Optimal Taxation with Capital Accumulation and Wage Bargaining

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Abstract

This paper examines optimal taxation in an economy where wages are determined by collective bargaining and some or all households save in capital. With centralized bargaining, wage settlement is strategically before investment, but with decentralized bargaining vice versa. The main findings are the following. If bargaining is centralized and the government can optimally tax wages, employment and consumption, then capital income should not be taxed in the limit. With decentralized bargaining, capital income taxation is necessary for aggregate production efficiency. Specific optimal tax rules are derived for both cases of bargaining.

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1 Introduction

This paper considers how taxes should be optimally determined in a unionized economy with capital accumulation. Palokangas (1987 and 2000, Ch. 4) shows that in a static general equilibrium framework, aggregate production efficiency can be maintained even in the presence of labour unions as long as the government can set specific wage and employment taxes. In this study, we examine whether this result also holds true in a dynamic general equilibrium framework, in which private agents save capital and there is a strategic interdependence between investors and labour unions.

In a dynamic model with investment, aggregate production efficiency takes lines up with the Judd-Chamley assertion: capital income should be taxed at a non-zero rate.[1] Because capital functions as an intermediate good, appearing only in the production but not in the utility function, it should not be taxed, if there are enough instruments to separate consumption and production decisions. Chamley (2001) shows that this assertion critically depends on the existence of a perfect bond market, in which private agents take the interest rate as given, households save in bonds, and firms can finance any amount of investment by issuing bonds. Instead a perfect bond market, we assume here that households own shares in firms directly. This establishes a conflict between workers in a firm and households who invest in the firm.

In this study, we assume that there are two groups of households. The *capitalists* save and earn all profits and a fixed proportion α of all wages. The *non-capitalists* earn the rest $(1 - \alpha)$ of total wages and consume all of their income. The model is then an extension of two special cases: for $\alpha = 0$, Judd's (1985) case in which the capitalists earn only profits and do not work, while the workers earn only wages and do not save; and for $\alpha = 1$, Chamley's (1986) model of a representative agent who saves and earns both wages and profits. We use parameter α as a measure of income distribution.

Optimal taxation in a unionized economy is sensitive to the timing of investment and wage settlement. For this reason, we examine two cases:

(a) If wages are bargained at the level of a single firm and, in particular, if wage contracts are not legally binding, then investment is strategically

[1] Judd (1985), Chamley (1986) and Correia (1996).

before wage settlement. Because the investment by the firm is a sunk cost and the firm might not invest big amounts any more for several years, the union neglects the effect through investment and the firm knows that some of the profit of its investment will be appropriated by the union. The investor is a Stackelberg leader and takes in its plans the parties' optimal responses in wage settlement into account.

(b) If bargaining over wages concerns a large number of firms simultaneously and, in particular, if wage contracts are extended to cover also non-organized employers and employees, then wage settlement is strategically before investment. Because some of the firms will invest in near future, the parties in bargaining take also the effect through investment into account. The investor is a Stackelberg follower and takes the wage as given. It can expect that wages are not revised immediately after investment to expropriate the profits of investment.

We call case (a) *decentralized* and case (b) *centralized* bargaining, for convenience. By the comparison of these two cases, we can examine the importance of labour market institutions in the design of optimal policy.

Unfortunately, the Nash bargaining over the wage cannot be introduced into a dynamic game in which capital stock evolves over time.[2] Hence, in case (b) we must content ourselves with the model of a monopoly union.

Lancing (1999) shows that when the capitalists' utility is logarithmic and the government faces a balanced-budget constraint, the steady-state optimal tax on capital income is generally non-zero. This is because with logarithmic utility agents' optimal decisions depend solely on the current rate of return, not any future rates of return or taxes, and the government is short of useful policy instruments, because promises about future tax rates do not influence current allocations. We prove that in a unionized economy Lancing's result holds only in the very unlikely special case that all of the following four

[2] Wage bargaining can be modelled through a game where two parties make alternately offers to each other to share a pie of exogenous size. In a stationary state, the outcome of such a game can be expressed by a geometric mean of the parties' utilities [Cf. Binmore et al. (1986)]. In case (a) above, capital stock and the size of the pie are exogenous for the parties of bargaining and the model of wage bargaining can be used. In case (b), the size of the pie varies with investment and therefore it is endogenous for the parties in bargaining. For this reason, it is not consistent to use the bargaining model in case (b).

conditions are simultaneously satisfied: wage contracts are centralized, the capitalists' preferences are logarithmic, there are constant returns to scale in production, and the capitalists earn no labour income.

In this study, we assume also that the government can commit indefinitely to optimal policy with a balanced budget. This admittedy strong assumption is made only for tractability. The case of non-commitment has been examined e.g. in Benhabib and Rustichini (1997) and Phelan and Stacchetti (2001), and the role of debt policy with optimal taxation in Chari and Kehoe (1999) and Ljungqvist and Sargent (2004). In this study, these two extensions would have excessively complicated the already very complex model.

So far, the literature on optimal taxation with labour unions and the accumulation of physical capital is very slim. Aronsson et al. (2001) examine a shift of income taxation from labour to capital. In contrast to this paper, they however assume a wage-setting monopoly union which maximizes the utility of the representative household in the economy. Koskela and von Thadden (2002) show that capital income should be taxed at a non-zero rate. In contrast to this paper, they however assume a perfect capital market and that a labour union takes capital stock as given.

The remainder of this paper is organized as follows. Section 2 specifies technology, preferences and taxation. Section 3 and 4 present a dynamic game with decentralized wage bargaining so that the strategic order of decisions is taxation, investment and wage settlement. Correspondingly, sections 5 and 6 specify a dynamic game with centralized wage bargaining so that the order is taxation, wage settlement and investment. Both games are solved by backward induction[3] and they result in optimal tax rules.

2 Production, investment and taxation

We aggregate all products in the economy into a single good which is chosen as the numeraire. There is a fixed number J of similar industries producing this good. In each industry j, the representative firm (hereafter firm j) produces its output Y_j from its capital K_j and labour L_j through technology

$$Y_j = F(K_j, L_j), \quad F_K > 0, \quad F_L > 0, \quad F_{LL} < 0, \quad F_{KK} < 0, \tag{1}$$

[3]The Stackelberg solution for dynamic games is from Basar and Olsder (1989).

where subscript K (L) denotes partial derivatives with respect to K_j (L_j).

