Aging in a Currency Union with Endogenous Growth

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Abstract

This paper considers the effects of aging in a currency union with endogenous growth. It is shown that mortality, labour supply and social security funding have no effect on inflation and pose therefore no problems for monetary integration. On the other hand, the demand for medical care speeds up inflation. Hence, if a currency union accepts a new member in which families have a relatively high need for medical care, its inflation rate will increase, but the growth rate and welfare will decrease everywhere in the enlarged union. There exists a growth-maximizing level for social security funding. The extension of funding beyond this level retards growth and welfare.
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1 Introduction

Since 2002, over 300 million Europeans in 12 countries have used Euro as a currency of exchange. The demographic variation among these countries is however large. Total Fertility Rate (TFR) is relatively high for some members of the EMU (e.g. 1.9 for Ireland, 1.7 for France and Finland) and relatively low for others (e.g. 1.3 for Germany and Greece and 1.2 for Italy and Spain).[1] Does this prose a problem for the stability of the EMU? For some new members of the EU, TFR is even lower (e.g. 1.2 for Estonia, Slovenia and Czech Republic, 1.1 for Latvia).[2] Should the EMU restrict the membership of these? This paper attempts to answer these questions.

Rising longevity affects national savings and intensifies the competition of resources between consumption, investment and the health needs of the elderly. Recent empirical work indicates that it may even slow down economic growth.[3] Zhang et.al. (2003) explain this through the assumption that aging changes the preferences of the median voter with regard to taxation for public education.[4] In this study, we offer an alternative explanation as follows. Because a currency union compels economies into a sub-optimal tax system by imposing a uniform inflation rate, its establishment may together with the consequences of aging retard growth.

[1] United Nations (2000).

[2] United Nations (2000).

[3] Using the cross-section data set in Barro and Wolf (1989), Zhang et.al. (2003) reports that as the initial life expectancy rises from below 60 to 69, the investment ratio and the growth rate increase substantially; but when the initial life expectancy rises to 70 and over, the investment ratio and the growth rate drop, although their levels are still higher than in countries with low life expectancy.

[4] The median voter is willing to increase the tax rate for public education at low longevity, but beyond some level of longevity, further declines in mortality may lead the median voter to lower the tax rate. Hence, the human capital accumulation may rise initially but may eventually fall.

Aging may have several and even opposing effects on growth. Steady mortality declines have raised the fraction of population living in retirement substantially in industrial countries. Consequently, the proportion of working population has been falling and is expected to fall much further in the next few decades. The return on individual investment in human capital depends positively on remaining active years. Postponing retirement age raises the proportion of working individuals and the returns on human capital, thus increasing the sustainable growth rate. Because increments in life expectancy do not affect optimal investment in human capital, but increase the proportion of retirees, they speed up growth only if they are accompanied by simultaneous increments in the working period.[5] Most developed countries have old-age social security programs, which will come under increasing pressure from mortality decline. Because many recent proposals call for replacing unfunded with fully funded social security, it is interesting to compare the effects of mortality decline across different social security systems.

In this study, we examine endogenous growth in a monetary economy by a model of a dynastic family which accumulates (human) capital through saving and holds money for transactions purposes. Each family contains both young (working) and elderly (retired) people. Given the discussion above, we characterize aging by the following parameters:

(*i*) Mortality declines. This is equivalent to the decrease in the rate of time preference [Cf. Blanchard and Fischer (1989), pp. 115-143].

(*ii*) The proportion of working population declines. This is equivalent to the decrease of the productivity of (human) capital.

[5] Cf. Echevarria (2003).

(iii) The demand for medical care increases.

Furthermore, we consider a fully funded social security system as one which families finance wholly by themselves, an unfunded system as one which the government finances wholly, and a partially funded system as one which the government and families finance together.

In this study, a key feature of the analysis is the central role assigned to the congestion of medical services. Following Palokangas (2003), we show that a specific form of congestion is necessary for persistent growth. Inflation has then two effects on the growth rate. First, its increase provides the government with more seigniorage and thereby helps to supply more medical care. This promotes private output, private saving, capital accumulation and economic growth. On the other hand, a higher inflation rate leads to higher transaction costs in the private sector, lower income, lower capital accumulation and slower growth. Where these two opposed effects exactly match, the inflation rate is optimal and the growth rate maximal. There exists a number of endogenously-growing economies, which produce the same composite good. On the assumption that there is no trade in factors of production, also the growth rates of economies may differ. A currency union compels the same inflation rate for all of its members.

The remainder of this paper is organized as follows. Section 2 introduces the institutional specifications on which the study is based. Section 3 presents the optimal behaviour of a private family, on which in section 4 the equilibrium of the private sector is based. Section 5 characterizes the structure of the public sector and section 6 constructs optimal policy rules for a rational government of a single economy. Finally, section 7 examines the effect of currency unions in this environment.

2 The setting

(i) Production and consumption. We aggregate all goods in the economy into a single good, the price of which is p. There are two assets, money and capital. There is a fixed number J of similar private families who save, invest in capital, hold money for transaction purposes and produce goods from capital. All private families benefit from medical care G. One unit of medical care is produced from one unit of the good. A fixed share $\beta \in [0,1]$ of medical care G is financed privately and the rest publicly through the government's budget. This characterized an unfunded social security system for $\beta = 0$, a fully funded system for $\beta = 1$ and a partially funded system for $\beta \in (0,1)$.

