Optimal Technology Policy with Imitation and Risk-Averting Households

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Abstract

A Schumpeterian growth model is constructed where R&D firms innovate to produce better versions of the products or imitate to copy existing innovations. Because firms cannot use their innovations or imitations as collateral, they finance their investment by issuing shares. Households save by purchasing these shares. The government affects the level of profits through competition policy. The main findings are the following. A small imitation subsidy slows down growth. In the first-best optimum collusion is socially optimal, but when the government cannot discriminate between innovation and imitation, it should promote product market competition.

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1 Introduction

This paper examines technology policy in an economy with innovation and imitation. Through the development of new products, an innovator achieves a temporary advantage earning monopoly profits. This advantage ends when an imitator succeeds in copying the innovation, enters the market and starts competing with the innovator. In this paper, we assume that (*a*) firms cannot borrow without collateral, and (*b*) they cannot use their immaterial property (e.g. innovations or imitations) as collateral. It is instructive to see how this specification affects technology policy.

There is already a large literature concerning technology policy and economic growth. First of all, Segerstrom (1991) presented a model characterized by the following properties: (*i*) Duopolists cooperate, so that there are monopoly prices in all industries. (*ii*) R&D firms are subject to constant returns to scale. (*iii*) R&D firms can both innovate and imitate in each industry. (*iv*) Outsiders can innovate a new quality of product at the same cost as the incumbents. He showed that innovation subsidies speeds up growth, but promote welfare only if innovative effort is initially large enough.

Later on, Segerstrom's (1991) model was challenged by the following papers. Walz (1995) replaced cooperation (*i*) by Cournot competition and found out that in some circumstances innovation subsidies may even retard economic growth. Davidson and Segerstrom (1998) substituted decreasing returns for constant returns to scale (*ii*) and showed that innovation subsidies promote growth but imitation subsidies do the opposite. Zeng (2001) obtained more or less similar results by rejecting (*iii*) and assuming that innovation improves product quality while imitation expands product variety.

Property (*iv*) in Segerstom's model leads to leapfrogging: innovations will always be performed by outsiders and the current industry leaders will be replaced. Several attempts have been made to eliminate this apparently unrealistic outcome. Aghion et al. (1997) and Aghion et al. (2001) constructed models where technological laggards must first catch up with the leading-edge technology before battling "neck-to-neck" for technological leadership in the future. They showed that imitation can promote growth by increasing the proportion of industries in which the more intense "neck-to-neck" competition takes place. Mukoyama (2003) constructed a model characterized by the following properties. Only leaders can conduct next-round innovation. Outsiders can become leaders by imitation. A newcomer and an incumbent compete in Bertrand manner, so that monopoly profits disappear after the entry of an imitator. In Mukoyama's model, subsidizing imitation may increase the economy-wide growth rate.

All papers referred above assume that R&D firms can borrow any amount of capital at a given market interest rate. In such a case, firms decide on R&D and households are protected from uncertainly through diversification in the market portfolio. Because this assumption is in contradiction with the whole literature of venture capital,[1] we prefer to assume that firms cannot borrow without collateral and immaterial property cannot be used as collateral. Firms must then finance their R&D through issuing shares and households purchasing these shares face the uncertainly associated with investment.

The remainder of this paper is organized as follows. Section 2 introduces the basic structure of the model. Sections 3 and 4 consider firms in production and R&D. Section 5 examines households deciding on saving, section 6 general equilibrium, and section 7 the prospects of public policy. Section 8 analyzes optimal public policy in two cases: the first best, in which the government can, and in the second best, in which it cannot discriminate between innovation and imitation. In both cases, optimal elasticity rules for subsidies and competition policy are presented.

2 The model

In this paper, we extend Wälde's (1999a, 1999b) model of risk-averting households as follows. (*i*) The sector of innovating firms is replaced by a large number of sectors (industries) in which firms both innovate and imitate. (*ii*) In R&D, constant returns are replaced by decreasing returns to scale. Following Mukoyama (2003), we eliminate leapfrogging through the assumption that only leaders can innovate. The model can then be characterized as follows:

(i) Labour is homogeneous and inelastically supplied. It is used in innovation, imitation or the production of the intermediate goods.

[1] A nice summary of this literature in given in Gompers and Lerner (1999).

(ii) Competitive firms produce the consumption good from a great number of intermediate goods according to Cobb-Douglas tehcnology. Each intermediate good is produced by a separate industry.

(iii) A R&D firm innovates or imitates. Its success depends on its own investment and some fixed factor of production (e.g. entrepreneurship).[2]

(iv) A successful innovator becomes a monopolistic producer of the latest technology in some industry until the technology is imitated.

(v) Imitation is necessary for an outsider to become a next innovator. A successful imitator enters the product market and starts competing with the previous leaders.

(vi) R&D firms finance their expenditure by issuing shares. The households save only in these shares. Each R&D firm distributes its profit among those who had financed it in proportion to their investment in the firm.

(vii) The government subsidizes innovation and imitation, and influences profits in industries with two or more producers by competition policy. These subsidies are financed by lump-sum taxes on the households.

3 Production

There is a great number of intermediate-good industries which are placed over limit $[0,1]$. The representative consumption-good firm makes its output y from all intermediate goods through technology

$$\log y = \int_0^1 \log[B_j x_j]dj, \quad x_j = \sum_{\kappa=1}^{a_j} x_{j\kappa}, \tag{1}$$

where B_j is the productivity parameter, a_j the number of firms in industry j, x_j the quantity of intermediate-good good j and $x_{j\kappa}$ the output of firm

[2]We assume these fixed factors to produce convexity in R&D technology. Cheng and Tao (1999) found out that the assumption of linear R&D technology leads to the counter-intuitive result that a government subsidy to imitative (innovative) R&D decreases (increases) imitative but increases (decreases) innovative effort. They showed that by replacing the linear R&D technology by sufficiently convex technology the result is reversed.

κ in industry j. The firm maximizes its profit $\Pi \doteq Py - \int_{j\in[0,1]} p_j x_j dj$ by its inputs x_j, taking its output price P and input prices $\{p_j\}$ as fixed. We normalize total consumption expenditure Py at unity. Because the firm is subject to constant returns to scale, we then obtain

$$Py = 1, \quad \Pi = 0, \quad p_j x_j = 0 \text{ for all } j. \tag{2}$$

Technological change is random. A successful innovator drives the other producers out of the market and takes over the whole industry. A successful imitator enters the market and starts competing with the earlier leaders. Hence, there are industries of two type: (*i*) one-leader industries, in which followers imitate, and (*ii*) industries with several innovating leaders.

Consider first one-leader industry j. Each new generation of good j provides exactly $\mu > 1$ times as many services as the product of the generation before it. The leader produces one unit of output from one labour unit and earns the profit $\pi_j \doteq (p_j - w)x_{j1}$, where w is the wage. Followers can manufacture the product one step behind on the quality ladder, produce $1/\mu$ units of output from one labour unit and earn the profit $\pi_j^f \doteq (p_j/\mu - w)x_{j1}$. To keep technological laggards away from the market, $\pi_j^f = 0$, the leader then sets the price p_j equal to μw.[3] From $p_j = \mu w$ and (2) it follows that

$$x_{j1} = \frac{1}{p_{j1}} = \frac{1}{\mu w} \text{ and } \pi_j = \Big(1 - \frac{1}{\mu}\Big)p_{j1}x_{j1} = 1 - \frac{1}{\mu} \doteq \pi \text{ for } a_j = 1. \tag{3}$$

Next, consider an industry j with two or more leaders. We assume that the κth leader anticipates the reaction of the other leaders by the function

$$p_j = \Phi(x_{j\kappa}, a_j, \varsigma), \quad \phi(a_j, \varsigma) \doteq -\frac{x_{j\kappa}}{\Phi}\frac{\partial\Phi}{\partial x_{j\kappa}} \in (0, \pi), \tag{4}$$

where the variable ς characterizes the government's competition policy, and maximizes its anticipated profit $\pi_{j\kappa} = p_j x_{j\kappa} - w x_{j\kappa} = [\Phi(x_{j\kappa}, a_j, \varsigma) - w]x_{j\kappa}$ by its output $x_{j\kappa}$. This, (2) and (4) yield the equilibrium conditions

$$w = p_j + x_{j\kappa}\frac{\partial\Phi}{\partial x_{j\kappa}} = [1 - \phi(a_j, \varsigma)]p_j, \quad x_{j\kappa} = \frac{1 - \phi(a_j, \varsigma)}{a_j w} \quad \text{and}$$
$$\pi_{j\kappa} = \phi(a_j, \varsigma)p_j x_{j\kappa} = \phi(a_j, \varsigma)/a_j \text{ for } a_j > 1. \tag{5}$$

[3] Cf. Grossman and Helpman (1991), chapter 4.

