  is decreasing. It is also immediate that  $g(1) = 0$  and  $g'(1) > 0$ . Next we show that  $g$  is not zero between  $h = 0$  and  $h = 1$ . If there were zeroes between  $h = 0$  and  $h = 1$  and if  $g$  attained strictly positive values, there should be at least two zeroes. Before the last zero  $g$  would reach a maximum and its derivative would be zero. Let us denote the last maximum of  $g$  (where it is positive) by  $k$ . Thus we know that

$$g'(k) = -ke^k + e^{k-1/k} \left( k + 1 + \frac{1}{k} \right) = 0, \quad (\text{A14})$$

and  $g(k) = e^k - 1 - ke^k + ke^{k-1/k} > 0$ . From these conditions we get

$$g(k) = e^k - 1 - ke^k + \frac{k^3 e^k}{1 + k + k^2} > 0 \quad (\text{A15})$$

which holds if and only if  $e^k - 1 - k - k^2 > 0$ . We have  $e^h - 1 - h - h^2 = 0$  at  $h = 0$  and  $e^h - 1 - h - h^2 < 0$  at  $h = 1$ . It is easy to see that  $e^h - 1 - h - h^2 < 0$  in interval  $h \in (0, 1]$ . Thus, the assumption that  $g(h) > 0$  at  $h \in (0, 1]$  leads to a contradiction, thus  $g(h) < 0$  at  $h \in (0, 1]$ , and (A11) has no solution at  $\gamma \in (0, 1)$ .

To show that equation  $1 - e^{-\gamma} - \gamma + \gamma e^{-1/\gamma} = 0$  has no solution at  $\gamma > 1$  it is enough to show that  $g'(h) = -he^h + e^{h-1/h}(h+1+1/h) > 0$  when  $h > 1$ , because  $g(1) = 0$  and  $g'(h) > 0$  at  $h = 1$ . The sign of  $g'(h)$  is positive if and only if  $v(h) \equiv 1 - e^{-1/h} - e^{-1/h}/h - e^{-1/h}/h^2 < 0$ . We see that  $v(1) < 0$  and  $\lim_{h \rightarrow \infty} v(h) = 0$ . Further,  $v'(h) = e^{-1/h}(1/h^3)(1 - 1/h)$  which is positive if  $h > 1$ . That is,  $v(h) < 0$  if  $h > 1$ , thus  $g'(h) = -he^h + e^{h-1/h}(h+1+1/h) > 0$  when  $h > 1$ , consequently (A11) has no solution at  $\gamma > 1$ .

We have shown that equation  $1 - e^{-\gamma} - \gamma + \gamma e^{-1/\gamma} = 0$  has exactly one strictly positive solution,  $\gamma = 1$ . If  $\gamma \in (0, 1)$ , then  $1 - e^{-\gamma} - \gamma + \gamma e^{-1/\gamma} < 0$ , and if  $\gamma > 1$ , then  $1 - e^{-\gamma} - \gamma + \gamma e^{-1/\gamma} > 0$ . Analogously, it is easy to see that there does exist a zero  $z$  such that  $g(z) = 0$  is a local maximum. In equilibrium  $\gamma = 1/\varphi$  which holds if and only if  $x = y$ . Combined with  $\gamma = 1$ , we conclude that two bargaining markets exist if and only if  $\theta = 1$ , for any  $x$  and  $y$  such that  $x = y$ .  $\square$

### Proof of Proposition 1

The proof of part (i) of the proposition is shown here. The proof of part (ii) is analogous and is left to the reader.

(i) In case  $\theta > \theta_3$  we show that if all the buyers wait and all the sellers search, there is a coalition of buyers and sellers who are better off in a market where buyers search and sellers wait, implying that the former market is not an equilibrium. On the other hand, if all the buyers search and all the sellers wait, a profitable deviating coalition cannot exist.