Unit labour cost for firm j is given by $v_j \doteq (1+\tau_W)w_j + \tau_L$, where w_j is the wage in firm j, τ_W the wage tax and τ_L the employment tax. Firm j takes its unit labour cost v_j and capital stock K_j as given and maximizes its profit $\pi_j = F(K_j, L_j) - v_j L_j - \mu K_j$ by labour input L_j, where the constant $\mu \in (0,1)$ is the rate of capital depreciation. This yields the profit function

$$\begin{aligned}
&\pi_j = \Pi(K_j, v_j) \doteq \max_{L_j}[F(K_j, L_j) - v_j L_j - \mu K_j] \text{ with the properties} \\
&\Pi_K \doteq \partial\Pi/\partial K_j = F_K - \mu, \quad \Pi_v \doteq \partial\Pi/\partial v_j = -L_j, \\
&\Pi_{KK}(K_j, v_j) \equiv 0 \;\Leftrightarrow\; \Pi(K_j, v_j) = \max_{\ell}[F(1,\ell) - v_j\ell - \mu]K_j = \Pi_K(v_j)K, \\
&v_j = F_L(K_j, L_j), \;\; w_j = [F_L(K_j, L_j) - \tau_L]/(1+\tau_W). \qquad (2)
\end{aligned}$$

The elasticity of the demand for labour with respect to unit labour cost v_j, when capital K_j is held constant, is given by

$$\varepsilon(K_j, L_j) \doteq \left|\frac{v_j}{L_j}\frac{\partial L_j}{\partial v_j}\right| \doteq -\frac{F_L(K_j, L_j)}{L_j F_{LL}(K_j, L_j)}. \qquad (3)$$

Each household is subject to the fixed cost b per unit of employment for the following reasons.[4] First, there are commuting costs in proportion to working days. Second, paid work decreases household's internal work which must be replaced by purchasing services from the goods market. We define labour income in industry j as wages minus the opportunity cost of employment, $W^j \doteq w_j L_j - bL_j = (w_j - b)L_j$. Noting (2), we obtain

$$\begin{aligned}
&W^j(L_j, K_j, \tau_W, \tau_L) = (w_j - b)L_j = \big\{[F_L(K_j, L_j) - \tau_L]/(1+\tau_W) - b\big\}L_j, \\
&\frac{\partial W^j}{\partial L_j} = \frac{F_{LL}L_j + F_L - \tau_L}{1+\tau_W} - b, \quad \frac{\partial W^j}{\partial K_j} = \frac{F_{KL}L_j}{1+\tau_W}. \qquad (4)
\end{aligned}$$

Total labour income in the economy is defined by

$$W \doteq \sum_j W^j = \sum_j (w_j - b)L_j. \qquad (5)$$

We assume that each capitalist invests only in a single industry, for tractability.[5] The representative capitalist in industry j (hereafter capitalist

[4]This is the simplest way of defining unemployment in the model.

[5]If capitalists invested in all industries, then the levels of investment for all industries would be 'bang-bang' controls and the model would be excessively complicated.

j) earns a fixed proportion α/J of total labour income in the economy, W, and takes this revenue as given. Its budget constraint is therefore given by

$$\dot{K}_j \doteq dK_j/dt = \alpha W/J + (1-\tau_K)\Pi(K_j, v_j) - (1+\tau_C)C_j, \tag{6}$$

where $\dot{K}_j$ is capital accumulation, $\alpha W/J$ exogenous labour income, $\Pi(K_j, v_j)$ profits, τ_K tax on capital income, C_j his consumption and τ_C consumption tax. Capitalist j's instantaneous utility is given by

$$U(C_j) \doteq \begin{cases} [C_j^{1-\sigma} - 1]/(1-\sigma) & \text{for } \sigma \in (0,1) \cup (1,\infty), \\ \log C_j & \text{for } \sigma = 1, \end{cases} \tag{7}$$

where the constant $1/\sigma$ is the intertemporal elasticity of substitution. The non-capitalists consume their entire income net of taxes, $(1-\alpha)W/\ (1+\tau_C)$, where $\tau_C > -1$ is the consumption tax. A representative non-capitalist's instantaneous utility is then given by the function

$$V\big((1-\alpha)W/(1+\tau_C)\big), \quad V' > 0, \quad V'' < 0. \tag{8}$$

The model integrates two common cases into the same framework. For $\alpha = 0$, the capitalists earn only profits and do not work [Cf. Judd (1985)]; and for $\alpha = 1$, all agents are similar [Cf. Chamley (1986)]. Hence, using the parameter α we can examine the effect of income distribution on optimal taxation in a unionized economy.

We assume that the whole population has the same constant rate of time preference, $\rho > 0$ and the social welfare function is a weighted average of the non-capitalists' and capitalists' utilities, (7) and (8):

$$\int_0^\infty \left[V\Big(\frac{1-\alpha}{1+\tau_C} W\Big) + \vartheta \sum_j U(C_j) \right] e^{-\rho t} dt, \tag{9}$$

where the constant $\vartheta > 0$ is the social weight of the capitalists. The fixed amount E of public expenditures is financed by taxing total consumption $\sum_j C_j + (1-\alpha)W/(1+\tau_C)$, profits $\sum_j \pi_j$, wages $\sum_j w_j L_j$ and employment $\sum_j L_j$. The government's budget constraint is therefore

$$E = \tau_C \left[\sum_j C_j + \frac{1-\alpha}{1+\tau_C} W \right] + \tau_K \sum_j \pi_j + \tau_W \sum_j w_j L_j + \tau_L \sum_j L_j, \tag{10}$$

where $\tau_K \leq 1$ is the tax on capital income, $\tau_W > -1$ the wage tax and τ_L the employment tax. We assume an upper limit $\upsilon \in [0, \infty)$ for the capital subsidy $-\tau_K$, for tractability, so that[6]

$$-\upsilon \leq \tau_K \leq 1. \tag{11}$$

Total capital accumulation $\sum_j \dot{K}_j$ is equal to total production $\sum_j F(K_j, L_j)$ minus the capitalists' consumption $\sum_j C_j$, the non-capitalists' consumption $(1-\alpha)W/(1+\tau_C)$, total employment costs $b\sum_j L_j$, public spending E and capital depreciation $\mu \sum_j K_j$:

$$\sum_j \dot{K}_j = \sum_j \Big[F(K_j, L_j) - C_j - \frac{1-\alpha}{1+\tau_C} W^j - bL_j - E - \mu K_j\Big]. \tag{12}$$

When (12) holds, the goods market is in equilibrium. Then, by Walras' law, the government budget is balanced and (10) holds as well.

3 Decentralized bargaining: agents

The workers in firm j are organized in union j. We consider a dynamic game where the strategic order of the players is (i) the government, which sets taxes, (ii) capitalist j, which invests, and finally (iii) firm j and union j, which bargain over the wage w_j. This game is solved by backward induction.

In wage bargaining, firm j attempts to maximize its owners' welfare and union j attempts to maximize its members' welfare. Because capital stock K_j is exogenous for firm j and union j, the outcome of bargaining can be obtained by maximizing the Generalized Nash product of the parties targets:

$$\begin{aligned}\Upsilon^j(L_j, K_j, \tau_W, \tau_L) &\doteq (W^j)^\delta \pi_j^{1-\delta} \\ &= \big\{[F_L(K_j, L_j) - \tau_L]/(1+\tau_W) - b\big\}^\delta L_j^\delta \, \Pi\big(K_j, F_L(K_j, L_j)\big)^{1-\delta},\end{aligned} \tag{13}$$

where the constant $\delta \in (0, 1]$ is the union's relative bargaining power. Because there is one-to-one correspondence from w_j to L_j through (2), the wage w_j can be replaced by employment L_j as the instrument of bargaining.