Output Y is produced from capital stock K according to

$$Y = \xi K, \tag{1}$$

where ξ is a parameter. The older people a family contains, the smaller proportion of these takes part in production and the lower ξ. The intertemporal utility function is given by

$$\begin{gathered}\int_0^\infty U(C,G)e^{-\rho t}dt = \int_0^\infty U(c,g)K^{1-\sigma}e^{-\rho t}dt \text{ with} \\ U(c,g) = (c+\alpha g)^{1-\sigma}/(1-\sigma), \;\; g \doteq G/K \text{ and } c \doteq C/K,\end{gathered} \tag{2}$$

where t is time, C consumption, G medical care, $\rho > 0$ the constant rate of time preference, $\sigma \in (0,1) \cup (1,\infty)$ the inverse of the constant intertemporal elasticity of substitution, and α the constant marginal rate of substitution between consumption and medical care when the level of instantaneous utility U is kept constant. The higher mortality, the higher ρ. The older people a family contains, the more medical care raises its welfare and the higher subjective price α it is ready to pay for medical care in terms of consumption.

(ii) Congestion. Congestion results from the existence of many families. As distinct from aggregate output Y and aggregate capital K, we denote a single family's output and capital by y and k. With congestion, a single family assumes it will get the more services $\widetilde{G}$ from medical care G the larger its share k/K of aggregate capital stock K. In line with Fischer and Turnovsky (1998), we specify this as follows:

$$\widetilde{G} \doteq (k/K)^{\delta}G = (k/K)^{\delta}gK = (k/K)^{\delta-1}gk \text{ with } 0 < \delta \leq 1, \tag{3}$$

where δ is a parameter. When congestion is *proportional*, $\delta = 1$, the family assumes that it receives medical services $\widetilde{G}$ in direct proportion to its capital stock k. When congestion is *partial*, $0 < \delta < 1$, the family assumes that it receives less $\widetilde{G}$ than in proportion to k. We ignore the case of no congestion, $\delta = 0$, where medical care G is a non-rival and non-excludable public good available equally to each family independent of the size of the economy.

The family takes medical care G, aggregate capital stock K and $g \doteq G/K$ as given. It perceives the true production function (1) and the true utility function (2), but so that macroeconomic variables G, K and Y are replaced by microeconomic variables $\widetilde{G}$, k and $y = \xi k$. Because the families are similar, the consumption-capital ratio c must be the same for the whole economy and a single family. Given (3), we then obtain a single family's perceived utility function as follows:

$$\int_0^{\infty} U(ck, \widetilde{G})e^{-\rho t}dt = \int_0^{\infty} \left[c + \alpha\left(\frac{k}{K}\right)^{\delta-1} g\right]^{1-\sigma} k^{1-\sigma}e^{-\rho t}dt. \tag{4}$$

(iii) Tax evasion. We assume that an family is able to hide its income at some cost in terms of the numeraire good.[6] Let py is the family's total income, qpy

[6]This assumption is needed to produce a distortion in the public sector, which gives the government the incentive to use seigniorage.

hidden income and $(1-q)py$ observed income, where $0 \le q \le 1$. We assume that the level of income does not affect the family's ability to conceal income, but that such activity is subject to increasing costs. The real administrative cost of hiding income, Z, is then linear homogeneous with respect to total real income y but increasing and strictly convex with respect to the ratio q of hidden to total income. With all profits revealed, $q = 1$, there is no administrative cost $Z = 0$. Given these assumptions, the following (real) cost function can be established:

$$Z = z(q)y, \quad z' > 0, \quad z'' > 0, \quad z(0) = 0, \quad z \doteq Z/(py), \tag{5}$$

where z is the ratio of administrative cost to total income.

(iv) Transaction technology. We introduce money as an intermediary good which reduces transaction costs.[7] One unit of transaction services is produced from one unit of the composite good. The requirement for real transaction services, V, is an increasing function of real expenditure

$$E \doteq ck + \dot{k} + \beta\widetilde{G}, \tag{6}$$

which consists of consumption ck, investment $\dot{k} = dk/dt$ and the private cost for medical services $\widetilde{G}$, $\beta\widetilde{G}$, and a non-increasing function of the real level of money balances, M/p, which is the ratio of the money supply M and the price level p, $V = \mathcal{V}(E, M/p)$. This function is also linearly homogeneous, i.e. a proportional increase in both real expenditure E and real money stock M/p increases V by the same proportion.

To obtain a stable demand function for money, we assume furthermore that $\mathcal{V}$ is strictly concave and thrice differentiable.[8] The function $\mathcal{V}$ can then

[7]This follows Palokangas (2003). Feenstra (1986) shows that under certain conditions, the approach of placing money in the utility function is equivalent to this approach.