*All buyers wait and all sellers search*

Let  $\psi = 1/\theta$  be the parameter of the Poisson process in the original market where buyers wait and sellers search, and let  $\alpha = \eta B/\mu S$  be the respective parameter in the new market where sellers wait and buyers search. Because  $\theta > \theta_3$ , then  $\psi < 1/\theta_3$ . A seller's utility in the original market is  $V_s^o$ , and his utility in the new, deviating market is  $V_s^n$ . The corresponding utilities for a buyer are  $V_b^o$  and  $V_b^n$ . A profitable deviation for a seller requires  $V_s^n > V_s^o$ :

$$\frac{1 - e^{-\alpha} - \alpha e^{-\alpha}}{1 - \delta \alpha e^{-\alpha}} > \frac{e^{-\psi}}{1 - \delta \psi e^{-\psi}}, \quad (\text{A16})$$

and condition  $V_b^n > V_b^o$  for deviating buyers is

$$\frac{e^{-\alpha}}{1 - \delta\alpha e^{-\alpha}} > \frac{1 - e^{-\psi} - \psi e^{-\psi}}{1 - \delta\psi e^{-\psi}}. \quad (\text{A17})$$

Let  $\alpha_1$  satisfy  $V_s^n = V_s^o$ :

$$\frac{1 - e^{-\alpha_1} - \alpha_1 e^{-\alpha_1}}{1 - \delta\alpha_1 e^{-\alpha_1}} = \frac{e^{-\psi}}{1 - \delta\psi e^{-\psi}}, \quad (\text{A18})$$

and let  $\alpha_2$  satisfy  $V_b^n = V_b^o$ :

$$\frac{e^{-\alpha_2}}{1 - \delta\alpha_2 e^{-\alpha_2}} = \frac{1 - e^{-\psi} - \psi e^{-\psi}}{1 - \delta\psi e^{-\psi}}. \quad (\text{A19})$$

Next we show that a deviating coalition exists if  $\alpha_1 < \alpha_2$ , in case we can choose  $\alpha \in (\alpha_1, \alpha_2)$  that satisfies (A16) and (A17). After that we show that  $\alpha_1 < \alpha_2$  for all  $\psi \in (0, 1/\theta_3]$ . Expression  $(1 - e^{-h} - he^{-h}) / (1 - \delta he^{-h})$  is increasing in  $h$ :

$$\frac{\partial \left( \frac{1 - e^{-h} - he^{-h}}{1 - \delta he^{-h}} \right)}{\partial h} = \frac{e^{-h} [(1 - \delta)h + \delta(1 - e^{-h})]}{(1 - \delta he^{-h})^2}, \quad (\text{A20})$$

and  $e^{-h} / (1 - \delta he^{-h})$  is decreasing in  $h$ :

$$\frac{\partial \left( \frac{e^{-h}}{1 - \delta he^{-h}} \right)}{\partial h} = \frac{-e^{-h} (1 - \delta e^{-h})}{(1 - \delta he^{-h})^2}. \quad (\text{A21})$$

In Figure 1,  $h$  is on the horizontal axis, and  $\psi$ ,  $\theta_3$ ,  $\alpha_1$  and  $\alpha_2$  are the values of our interest. We see that if curve  $(1 - e^{-h} - he^{-h}) / (1 - \delta he^{-h})$  is steeper than curve  $e^{-h} / (1 - \delta he^{-h})$ , then  $\alpha_1 < \alpha_2$ , and we can choose  $\alpha \in (\alpha_1, \alpha_2)$  that satisfies (A16) and (A17). If  $\alpha_1 > \alpha_2$ , a deviating coalition does not exist.

The sum of the right-hand sides of (A20) and (A21) is equal to

$$\frac{e^{-h} (1 - \delta) (h - 1)}{(1 - \delta he^{-h})^2}, \quad (\text{A22})$$

and we note that curve  $(1 - e^{-h} - he^{-h}) / (1 - \delta he^{-h})$  is steeper than curve

$e^{-h} / (1 - \delta he^{-h})$  if  $h > 1$ . That is, if  $\psi$  is larger than one, then  $\alpha_1 < \alpha_2$ . However, the value of  $\psi$  is assumed to be less than  $1/\theta_3$  which is smaller than one. Do we have  $\alpha_1 < \alpha_2$  for all  $\psi < 1$ ? In order to have  $\alpha_1 > \alpha_2$ , we should have  $\alpha_1 = \alpha_2$  for some value of  $\psi$ . Next we show that this is not possible.