[6] Otherwise, the subsidy $-\tau_K$ could get an infinite value in the government's optimal optimal policy.

Noting this, (2), (3), (4) and (13), we obtain that employment and the unit labour cost are determined by

$$
\begin{aligned}
& L_j(K_j,\tau_W,\tau_L) \doteq \arg\max_{L_j} \Upsilon^j(L_j,K_j,\tau_W,\tau_L) \quad \Leftrightarrow \\
& [\tau_L+(1+\tau_W)b]/v_j = 1+\big[(1+\tau_W)(1/\delta-1)W_j/\pi_j-1\big]/\varepsilon(K_j,L_j), \\
& v_j(K_j,\tau_W,\tau_L) \doteq F_L(K_j,L_j(K_j,\tau_W,\tau_L)). \qquad (14)
\end{aligned}
$$

We define the elasticity of unit labour cost v_j with respect to capital stock K_j, when taxes (τ_W,τ_L) are kept constant, as follows:

$$
\epsilon(K_j,\tau_W,\tau_L) \doteq \frac{K_j}{v_j}\frac{\partial v_j}{\partial K_j}. \qquad (15)
$$

Capitalist j as chooses his consumption C_j to maximize the flow of utility starting at time zero, $\int_0^\infty U(C_j)e^{-\rho t}dt$, where t is time and $\rho>0$ the rate of time preference, subject to capital accumulation (6) and the firm's and the union's response (4) and (14), taking unit labour cost in the industry, v_j, and his own labour income $\alpha W/J$ as given. This maximization yields the Euler equation (see Appendix A)

$$
\frac{\dot{C}_j}{C_j} = \left\{(1-\tau_K)\left[F_K(K_j,L_j)-\mu-\frac{L_j}{K_j}v_j\epsilon(K_j,\tau_W,\tau_L)\right]-\rho\right\}\frac{1}{\sigma}. \qquad (16)
$$

Capitalist j can smooth its flow of consumption through investment in capital K_j. The Euler equation (16) shows how this consumption evolves.

4 Taxation with decentralized bargaining

Because there is perfect symmetry throughout industries $j=1,...,J$, we obtain $K_j=K$, $L_j=L$, $C_j=C$, $W^j=W$, $\xi_j=\xi$ and $\phi_j=\phi$. Noting this and (2), capital accumulation (12) and the Euler equation (16) become:

$$
\begin{aligned}
\dot{K} &= F(K,L)-C-(1-\alpha)W/(1+\tau_C)-bL-E/J-\mu K, \qquad (17)\\
\frac{\dot{C}}{C} &= \left\{(1-\tau_K)\left[F_K(K,L)-\mu-\frac{L}{K}v\epsilon\right]-\rho\right\}\frac{1}{\sigma}. \qquad (18)
\end{aligned}
$$

The government sets taxes $(\tau_C,\tau_L,\tau_W,\tau_K)$ to maximize social welfare (9) subject to the development of capital (17) and consumption (18) as well as constraints (11) on capital taxation. In Appendix B, we show that this maximization yields the steady-state conditions,

$$\vartheta C^{-\sigma} = \vartheta U' = V'\Big(\frac{1-\alpha}{1+\tau_C}W\Big), \quad \upsilon = F_L(K^*, L^*) = b, \quad F_K(K^*, L^*) = \rho + \mu, \tag{19}$$

and the adjustment of capital taxation,

$$\tau_K = -\upsilon \text{ for } K < K^*, \quad \tau_K = 1 \text{ for } K > K^*, \tag{20}$$

where L^* and K^* are the optimal employment and the optimal capital stock in the steady-state. The results (19) and (20) can be explained as follows.

With a higher wage tax τ_W, the labour union substitutes the wage by employment and the wage decreases. As a consequence, profits and capital income increase. This mechanism allows the government to efficiently distribute income so that at the optimum the ratio of the marginal utility of income for the non-capitalists and capitalists, V'/U', is equal to their relative social weight ϑ. The employment tax τ_L decreases the demand for labour, L. It should be set so that the marginal product of labour, F_L, is equal to the opportunity cost of employment, b. In the steady state, with these two optimal tax rules, the optimal capital stock K^* is determined so that marginal product of capital, F_K, is equal to the rate of time preference, ρ, plus the rate of capital depreciation, μ, and the marginal product of labour, F_L, is equal to the opportunity cost of employment, b. The government must encourage (discourage) investment as long as capital K is above (below) K^*.

With constant returns to scale, conditions $F_K(1, L/K) = F_K(K, L) = \rho + \mu$ and $F_L(1, L/K) = F_L(K, L) = b$ in (19) hold only if they accidentally define the same employment-capital ratio L/K. Hence, we conclude:[7]

Proposition 1 *Aggregate production efficiency can be maintained only when there are decreasing returns to scale in production.*

Inserting $\upsilon = F_L = b$ from (19) into the equilibrium condition (14), we can explicitly solve for the optimal employment subsidy:

[7] Because of the steady-state condition $\vartheta C^{-\sigma} = V'$ and the existence of fixed public expenditures E, there cannot be endogenous growth with constant returns to scale.

Proposition 2 *Employment should be subsidized at the rate*

$$-\tau_L = \{\tau_W + [1 + (1 - 1/\delta)(1 + \tau_W)W/\Pi]/\varepsilon\}v,$$

where τ_W is the wage tax, v unit labour cost, W/Π the ratio of labour income to profits, ε the elasticity of the demand for labour with respect to unit labour cost [Cf. (3)*], and δ union relative bargaining power.*

This subsidy changes the slope of the labour demand function and eliminates the effect of union power so that in equilibrium $F_L = b$ holds. In the special case of a monopoly union, $\delta = 1$, we obtain the classical rule that with a monopoly total subsidies relative to the price, $(-\tau_L - \tau_W v)/v$, must be equal to the inverse of the elasticity of the demand, $1/\varepsilon$. If relative union bargaining power δ is decreased below unity, then a smaller subsidy $-\tau_L$ is needed to eliminate the effect of union power, $\partial(-\tau_L)/\partial\delta > 0$.

In Appendix B, we show furthermore the following result:

Proposition 3 *With decentralized wage bargaining, capital income should be subsidized at the rate $-\tau_K = \left(1 - \frac{vL}{\rho K}\epsilon\right)^{-1} - 1 > 0$, where v is unit labour cost, L/K the labour-capital ratio, ρ the households' rate of time preference and ϵ is the elasticity of unit labour cost with respect to capital (Cf.* (15)*).*

After the capitalist has installed the machines, the union claims higher wages in bargaining and expropriates part of the profit from the investment. Because the capitalist observes this, he invests less and capital stock converges to a lower level. The subsidy $-\tau_K$ eliminates this effect. The more elastic unit labour cost is with respect to capital stock (i.e. the higher ϵ), the higher subsidy $-\tau_K$ is needed to keep capital stock at the socially optimal level.