We assume that the more competitors in industry j (i.e. the higher a_j), the lower profits $\pi_{j\kappa}$. This and (5) yield

$$\pi_{j\kappa}\big|_{a_j>2} < \pi_{j\kappa}\big|_{a_j=2}. \tag{6}$$

Anyone investing in firms attempts to maximize its expected profit. The first leader in an industry is an innovator. If one invests in imitation to enter an industry with one leader, then its prospective profit is $\pi_{j\kappa}\big|_{a_j=2}$, but if it invests (with the same cost) in imitatation to enter an industry with more than two leaders, then its prospective profit is $\pi_{j\kappa}\big|_{a_j>2}$. Hence, given (6), investors invest in imitation only in one-leader industries. We summarize:

Proposition 1 *An industry has one or two leaders. On one-leader industries the followers imitate, but in two-leader industries the leaders innovate.*

We denote the set of one-leader industries by $\Theta \subset [0,1]$, the relative proportion of one-leader industries (two-leader industries), α (β) by

$$\alpha = \int_{j\in\Theta} dj, \quad \beta \doteq \int_{j\notin\Theta} dj = 1-\alpha. \tag{7}$$

Noting this, (3) and (5), we obtain the following result:

Proposition 2 *The expected profit is π for the monopoly in industry $j \in \Theta$, and $\theta = \phi(2,\varsigma)/2 \in (0,\, \pi/2)$ for both firms in industry $j \notin \Theta$. The output is $x_{j1} = x_\alpha \doteq 1/(\mu w)$ for one-leader industry $j \in \Theta$, and $2x_{j\kappa} = x_\beta \doteq [1-\phi(2,\varsigma)]/w = (1-2\theta)/w$ for two-leader industry $j \notin \Theta$.*

Employment is higher in two-leader than one-leader industries, $x_\beta > x_\alpha$. There is collusion for $\theta = \pi/2$ and Bertrand competition for $\theta = 0$. The less competition, the higher θ. Hence, θ can be used as a measure for the intensity of product market competition (PMC).[4] The government determines θ through its competition policy ς.

[4]Kanniainen and Stenbacka (2000) define the relative difference of the imitator's to the innovator's profit as a measure of patent width.

Summing up throughout firms and industries and noting propositions 1 and 2, (1), (3) and (7), we obtain employment in production, x, as follows:

$$x \doteq \int_0^1 \sum_{\kappa=1}^{a_j} x_{j\kappa} dj = \alpha x_\alpha + (1-\alpha) x_\beta = \frac{\varphi}{w}, \quad \varphi(\alpha,\theta) \doteq (2\theta - \pi)\alpha + 1 - 2\theta,$$
$$\frac{\partial \varphi}{\partial \theta} < 0, \quad x_\alpha = \frac{x}{\mu\varphi}, \quad x_\beta = (1-2\theta)\frac{x}{\varphi}, \quad y = B x_\alpha^\alpha x_\beta^{1-\alpha} = \chi(\alpha,\theta) x B,$$
$$\frac{\partial}{\partial \theta}\left(\frac{x_\alpha}{x}\right) > 0, \quad \frac{\partial}{\partial \theta}\left(\frac{x_\beta}{x}\right) < 0, \quad \chi(\alpha,\theta) \doteq \frac{(1-2\theta)^{1-\alpha}}{\mu^\alpha \varphi(\alpha,\theta)}, \quad \arg\max_\theta \chi = \frac{\pi}{2},$$
$$\frac{\partial \chi}{\partial \theta} > 0 \;\Leftrightarrow\; \theta < \frac{\pi}{2}, \quad \log B = \int_0^1 \log B_j dj, \tag{8}$$

where φ is wage expenditure, B the average level of productivity, x employment, xB efficient labour input and χ the productivity of efficient labour in production. More intense PMC (i.e. a smaller θ) increases employment x and total wages $wx = \varphi$ in production, $\partial\varphi/\partial\theta = 2(\alpha - 1) < 0$. Because innovating two-leader industries $j \notin \Theta$ employ more than imitating one-leader industries $j \in \Theta$, less frequent imitation (i.e. a smaller α) increases employment x and total wages φ in production, $\partial\varphi/\partial\alpha = 2\theta - \pi < 0$. The rest of the results can be rephrased as follows:

Proposition 3 *The productivity of efficient labour in the production of the consumption good, χ, is maximized with collusion $\theta = \pi/2$. The increase in PMC (i.e. a smaller θ) causes inefficiency, $\partial\chi/\partial\theta < 0$.*

Proposition 3 can be explained as follows. The problem is the maximization of total output $y = B x_\alpha^\alpha x_\beta^{1-\alpha}$ subject to the allocation of labour between innovation and imitation, $x = \alpha x_\alpha + (1-\alpha)x_\beta$, where total employment in production, x, is kept constant. Output y is at the maximum, if all industries employ the same amount of labour, $x_\alpha = x_\beta$, and this is possible only if two-leader industries collude and behave as if they were monopolies. An increase in PMC (i.e. a decrease in θ) transfers labour from one-leader into two-leader industries (i.e. x_α falls and x_β rises). The greater the difference $x_\beta - x_\alpha$, the lower total output y for given x.

4 Research

According to proposition 1, there are R&D firms of three type: a follower, which we call firm 0, and the first (second) leader, which we call firm 1 (2). R&D firm $\kappa \in \{0,1,2\}$ in industry j employs $l_{j\kappa}$ labour units. Its technology is a random variable but the probability of its success in one unit of time is a function of both its input $l_{j\kappa}$ and some fixed factor (e.g. entrepreneurship).

In two-leader industry $j \notin \Theta$ firms 1 and 2 innovate, no firm imitates, and the technological change of firm $\kappa \in \{1,2\}$ is characterized by a Poisson process $q_{j\kappa}$ in which the arrival rate of innovations is given by

$$\Lambda_{j\kappa} = \lambda l_{j\kappa}^{\xi} \text{ for } j \notin \Theta \text{ and } \kappa \in \{1,2\}, \tag{9}$$

where $\xi \in (0,1)$ and $\lambda > 0$ are constants. During a short time interval $d\nu$, there is an innovation $dq_{j\kappa} = 1$ in firm κ with probability $\Lambda_{j\kappa}d\nu$, and no innovation $dq_{j\kappa} = 0$ with probability $1 - \Lambda_{j\kappa}d\nu$.

In one-leader industry $j \in \Theta$ firm 0 imitates, no firm innovates, and technological change is characterized by a Poisson process Q_j in which the arrival rate of imitations is given by

$$\Gamma_j = \gamma l_{j0}^{\xi} \text{ for } j \in \Theta, \tag{10}$$

where $\gamma > 0$ is a constant. During a short time interval $d\nu$, there is an imitation $dQ_j = 1$ with probability $\Gamma_j d\nu$, and no imitation $dQ_j = 0$ with probability $1 - \Gamma_j d\nu$. We assume that it is so much easier to imitate than innovate that with the same input $l_{j\kappa}$ the productivity of imitation relative to innovation, γ/λ, is greater than $2\mu^{\sigma}$:

$$\gamma > 2\lambda\mu^{\sigma}. \tag{11}$$

The invention of a new technology in industry j raises the number of technology in that industry, t_j, by one and the level of productivity, $B_j^{t_j}$, by $\mu > 1$. Given this and (8), the average productivity in the economy, B, is a function of the technologies of all industries, $\{t_k\}$, as follows:

$$\log B^{\{t_k\}} \doteq \int_0^1 \log B_j^{t_j} dj, \quad B^{t_j+1}/B_j^{t_j} = \mu. \tag{12}$$

The arrival rate of innovations in industry $j \notin \Theta$ is the sum of the arrival rates of both firms in the industry, $\Lambda_{j1} + \Lambda_{j2}$. The average growth rate of B_j due to technological change in industry j in the stationary state is then $E\big[\log B_j^{t_j+1} - \log B_j^{t_j}\big] = (\Lambda_{j1} + \Lambda_{j2}) \log \mu$, where E is the expectation operator.[5] Because only industries $j \notin \Theta$ innovate, then, noting (9), the average growth rate of B in the stationary state is given by

$$
\begin{aligned}
g &\doteq \int_{j\notin\Theta} E\big[\log B_j^{t_j+1} - \log B_j^{t_j}\big] dj = (\log \mu) \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj \\
&= (\lambda \log \mu) \int_{j\notin\Theta} (l_{j1}^{\xi} + l_{j2}^{\xi}) dj. \qquad (13)
\end{aligned}
$$

This is used as a proxy of the growth rate of the economy.