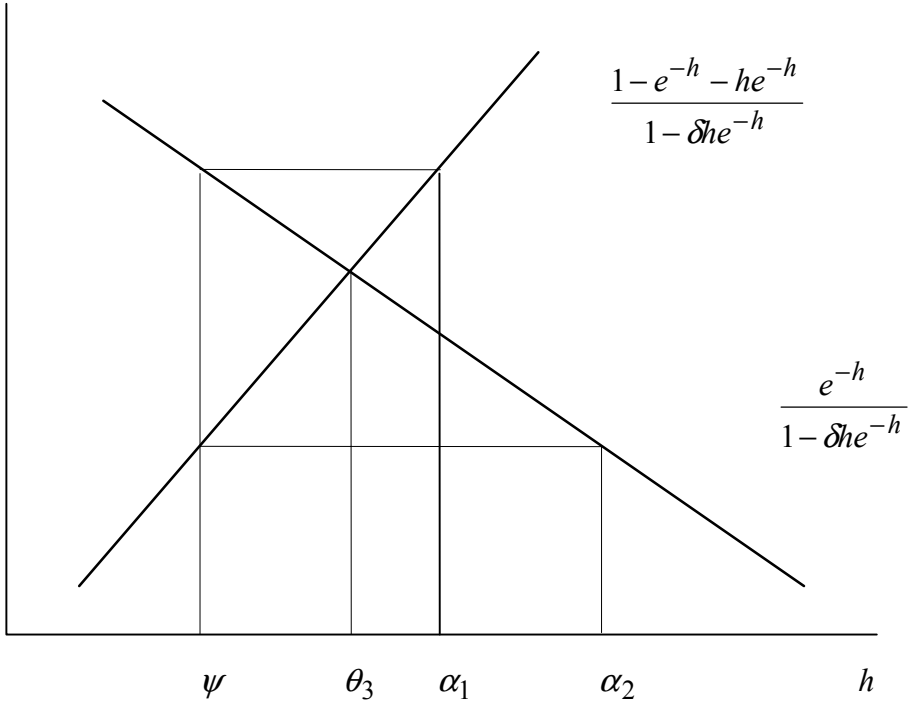


Figure 1: A deviating coalition exists.

Let  $\alpha$  satisfy

$$\frac{1 - e^{-\alpha} - \alpha e^{-\alpha}}{1 - \delta \alpha e^{-\alpha}} = \frac{e^{-\psi}}{1 - \delta \psi e^{-\psi}} \quad (\text{A23})$$

and

$$\frac{e^{-\alpha}}{1 - \delta \alpha e^{-\alpha}} = \frac{1 - e^{-\psi} - \psi e^{-\psi}}{1 - \delta \psi e^{-\psi}}. \quad (\text{A24})$$

By summing (A23) and (A24) we get

$$\frac{1 - \alpha e^{-\alpha}}{1 - \psi e^{-\psi}} = \frac{1 - \delta \alpha e^{-\alpha}}{1 - \delta \psi e^{-\psi}} \quad (\text{A25})$$

which holds for an arbitrary value of  $\delta$  only if  $\alpha e^{-\alpha} = \psi e^{-\psi}$ . Equations (A23) and (A24) then yield

$$e^{-\alpha} = 1 - e^{-\psi} - \psi e^{-\psi}, \quad (\text{A26})$$

$$e^{-\psi} = 1 - e^{-\alpha} - \alpha e^{-\alpha}. \quad (\text{A27})$$

Obviously,  $\alpha = \psi = \theta_3$  solves (A26) and (A27). Are there other solutions? Solving  $\alpha$

from (A26) yields  $\alpha = -\ln(1 - e^{-\psi} - \psi e^{-\psi})$ , and using this in (A27) we get

$$\frac{-\psi e^{-\psi}}{1 - e^{-\psi} - \psi e^{-\psi}} = \ln(1 - e^{-\psi} - \psi e^{-\psi}). \quad (\text{A28})$$

Subtracting the derivative of the left-hand side from the derivative of the right-hand side equals

$$\frac{\psi e^{-\psi}}{1 - e^{-\psi} - \psi e^{-\psi}} - \frac{e^{-\psi}(e^{-\psi} + \psi - 1)}{(1 - e^{-\psi} - \psi e^{-\psi})^2} \quad (\text{A29})$$

which has the same sign as  $1 - e^{-\psi} - \psi e^{-\psi} - \psi^2 e^{-\psi}$ . This equals zero at  $\psi = 0$ , and

$$\frac{\partial(1 - e^{-\psi} - \psi e^{-\psi} - \psi^2 e^{-\psi})}{\partial\psi} = \psi e^{-\psi}(\psi - 1) \quad (\text{A30})$$