5 Centralized bargaining: agents

Because Nash bargaining cannot be consistently integrated into a model where capital stock K_j and consequently income evolves over time, we must content ourselves with the model of a monopoly union. We consider a dynamic game where capitalist j is the follower and union j, which sets the wage w_j, is the leader, taking capitalist j's investment behaviour as given.

Capitalist j then chooses his consumption to maximize the flow of his utility starting at time zero, $\int_0^\infty U(C_j)e^{-\rho t}dt$, where t is time and $\rho > 0$ the rate of time preference, subject to capital accumulation (6) and the functions (4), taking unit labour cost in the industry, v_j, and his own labour income $\alpha W/J$ as given. This maximization yields the Euler equation (see Appendix C)

$$\dot{C}_j/C_j = [(1-\tau_K)\Pi_K(K_j, v_j) - \rho]/\sigma. \tag{21}$$

In the very special case where $\sigma = 1$, $\Pi(v_j, K_j) = \Pi_K(v_j)K_j$ and $\alpha = 0$, the equations (6) and (21) take the form

$$\dot{K}_j/K_j = (1-\tau_K)\Pi_K(v_j) - (1+\tau_C)C_j/K_j, \quad \dot{C}_j/C_j = (1-\tau_K)\Pi_K(v_j) - \rho.$$

In this system, the co-state variable C_j jumps to the level $[\rho/(1+\tau_C)]K_j$, which maintains the steady state $\dot{K}_j/K_j = \dot{C}_j/C_j$. This reconstructs Lancing's (1999) assertion as follows:

Proposition 4 *If* (i) *the capitalists' preferences are logarithmic,* $\sigma = 1$, (ii) *there are constant returns to scale in production,* $\Pi(v_j, K_j) = \Pi_K(v_j)K_j$, (iii) *the capitalists earn no labour income,* $\alpha \equiv 0$, *and* (iv) *wage bargaining is centralized, then the capitalists' optimal decisions depend solely on the current rates of return or tax rates, not on future rates of return or tax rates.*

In the case of this proposition, the government would be short of useful policy instruments, because promises about future tax rates do not influence current allocations. To exclude this very special case, we assume that at least one of the conditions $(i) - (iii)$ in proposition 4 is not true.

We assume that union j maximizes the value of the flow of labour income W^j, discounted by the households' rate of time preference, ρ.[8] Given (4), this target is written as

$$\int_0^\infty W^j(L_j, K_j, \tau_W, \tau_L)e^{-\rho t}dt. \tag{22}$$

Union j sets its wage w_j to maximize its welfare (22), given the capitalist's Euler equation (21) and capital accumulation (6) and the firm's responses

[8] In the steady state, in which the marginal utility of income is constant for households, this maximization is equivalent to the maximization of the union members' welfare.

(4). Because there is a one-to-one correspondence from w_j to L_j through (2), the wage w_j can be replaced by employment L_j as the union's policy instrument. The union then maximizes by L_j the Hamiltonian

$$\begin{aligned} H_j =& W^j(L_j, K_j, \tau_W, \tau_L) + \xi_j\big\{(1-\tau_K)[F_K(K_j, L_j) - \mu] - \rho\big\}C_j/\sigma \\ &+ \phi_j\Big\{\alpha_j W^j(L_j, K_j, \tau_W, \tau_L) + \alpha_j \sum_{k\neq j} W^k + (1-\tau_K)\Pi(K_j, F_L(K_j, L_j)) \\ &\qquad - (1+\tau_C)C_j\Big\}, \end{aligned} \tag{23}$$

where the shadow prices (ξ_j, ϕ_j) for consumption C_j and capital K_j evolve according to

$$\begin{aligned} &\dot{\xi}_j = \rho\xi_j - \partial H_j/\partial C_j = \rho\xi_j + \big\{\rho - (1-\tau_K)[F_K(K_j, L_j) - \mu]\big\}\frac{\xi_j}{\sigma} + (1+\tau_C)\phi_j, \\ &\quad \lim_{t\to\infty} \xi_j C_j e^{-\rho t} = 0, \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{\phi}_j =& \rho\phi_j - \partial H_j/\partial K_j = [\rho - (1-\tau_K)(\Pi_K + \Pi_v F_{KL})]\phi_j - (1+\alpha_j\phi_j)W_K \\ &- (1-\tau_K)F_{KK}C_j\xi_j/\sigma, \quad \lim_{t\to\infty} \phi_j K_j e^{-\rho t} = 0. \end{aligned} \tag{25}$$

Noting (2) and (4), we obtain the first-order condition for employment L_j as

$$\begin{aligned} \partial H_j/\partial L_j &= (1+\alpha_j\phi_j)W_L + (1-\tau_K)[F_{KL}\xi_j C_j/\sigma + \Pi_v F_{LL}\phi_j] \\ &= (1+\alpha_j\phi_j)\big\{(1+\tau_W)^{-1}\big[F_{LL}(K_j, L_j)L_j + F_L(K_j, L_j) - \tau_L\big] - b\big\} \\ &\quad + (1-\tau_K)\big[F_{KL}(K_j, L_j)\xi_j C_j/\sigma - L_j F_{LL}(K_j, L_j)\phi_j\big] = 0. \end{aligned} \tag{26}$$

This defines employment L_j (and hence also the wage w_j) as a function of capital stock K_j, the shadow prices for consumption and capital, (ξ_j, ϕ_j), and taxes (τ_L, τ_W, τ_K). The condition (26) together with the development of the shadow prices (ξ_j, ϕ_j) for consumption C_j and capital K_j determine the union's behaviour uniquely for given taxes $(\tau_C, \tau_K, \tau_L, \tau_W)$.

6 Taxation with centralized bargaining

Because there is perfect symmetry throughout industries $j = 1, ..., J$, we obtain $K_j = K$, $L_j = L$, $C_j = C$, $W^j = W$, $\xi_j = \xi$ and $\phi_j = \phi$. Noting this and (2), capital accumulation (12), the Euler equation (21) and the union's equilibrium conditions (24)-(26) take the form

$$\dot{K} = F(K, L) - C - (1-\alpha)W/(1+\tau_C) - bL - E/J - \mu K, \tag{27}$$

$$\dot{C}/C = [(1-\tau_K)[F_K(K,L)-\mu]/\sigma - \rho/\sigma, \tag{28}$$

$$\dot{\xi} = \{\rho + \rho/\sigma - (1-\tau_K)[F_K(K,L)-\mu]/\sigma\}\xi + (1+\tau_C)\phi, \quad \lim_{t\to\infty} \xi C e^{-\rho t} = 0, \tag{29}$$

$$\begin{aligned}\dot{\phi} = {} & \{\rho - (1-\tau_K)[F_K(K,L) - F_{KL}(K,L)L - \mu]\}\phi - (1-\tau_K)F_{KK}C\xi/\sigma \\ & - (1+\tau_W)^{-1}(1+\alpha\phi)F_{KL}L, \quad \lim_{t\to\infty}\phi K e^{-\rho t} = 0,\end{aligned} \tag{30}$$

$$\begin{aligned}& (1+\alpha\phi)\{(1+\tau_W)^{-1}[F_{LL}(K,L)L + F_L(K,L) - \tau_L] - b\} \\ & + (1-\tau_K)[F_{KL}(K,L)\xi C/\sigma - LF_{LL}(K,L)\phi] = 0.\end{aligned} \tag{31}$$

We extend the dynamic game in the preceding section so that the government is the Stackelberg leader over both the investor and the parties in wage bargaining. It then sets taxes τ_C, τ_K, τ_L and τ_W to maximize social welfare (9) subject to the dynamics of the economy (27)-(31) and the constraints for the capital tax (11). In Appendix D, we show that results (19) and (20) as well as proposition 1 holds also in the case of centralized bargaining.