Total employment in R&D is given by

$$
l \doteq \int_{j\notin\Theta} (l_{j1} + l_{j2}) dj + \int_{j\in\Theta} l_j dj. \qquad (14)
$$

There exists a fixed number N of households, each supplying one labour unit. Total labour supply N is equal to inputs in production, x, and R&D, l:

$$
N = x + l. \qquad (15)
$$

The government subsidizes R&D expenditures, but possibly at different rates in innovating and imitating industries. Given proposition 2, we obtain total expenditures from these subsidies as follows:

$$
R \doteq \tau_\alpha \int_{j\in\Theta} w l_{j0} dj + \tau_\beta \int_{j\notin\Theta} (w l_{j1} + w l_{j2}) dj, \qquad (16)
$$

where wl_{j0} is expenditure on imitation in firm 0 industry $j \in \Theta$, $wl_{j\kappa}$ expenditure on innovation in firm $\kappa \in \{1, 2\}$ in industry $j \notin \Theta$ and $\tau_\alpha \in (-\infty, 1)$ $\big(\tau_\beta \in (-\infty, 1)\big)$ is the subsidy to imitation (innovation). If the government cannot discriminate between innovation and imitation, then $\tau_\alpha = \tau_\beta$.

In industry $j \in \Theta$ firm 0 and in industry $j \notin \Theta$ firms 1 and 2 issue shares to finance their labour expenditure in R&D, net of government subsidies.

[5] For this, see Aghion and Howitt (1998), p. 59.

Because the households invest in these shares, we obtain

$$\sum_{\iota=1}^{N} S_{\iota j0} = (1-\tau_\alpha) w l_{j0} \text{ for } j \in \Theta,$$
$$\sum_{\iota=1}^{N} S_{\iota j\kappa} = (1-\tau_\beta) w l_{j\kappa} \text{ for } \kappa \in \{1,2\} \text{ and } j \notin \Theta, \tag{17}$$

where wl_{j0} is the imitation expenditure of firm 0 in industry $j \in \Theta$ and τ_α the subsidy to it, $wl_{j\kappa}$ the innovation expenditure of firm $\kappa \in \{1,2\}$ in industry $j \notin \Theta$ and τ_β subsidy to it, and $S_{\iota j\kappa}$ the household ι's investment in firm $\kappa \in \{0,1,2\}$ in industry j. Household ι's relative investment shares in the firms can be defined as follows:

$$i_{\iota j0} \doteq \frac{S_{\iota j0}}{(1-\tau_\alpha) w l_j} \text{ for } j \in \Theta; \quad i_{\iota j\kappa} \doteq \frac{S_{\iota j\kappa}}{(1-\tau_\beta) w l_{j\kappa}} \text{ for } j \notin \Theta. \tag{18}$$

We denote household ι's income by A_ι. Total income throughout all households $\iota \in \{1,...,N\}$ is then equal to income earned in the production of consumption goods, Py, plus income earned in R&D, wl, minus tax revenue R. Since total consumption expenditure Py is normalized at unity, this yields

$$\sum_{\iota=1}^{N} A_\iota = Py + wl - R = 1 + wl - R. \tag{19}$$

5 Households

The utility for a risk-averting household $\iota \in \{1,...,N\}$ from an infinite stream of consumption beginning at time T is given by

$$U(C_\iota, T) = E \int_T^\infty C_\iota^\sigma e^{-\rho(\nu - T)} d\nu \text{ with } 0 < \sigma < 1 \text{ and } \rho > 0, \tag{20}$$

where ν is time, E the expectation operator, C_ι the index of consumption, ρ the rate of time preference and $1/(1-\sigma)$ is the constant relative risk aversion.

Because investment in these shares is the only form of saving in the model, the budget constraint of household ι is given by

$$A_\iota = PC_\iota + \int_{j\in\Theta} S_{\iota j0} dj + \int_{j\notin\Theta} (S_{\iota j1} + S_{\iota j2}) dj, \tag{21}$$

where A_ι is the household's total income, C_ι its consumption, P the consumption price, $S_{\iota j\kappa}$ the household's investment in firm κ in industry j. When household ι has financed a successful R&D firm, it acquires the right to the firm's profit in proportion to its relative investment share. Hence, we define:

$s_{\iota j\kappa}$ household ι's true profit from firm κ in industry j when the uncertainty in R&D is taken into account;

$i_{\iota j\kappa}$ household ι's investment share in firm κ in industry j [Cf. (18)];

$\pi i_{\iota j\kappa}$ household ι's expected profit from firm κ in industry $j \notin \Theta$ after an innovation;

$\theta i_{\iota j0}$ household ι's expected profit from industry $j \in \Theta$ after an imitation.

The changes in the profits of firms in industry j are functions of the increments (dq_{j1}, dq_{j2}, dQ_j) of Poisson processes (q_{j1}, q_{j2}, Q_j) as follows:[6]

$$\begin{aligned} ds_{\iota j\kappa} &= (\pi i_{\iota j\kappa} - s_{\iota j\kappa})dq_{j\kappa} - s_{\iota j\kappa}dq_{j(\zeta\neq\kappa)} \text{ when } j \notin \Theta; \\ ds_{\iota j0} &= (\theta i_{\iota j0} - s_{\iota j0})dQ_j \text{ when } j \in \Theta. \end{aligned} \tag{22}$$

These functions can be explained as follows. Consider first industry $j \notin \Theta$ in which there are two innovating leaders $\kappa \in \{1, 2\}$. If a household invests in firm κ, then, in the advent of a success for the firm, $dq_{j\kappa} = 1$, the amount of its share holdings rises up to $\pi i_{\iota j\kappa}$, $ds_{\iota j\kappa} = \pi i_{\iota j\kappa} - s_{\iota j\kappa}$, but in the advent of success for the other firm $\zeta \neq \kappa$ in the industry, its share holdings in the firm fall down to zero, $ds_{\iota j\kappa} = -s_{\iota j\kappa}$. Next, consider industry $j \in \Theta$ in which firm 0 imitates. If a household invests in that firm, then, in the advent of a success for the firm, $dQ_j = 1$, the amount of its share holdings rises up to $\theta i_{\iota j0}$, $ds_{\iota j0} = \theta i_{\iota j0} - s_{\iota j0}$.

The total income of household ι, A_ι, consists of its wage income w (the household supplies one labour unit) and its profits $s_{\iota j1}$ and $s_{\iota j2}$ from the leaders 1 and 2 in each industry j, minus its share $1/N$ in the government's expenditures R. Given this and proposition 2, we obtain

$$A_\iota = w + \int_{j\in\Theta} s_{\iota j1}dj + \int_{j\notin\Theta}(s_{\iota j1} + s_{\iota j2})dj - \frac{R}{N}. \tag{23}$$

[6]This extends the idea of Wälde (1999a, 1999b).

We denote:

$\Omega\big(\{s_{\iota k \upsilon}\},\{t_k\}\big)$ the value of receiving profits $s_{\iota k \upsilon}$ from all firms υ in all industries k using current technology t_k.

$\Omega\big(\pi i_{\iota j\kappa},0,\{s_{\iota(k\neq j)\upsilon}\},t_j+1,\{t_{k\neq j}\}\big)$ the value of receiving the profit $\pi i_{\iota j\kappa}$ from firm κ in industry $j\notin\Theta$ using technology t_j+1, but receiving no profits from the other firm which was a leader in that industry when technology t_j was used, and receiving profits $s_{\iota(k\neq j)\upsilon}$ from all firms υ in other industries $k\neq j$ with current technology t_k.

$\Omega\big(\theta i_{\iota j1},\theta i_{\iota j2},\{s_{\iota(k\neq j)\upsilon}\},\{t_k\}\big)$ the value of receiving profits $\theta i_{\iota j\kappa}$ from firms $\kappa\in\{1,2\}$ in industry $j\in\Theta$, but receiving profits $s_{\iota(k\neq j)\upsilon}$ from all firms υ in the other industries $k\neq j$ with current technology t_k.