which is negative if  $\psi < 1$ . At  $\psi = 1.2 (> \theta_3)$  we have  $1 - e^{-\psi} - \psi e^{-\psi} - \psi^2 e^{-\psi} \approx -0.0096$ , and we conclude that  $1 - e^{-\psi} - \psi e^{-\psi} - \psi^2 e^{-\psi} < 0$  for  $\psi \in (0, \theta_3]$ . Thus the left-hand side of (A28) can cut the right-hand side of (A28) only from below in interval  $\psi \in (0, \theta_3]$ , therefore (A28) has a unique solution in that interval. We conclude that (A26) and (A27) have a unique solution, namely  $\alpha = \psi = \theta_3$ . That is, there does not exist  $\psi$  smaller than  $\theta_3$  and a corresponding  $\alpha$  larger than  $\theta_3$  that satisfy (A23) and (A24). Because of that and because  $\alpha_1 < \alpha_2$  for  $\psi$  larger than one, we conclude that  $\alpha_1 < \alpha_2$  for all  $\psi \in (0, 1/\theta_3]$ , and therefore a deviating coalition exists for all  $\psi \in (0, 1/\theta_3]$ .  $\square$

*All buyers search and all sellers wait*

This is a reverse case to the above. The Poisson parameter in the original market is  $\theta$  which is larger than  $\theta_3$ , and the Poisson parameter in the new market is  $\omega$  which is smaller than  $\theta_3$ . A profitable deviation requires that  $\omega$  is smaller than  $\omega_1$  which satisfies  $V_s^n = V_s^o$ , given by

$$\frac{e^{-\omega_1}}{1 - \delta\omega_1 e^{-\omega_1}} = \frac{1 - e^{-\theta} - \theta e^{-\theta}}{1 - \delta\theta e^{-\theta}}, \quad (\text{A31})$$

and larger than  $\omega_2$  which satisfies  $V_b^n = V_b^o$ , given by

$$\frac{1 - e^{-\omega_2} - \omega_2 e^{-\omega_2}}{1 - \delta\omega_2 e^{-\omega_2}} = \frac{e^{-\theta}}{1 - \delta\theta e^{-\theta}} \quad (\text{A32})$$

Proceeding like in the above, it can be shown that  $\omega_1 < \omega_2$  for all  $\theta$  larger than  $\theta_3$ , and a deviating coalition does not exist.  $\square$

ii) In case  $\theta < 1/\theta_3$  the proof goes similarly as in case  $\theta > \theta_3$ , one just has to replace buyers for sellers and sellers for buyers.  $\square$

## Proof of Proposition 2

The logic of the proof is similar to that of Proposition 1. The proof is shown only for part (i).

(i) In case  $\theta > 1$  we show that if all the buyers wait and all the sellers search, there is a coalition of buyers and sellers who are better off in a market where buyers search and sellers wait, implying that the former market is not an equilibrium. On the other hand, if all the buyers search and all the sellers wait, a profitable deviating coalition cannot exist.

*All buyers wait and all sellers search*

The Poisson parameter in the original market is  $\psi = 1/\theta$ , and the utilities are  $V_b^o$  for a buyer and  $V_s^o$  for a seller. The corresponding parameter in the deviating market where buyers search and sellers wait is  $\alpha = \eta B/\mu S$ , and the utilities are  $V_b^n$  and  $V_s^n$ . Consider a deviating coalition where sellers wait, buyers search, and the ratio of buyers to sellers is  $\alpha$ . Profitable deviation requires that  $V_s^n > V_s^o$ , and  $V_b^n > V_b^o$ , that is,

$$\frac{\alpha(1 - e^{-\alpha})}{(2 - \delta - \delta e^{-\alpha})\alpha + \delta(1 - e^{-\alpha})} > \frac{1 - e^{-\psi}}{(2 - \delta - \delta e^{-\psi})\psi + \delta(1 - e^{-\psi})}, \quad (\text{A33})$$

$$\frac{1 - e^{-\alpha}}{(2 - \delta - \delta e^{-\alpha})\alpha + \delta(1 - e^{-\alpha})} > \frac{\psi(1 - e^{-\psi})}{(2 - \delta - \delta e^{-\psi})\psi + \delta(1 - e^{-\psi})}. \quad (\text{A34})$$