Equations $\dot{C} = 0$, (28) and (19) imply $0 = (1-\tau_K)\Pi_K - \rho = -\tau_K\rho$. This yields $\tau_K = 0$ and the following result:

Proposition 5 *The steady-state capital-income tax $\tau_K = 0$ should be zero.*

The explanation of this result is the following. The government balances its budget by the consumption tax τ_C. Because taxes on wages, labour and consumption are sufficient to redistribute income, to achieve the optimal production efficiency and to balance the government's budget, the tax rate on capital income, τ_K, should be zero. Any deviation from this zero tax rate would distort aggregate production efficiency.

Noting (2) and (27)-(19), we show the following tax rule (Appendix E):

Proposition 6 *The optimal employment subsidy is given by*

$$-\tau_L = \tau_W v + \frac{F_{KL}^2 - F_{KK}F_{LL}}{\sigma\rho F_{KL}/(cK) + F_{KK}/L}, \tag{32}$$

where v is the unit labour cost, τ_W the wage tax and $c \doteq (1+\tau_C)C/K$ the capitalists' consumption expenditure per unit of capital.

The application of this rule presupposes information on the shape of the production function $F(K,L)$. We specify the rule for Cobb-Douglas technology

$$F(K,L) \doteq AK^{\varphi}L^{1-1/\varepsilon}, \quad \varphi > 0, \;\; \varphi + 1 - 1/\varepsilon < 1, \;\; A > 0, \tag{33}$$

where A is a constant, the parameter ε the elasticity of employment with respect to unit labour cost [Cf. (3)] and the parameter φ the elasticity of output F with respect to capital K, when employment L is kept constant. Given (33), we obtain $F_{LL} = -F/(\varepsilon L)$, $F_{KK} = (\varphi - 1)F_K/K$ and $F_{KL} = \varphi F_L/K = (1 - 1/\varepsilon)F_K/L$. Inserting these and $F_L = v$ from (19) into (32) yields a corollary for proposition 6:

Proposition 7 *With* (33)*, the optimal employment tax is given by*

$$-\tau_L = \tau_W v + \frac{(\varphi - 1/\varepsilon)v}{(1 - 1/\varepsilon)\sigma\rho/c + \varphi - 1}.$$

7 Conclusions

This paper examines optimal taxation in a unionized economy. Workers in each industry form a union, which raises their wage above the opportunity cost of employment. Decentralized wage bargaining covers only a single firm, but centralized wage bargaining a large number of firms. Some (or all) households specified as capitalists save and earn a fixed proportion of all wages, while the others specified as non-capitalists spend all of their income. The government can tax consumption, employment, wages and capital income. The main findings of this paper are the following.

The wage subsidy redistributes income between capitalists and others. Its increase makes labour unions to substitute employment by the wage and wages increase. As a consequence, profits and capital income decrease. Through this mechanism, the government can efficiently distribute income so that at the optimum the ratio of the marginal utility of income for the non-capitalists and capitalists is equal to their relative social weight. The government must subsidize employment to keep the marginal product of labour equal to the opportunity cost of employment. Because aggregate production efficiency requires that the marginal product of labour equals the opportunity cost of employment, wage and employment subsidies together must be positive to maintain incentives to supply labour. The consumption tax is needed to balance the government's budget.

Optimal public policy is possible only if there are decreasing returns to scale. In the steady state, the marginal product of capital is equal to the rate of time preference plus the rate of capital depreciation. Because with constant returns to scale this steady-state condition fixes the capital-labour ratio, the marginal product of labour and the opportunity cost of employment can be equalized only in the case of decreasing returns to scale.

Zero taxation of capital income in the limit does not apply to the unionized economy with decentralized bargaining, but applies if there is centralized bargaining. With decentralized bargaining, capital accumulation increases employment for given wages. This strengthens the union's position in bargaining and raises wages. Because the capitalist takes the effect of capital accumulation on wages into account in his investment plans, production efficiency cannot be maintained without a subsidy to capital income. With centralized wage bargaining, aggregate production efficiency can be maintained by the consumption tax and subsidies to wages and employment. Because these three instruments are sufficient to redistribute income, to achieve production efficiency and to balance the government's budget, the tax rate on capital income should be zero. Any deviation from this zero tax rate would distort aggregate production efficiency.

These tax rules hold for any proportion of wages earned by the capital-saving households. This includes also the Judd case in which only capitalists save, as well as the Chamley case in which all households are similar.

Appendix

A. The Euler equation in the case of decentralized bargaining

Noting (2), (7) and (14), we obtain the Hamiltonian for the capitalist's maximization in the case of decentralized bargaining as follows:

$$\mathcal{H}^{Cj} = U(C_j) + \lambda_j \big[\alpha W/J + (1-\tau_K)\Pi\big(K_j, v_j(K_j, \tau_W, \tau_L)\big) - (1+\tau_C)C_j\big],$$

where the co-state variable λ evolves according to

$$\dot{\lambda}_j = \rho\lambda_j - \frac{\partial \mathcal{H}^{Cj}}{\partial K_j} = \left[\rho - (1-\tau_K)\left(F_K - \mu - L_j\frac{\partial v_j}{\partial K_j}\right)\right]\lambda_j,$$
$$\lim_{t\to\infty} \lambda_j K_j e^{-\rho t} = 0. \tag{34}$$

The first-order condition for capitalist j's optimization is given by

$$C_j^{-\sigma} = U'(C_j) = (1+\tau_C)\lambda_j. \tag{35}$$

Noting (14), (15), (34) and (35), we obtain

$$\frac{\dot C_j}{C_j} = -\frac{1}{\sigma}\frac{\dot\lambda_j}{\lambda_j} = \left\{(1-\tau_K)\left[F_K(K_j,L_j) - \mu - L_j\frac{\partial v_j}{\partial K_j}\right] - \rho\right\}\frac{1}{\sigma}$$
$$= \big\{(1-\tau_K)\big[F_K(K_j,L_j) - \mu - (L_j/K_j)v_j\epsilon(K_j,\tau_W,\tau_L)\big] - \rho\big\}/\sigma. \quad (36)$$