Household ι maximizes its utility (20) by its investment $\{S_{\iota j\kappa}\}$ subject to the stochastic processes (22), its budget constraint (21), the composition of its income, (23), and the determination of its relative investment shares, (18), given the arrival rates $\{\Lambda_{j\kappa},\Gamma_j\}$, the wage w, the consumption price P, the subsidies (τ_α,τ_β) and the government's expenditures R. This maximization leads to the Bellman equation[7]

$$\rho\Omega\big(\{s_{\iota k\upsilon}\},\{t_k\}\big)=\max_{S_{\iota j}\,\geq\,0\text{ for all }j}\Xi_\iota, \tag{24}$$

where

$$\begin{aligned}\Xi_\iota \doteq{}& C_\iota^\sigma+\int_{j\in\Theta}\Gamma_j\Big[\Omega\big(\theta i_{\iota j1},\theta i_{\iota j1},\{s_{\iota(k\neq j)\upsilon}\},\{t_k\}\big)-\Omega\big(\{s_{\iota k\upsilon}\},\{t_k\}\big)\Big]dj\\&+\int_{j\notin\Theta}\sum_{\kappa=1,2}\Lambda_{j\kappa}\Big[\Omega\big(\pi i_{\iota j\kappa},0,\{s_{\iota(k\neq j)\upsilon}\},t_j+1,\{t_{k\neq j}\}\big)-\Omega\big(\{s_{\iota k\upsilon}\},\{t_k\}\big)\Big]dj.\end{aligned} \tag{25}$$

Because $\partial C_\iota/\partial S_{\iota j\kappa}=-1/P$ by (21), the first-order conditions are given by

$$\Lambda_{j\kappa}\frac{d}{dS_{\iota j\kappa}}\Big[\Omega\big(\pi i_{\iota j\kappa},0,\{s_{\iota(k\neq j)\upsilon}\},t_j+1,\{t_{k\neq j}\}\big)-\Omega\big(\{s_{\iota k\upsilon}\},\{t_k\}\big)\Big]=\frac{\sigma}{P}C_\iota^{\sigma-1}\quad\text{for } j\notin\Theta\text{ and }\kappa\in\{1,2\}, \tag{26}$$

$$\Gamma_j\frac{d}{dS_{\iota j0}}\Big[\Omega\big(\theta i_{\iota j1},\theta i_{\iota j2},\{s_{\iota(k\neq j)\upsilon}\},\{t_k\}\big)-\Omega\big(\{s_{\iota k\upsilon}\},\{t_k\}\big)\Big]=\frac{\sigma}{P}C_\iota^{\sigma-1}\quad\text{for } j\in\Theta. \tag{27}$$

[7] Cf. Dixit and Pindyck (1994).

6 General equilibrium

The study focuses entirely on the households' stationary equilibrium in which the allocation of resources is invariable across technologies, and ignores the behaviour of the system during the transitional period before the equilibrium is reached. We try the solution that for each household ι the propensity to consume, h_ι, and the subjective interest rate r_ι are independent of income A_ι, i.e. $PC_\iota = h_\iota A_\iota$ and $\Omega = C_\iota^\sigma / r_\iota$. Inserting these tentative solutions into (24)-(27), and noting the equilibrium conditions of the labour market (8)-(15), we obtain $h_j = h$ and the following results (see Appendix A):

$$\begin{array}{ll} l_{j\kappa} = \ell_\beta \ \text{ for } j \notin \Theta, \\ l_{j0} = \ell_\alpha \ \text{ for } j \in \Theta, \end{array} \quad \frac{\ell_\alpha}{\ell_\beta} = \left[\frac{(1-\tau_\beta)\theta\gamma}{(1-\tau_\alpha)\pi\lambda\mu^\sigma} \right]^{1/(1-\xi)} \doteq \psi, \tag{28}$$

$$\Lambda_{j1} + \Lambda_{j2} = \frac{g}{(1-\alpha)\log\mu}, \quad l = \delta(\alpha, \psi) g^{1/\xi}, \quad \frac{\partial \delta}{\partial \alpha} > 0, \quad \frac{\partial \delta}{\partial \psi} > 0, \tag{29}$$

$$\rho + \frac{1-\mu^\sigma}{\log\mu} g = \frac{\mu^\sigma h z}{1-\tau_\beta}, \tag{30}$$

$$g(\alpha, \theta, \tau_\alpha, \tau_\beta), \quad \frac{\partial g}{\partial \tau_\alpha} < 0 \ \Leftrightarrow \ \frac{\partial g}{\partial \tau_\beta} > 0 \ \Leftrightarrow \ \frac{\partial g}{\partial \theta} > 0$$
$$\text{for } \tau_\alpha \approx 0, \ \tau_\beta \approx 0 \text{ and } wl < 1 + (\psi/2 - 1)\alpha, \tag{31}$$

where $z \doteq (\pi/w)\lambda\ell_\beta^{\xi-1}$ is the rate of return to investment in innovative R&D.

Result (28) says that with a higher imitation subsidy τ_α, a lower innovation subsidy τ_β or a higher profit θ in two-leader industries $j \notin \Theta$, R&D firms spend relatively more in imitation (i.e. a higher ℓ_α/ℓ_β). With a uniform R&D subsidy $\tau_\alpha = \tau_\beta = \tau$, the relative investment in imitation is independent of taxation. Result (29) says that the more there is imitation (i.e. the bigger α) or the more an imitating firm invests relative to an innovating firm (i.e. the bigger $\ell_\alpha/\ell_\beta \doteq \psi$), the larger R&D input l is needed to produce a given growth rate g. The effect of taxation on the average growth rate g is unfortunately ambiguous [Cf. (31)]. First, a higher growth rate g increases both the demand for labour in R&D, l, and the wage w, which decreases the right-hand side of the equation (30). Second, a higher growth rate decreases a household's discount factor $\rho + \frac{1-\mu^\sigma}{\log\mu} g$ and the left-hand side of (30). We assume that the first effect dominates over the second and conclude:

Proposition 4 *If the proportion of imitating industries, α, is kept constant,* *then, on rather general conditions,*[8] *a small innovation subsidy* τ_β *promotes growth,* $\partial g/\partial\tau_\beta > 0$*, but a small imitation subsidy* τ_α *and an increase in PMC (i.e. a lower* θ*) slow down growth.*

The imitation subsidy encourages R&D firms to transfer their resources from innovation to imitation, which hampers growth. PMC increases employment in production, the wage and the cost of R&D, which deters growth. Imitation increases the proportion of industries with "neck-to-neck" competition. In Aghion et al. (1997) and Aghion et al. (2001), firms innovate to escape "neck-to-neck" competition and consequently imitation increases growth. In our model, investment is financed by the issue of shares. Because household even out risk by diversifying their portfolios, they hold shares in both firms competing "neck-to-neck". Hence, only the sum of R&D investment in these two firms taken together is relevant for becoming the owner of the new technology and households have no reason to support one firm against the other. Because PMC decreases the profits of both of the competing firms, it decreases incentives to invest in these.

To close the system, we must now specify how the proportion of imitating industries, α, is determined. When innovation occurs in an industry, this industry switches from the group of two-leader industries to that of one-leader industries, and when imitation occurs in an industry, this industry switches from one-leader industries to two-leader industries. In a steady-state equilibrium, every time a new superior-quality product is discovered in some industry, imitation must occur in some other industry.[9] Hence, the rate at which industries leave the group of two-leader industries, $\beta(\Lambda_{j1}+\Lambda_{j2})d\nu$, is equal to the rate at which industries leave the group of one-leader industries, $\alpha\Gamma_j d\nu$. From this, (7) and (28) it follows that

$$2\beta\lambda\ell_\beta^\xi = \alpha\gamma\ell_\alpha^\xi = \alpha\gamma\psi^\xi\ell_\beta^\xi, \quad \alpha/(1-\alpha) = \alpha/\beta = 2\lambda\psi^{-\xi}/\gamma$$

and

$$\psi = \Psi(\alpha) \doteq \Big(\frac{2\lambda}{\gamma}\Big)^{1/\xi}\Big(\frac{1}{\alpha}-1\Big)^{1/\xi} \text{ with } \Psi' < 0. \tag{32}$$

[8] Because $\alpha \in [0,1]$, the ratio of R&D expenditure to total consumption expenditure, $wl = wl/(Py)$, is a small number and the imitating firms cannot invest very much more than innovating firms (i.e. $\psi = \ell_\alpha/\ell_\beta$ cannot be very much greater than one), the condition $wl < 1 + (\psi/2-1)\alpha$ in (31) holds very likely.

[9] Cf. Segerstrom (1991), p. 817.

This result says that the more an imitating firm invests relative to an innovating firm (i.e. the larger $\ell_\alpha/\ell_\beta \doteq \psi$), the shorter time industries spend in the imitating stage and the smaller the proportion α of imitating industries.

7 The government

The symmetry across the households $\iota = 1, ..., n$ yields $C_\iota = y/N$. Noting $C_\iota = y/N$, (8), (15), (29) and (32), a single household's consumption relative to the level of productivity, c, can be written as follows:

$$c(g,\alpha,\theta) \doteq \frac{C_\iota}{B^{\{t_k\}}} = \frac{y}{NB^{\{t_k\}}} = \frac{x}{N}\chi = \chi(\alpha,\theta)\left[1 - \frac{1}{N}\delta(\alpha,\Psi(\alpha))g^{1/\xi}\right],$$
$$\frac{\partial c}{\partial g} = -\frac{\chi}{N}\frac{l}{\xi g} = -\frac{cl}{\xi x g} < 0. \tag{33}$$

Given this, a single household's utility function (20) takes the form

$$U(C_\iota, T) = E\int_T^\infty c(g,\alpha,\theta)^\sigma \left(B^{\{t_k\}}\right)^\sigma e^{-\rho(\nu - T)} d\nu. \tag{34}$$

The government controls the level of profits in two-leader industries, θ, by competition policy and the growth rate g and the proportion of imitating industries, α, by subsidies $(\tau_\alpha, \tau_\beta)$. Hence, we can define θ, g and α as the government's instruments in welfare maximization. We consider two cases:

(a) *First-best policy.* Where the government can discriminate between innovation and imitation, $\tau_\alpha \neq \tau_\beta$, it controls (g, α) by subsidies $(\tau_\alpha, \tau_\beta)$ and maximizes social welfare (34) by the growth rate g, the proportion of imitating industries, α, and the profit in two-leader industries, θ.