[8]Thrice differentiability is needed for the differentiability of the elasticity ε in (15).

be transformed into the form $V = v(m)E$, where $m \doteq M/E$ is the money-expenditure ratio and $v(m) \doteq \mathcal{V}(1, m)$ is a thrice differentiable function with $v'' > 0$. Finally, we assume that there is a bliss point $\bar{m}$ for the money-expenditure ratio with $m \leq \bar{m}$ and $v(\bar{m}) = v'(\bar{m}) = 0$. Defining the rate of investment $\phi \doteq \dot{k}/k \geq 0$, we can summarize the transaction technology as:

$$\begin{aligned} &0 \leq m \doteq M/E \leq \bar{m}, \quad 0 \leq v(m) = V/E < 1, \\ &v' \leq 0, \quad v'' > 0, \quad v(\bar{m}) = v'(\bar{m}) = 0. \end{aligned} \tag{7}$$

3 The families

In the family's steady state, the ratios $c = C/K$, $g = G/K$ as well as the ratio of medical services (3) to private consumption $C = ck$, i.e. $(k/K)^{\delta-1}g/c$, must be constant. This means that the term $k^{\delta-1}$ must be constant as well. Capital stock k and output y are then constant for $0 < \delta < 1$. If $\delta = 1$, k is undetermined, the rate of investment $\phi = \dot{k}/k$ can be positive and output can grow at a positive rate. We summarize:

Proposition 1 *Persistent growth is possible only with proportional congestion $\delta = 1$. With partial congestion, $0 < \delta < 1$, there is no such growth.*

With proportional congestion, the average product of capital, y/k, is constant, there is no equilibrium for capital stock and capital grows indefinitely. With partial congestion, the average product of capital is decreasing, there is an equilibrium for capital stock and there is no growth in capital.

An family's budget constraint is given by

$$(1 - x)(1 - q)y + qy = E + iM/p + V + Z, \tag{8}$$

where y is its output (= total real income), qy its hidden income, $(1-q)y$ its revealed income, x the tax rate, E its real expenditure, M its money supply, iM/p the depreciation in its real balances M/p due to the inflation rate i, V its purchase of transaction services and Z its costs in tax evasion. Given (3), (5), (6), (7), $\delta = 1$, $y = \xi k$ and $\phi \doteq \dot{k}/k$, the constraint (8) takes the form

$$\begin{aligned}[(1-x)(1-q)+q-z(q)]\xi &= [(1-x)(1-q)+q-z(q)]y/k \\ &= (1+im+v)E/k = [1+im+v(m)](c+\phi+\beta g). \end{aligned} \tag{9}$$

An family maximizes its utility (4) subject to its budget constraint (9) and its capital accumulation $\phi \doteq \dot{k}/k$ by the consumption-capital ratio c, the rate of investment, ϕ, the ratio of hidden to total income, q, and the money-expenditure ratio m, given the inflation rate i, the tax rate x, medical care G, the aggregate capital stock K and $g = G/K$.

Since our purpose is to examine public policy in an economy with persistent growth, then, given proposition 1, we can focus wholly on the case $\delta = 1$. Given $\delta = 1$, $\widetilde{G} = G$ (4) and (9), the family maximizes its utility

$$\int_0^\infty U(c,g)k^{1-\sigma}e^{-\rho t}dt = \int_0^\infty \frac{e^{-\rho t}}{1-\sigma}(c+\alpha g)^{1-\sigma}k^{1-\sigma}dt$$

subject to the budget constraint (9) and capital accumulation $\phi \doteq \dot{k}/k$ by variables c, ϕ, z, q and m, given i, x, G, K and $g \doteq G/K$. The Lagrangean of this maximization is given by

$$\begin{aligned}\Psi = {} & (c+\alpha g)^{1-\sigma}k^{1-\sigma}/(1-\sigma) + \mu\phi k \\ & + \omega\big\{[(1-x)(1-q)+q-z(q)]\xi - [1+im+v(m)](c+\phi+\beta g)\big\}, \end{aligned} \tag{10}$$

where ω is a Lagrangean multiplier and variable μ evolves such that

$$\dot{\mu} = \rho\mu - \partial\Psi/\partial k = (\rho-\phi)\mu - (c+\alpha g)^{1-\sigma}k^{-\sigma}, \quad \lim_{t\to\infty}\mu k e^{-\rho t} = 0. \tag{11}$$

4 The equilibrium of the private sector

The maximization of the Langrangean (10) by q leads, by duality and by the properties of the function (5), to the definition

$$\pi(x) \doteq \max_q [(1-x)(1-q) + q - z(q)], \quad \pi' < 0, \quad \pi'' > 0. \tag{12}$$

Given this definition, we obtain the tax base T and the elasticity of the tax base with respect to the tax rate x, when capital k and medical care intensity g are kept constant, as follows:

$$T = (1-q)y = -\pi'(x)\xi k, \quad \eta(x) \doteq -\frac{x}{T}\frac{\partial T}{\partial x} = -x\frac{\pi''(x)}{\pi'(x)} > 0. \tag{13}$$

Given (4), the maximization of the Lagrangean (10) by m yields

$$v'(m) = -i. \tag{14}$$

From this and (7) it follows that the money-expenditure ratio, m, is a function of the inflation rate i only:

$$m(i), \;\; m' \doteq \frac{dm}{di} = -\frac{1}{v''} < 0, \;\; \varepsilon(i) \doteq -\frac{im'}{m} = \begin{cases} > 0 & \text{for } m < \bar{m}, \\ = 0 & \text{for } m = \bar{m}, \end{cases} \tag{15}$$

where ε is the elasticity of the demand for money with respect to the inflation rate i, when real expenditure E is kept constant.