Variables K_j and C_j are governed by the system (6) and (36), the dynamics of which is as follows. Because $\partial\dot K_j/\partial C_j < 0$,

$$\left.\frac{\partial \dot K_j}{\partial K_j}\right|_{\dot C_j=0} = (1-\tau_K)\left(F_K - \mu - \frac{L_j}{K_j}v_j\epsilon\right)_{\dot C_j=0} = \rho > 0, \quad \left.\frac{\partial \dot C_j}{\partial C_j}\right|_{\dot C_j=0} = 0,$$

we obtain

$$\left[\frac{\partial \dot K_j}{\partial K_j} + \frac{\partial \dot C_j}{\partial C_j}\right]_{\dot C_j=0} > 0, \quad 0 = \left[\frac{\partial \dot K_j}{\partial K_j}\frac{\partial \dot C_j}{\partial C_j}\right]_{\dot C_j=0} < \frac{\partial \dot K_j}{\partial C_j}\frac{\partial \dot C_j}{\partial K_j} \quad \Leftrightarrow \quad \frac{\partial \dot C_j}{\partial K_j} < 0.$$

This means that there exists a saddle-point solution for $\partial\dot C_j/\partial K_j < 0$, but otherwise the system is globally stable. Hence, the capitalist's equilibrium is indeterminate for $\partial\dot C_j/\partial K_j \geq 0$. Only if $\partial\dot C_j/\partial K_j < 0$, the capitalist has a unique solution where the co-state variable C_j (which represents λ_j) jumps onto the saddle path which leads to the steady state in which K_j, C_j and λ_j are constants, and $\lim_{t\to\infty}\lambda_j K_j e^{-\rho t} = 0$ holds. We assume the latter.

B. Optimal policy in the case of decentralized bargaining

Because there is a one-to-one correspondence from (τ_L,τ_W) to W and L through (4) and (14), employment and wage taxes (τ_L,τ_W) can be replaced by labour income W and employment L as control variables. The Hamiltonian and Lagrangean corresponding to the government's maximization are

$$\begin{aligned}\mathcal{H}^G &= V\big((1-\alpha)W/(1+\tau_C)\big) + \vartheta U(C)\\ &\quad + \eta\big\{(1-\tau_K)[F_K(K,L) - \mu - (L/K)v\epsilon] - \rho\big\}C/\sigma\\ &\quad + \kappa\big[F(K,L) - C - (1-\alpha)W/(1+\tau_C) - bL - E - \mu K\big],\\ \mathcal{L}^G &= \mathcal{H}^G + \nu_1[\tau_K + \upsilon] + \nu_2[1-\tau_K],\end{aligned} \quad (37)$$

where the functions L and $\partial v_j/\partial K_j$ are given by (14) and the co-state variables evolve κ and η according to

$$\dot\kappa = \rho\kappa - \frac{\partial\mathcal{L}^G}{\partial K}, \ \lim_{t\to\infty} K\kappa e^{-\rho t} = 0, \ \dot\eta = \rho\eta - \frac{\partial\mathcal{L}^G}{\partial C}, \ \lim_{t\to\infty} C\eta e^{-\rho t} = 0. \quad (38)$$

Noting (37), the first-order condition for τ_K is given by

$$\begin{aligned}\partial \mathcal{L}^G/\partial \tau_K &= \partial \mathcal{H}^G/\partial \tau_K + \nu_1 - \nu_2 \\ &= [\mu + (L/K)v\epsilon - F_K](C/\sigma)\eta + \nu_1 - \nu_2 = 0. \end{aligned} \tag{39}$$

Assume first $-v < \tau_K < 1$, so that $\nu_1 = \nu_2 = 0$. Because $C > 0$ by (35), from (39) it follows that $\eta = 0$. Noting $\eta = 0$ and (37), the first-order conditions for L and W are

$$V' = \kappa, \quad v = F_L(K, L) = b. \tag{40}$$

Because $\partial^2 \mathcal{H}^G/\partial \tau_K^2 \equiv 0$, we have to solve τ_K through the generalized Legendre-Clebsch conditions:[9]

$$\begin{aligned}&\frac{\partial}{\partial \tau_K}\Big(\frac{d^p}{dt^p}\frac{\partial \mathcal{H}^G}{\partial \tau_K}\Big) = 0 \text{ for any odd integrer } p, \\ &(-1)^q \frac{\partial}{\partial \tau_K}\Big(\frac{d^{2q}}{dt^{2q}}\frac{\partial \mathcal{H}^G}{\partial \tau_K}\Big) \geq 0 \text{ for any integrer } q,\end{aligned} \tag{41}$$

where t is time.

Differentiating the first of equations (39) with respect to time t and noting (7), (16), (37), (38), $\eta = 0$ and $\nu_1 = \nu_2 = 0$, we obtain

$$\begin{aligned}\frac{d}{dt}\Big(\frac{\partial \mathcal{H}^G}{\partial \tau_K}\Big) &= \Big(\mu + \frac{L}{K}v\epsilon - F_K\Big)\frac{C}{\sigma}\dot{\eta}\big|_{\eta=0} = -\Big(\mu + \frac{L}{K}v\epsilon - F_K\Big)\frac{C}{\sigma}\frac{\partial \mathcal{H}^G}{\partial C} \\ &= -\Big(\mu + \frac{L}{K}v\epsilon - F_K\Big)\frac{C}{\sigma}(\vartheta C^{-\sigma} - \kappa) = 0, \quad \frac{\partial}{\partial \tau_K}\frac{d}{dt}\Big(\frac{\partial \mathcal{H}^G}{\partial \tau_K}\Big) = 0.\end{aligned} \tag{42}$$

Given these, we furthermore obtain

$$\begin{aligned}&\frac{d^2}{dt^2}\Big(\frac{\partial \mathcal{H}^G}{\partial \tau_K}\Big) = \Big(\mu + \frac{L}{K}v\epsilon - F_K\Big)\vartheta C^{-\sigma}\dot{C} = 0, \\ &\frac{\partial}{\partial \tau_K}\frac{d^2}{dt^2}\Big(\frac{\partial \mathcal{H}^G}{\partial \tau_K}\Big) = \Big(\mu + \frac{L}{K}v\epsilon - F_K\Big)^2 \vartheta C^{-\sigma}\frac{C}{\sigma} > 0.\end{aligned} \tag{43}$$

Results (42) and (43) satisfy the Clebsch-Legendre conditions (41). From (18) and (43) it follows that

$$0 = \sigma \dot{C}/C = (1 - \tau_K)[F_K - \mu - (L/K)v\epsilon] - \rho. \tag{44}$$

[9]Cf. Bell and Jacobson (1975), p. 12-19.

From (40) and (42) it follows that

$$\vartheta C^{-\sigma} = \kappa = V'. \tag{45}$$

Differentiating $\vartheta C^{-\sigma} = \kappa$ with respect to time t and noting $\eta = 0$, (2), (37), (38) and $\dot{C} = 0$ by (43) yield $\dot{\kappa} = (\rho + \mu - F_K)\kappa$ and

$$\rho = F_K(K, L) - \mu. \tag{46}$$

Results (40) and (46) yield (19).