(b) *Second-best policy.* Where the government cannot discriminate between innovation and imitation, $\tau_\alpha = \tau_\beta = \tau$, it controls (g, α) by the subsidy τ and competition policy θ, observes the negative relationship of α and β through (32) and maximizes (34) by g and α.

We denote:

$\Upsilon(\{t_k\})$ the value of each industry k using current technology t_k.

$\Upsilon\big(t_j + 1, \{t_{k\neq j}\}\big)$ the value of industry j using technology $t_j + 1$, but other industries $k \neq j$ using current technology t_k.

Both maximization problems (a) and (b) lead to the same Bellman equation

$\rho\Upsilon(t)=\max\mathcal{F}$, where

$$\mathcal{F}\doteq c(g,\alpha,\theta)^{\sigma}\big(B^{\{t_k\}}\big)^{\sigma}+\int_{j\notin\Theta}\Lambda_j\big[\Upsilon\big(t_j+1,\{t_{k\neq j}\}\big)-\Upsilon\big(\{t_k\}\big)\big]dj$$
$$=c(g,\alpha,\theta)^{\sigma}\big(B^{\{t_k\}}\big)^{\sigma}+\frac{g}{(1-\alpha)\log\mu}\int_{j\notin\Theta}\big[\Upsilon\big(t_j+1,\{t_{k\neq j}\}\big)-\Upsilon\big(\{t_k\}\big)\big]dj. \tag{35}$$

Noting (8), (33) and (35), we obtain the following result:

Proposition 5 *In the first-best case* (a)*, collusion is socially optimal,*

$$\theta=\arg\max_{\theta}\mathcal{F}=\arg\max_{\theta}c=\arg\max_{\theta}\chi=\frac{\pi}{2}.$$

In the second-best case (b)*, there must not be collusion,* $\theta<\pi/2$*, and PMC (i.e. a smaller* θ*) decreases consumption,* $\partial c/\partial\theta>0$*.*

Product market competition causes inefficiency and decreases output in the production of consumption goods [cf. proposition 3].

In the second-best case (b) with $\tau_\alpha=\tau_\beta$, inserting (32) into (28) yields

$$\theta(\alpha)\doteq\pi\lambda\mu^{\sigma}\psi^{1-\xi}/\gamma=\pi\lambda\mu^{\sigma}\Psi(\alpha)^{1-\xi}/\gamma\ \text{ with }\theta'<0. \tag{36}$$

This says that the higher the imitator's profit obtain after a successful imitation, the more there are incentives to imitate, the shorter time industries spend in the imitating stage and the smaller the proportion α of imitating industries. Noting the function (36), we define the elasticity of total consumption c with respect to the proportion of imitating industries, α, when the arrival rate of innovations, g, is kept constant, as follows:[10]

$$\eta(g,\alpha,\theta)\doteq-\frac{\alpha}{c}\frac{dc}{d\alpha}=\begin{cases}-\frac{\alpha}{c}\frac{\partial c}{\partial\alpha} & \text{with }\tau_\alpha\neq\tau_\beta,\\ -\frac{\alpha}{c}\left[\frac{\partial c}{\partial\alpha}+\frac{\partial c}{\partial\theta}\frac{d\theta}{d\alpha}\right] & \text{with }\tau_\alpha=\tau_\beta=\tau.\end{cases} \tag{37}$$

Hence, noting proposition 5, we obtain the following result:

Proposition 6 *A shift from the first-best to the second-best optimum can be expressed by a discrete increase in the elasticity* η*:*

$$\eta\Big(g,\alpha,\frac{\pi}{2}\Big)=-\frac{\alpha}{c}\frac{\partial c}{\partial\alpha}<-\frac{\alpha}{c}\Big[\frac{\partial c}{\partial\alpha}+\frac{\partial c}{\partial\theta}\frac{d\theta}{d\alpha}\Big]=\eta\big(g,\alpha,\theta(\alpha)\big).$$

In other respects, cases (a) and (b) can be examined in the same framework.

[10]With this definition, the elasticity η will be positive under optimal public policy.

8 Optimal policy

The government chooses the average growth rate g and the proportion of one-leader industries, $\alpha \in [0,1]$, to maximize social welfare (34). Noting (33), the first-order conditions for g and α are given by

$$\frac{\partial \mathcal{F}}{\partial g} = \sigma c^{\sigma-1}\left(B^{\{t_k\}}\right)^{\sigma}\frac{\partial c}{\partial g} + \frac{1}{(1-\alpha)\log\mu}\int_{j\notin\Theta}\left[\Upsilon\left(t_j+1,\{t_{k\neq j}\}\right) - \Upsilon\left(\{t_k\}\right)\right]dj = 0, \tag{38}$$

$$\frac{\partial \mathcal{F}}{\partial \alpha} = \sigma c^{\sigma-1}\left(B^{\{t_k\}}\right)^{\sigma}\frac{dc}{d\alpha} + \frac{g}{(1-\alpha)^2\log\mu}\int_{j\notin\Theta}\left[\Upsilon\left(t_j+1,\{t_{k\neq j}\}\right) - \Upsilon\left(\{t_k\}\right)\right]dj = 0. \tag{39}$$

We try the solution

$$\Upsilon\left(\{t_k\}\right) \doteq \vartheta c^{\sigma}\left(B^{\{t_k\}}\right)^{\sigma}, \tag{40}$$

where ϑ is independent of the endogenous variables of the system. Noting this and (12), we obtain the optimal growth rate g^* and the optimal proportion of imitating industries, α^*, as follows (see Appendix B):

$$g^* = \frac{\rho\sigma\log\mu}{(\mu^{\sigma}-1)(\sigma+\xi x/l)}, \quad \alpha^* = \frac{\eta}{\eta + l/x}. \tag{41}$$

We can rephrase these results as follows:

Proposition 7 *A small relative proportion of workers in R&D, l/x, and a low rate of relative risk aversion (i.e. a small $1/(1-\sigma)$ and a small σ) predict slow economic growth. A large elasticity of consumption with respect to the proportion of imitating industries, η, and a small relative proportion of workers in R&D, l/x, predict a high proportion α of imitating industries.*

Finally, given (6) and (41), we obtain the following result:

Proposition 8 *A shift from the first-best with $\tau_\alpha \neq \tau_\beta$ to the second-best with $\tau_\alpha = \tau_\beta$ increases the proportion α of imitating industries.*

Hence, PMC (i.e. the decrease of θ below the level of collusion) is a strategic substitute for discriminating R&D subsidies. It and the imitation tax both decrease the expected rate of return for imitation and the proportion α.

Inserting $g = g^*$ from (41) into (30) yields the following result:

Proposition 9 *The welfare-maximizing subsidy to innovation is given by*

$$\tau_\beta^* = 1 - \frac{\mu^\sigma}{\rho}\Big(\frac{\sigma}{\xi}\frac{l}{x} + 1\Big)hz.$$

The innovation subsidy should be the higher, the lower the propensity to consume h, the average rate of return to innovative investment, z, or the relative proportion of workers in R&D, l/x, or the more elastic consumption is with respect to imitation (i.e. the higher η).

Inserting (32) and (41) into (28), and noting proposition 5, we obtain:

Proposition 10 *If the government can discriminate between innovation and imitation, $\tau_\alpha \neq \tau_\beta$, the welfare-maximizing subsidy to imitation is given by*

$$\tau_\alpha^* = 1 - (1 - \tau_\beta)\mu^{-\sigma}\Big(\frac{\gamma}{2\lambda}\Big)^{1/\xi}\Big(\frac{l}{x\eta}\Big)^{1-1/\xi}.$$

The imitation subsidy should be the higher relative to the innovation subsidy, the lower the relative proportion of workers in R&D, l/x, or the more elastic consumption is with respect to imitation (i.e. the higher η).