The maximization of the Lagrangean (10) by c and ϕ yields

$$\partial\Psi/\partial c = (c+\alpha g)^{-\sigma}k^{1-\sigma} - [1 + im + v(m)]\omega = 0,$$
$$\partial\Psi/\partial\phi = \mu k - [1 + im + v(m)]\omega = 0. \tag{16}$$

Output $y = \xi k$, consumption ck and medical services $\tilde{G} = gk$ are now in fixed proportion to capital stock k, which is the family's only state variable.

Consequently, the system jumps immediately to the steady state and there are no transitional dynamics. Given the first-order conditions (16), we obtain

$$\mu = (1 + im + v)\omega/k = (c + \alpha g)^{-\sigma} k^{-\sigma}.$$

This implies that terms $k^{-\sigma}$ and μ grow at the same rate, $\dot{\mu}/\mu = -\sigma\,\dot{k}/k = -\sigma\phi$. This, (11) and (14) produce

$$\rho + (\sigma - 1)\phi = (c + \alpha g)^{1-\sigma} k^{-\sigma}/\mu = c + \alpha g. \tag{17}$$

Inserting $\delta = 1$, (12) into (9) and solving for ϕ, we obtain

$$\dot{k}/k = \phi = \xi\pi(x)/[1 + im + v(m)] - c - \beta g. \tag{18}$$

A balanced-growth equilibrium exists in the model, because the model is proportional to the state variable k. Inserting (17) into (18) yields:

$$\phi = \Phi(x, i, g, \rho, \xi) \doteq \frac{1}{\sigma}\Big[\frac{\xi\pi(x)}{1 + im + v(m)} + (\alpha - \beta)g - \rho\Big]. \tag{19}$$

Because in equilibrium the consumption-capital ratio c, the rate of investment ϕ and the money-expenditure ratio m are kept constant, given (7), money supply M and the price level p grow at the same rate, $\dot{p}/p = \dot{M}/M$. Hence, by increasing the quantity of money per family, M, at a fixed rate, the government can control the inflation rate $i \doteq \dot{p}/p = \dot{M}/M$.

5 Governments

Because the families are similar, aggregate capital is given by $K = Jk$, where J is the number of families. Since the public expenditures on medical care, $(1 - \beta)G$, are financed by taxes xT and seigniorage iM/p from all J families, the government's budget constraint is $(iM/p + xT)J = (1-\beta)G$. Given (7), (12), (13), (15), (19), $g = G/K$ and $K = Jk$, this constraint also reads as:

$$\Upsilon(i,g,x,\xi) \doteq \Big[\Big(i\frac{M}{p} + xT\Big)J - (1-\beta)G\Big]\frac{1}{\xi K} = (c+\phi)i\frac{m}{\xi} - x\pi' - (1-\beta)\frac{g}{\xi}$$
$$= \frac{\pi(x)im(i)}{1+im+v(m)} - x\pi'(x) - (1-\beta)\frac{g}{\xi} = 0. \tag{20}$$

We assume that the economy is on the increasing part of the Laffer curve. This means that if the government budget is initially balanced, $\Upsilon = 0$, and the inflation rate i is kept constant, then an increase in the tax rate x produces a budget surplus $\Upsilon > 0$. Given (12), (13) and (20), this implies

$$\frac{\partial \Upsilon}{\partial x} = \pi'\Big[\frac{im}{1+im+v} - 1 + \eta\Big] > 0, \quad 1 > \eta + \frac{im}{1+im+v}. \tag{21}$$

The government maximizes welfare by the tax rate x, the inflation rate i and medical care intensity $g \doteq G/K$, given the budget constraint (20). We can equivalently express the budget constraint (20) in terms of the tax rate and assume that the government maximizes welfare by i and g, given this tax function. Differentiating (20) totally, and noting (13), (15) and (21), we obtain the tax rate as the following function of the other policy variables (i,g) and the parameters ξ and β:

$$x(i,g,\xi,\beta), \quad \frac{\partial x}{\partial \xi} = (\beta-1)\frac{g}{\xi^2}\Big/\frac{\partial \Upsilon}{\partial x} < 0, \quad \frac{\partial x}{\partial \beta} = -\frac{g}{\xi}\Big/\frac{\partial \Upsilon}{\partial x} < 0,$$
$$\frac{\partial x}{\partial g} = -\frac{\partial \Upsilon}{\partial g}\Big/\frac{\partial \Upsilon}{\partial x} = \frac{1-\beta}{\xi\pi'}\Big[\frac{im}{1+im+v} - 1 + \eta\Big]^{-1} > 0,$$
$$\frac{\partial x}{\partial i} = -\frac{\partial \Upsilon}{\partial i}\Big/\frac{\partial \Upsilon}{\partial x} = \Big[\frac{m}{(1+im+v)^2} - \frac{m+im'}{1+im+v}\Big]\pi\Big/\frac{\partial \Upsilon}{\partial x}$$
$$= \frac{m(i)\pi(x)}{[1+im+v(m)]\pi'(x)}\,\frac{\varepsilon(i) + im(i)/[1+im+v(m)] - 1}{\eta(x) + im(i)/[1+im+v(m)] - 1}. \tag{22}$$