From (40) and (46) it follows that the value (K^*, L^*) for (K, L) in the steady state is determined by two equations $F_K(K^*, L^*) = \rho + \mu$ and $F_L(K^*, L^*) = b$. Noting (51) and (52), we obtain the following. If $\eta > 0$ ($\eta < 0$), then the capital subsidy (tax) should be raised to the maximum, $-\tau_W = \eta$ ($\tau_W = 1$), so that the capitalist accumulates (exhausts) capital, $\dot{K} > 0$ ($\dot{K} > 0$). Since the system ends up with a steady state in which K, C, χ and κ are constants, $\lim_{t\to\infty} K\eta e^{-\rho t} = 0$ and $\lim_{t\to\infty} C\kappa e^{-\rho t} = 0$ hold. Finally, given (15), (44) and (46), we obtain

$$\rho = (1 - \tau_K)\Big(\rho - \frac{L}{K} v\epsilon\Big) = (1 - \tau_K)\Big(\rho - \frac{L}{K} v\epsilon\Big), \quad -\tau_K = \Big(1 - \frac{vL}{\rho K}\epsilon\Big)^{-1} - 1.$$

C. The Euler equation in the case of centralized bargaining

The Hamiltonian corresponding to the capitalist's maximization in the case of centralized bargaining is given by

$$H^{Cj} = U(C_j) + \theta_j\big[\alpha W/J + (1 - \tau_K)\Pi(K_j, v_j) - (1 + \tau_C)C_j\big],$$

where the co-state variable θ_j evolves according to

$$\dot{\theta}_j = \rho\theta_j - \frac{\partial H^{Cj}}{\partial K_j} = [\rho - (1 - \tau_K)\Pi_K(K_j, v_j)]\theta_j, \quad \lim_{t\to\infty} \theta_j K_j e^{-\rho t} = 0. \tag{47}$$

The first-order condition for the capitalist's maximization is given by $C_j^{-\sigma} = U'(C_j) = (1 + \tau_C)\theta_j$. Noting this, we can transform the constraint (47) into the capitalist's Euler equation

$$\dot{C}_j/C_j = -(1/\sigma)\dot{\theta}_j/\theta_j = [(1 - \tau_K)\Pi_K(K_j, v_j) - \rho]/\sigma. \tag{48}$$

Variables K_j and C_j are governed by (6) and (48). When there are decreasing returns to scale in production, $\Pi_{KK} < 0$, the dynamics is as follows. Because $\partial \dot{K}_j/\partial K_j = (1-\tau_K)\Pi_K > 0$, $\partial \dot{K}_j/\partial C_j < 0$, $\partial \dot{C}_j/\partial K_j = (1-\tau_K)\Pi_{KK}C_j/\sigma$ < 0 and $[\partial \dot{C}_j/\partial C_j]_{\dot{C}_j=0} = 0$, we obtain

$$\frac{\partial \dot{K}_j}{\partial K_j} + \frac{\partial \dot{C}_j}{\partial C_j}\bigg|_{\dot{C}_j=0} > 0, \quad \frac{\partial \dot{K}_j}{\partial K_j}\frac{\partial \dot{C}_j}{\partial C_j}\bigg|_{\dot{C}_j=0} < \frac{\partial \dot{K}_j}{\partial C_j}\frac{\partial \dot{C}_j}{\partial K_j},$$

and there is a saddle-point solution. Hence, the co-state variable C_j (which represents θ_j) jumps onto the saddle path which leads to the steady state in which K_j, C_j and θ_j are constants, and $\lim_{t\to\infty}\theta_j K_j e^{-\rho t} = 0$ holds.

With constant returns to scale $\Pi_{KK} \equiv 0$, from (2), (6) and (47) it follows that

$$\left[\frac{\dot{K}_j}{K_j} + \frac{\dot{\theta}_j}{\theta_j} - \rho\right]_{\dot{K}_j=0} = \left[\alpha_j \sum_k W^k - (1+\tau_C)\frac{C_j}{K_j}\right]_{\dot{K}_j=0} = (\tau_K - 1)\Pi_K < 0.$$

This as well implies the transversality condition $\lim_{t\to\infty} K_j\theta_j e^{-\rho t}dt = 0$.

D. Optimal policy in the case of centralized bargaining

Because there is a one-to-one correspondence from (τ_L, τ_W) to W and L through (4) and (26), employment and wage taxes (τ_L, τ_W) can be replaced by labour income W and employment L as control variables. Noting (7), the Hamiltonian and the Lagrangean corresponding to the government's maximization in the case of centralized bargaining are given by

$$\begin{aligned}\mathcal{H} &= V\Big(\frac{1-\alpha}{1+\tau_C}JW\Big) + \vartheta JU(C) + \gamma\big\{(1-\tau_K)[F_K(K,L)-\mu] - \rho\big\}C/\sigma \\ &\quad + \chi\big\{F(K,L) - C - (1-\alpha)W/(1+\tau_C) - bL - \mu K - E/J\big\},, \\ \mathcal{L}^G &= \mathcal{H} + \varphi_1[\tau_K + \upsilon] + \varphi_2[1-\tau_K], \end{aligned} \tag{49}$$

where the co-state variables χ and γ evolve according to

$$\begin{aligned}\dot{\gamma} &= \rho\gamma - \partial\mathcal{H}/\partial C = [\rho + \rho/\sigma - (1-\tau_K)(F_K - \mu)/\sigma]\gamma + \chi - \vartheta JC^{-\sigma}, \\ \dot{\chi} &= \rho\chi - \partial\mathcal{H}/\partial K = \big[\rho + \mu - F_K(K,L)\big]\chi - (1-\tau_K)F_{KK}C\gamma/\sigma, \\ &\lim_{t\to\infty}\chi K e^{-\rho t} = 0, \ \lim_{t\to\infty}\gamma C e^{-\rho t} = 0, \end{aligned} \tag{50}$$

and variables φ_1 and φ_2 satisfy the Kuhn-Tucker conditions

$$\varphi_1[\tau_K + \eta\upsilon] = 0, \quad \varphi_1 \geq 0, \;\; \varphi_2[1 - \tau_K] = 0, \quad \varphi_2 \geq 0. \tag{51}$$

Noting (49), the first-order condition for τ_K is given by

$$\partial\mathcal{H}/\partial\tau_K = (\mu - F_K)C\gamma/\sigma + \varphi_1 - \varphi_2 = 0. \tag{52}$$