In the case of second-best policy with $\tau_\alpha = \tau_\beta$, noting (28), (32) and (41), the optimal level of profits in the two-leader industries $j \notin \Theta$ is given by

$$\theta^* = 2^{1/\xi - 1}\mu^\sigma \pi\Big(\frac{\lambda}{\gamma}\Big)^{1/\xi}\Big(\frac{l}{x\eta}\Big)^{1/\xi - 1}.$$

When a R&D firm succeeds in imitation and enter the market as the second leader, the profit in the industry falls from π to θ. Therefore, we can define the relative fall of profit in the advent of imitation as follows:

$$b \doteq \frac{\pi - \theta^*}{\pi} = 1 - \frac{\theta^*}{\pi} = 1 - 2^{1/\xi - 1}\Big(\frac{\lambda}{\gamma}\Big)^{1/\xi}\mu^\sigma\Big(\frac{l}{x\eta}\Big)^{1/\xi - 1}. \tag{42}$$

The increase in PMC (i.e. a decrease in θ) is welfare enhancing (reducing) for $\theta \geq \theta^*$ and $(\pi - \theta)/\pi \leq b$ $\big(\theta \leq \theta^*$ and $(\pi - \theta)/\pi \geq b\big)$. We rephrase our last result as follows:

Proposition 11 *If the government cannot discriminate between innovation and imitation, $\tau_\alpha = \tau_\beta$, then it should increase (decrease) PMC as long as* *in the advent of imitation profits fall less (more) than* $100 \cdot b$ *percentages, where* b *is defined by* (42)*. The easier it is to imitate (i.e. the higher* γ*), the lower the relative proportion of workers in R&D,* l/x*, or the more elastic consumption is with respect to imitation (i.e. the higher* η*), the higher* b *and the more likely PMC increases welfare.*

9 Conclusions

This paper examines an economy where growth is generated by creative destruction: a firm creating the newest technology by a successful R&D project crowds out the other firms with older technologies from the market so that the latter lose their value. A research firms can innovate to produce better versions of the products or imitate to copy existing innovations. Firms finance their R&D by issuing shares, and households save only in these shares. The government subsidizes R&D and promotes or hampers product market competition (PMC). The main findings of this paper are as follows.

There are at most two leaders in an industry. If a firm attempted to enter an industry with two leaders already, then its prospective profits would be lower than in industries with one leader only. In one-leader industries outsiders imitate and in two-leader industries both leaders innovate. For a given proportion of imitating industries, a small innovation subsidy promotes growth, but a small imitation subsidy and an increase in PMC (i.e. a lower θ) slow down growth. The imitation subsidy encourages R&D firms to transfer their resources from innovation to imitation, which hampers growth.

PMC increases employment in production, the wage and the cost of R&D, which discourages growth. Imitation increases the proportion of industries with "neck-to-neck" competition. In Aghion et al. (1997) and Aghion et al. (2001), firms innovate to escape "neck-to-neck" competition and consequently imitation increases growth. In this study, R&D is financed by the issue of shares. Because household even out risk by diversifying portfolios, they hold shares in both firms in "neck-to-neck" competition. Hence, only the sum of R&D investment in these firms taken together is relevant for becoming the owner of the new technology and households have no reason to support one firm against the other. Because PMC decreases the profits of both firms, it decreases incentives to invest in these.

This paper examines also optimal policy in two cases: the first best, in which case the government can discriminate between innovation and imitation; and the second best, in which case imitation and innovation expenditures are subject to the same subsidy rate. In the first-best case collusion is socially optimal. A shift from the first-best to the second-best optimum involves a discrete increase in the proportion of imitating industries. Because PMC decreases profits in two-leader industries and thereby discourages imitation, it is a strategic substitute for a tax to imitation.

If the governments are rational, a small relative proportion of workers in R&D or a low rate of relative risk aversion predicts slow economic growth. A large elasticity of consumption with respect to the proportion of imitating industries or a small relative proportion of workers in R&D predict a high proportion of imitating industries. Innovation should be subsidized the more, the lower the propensity to consume, the average rate of return to innovative investment, or the relative proportion of workers in R&D, or the more elastic consumption is with respect to imitation.

If the government can discriminate between innovation and imitation, then imitation should be the more subsidized relative to innovation, the lower the relative proportion of workers in R&D, or the more elastic consumption with respect to imitation. If such discrimination is impossible, then the government should increase PMC as long as the resulting fall in profits in the advent of imitation is small enough. The lower the relative proportion of workers in R&D or the more elastic consumption is with respect to imitation, the more likely PMC increases welfare.

Appendix

A. Results (28)-(31)

Let us denote variables depending on technology t_k by superscript t_k. Since according to (23) income $A_\iota^{\{t_k\}}$ depends directly on variables $\{s_{\iota k}^{t_k}\}$, we denote $A_\iota^{\{t_k\}}(\{s_{\iota k}^{t_k}\})$. Assuming that h_ι is invariant across technologies yields

$$P^{\{t_k\}}C_\iota^{\{t_k\}} = h_\iota A_\iota^{\{t_k\}}(\{s_{\iota k}^{t_k}\}). \tag{43}$$

The share in the next innovator t_j+1 is determined by investment under the present technology t_j, $s^{t_j+1}_{\iota j\kappa} = \pi i^{t_j}_{\iota j\kappa}$ for $j \notin \Theta$. The share in the next imitator is determined by investment under the same technology t_j, $s^{t_j}_{\iota j\kappa} = \theta i^{t_j}_{\iota j\kappa}$ for $j \in \Theta$. The value functions are then given by

$$\begin{aligned}
&\Omega\big(\{s_{\iota k v}\},\{t_k\}\big) = \Omega\big(\theta i_{\iota j1}, \theta i_{\iota j2}, \{s_{\iota(k\neq j)v}\},\{t_k\}\big) = \frac{1}{r_\iota}\big(C_\iota^{\{t_k\}}\big)^\sigma, \\
&\Omega\big(\pi i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)v}\}, t_j+1, \{t_{k\neq j}\}\big) = \frac{1}{r_\iota}\big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^\sigma. \qquad (44)
\end{aligned}$$

Given this, we obtain

$$\frac{\partial\Omega\big(\{s_{\iota k v}\},\{t_k\}\big)}{\partial S^{t_j}_{\iota j}} = 0. \qquad (45)$$

From (18), (23), (43), (44), $s^{t_j+1}_{\iota j\kappa} = \pi i^{t_j}_{\iota j\kappa}$ for $j \notin \Theta$, and $s^{t_j}_{\iota j\kappa} = \theta i^{t_j}_{\iota j\kappa}$ for $j \in \Theta$ it follows that

$$\frac{\partial s^{t_j+1}_{\iota j\kappa}}{\partial i^{t_j}_{\iota j\kappa}} = \pi \ \text{ for } j\notin\Theta, \quad \frac{\partial s^{t_j}_{\iota j0}}{\partial i^{t_j}_{\iota j0}} = \theta \ \text{ for } j\in\Theta, \quad \frac{\partial A_\iota^{t_j+1,\{t_{k\neq j}\}}}{\partial s^{t_j+1}_{\iota j\kappa}} = \frac{\partial A_\iota^{\{t_k\}}}{\partial s^{t_j}_{\iota j\kappa}} = 1,$$

$$\frac{\partial i^{t_j}_{\iota j0}}{\partial S^{t_j}_{\iota j0}} = \frac{1}{(1-\tau_\alpha)w^{\{t_k\}}l^{\{t_k\}}_{j0}} \ \text{ for } j\in\Theta, \quad \frac{\partial i^{t_j}_{\iota j\kappa}}{\partial S^{t_j}_{\iota j\kappa}} = \frac{1}{(1-\tau_\beta)w^{\{t_k\}}l^{\{t_k\}}_{j\kappa}} \ \text{ for } j\notin\Theta,$$

$$\begin{aligned}
&\frac{\partial\Omega\big(\pi i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)v}\}, t_j+1, \{t_{k\neq j}\}\big)}{\partial S^{t_j}_{\iota j\kappa}} \\
&= \frac{\sigma}{r_\iota}\big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^{\sigma-1} \underbrace{\frac{\partial C_\iota^{t_j+1,\{t_{k\neq j}\}}}{\partial A_\iota^{t_j+1,\{t_{k\neq j}\}}}}_{h_\iota/P^{t_j+1,\{t_{k\neq j}\}}} \underbrace{\frac{\partial A_\iota^{t_j+1,\{t_{k\neq j}\}}}{\partial s^{t_j+1}_{\iota j\kappa}}}_{=1} \underbrace{\frac{\partial s^{t_j+1}_{\iota j\kappa}}{\partial i^{t_j}_{\iota j\kappa}}}_{=\pi} \frac{\partial i^{t_j}_{\iota j\kappa}}{\partial S^{t_j}_{\iota j\kappa}} \\
&= \frac{\pi\sigma h_\iota\big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^{\sigma-1}}{r_\iota P^{t_j+1,\{t_{k\neq j}\}}} \frac{\partial i^{t_j}_{\iota j\kappa}}{\partial S^{t_j}_{\iota j\kappa}} = \frac{\pi h_\iota\sigma\big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^{\sigma-1}}{(1-\tau_\beta)r_\iota w^{\{t_k\}}P^{t_j+1,\{t_{k\neq j}\}}l^{\{t_k\}}_{j\kappa}} \quad \text{for } j\notin\Theta, \qquad (46)
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial\Omega\big(\theta i_{\iota j1}, \theta i_{\iota j2}, \{s_{\iota(k\neq j)v}\},\{t_k\}\big)}{\partial S^{t_j}_{\iota j0}} = \frac{\sigma}{r_\iota}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1} \underbrace{\frac{\partial C_\iota^{\{t_k\}}}{\partial A_\iota^{\{t_k\}}}}_{=h_\iota/P^{\{t_k\}}} \underbrace{\frac{\partial A_\iota^{\{t_k\}}}{s^{t_j}_{\iota j0}}}_{=1} \underbrace{\frac{s^{t_j}_{\iota j0}}{\partial i^t_{\iota j0}}}_{=\theta} \frac{\partial i^t_{\iota j0}}{\partial S^t_{\iota j0}} \\
&= \frac{\theta\sigma h_\iota}{r_\iota P^{\{t_k\}}}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}\frac{\partial i^t_{\iota j0}}{\partial S^t_{\iota j0}} = \frac{\theta h_\iota\sigma\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}}{(1-\tau_\alpha)r_\iota w^{\{t_k\}}P^{\{t_k\}}l^{\{t_k\}}_{j0}} \quad \text{for } j\in\Theta. \qquad (47)
\end{aligned}$$