Given (13), (18), (19) and (22), the growth rate can then be specified as a function of the policy variables (i, g) and the parameters (ρ, ξ, α) (Appendix):

$$\dot{k}/k = \phi(i,g,\rho,\xi,\alpha) \doteq \Phi(x(i,g,\xi), i, g, \rho, \xi, \alpha),$$
$$\frac{\partial\phi}{\partial\beta} > 0 \;\Leftrightarrow\; \eta(x) > \frac{v(m(i))}{1+im+v(m)},$$
$$\frac{\partial\phi}{\partial g} = \frac{(1-\beta)/\sigma}{im+[\eta(x)-1][1+im+v(m)]} + \frac{\alpha-\beta}{\sigma},$$
$$\frac{\partial\phi}{\partial i} = \frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial i} + \frac{\partial\Phi}{\partial i} = \frac{\pi(x)m(i)\xi/\sigma}{[1+im+v(m)]^2}\,\frac{\eta(x)-\varepsilon(i)}{1-\eta(x)-im(i)/[1+im+v(m)]},$$
$$\frac{\partial^2\phi}{\partial g^2} < 0 \;\Leftrightarrow\; \left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0} > 0, \quad \frac{\partial\phi}{\partial\xi} > 0, \;\; \frac{\partial\phi}{\partial\alpha} > 0, \;\; \frac{\partial\phi}{\partial\rho} < 0,$$
$$\frac{\partial^2\phi}{\partial g\partial\rho} \equiv 0, \quad \frac{\partial^2\phi}{\partial i\partial\rho} \equiv 0, \quad \frac{\partial^2\phi}{\partial i\partial\alpha} \equiv 0, \quad \frac{\partial^2\phi}{\partial g\partial\alpha} = \frac{1}{\sigma} > 0,$$
$$\left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0} \Big/ \left.\frac{\partial^2\phi}{\partial i\partial\xi}\right|_{\partial\phi/\partial i=0} = \frac{\partial^2\phi}{\partial g^2} \Big/ \frac{\partial^2\phi}{\partial g\partial\xi},$$
$$\left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0} \Big/ \left.\frac{\partial^2\phi}{\partial i\partial\beta}\right|_{\partial\phi/\partial i=0} = \frac{\partial^2\phi}{\partial g^2} \Big/ \frac{\partial^2\phi}{\partial g\partial\beta}. \tag{23}$$

The government maximizes the representative family's utility (4) by g and i, given the accumulation of capital (23) and the family's reaction function (17). It is equivalent to maximizing the Hamiltonian

$$\Lambda = [\rho + (\sigma-1)\phi(i,g,\rho,\xi)]^{1-\sigma}k^{1-\sigma}/(1-\sigma) + \lambda\phi(i,g,\rho,\xi)k \tag{24}$$

by g and i, where variable λ evolves according to

$$\dot{\lambda} = \rho\lambda - \partial\Lambda/\partial k = (\rho-\phi)\lambda - [\rho+(\sigma-1)\phi]^{1-\sigma}k^{-\sigma}, \;\; \lim_{t\to\infty}\mu k e^{-\rho t} = 0. \tag{25}$$

6 Public policy

Because the model of an family contains only one state variable k and is linearly homogeneous with respect to this, the system jumps immediately to the steady state in which c, ϕ, g, $\partial\phi/\partial i$, $\partial\phi/\partial g$ and $\partial c/\partial\phi$ are constants. Given (24), this means that λ and $k^{-\sigma}$ must grow at the same rate, $\dot{\lambda}/\lambda =$

$-\sigma\dot{k}/k = -\sigma\phi$. Inserting this into (25) and noting (17), we obtain $\lambda k^\sigma = [\rho + (\sigma-1)\phi]^{-\sigma}$. This and (24) yield

$$\partial\Lambda/\partial\phi = [\rho + (\sigma-1)\phi]^{-\sigma} k^{1-\sigma}(\sigma-1) + \lambda k = \sigma\lambda k > 0,$$

which implies

$$\arg\max_{i,g} \Lambda = \arg\max_{i,g} \phi(i, g, \rho, \xi, \alpha). \tag{26}$$

This result can be rephrased as follows:[9]

Proposition 2 *A rational government attempts to maximize the growth rate of its economy by its policy variables* (i, g)*.*

In an endogenous growth model, the congestion of medical care must be proportional (see proposition 1). This implies that private income is in fixed proportion to capital, and that the government instruments i and g affect welfare only through the growth rate of the economy.