Let  $\alpha_1$  satisfy  $V_s^n = V_s^o$ , and let  $\alpha_2$  satisfy  $V_b^n = V_b^o$ . Expression

$$\frac{1 - e^{-h}}{(2 - \delta - \delta e^{-h})h + \delta(1 - e^{-h})} \quad (\text{A35})$$

is decreasing in  $h$ , and expression

$$\frac{h(1 - e^{-h})}{(2 - \delta - \delta e^{-h})h + \delta(1 - e^{-h})} \quad (\text{A36})$$

is increasing in  $h$ . As in the proof of part (i) of Proposition 1, a deviating coalition exists if  $\alpha_1 < \alpha_2$ . The curve given by (A36) is steeper than the curve given by (A35) if  $2(1 - \delta)(1 - e^{-h} - he^{-h} - h^2e^{-h}) < 0$ , which holds if  $h$  is sufficiently small. Thus  $\alpha_1 < \alpha_2$  if  $\psi$  is sufficiently close to one. As in the proof above,  $\alpha_1 > \alpha_2$  requires that  $\alpha_1 = \alpha_2$  for some value of  $\psi$ . We show that the latter cannot happen. Let  $\alpha$  satisfy

$$\frac{\alpha(1 - e^{-\alpha})}{(2 - \delta - \delta e^{-\alpha})\alpha + \delta(1 - e^{-\alpha})} = \frac{1 - e^{-\psi}}{(2 - \delta - \delta e^{-\psi})\psi + \delta(1 - e^{-\psi})}, \quad (\text{A37})$$

$$\frac{1 - e^{-\alpha}}{(2 - \delta - \delta e^{-\alpha})\alpha + \delta(1 - e^{-\alpha})} = \frac{\psi(1 - e^{-\psi})}{(2 - \delta - \delta e^{-\psi})\psi + \delta(1 - e^{-\psi})}. \quad (\text{A38})$$

Dividing the left-hand side of (A37) by the left-hand side of (A38), and doing the same to right-hand sides gives

$$\alpha = 1/\psi. \quad (\text{A39})$$

Adding the left-hand side of (A37) to that of (A38), and doing the same to the right-hand sides leads to

$$\psi (1 - e^{-\alpha} - \alpha e^{-\alpha}) = \alpha (1 - e^{-\psi} - \psi e^{-\psi}). \quad (\text{A40})$$

Obviously,  $\alpha = \psi = 1$  solves (A40). If this is the only solution, then  $\alpha_1 < \alpha_2$  for all  $\psi$  smaller than one. Because  $\alpha = 1/\psi$ , we look for solutions to equation of form

$$h (1 - e^{-1/h} - (1/h)e^{-1/h}) = (1/h) (1 - e^{-h} - he^{-h}). \quad (\text{A41})$$

At  $h = 1$  the left-hand side is decreasing and the right-hand side is increasing. If another strictly positive solution exists, the derivative of the right-hand side must be smaller than the derivative of the left-hand side. It is however easy to show that in any strictly positive solution to (A41), the derivative of the right-hand side is larger than the derivative of the left-hand side, thus  $h = 1$  is the only solution. Then  $\alpha_1 < \alpha_2$  for all  $\psi$  smaller than one, and we can choose  $\alpha \in (\alpha_1, \alpha_2)$  such that  $V_s^n > V_s^o$  and  $V_b^n > V_b^o$ .

If all sellers wait and all buyers search in the original market, the same logic is applied to demonstrate that a deviating coalition does not exist.  $\square$

(ii) For  $\theta < 1$ , the proof is analogous to case  $\theta > 1$  and is left to the reader.  $\square$

## References

Burdett, K., Coles, M., Kiyotaki, N. and R. Wright, 1995, Buyers and sellers: should I stay or should I go? AEA Papers and Proceedings, May, 281 - 286.

Herreiner, D., 1999, The decision to seek or to be sought? Rheinische Friedrich-Wilhelms-Universität Bonn, Discussion Paper B-462.

Kultti, K., 1999, Equivalence of auctions and posted prices, Games and Economic Behavior 27, 106-113.

Kultti, K., Miettunen, A. and J. Virrankoski, 2003, Physical search, University of Helsinki, Department of Economics Discussion Paper No. 573:2003.

Kultti, K. and T. Takalo, 1999, Equilibrium in auction and bargaining markets when agents can wait and search, *Bulletin of Economic Research* 51, 171 - 205.

Lu, X. and R.P. McAfee, 1996, The evolutionary stability of auctions over bargaining, *Games and Economic Behavior* 15, 228-254.