We focus on a stationary equilibrium where the average growth rate g and the allocation of labour, $(l_{j\kappa}, x)$, are invariant across technologies. Given (2), (8), (12) and (15), this implies

$$\begin{aligned}
&l_{j\kappa}^{\{t_k\}} = l_{j\kappa}, \quad x^{\{t_k\}} = x = N - l, \quad w^{\{t_k\}} = w = x/\varphi, \\
&\frac{P^{\{t_k\}}}{P^{t_j+1,\{t_{k\neq j}\}}} = \frac{C_\iota^{t_j+1,\{t_{k\neq j}\}}}{C_\iota^{\{t_k\}}} = \frac{A_\iota^{t_j+1,\{t_{k\neq j}\}}}{A_\iota^{\{t_k\}}} = \frac{y^{t_j+1,\{t_{k\neq j}\}}}{y^{\{t_k\}}} = \frac{B^{t_j+1,\{t_{k\neq j}\}}}{B^{\{t_k\}}} = \mu.
\end{aligned} \tag{48}$$

Inserting (25), (43), (44), (48) and $g \doteq \int_{j\notin\Theta} l_j dj$ into equation (24) yields

$$\begin{aligned}
0 &= \Big[\rho + \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj + \int_{j\in\Theta} \Gamma_j dj\Big] \Omega\big(\{s_{\iota k v}\}, \{t_k\}\big) - \big(C_\iota^{\{t_k\}}\big)^\sigma \\
&\quad - \int_{j\notin\Theta} \sum_{\kappa=1,2} \Lambda_{j\kappa} \Omega\big(\pi i_{\iota j\kappa}, 0, \{s_{\iota(k\neq j)v}\}, t_j + 1, \{t_{k\neq j}\}\big) dj \\
&\quad - \int_{j\in\Theta} \Gamma_j \Omega\big(\theta i_{\iota j1}, \theta i_{\iota j2}, \{s_{\iota(k\neq j)v}\}, \{t_k\}\big) dj \\
&= \Big[\rho + \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj\Big] \frac{\big(C_\iota^{\{t_k\}}\big)^\sigma}{r_\iota} - \big(C_\iota^{\{t_k\}}\big)^\sigma \\
&\quad - \int_{j\notin\Theta} \sum_{\kappa=1,2} \frac{\Lambda_{j\kappa}}{r_\iota} \big(C_\iota^{\{t_j+1\},\{t_{k\neq j}\}}\big)^\sigma dj \\
&= \Big[\rho + \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj\Big] \frac{\big(C_\iota^{\{t_k\}}\big)^\sigma}{r_\iota} - \big(C_\iota^{\{t_k\}}\big)^\sigma - \int_{j\notin\Theta} \sum_{\kappa=1,2} \Lambda_{j\kappa} \frac{\mu^\sigma}{r_\iota} \big(C_\iota^{\{t_k\}}\big)^\sigma dj \\
&= \frac{1}{r_\iota} \big(C_\iota^{\{t_k\}}\big)^\sigma \Big[\rho + (1 - \mu^\sigma) \int_{j\notin\Theta} (\Lambda_{j1} + \Lambda_{j2}) dj - r_\iota\Big] \\
&= \frac{1}{r_\iota} \big(C_\iota^{\{t_k\}}\big)^\sigma \Big[\rho - r_\iota + \frac{1 - \mu^\sigma}{\log\, \mu} g\Big].
\end{aligned}$$

This equation is equivalent to

$$r_\iota = \rho + \frac{1 - \mu^\sigma}{\log\, \mu} g. \tag{49}$$

Because there is symmetry throughout all households ι, their propensity to consume is equal, $h_\iota = h$. From $h_\iota = h$, (16), (17), (19), (21), (23) and

(43) it follows that

$$\begin{aligned} wl - R &= w\int_{j\in\Theta} l_{j0}dj + w\int_{j\notin\Theta}(l_{j1}+l_{j2})dj - R \\ &= (1-\tau_\alpha)w\int_{j\in\Theta} l_{j0}dj + (1-\tau_\beta)w\int_{j\notin\Theta}(l_{j1}+l_{j2})dj \\ &= \sum_{\iota=1}^{N}\left[\int_{j\in\Theta} S_{\iota j0}dj + \int_{j\notin\Theta}(S_{\iota j1}+S_{\iota j2})dj\right] = \sum_{\iota=1}^{N}(A_\iota - PC_\iota) \\ &= (1-h)\sum_{\iota=1}^{N}A_\iota = (1-h)(1+wl-R). \end{aligned}$$

Solving for the propensity to consume, we obtain

$$h_\iota = h = \frac{1-R}{1+wl}. \tag{50}$$

Given (8) and (15), we obtain the wage

$$w = \frac{\varphi}{x} = \frac{\varphi(\alpha,\theta)}{N-l}. \tag{51}$$

Inserting (9), (10), (45), (46) and (47) into (26) and (27), we obtain

$$\begin{aligned} &\frac{\pi h\lambda\sigma\mu^\sigma\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}l_{j\kappa}^{\xi-1}}{(1-\tau_\beta)\Big(\rho+\frac{1-\mu^\sigma}{\log\mu}g\Big)wP^{\{t_k\}}} = \frac{\sigma\pi h_\iota\Lambda_{j\kappa}\big(C_\iota^{t_j+1,\{t_{k\neq j}\}}\big)^{\sigma-1}}{(1-\tau_\beta)r_\iota wl_{nj}P^{t_{t_j+1},\{t_{k\neq j}\}}} \\ &= \Lambda_{j\kappa}\frac{d}{dS_{\iota j\kappa}}\Omega\big(\pi i_{\iota j},\{s_{\iota(k\neq j)}\},t_j+1,\{t_{k\neq j}\}\big) = \frac{\sigma}{P^{\{t_k\}}}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1} \\ &\text{for } j\notin\Theta \text{ and } \kappa\in\{1,2\}, \end{aligned} \tag{52}$$

$$\begin{aligned} &\frac{\theta h\gamma\sigma\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}l_{j0}^{\xi-1}}{(1-\tau_\alpha)\Big(\rho+\frac{1-\mu^\sigma}{\log\mu}g\Big)wP^{\{t_k\}}} = \frac{\sigma\theta h_\iota\Gamma_j\big(C_\iota^{\{t_k\}}\big)^{\sigma-1}}{(1-\tau_\alpha)r_\iota wl_{mj}P^{\{t_k\}}} \\ &= \Gamma_j\frac{d}{dS_{\iota j0}}\Omega\big(\{\theta i_{\iota j1},\theta i_{\iota j2},\{s_{\iota m(k\neq j)}\},\{t_k\}\big) = \frac{\sigma}{P^{\{t_k\}}}\big(C_\iota^{\{t_k\}}\big)^{\sigma-1} \text{ for } j\in\Theta. \end{aligned} \tag{53}$$

Equations (52) and (53) yield

$$\begin{aligned} &l_{j\kappa} = \ell_\beta \text{ for } j\notin\Theta, \quad \frac{\ell_\alpha}{\ell_\beta} = \left[\frac{(1-\tau_\beta)\theta\gamma}{(1-\tau_\alpha)\pi\lambda\mu^\sigma}\right]^{1/(1-\xi)} \doteq \psi(\theta,\tau_\alpha,\tau_\beta), \\ &l_{j0} = \ell_\alpha \text{ for } j\in\Theta, \\ &\partial\psi/\partial\theta>0,\quad \partial\psi/\partial\tau_\alpha<0,\quad \partial\psi/\partial\tau_\beta>0,\quad [\partial\psi/\partial\tau]_{\tau_\alpha=\tau_\beta=\tau}=0. \end{aligned} \tag{54}$$