Because (23) yields that

$$(\partial/\partial\rho)\max_{i,g}\phi < 0, \quad (\partial/\partial\xi)\max_{i,g}\phi > 0, \quad (\partial/\partial\alpha)\max_{i,g}\phi > 0,$$

we obtain the following result:

Proposition 3 *A decline in mortality (i.e. a lower* ρ*) and an increase in labour supply (i.e. a bigger* ξ*) promote growth and welfare. Increased demand for medical care (i.e. a bigger* α*) speeds up economic growth.*

A decline in mortality or higher labour supply enable a higher savings rate, which boosts capital accumulation and growth. The higher the relative

[9] This proposition is the same as in Palokangas (2003).

weight of medical care and lower the relative weight of consumption in a family's preferences, the less incentives the family has to consume and the more to save. A higher savings rate boosts capital accumulation and growth.

The first-order and second-order conditions corresponding to (26) are

$$\frac{\partial \phi}{\partial i} = 0, \ \ \frac{\partial \phi}{\partial g} = 0, \ \ \frac{\partial^2 \phi}{\partial i^2} < 0, \ \ \frac{\partial^2 \phi}{\partial g^2} < 0, \ \ \mathcal{J} \doteq \frac{\partial^2 \phi}{\partial i^2}\frac{\partial^2 \phi}{\partial g^2} - \Big(\frac{\partial^2 \phi}{\partial i \partial g}\Big)^2 > 0. \tag{27}$$

Given this and (23), we reconstruct a result from Palokangas (1997; 2003):

Proposition 4 ***(Ramsey rule)*** *A rational government chooses he inflation rate i so that the elasticity of money holdings with respect to the inflation rate, ε, is equal to the elasticity of the tax base with respect to the tax rate, η.*

Since the inflation rate is equivalent to a tax on money, the elasticity of tax revenue with respect to the tax rate, $1 - \eta$, must be equal to the elasticity of seigniorage with respect to the inflation rate, $1 - \varepsilon$, which yields $\eta = \varepsilon$.

The optimal funding of social security is given by $\partial \Lambda / \partial \beta = \partial \phi / \partial \beta = 0$. Given (23), this leads to the following result:

Proposition 5 *To promote growth and welfare, private finance in medical care should be increased (decreased) as long as the elasticity of the tax base, η, is greater (lower) than the share of transaction cost in total private expenditure, $v/(1 + im + v)$.*

The unfunded social security system causes deadweight loss through distorting taxation, but the fully funded system involves transaction costs. The proportion of funding in medical care should be determined by a trade-off between these two losses.

From (23) and (27) it follows that

$$\frac{\partial^2\phi}{\partial i\partial g} < 0, \quad \frac{\partial^2\phi}{\partial g\partial\rho} = \frac{\partial^2\phi}{\partial i\partial\rho} = \frac{\partial^2\phi}{\partial i\partial\alpha} = 0, \quad \frac{\partial^2\phi}{\partial g\partial\alpha} > 0. \tag{28}$$

Differentiating the first-order conditions $\partial\phi/\partial i = 0$ and $\partial\phi/\partial g = 0$ totally and noting (23), (27) and (28), we obtain the partial derivatives

$$\frac{\partial i}{\partial\rho} \equiv 0, \quad \frac{\partial i}{\partial\alpha} = \frac{1}{\mathcal{J}}\frac{\partial^2\phi}{\partial i\partial g}\frac{\partial^2\phi}{\partial g\partial\alpha} > 0, \quad \frac{\partial i}{\partial\xi} = \frac{1}{\mathcal{J}}\left[\frac{\partial^2\phi}{\partial g\partial i}\frac{\partial^2\phi}{\partial g\partial\xi} - \frac{\partial^2\phi}{\partial i\partial\xi}\frac{\partial^2\phi}{\partial g^2}\right] \equiv 0,$$
$$\frac{\partial i}{\partial\beta} = \frac{1}{\mathcal{J}}\left[\frac{\partial^2\phi}{\partial g\partial i}\frac{\partial^2\phi}{\partial g\partial\beta} - \frac{\partial^2\phi}{\partial i\partial\beta}\frac{\partial^2\phi}{\partial g^2}\right] \equiv 0.$$

These results can be rephrased as follows:

Proposition 6 *Mortality (i.e. ρ), labour supply (i.e. ξ) and the funding of social security (i.e. β) have no effect on the inflation rate i. A higher demand for medical care (i.e. a bigger α) speeds up inflation.*

A decline in mortality, higher labour supply or greater funding of social security boost saving and economic growth, but do not affect public finance. With a higher demand for medical care, the inflation tax should be raised to collect more seigniorage.

7 Currency unions

Finally, we examine the consequences of monetary integration. Proposition 6 has then the following corollary:

Proposition 7 *Mortality (i.e. ρ) and labour supply (i.e. ξ) have no effect on monetary integration.*

Let there be two economies, labelled 1 and 2. The economies are similar, except that economy 1 has a larger demand for medical care (i.e. $\alpha_1 > \alpha_2$).