Assume first $-\upsilon < \tau_K < 1$, so that $\varphi_1 = \varphi_2 = 0$. Because $\partial^2\mathcal{H}/\partial(\tau_K)^2 \equiv 0$, we have to solve for τ_K through the generalized Clebsch-Legendre conditions (41). Because $C > 0$ and $(F_K - \mu)_{\dot{C}=0} > 0$ by (28), equation (52) yields $\gamma = 0$. Differentiating (52) with respect to time t and noting (2), (21), (28), (50) and $\gamma = 0$, we see that the Clebsch-Legendre conditions (41) hold:

$$\frac{d}{dt}\left(\frac{\partial\mathcal{H}}{\partial\tau_K}\right) = (\mu - F_K)\frac{C}{\sigma}\dot{\gamma}\big|_{\gamma=0} = \Pi_K\frac{C}{\sigma}[\vartheta JC^{-\sigma} - \chi] = 0, \tag{53}$$

$$\frac{\partial}{\partial\tau_K}\frac{d}{dt}\left(\frac{\partial\mathcal{H}}{\partial\tau_K}\right) = 0, \quad \frac{d^2}{dt^2}\left(\frac{\partial\mathcal{H}}{\partial\tau_K}\right) = -\Pi_K\frac{C}{\sigma}[\dot{\chi} + \sigma\vartheta JC^{-\sigma-1}\dot{C}] = 0, \tag{54}$$

$$\frac{\partial}{\partial\tau_K}\frac{d^2}{dt^2}\left(\frac{\partial\mathcal{H}}{\partial\tau_K}\right) = -\Pi_K\frac{C}{\sigma}\left[\frac{\partial\dot{\chi}}{\partial\tau_K} + \frac{\sigma\vartheta J}{C^{\sigma+1}}\frac{\partial\dot{C}}{\partial\tau_K}\right] = (\Pi_K)^2\frac{\vartheta}{\sigma^2}JC^{1-\sigma} > 0. \tag{55}$$

Given (53), we obtain

$$\chi = \vartheta JC^{-\sigma}. \tag{56}$$

From $\gamma = 0$, (2), (28), (50), (54) and (56) it follows that

$$\begin{aligned} 0 = \dot{\chi} + \sigma\vartheta JC^{-\sigma-1}\dot{C} &= (\rho + \mu - F_K)\chi + \vartheta JC^{-\sigma}[(1 - \tau_K)\Pi_K - \rho] \\ &= (\rho - \Pi_K)\vartheta JC^{-\sigma} + \vartheta JC^{-\sigma}[(1 - \tau_K)\Pi_K - \rho] = -\vartheta JC^{-\sigma}\tau_K\Pi_K, \end{aligned}$$

which is equivalent to $\tau_K = 0$.

Noting $\gamma = 0$ and (49), the first-order conditions for W and L are

$$\partial\mathcal{H}/\partial W = (1 - \alpha)(JV' - \chi) = 0, \;\; \partial\mathcal{H}/\partial L = \chi(F_L - b) = 0. \tag{57}$$

Given (56) and (57), we obtain $\vartheta C^{-\sigma} = \chi/J = V'$ and

$$F_L = b, \quad \vartheta C^{-\sigma} = V', \tag{58}$$

and the adjustment rules (20). The system (12) and (21) produces a steady state in which K and C are kept constant. Given $\dot{C} = 0$, (2), (21) and $\tau_K = 0$ in (58), we obtain

$$\rho = F_K(K, L) - \mu = \Pi_K. \tag{59}$$

Equations (58) and (59) yield (19).

From (59) and (58) it follows that the value (K^*, L^*) for (K, L) in the steady state with $\gamma = 0$ is determined by two equations $F_K(K^*, L^*) = \rho + \mu$ and $F_L(K^*, L^*) = b$. Noting (51) and (52), we obtain the following. If $\gamma > 0$ ($\gamma < 0$), then the capital subsidy (tax) should be raised to the maximum, $-\tau_W = \upsilon$ ($\tau_W = 1$), so that the capitalist accumulates (exhausts) capital, $\dot{K} > 0$ ($\dot{K} > 0$). Since the system ends up with a steady state in which K, C, χ and γ are constants, $\lim_{t\to\infty} K\chi e^{-\rho t} = 0$ and $\lim_{t\to\infty} C\gamma e^{-\rho t} = 0$ hold.

E. The optimal employment tax in the case of centralized bargaining

In the steady state, consumption C, capital stock K and shadow prices ξ and ϕ are constant. Noting (19) and $\tau_K = 0$, conditions (28)-(31) then become $\dot{C} = 0$ and

$$\dot{\xi} = \rho\xi + (1 + \tau_C)\phi = 0, \quad \lim_{t\to\infty} \phi K e^{-\rho t} = 0, \tag{60}$$

$$\dot{\phi} = LF_{KL}[\phi - (1 + \tau_W)^{-1}(1 + \alpha\phi)] - F_{KK}C\xi/\sigma = 0, \ \lim_{t\to\infty} \phi K e^{-\rho t} = 0, \tag{61}$$

$$0 = (1 + \alpha\phi)\big[(1 + \tau_W)^{-1}(LF_{LL} + b - \tau_L) - b\big] + F_{KL}\xi C/\sigma - LF_{LL}\phi. \tag{62}$$

Equation (60) yields $\xi = -(1 + \tau_C)\phi/\rho$. Inserting this into (61), we obtain

$$1 + \alpha\phi = (1 + \tau_W)\left[\phi - \frac{\xi F_{KK}C}{\sigma L F_{KL}}\right] = (1 + \tau_W)\phi\left[1 + (1 + \tau_C)\frac{F_{KK}C}{\sigma\rho L F_{KL}}\right].$$

Inserting this, $\xi = -(1+\tau_C)\phi/\rho$ and $\upsilon = F_L = b$ [Cf. (2) and (19)] into (62), and defining $c \doteq (1 + \tau_C)C/K$, we furthermore obtain

$$\begin{aligned} 0 &= \frac{1 + \alpha\phi}{\phi}\left[\frac{LF_{LL} + b - \tau_L}{1 + \tau_W} - b\right] + \frac{\xi C}{\sigma\phi}F_{KL} - LF_{LL} \\ &= \left[1 + (1 + \tau_C)\frac{F_{KK}C}{\sigma\rho L F_{KL}}\right]\big[LF_{LL} - \tau_L - \tau_W b\big] - (1 + \tau_C)\frac{C}{\sigma\rho}F_{KL} - LF_{LL} \end{aligned}$$

$$= (1+\tau_C)\frac{F_{KK}F_{LL}}{\sigma\rho F_{KL}}C - (1+\tau_C)\frac{CF_{KL}}{\sigma\rho} - \left[1 + (1+\tau_C)\frac{F_{KK}C}{\sigma L F_{KL}}\right](\tau_L + \tau_W b)$$
$$= \frac{(1+\tau_C)C}{\sigma\rho F_{KL}}\left(F_{KK}F_{LL} - F_{KL}^2\right) - \left[1 + (1+\tau_C)\frac{F_{KK}C}{\sigma\rho L F_{KL}}\right](\tau_L + \tau_W v)$$
$$= \frac{(1+\tau_C)C}{\sigma\rho F_{KL}}\left\{F_{KK}F_{LL} - F_{KL}^2 - \left[\frac{\sigma\rho F_{KL}}{(1+\tau_C)C} + \frac{F_{KK}}{L}\right](\tau_L + \tau_W v)\right\}$$
$$= \frac{(1+\tau_C)C}{\sigma\rho F_{KL}}\left\{F_{KK}F_{LL} - F_{KL}^2 - \left[\frac{\sigma\rho F_{KL}}{cK} + \frac{F_{KK}}{L}\right](\tau_L + \tau_W v)\right\}$$

and (32).

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