The total cost of innovating firm κ is the wage times labour input in innovation, $wl_{j\kappa}$. Because in the advent of innovation that firm obtains the monopoly profit π, the expected revenue for innovation is the profit times the arrival rate of innovations, $\pi\Lambda_{j\kappa}$. Dividing this by total cost $wl_{j\kappa}$ yields the rate of return to investment in innovating firm κ, $\pi\Lambda_{j\kappa}/(wl_{j\kappa})$. Noting this, (7), (9) and (54), we obtain the rate of return to innovative R&D as:

$$z = \frac{\pi\Lambda_{j\kappa}}{wl_{j\kappa}} = \frac{\pi}{w}\lambda l_{j\kappa}^{\xi-1} = \frac{\pi}{w}\lambda \ell_{\beta}^{\xi-1} \text{ for all innovative firms } \kappa \notin \Theta. \tag{55}$$

From (7), (9), (13), (14), (16), (50), (51) and (55), it follows that

$$\begin{aligned}
& l = \alpha\ell_\alpha + 2(1-\alpha)\ell_\beta = [\alpha\psi + 2(1-\alpha)]\ell_\beta, \\
& R = \tau_\alpha w\ell_\alpha + 2\tau_\beta w\ell_\beta = (\tau_\alpha\psi + 2\tau_\beta)w\ell_\beta = \frac{\tau_\alpha\psi + 2\tau_\beta}{\alpha\psi + 2(1-\alpha)}wl, \\
& h = \frac{1}{1+wl}\Big[1 - \frac{\tau_\alpha\psi + 2\tau_\beta}{\alpha\psi + 2(1-\alpha)}wl\Big], \\
& g \doteq (\log\mu)(1-\alpha)(\Lambda_{j1} + \Lambda_{j2}) = (2\lambda\log\mu)(1-\alpha)\ell_\beta, && (56) \\
& \frac{l}{g^{1/\xi}} = \frac{l/\ell_\beta}{(2\lambda\log\mu)^{1/\xi}(1-\alpha)^{1/\xi}} = \frac{\alpha\psi/(1-\alpha)+2}{(2\lambda\log\mu)(1-\alpha)^{1/\xi-1}} \doteq \delta(\alpha,\psi), \\
& \partial\delta/\partial\alpha > 0, \ \ \partial\delta/\partial\psi > 0, && (57) \\
& \rho + \frac{1-\mu^\sigma}{\log\,\mu}g = \frac{\pi h\lambda\mu^\sigma l_{j\kappa}^{\xi-1}}{(1-\tau_\beta)w} = \frac{h\mu^\sigma z}{1-\tau_\beta} \\
& = \frac{\pi}{1-\tau_\beta}\frac{\lambda\mu^\sigma}{1+wl}\left[\frac{1}{w} - \frac{\tau_\alpha\psi + 2\tau_\beta}{\alpha\psi + 2(1-\alpha)}l\right]\big[\alpha\psi + 2(1-\alpha)\big]^{1-\xi} l^{\xi-1} \\
& = \frac{\pi\lambda\mu^\sigma}{1-\tau_\beta}\frac{\big[\alpha\psi + 2(1-\alpha)\big]^{1-\xi} l^{\xi-1}}{1+\varphi(\alpha,\theta)l/(N-l)}\left[\frac{N-l}{\varphi(\alpha,\theta)} - \frac{\tau_\alpha\psi(\theta,\tau_\alpha,\tau_\beta) + 2\tau_\beta}{\alpha\psi(\theta,\tau_\alpha,\tau_\beta) + 2(1-\alpha)}l\right] \\
& \doteq \nabla(l,\alpha,\theta,\tau_\alpha,\tau_\beta) \text{ with } \frac{\partial\nabla}{\partial l} < 0, \ \frac{\partial\nabla}{\partial\tau_\beta} > 0, \ \frac{\partial\nabla}{\partial\tau_\alpha} < 0 \text{ and } \frac{\partial\nabla}{\partial\theta} > 0 \\
& \text{for } \tau_\alpha \approx 0, \ \tau_\beta \approx 0 \text{ and } wl < 1 + (\psi/2-1)\alpha. && (58)
\end{aligned}$$

Equations (54) and (56)-(58) yield (28)-(30). Noting (57), equation (58) defines $g(\alpha,\theta,\tau_\alpha,\tau_\beta)$. Differentiating (58) totally and noting (57), we obtain

$$\Big[\underbrace{\frac{1-\mu^\sigma}{\log\,\mu}}_{-} - \underbrace{\frac{\partial\nabla}{\partial l}}_{-}\delta\Big]dg = \underbrace{\frac{\partial\nabla}{\partial\tau_\alpha}}_{-}d\tau_\alpha + \underbrace{\frac{\partial\nabla}{\partial\tau_\beta}}_{+}d\tau_\beta,$$

$$\frac{\partial g}{\partial\tau_\alpha} < 0 \ \ \Leftrightarrow \ \ \frac{\partial g}{\partial\tau_\beta} > 0 \ \ \Leftrightarrow \ \ \frac{\partial g}{\partial\theta} > 0.$$

B. Results (41)

Noting (12) and (40), we then obtain

$$\Upsilon\big(t_j+1,\{t_{k\neq j}\}\big)=\vartheta c^{\sigma}\big(B^{t_j+1,\{t_k\}}\big)^{\sigma}=\vartheta\mu^{\sigma}c^{\sigma}\big(B^{\{t_k\}}\big)^{\sigma}=\mu^{\sigma}\Upsilon\big(\{t_k\}\big). \quad (59)$$

Inserting (40) and (59) into the Bellman equation (35), we obtain

$$\begin{aligned}0&=c^{\sigma}\big(B^{\{t_k\}}\big)^{\sigma}+\frac{g/(1-\alpha)}{\log\mu}\int_{j\notin\Theta}\Big[\Upsilon\big(t_j+1,\{t_{k\neq j}\}\big)-\Upsilon\big(\{t_k\}\big)\Big]dj-\rho\Upsilon\big(\{t_k\}\big)\\&=\Upsilon\big(\{t_k\}\big)[1/\vartheta-\rho+(\mu^{\sigma}-1)g/(\log\mu)]\end{aligned}$$

and

$$1/\vartheta=\rho-(\mu^{\sigma}-1)g/(\log\mu)<\rho. \quad (60)$$

Given (33), (37)-(40), (59) and (60), we obtain

$$\begin{aligned}\frac{\partial\mathcal{F}}{\partial g}&=\sigma c^{\sigma-1}\big(B^{\{t_k\}}\big)^{\sigma}\frac{\partial c}{\partial g}+\frac{\mu^{\sigma}-1}{\log\mu}\Upsilon\big(\{t_k\}\big)=\Big(\frac{\sigma}{\vartheta c}\frac{\partial c}{\partial g}+\frac{\mu^{\sigma}-1}{\log\mu}\Big)\Upsilon\big(\{t_k\}\big)\\&=\Big(\frac{\mu^{\sigma}-1}{\sigma\log\mu}-\frac{l}{\vartheta\xi xg}\Big)\sigma\Upsilon\big(\{t_k\}\big)=\Big[\frac{\mu^{\sigma}-1}{\sigma\log\mu}-\Big(\rho-\frac{\mu^{\sigma}-1}{\log\mu}g\Big)\frac{l}{\xi xg}\Big]\sigma\Upsilon\big(\{t_k\}\big)\\&=0, \quad (61)\\\frac{\partial\mathcal{F}}{\partial\alpha}&=\sigma c^{\sigma-1}\big(B^{\{t_k\}}\big)^{\sigma}\frac{dc}{d\alpha}+\frac{(\mu^{\sigma}-1)g}{(1-\alpha)\log\mu}\Upsilon\big(\{t_k\}\big)\\&=\Big(\frac{\sigma}{c\vartheta}\frac{dc}{d\alpha}+\frac{\mu^{\sigma}-1}{\log\mu}\frac{g}{1-\alpha}\Big)\Upsilon\big(\{t_k\}\big)=\Big(\frac{1}{c}\frac{dc}{d\alpha}+\frac{\mu^{\sigma}-1}{\sigma\log\mu}\frac{\vartheta g}{1-\alpha}\Big)\frac{\sigma}{\vartheta}\Upsilon\big(\{t_k\}\big)\\&=\Big[-\frac{\eta}{\alpha}+\frac{l}{(1-\alpha)x}\Big]\frac{\sigma}{\vartheta}\Upsilon\big(\{t_k\}\big)=0. \quad (62)\end{aligned}$$

Noting (61), we obtain

$$g=\frac{\rho\sigma\log\mu}{(\mu^{\sigma}-1)(\sigma+\xi x/l)}.$$

Given (62), consumption must be negatively associated with the proportion of imitating industries, $\partial c/\partial\alpha<0$. Noting (37) and (62), we obtain $\eta>0$,

$$\frac{\alpha}{1-\alpha}=-\frac{x}{l}\frac{\alpha}{c}\frac{\partial c}{\partial\alpha}=\frac{x}{l}\eta \quad\text{and}\quad \alpha=\frac{\eta}{\eta+l/x}.$$

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