Economy 1 has then a lower inflation rate than economy 2,

$$i_1 < i_2. \tag{29}$$

Each economy can exercise fiscal and monetary policy independently of the other, and the growth rates of each may also differ.[10] On the assumption that in both economies fiscal policy is chosen to maximize the welfare of the representative family, we can define the economy-specific growth rates as:

$$\phi^1 \doteq \{\phi| \ \xi = \xi_1, \ \text{ fiscal policy optimized in economy 1}\},$$
$$\phi^2 \doteq \{\phi| \ \xi = \xi_2, \ \text{ fiscal policy optimized in economy 2}\}.$$

According to proposition 2, the independent monetary policy of economy j should maximize its growth rate ϕ^j. This yields

$$i_j \doteq \arg\max_i \phi^j \ \text{ for } j = 1, 2. \tag{30}$$

Now assume that the two economies form a currency union so that a common central bank will set a common inflation rate ι for them. This central bank maximizes a target $\mathcal{W}(\phi^1, \phi^2)$, which is a differentiable and increasing function of the growth rates of economies ϕ^1 and ϕ^2. The maximization yields the first-order condition

$$\frac{d\mathcal{W}}{d\iota} = \frac{\partial \mathcal{W}}{\partial \phi^1} \frac{\partial \phi^1}{\partial i}\bigg|_{i=\iota} + \frac{\partial \mathcal{W}}{\partial \phi^2} \frac{\partial \phi^2}{\partial i}\bigg|_{i=\iota} = 0.$$

Given this, the partial derivatives $[\partial\phi^1/\partial i]_{i=\iota}$ and $[\partial\phi^2/\partial i]_{i=\iota}$ must have different signs. In economy j with $[\partial\phi^1/\partial i]_{i=\iota} > 0$, the inflation rate i must be increased from ι to attain the growth-maximizing level i_j with

[10] This property is mainly due to the assumption of a small economy and the exclusion of direct foreign investment from the model, but it helps in analysing monetary integration.

$[\partial\phi^1/\partial i]_{i=i_j} = 0$, and in economy j with $[\partial\phi^1/\partial i]_{i=\iota} < 0$, i must be decreased from ι to attain i_j. Given (29), this is true only when $i_1 < \iota < i_2$. We have thus obtained our final result:

Proposition 8 *The establishment of the currency union by two economies will increase (decrease) the inflation rate in the economy with a larger (smaller) demand for medical care, $i_1 < \iota$ ($i_2 > \iota$), and decrease the growth rate and welfare in both economies, $\phi^j\big|_{i=\iota} < \max_i \phi^j = \phi^j\big|_{i=i_j}$ for $j = 1, 2$.*

If institutional differences of potential members are too large, the establishment of a currency union will slow down economic growth and increase the inflation rate in the union.

8 Conclusions

This paper examines economies in which the main engine of growth is the existence of a lower limit for the marginal product of (human) capital. Money is introduced as a substitute for transaction services and seigniorage as a substitute for distorting taxation. The governments produce public services (e.g. medical care) from private sector output and finance this by taxation and seigniorage. In this setting, the establishment of a currency union affects growth and welfare through unifying the inflation rate throughout the member economies. Aging is characterized by three shocks: (i) a decline in mortality, (ii) a decline in the productivity of capital (through smaller working population) and (iii) an increase in the demand for medical care. We are also interested in how the structure of the social security system affects the outcome of aging. The main findings are the following.

A rational government in any member economy attempts to maximize the growth rate of its economy by taxation and seigniorage. This is because in an endogenous growth model, the congestion of medical care must be proportional. In such a case, private income is in fixed proportion to capital and the government's policy instruments affect welfare only through the growth rate of the economy. A decline in mortality and an increase in labour supply raise savings and thereby promote growth and welfare. The higher the relative weight of medical care and lower the relative weight of consumption in a family's preferences, the less incentives the family has to consume and the more to save. A higher savings rate boosts capital accumulation and growth.

To even out the deadweight loss in public finance, a rational government in any member economy chooses he inflation rate so that the elasticity of money holdings with respect to the inflation rate is equal to the elasticity of the tax base with respect to the tax rate. To promote growth and welfare, private finance in medical care should be increased (decreased) as long as the elasticity of the tax base is greater (lower) than the share of transaction cost in total private expenditure. The unfunded social security system causes deadweight loss through distorting taxation, but the fully funded system involves transaction costs. Hence, the proportion of funding in medical care should be determined by a trade-off between these two losses.

A decline in mortality, higher labour supply or greater funding of social security boost saving and economic growth, but do not affect public finance and consequently does not have any effect on seigniorage. This means that mortality and labour supply have no effect on monetary integration. Economies with different rates of mortality and different proportions of working population can have the same inflation rate without any deadweight loss in public finance. With a higher demand for medical care, the inflation tax should be raised to collect more seigniorage. This means that the establishment of the currency union by two economies will increase (decrease) the inflation rate in the economy with a larger (smaller) demand for medical care. This compels sub-optimal taxation and public policy for both economies, which also decreases the growth rate and welfare in both economies. Hence, the need for medical care seems to the only characteristics of aging which influences the possibilities of monetary integration. If differences in it among potential members are too large, the establishment of a currency union will slow down economic growth and increase the inflation rate in the union.

Appendix

From (13), (18), (19) and (22) it follows that

$$
\begin{aligned}
\dot{k}/k &= \phi(i,g,\rho,\xi,\alpha) \doteq \Phi(x(i,g,\xi),i,g,\rho,\xi,\alpha),\\
\frac{\partial\phi}{\partial g} &= \frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial g} + \frac{\partial\Phi}{\partial g} = \frac{\xi\pi'/\sigma}{1+im+v}\frac{\partial x}{\partial g} + \frac{\alpha-\beta}{\sigma}\\
&= \frac{(1-\beta)/\sigma}{im+[\eta(x)-1][1+im+v(m)]} + \frac{\alpha-\beta}{\sigma},\\
\frac{\partial\phi}{\partial i} &= \frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial i} + \frac{\partial\Phi}{\partial i} = \frac{\xi/\sigma}{1+im+v}\left[\pi'\frac{\partial x}{\partial i} - \frac{\pi m}{1+im+v}\right]\\
&= \frac{\pi m\xi/\sigma}{(1+im+v)^2}\left\{\frac{\varepsilon+im/(1+im+v)-1}{\eta+im/(1+im+v)-1} - 1\right\}\\
&= \frac{\pi(x)m(i)\xi/\sigma}{[1+im+v(m)]^2}\,\frac{\eta(x)-\varepsilon(i)}{1-\eta(x)-im(i)/[1+im+v(m)]},\\
\frac{\partial\phi}{\partial\xi} &= \frac{\partial\Phi}{\partial\xi} + \frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial\xi} > 0, \quad \frac{\partial\phi}{\partial\alpha} = \frac{\partial\Phi}{\partial\alpha} = \frac{g}{\sigma} > 0, \quad \frac{\partial\phi}{\partial\rho} = -\frac{1}{\sigma} < 0,\\
\frac{\partial\phi}{\partial\beta} &= \frac{\partial\Phi}{\partial\beta} + \frac{\partial\Phi}{\partial x}\frac{\partial x}{\partial\beta} = -\frac{g}{\sigma} - \frac{g/\sigma}{1+im+v}\Big/\left[\frac{im}{1+im+v} - 1 + \eta\right]\\
&= \frac{g}{\sigma}\,\frac{\eta - v/(1+im+v)}{1-\eta-im/(1+im+v)} > 0 \;\Leftrightarrow\; \eta > \frac{v}{1+im+v},
\end{aligned}
$$

$$\frac{\partial^2\phi}{\partial g^2} = \frac{-(1+im+v)\eta'}{\sigma\{im+[\eta(x)-1][1+im+v(m)]\}^2}\,\frac{\partial x}{\partial g} < 0 \;\;\Leftrightarrow$$
$$\frac{\partial^2\phi}{\partial g\partial\xi} = \frac{-(1+im+v)\eta'}{\sigma\{im+[\eta(x)-1][1+im+v(m)]\}^2}\,\frac{\partial x}{\partial \xi} > 0 \;\;\Leftrightarrow$$
$$\frac{\partial^2\phi}{\partial g\partial\beta} = \frac{-(1+im+v)\eta'}{\sigma\{im+[\eta(x)-1][1+im+v(m)]\}^2}\,\frac{\partial x}{\partial \beta} > 0 \;\;\Leftrightarrow$$
$$\left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0} = \frac{\pi m\xi/\sigma}{(1+im+v)^2}\,\frac{\eta'(x)}{1-\eta-im/(1+im+v)}\,\frac{\partial x}{\partial g} > 0 \;\;\Leftrightarrow$$
$$\left.\frac{\partial^2\phi}{\partial i\partial\xi}\right|_{\partial\phi/\partial i=0} = \frac{\pi m\xi/\sigma}{(1+im+v)^2}\,\frac{\eta'(x)}{1-\eta-im/(1+im+v)}\,\frac{\partial x}{\partial \xi} < 0 \;\;\Leftrightarrow$$
$$\left.\frac{\partial^2\phi}{\partial i\partial\beta}\right|_{\partial\phi/\partial i=0} = \frac{\pi m\xi/\sigma}{(1+im+v)^2}\,\frac{\eta'(x)}{1-\eta-im/(1+im+v)}\,\frac{\partial x}{\partial \beta} < 0 \;\;\Leftrightarrow\;\; \eta' > 0,$$
$$\frac{\partial^2\phi}{\partial g\partial\rho} \equiv 0, \quad \frac{\partial^2\phi}{\partial i\partial\rho} \equiv 0, \quad \frac{\partial^2\phi}{\partial i\partial\alpha} \equiv 0, \quad \frac{\partial^2\phi}{\partial g\partial\alpha} = \frac{1}{\sigma} > 0,$$
$$\left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0} \Big/ \left.\frac{\partial^2\phi}{\partial i\partial\xi}\right|_{\partial\phi/\partial i=0} = \frac{\partial x}{\partial g}\Big/\frac{\partial x}{\partial \xi} = \frac{\partial^2\phi}{\partial g^2}\Big/\frac{\partial^2\phi}{\partial g\partial\xi},$$
$$\left.\frac{\partial^2\phi}{\partial g\partial i}\right|_{\partial\phi/\partial i=0} \Big/ \left.\frac{\partial^2\phi}{\partial i\partial\beta}\right|_{\partial\phi/\partial i=0} = \frac{\partial x}{\partial g}\Big/\frac{\partial x}{\partial \beta} = \frac{\partial^2\phi}{\partial g^2}\Big/\frac{\partial^2\phi}{\partial g\partial\beta}.$$

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