Optimal Pollution, Endogenous Population, and Sustainability*

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Abstract

This paper developes a model with optimal pollution and endogenous population. The positive Malthusian check increases mortality if pollution increases. The optimal path is demographically sustainable if it provides non-decreasing consumption for a non-decreasing population. It is found that demographic sustainability without technical progress is possible if population and consumption are constants. Technical progress does not necessarily lead to demographically sustainable growth.

Journal of Economic Literature: Q01, Q53, Q54, J11

Keywords: sustainable development, mortality, pollution

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1 Introduction

Is it possible that the current utility maximization takes place at the cost of human lives? Is it possible even that this maximization ultimately takes the mankind to extinction? These possibilities are indeed *implied* in the papers of Dasgupta and Heal (1974), Solow (1974), Stiglitz (1974), Krautkraemer (1985), and Pezzey and Withagen (1998) who argue that the scarcity of natural resources may lead to ever-decreasing per capita consumption. Per capita consumption may also decrease if excessive pollution hurts production and compromises life-supporting systems as is argued in Keeler *et al.* (1971), Plorde (1972), Foster (1973), and Smulders and Gradus (1996). Ever-decreasing per capita consumption naturally implies that mortality is to increase and the mankind is ultimately to extinct even if these demographic consequences are not explicitly considered in the models.

In this paper, we argue that population is endogenous to environment by assuming the positive Malthusian check which states that mortality increases if population is not environmentally supported (Malthus 1914). The positive check may step in because of insufficient production of food or because of continuous concentration of pollutants. We focus on the pollutants because an emerging evidence maintains that the positive check may already be in work. This evidence consist of medical and econometric studies performed by individual researchers and international organizations and the main argument is that there already exists a statistically significant increase in mortality due to urban air pollution, and that the climate change may induce such an increase in the closest future. Thus, we introduce an optimal pollution model with endogenous population. In spite of our emphasis on pollutants, the results can be generalized to natural resources since running down resources can be seen as a form of pollution in the extended sense (Keeler *et al.* 1971).

In the earlier literature, a path is denoted as sustainable if it provides non-decreasing per capita consumption or non-decreasing utility (Pezzey 1992). Endogenous population calls for a redefinition since non-decreasing per capita consumption (utility) must be provided for non-decreasing population. With endogenous population, the main prediction of the model also changes. Earlier, several authors have argued that the utility maximizing optimal path is not necessarily sustainable. On the contrary, along the optimal path the per capita consumption typically first increases and then decreases (see for example Dasgupta and Heal 1974 and Pezzey and Withagen 1998). However, in the presence of the positive check there is a trade-off between per capita consumption and population. Therefore, the utility maximizing path may exhibit increasing per capita consumption, both during the transitional period and in the steady state, since it may well be optimal to provide increasing consumption for some at the cost of the lives of others.

The plan of the paper is the following: Section 2 reviews the evidence for the positive Malthusian check. Sections 3-4 give the model and its sustainability implications together with a parametric example. To concentrate on population, we choose the simplest possible model, called "the prototype of pollution" by Tahvonen and Salo (1996). Even so, endogenous population tends to make the model "murky" (Solow 1974) but excessive complexity can be avoided by modelling in virtual time in the lines with Uzawa (1968). Section 5 discusses the role of technical progress which in our model is not as positive as usually suggested. Section 6 closes the paper.

2 The Positive Check, Recent Evidence

Mortality that has been induced by air pollution has been debated since the local smog in Meuse Valley (Belgium) 1930 and in London 1952 took the lives of 60 and 4000 people (Nemery *et al.* 2001 and Logan 1953). Air pollution consists of several components of which particulate matter (PM) and ozone are the most dangerous (WHO 2004b).[1] Air pollution increases mortality mainly through increase in respiratory and cardiovascular diseases and lung cancer (Samet *et al.* 2000), but increase in skin cancer prevalence is also reported (Brunekreef and Holgate 2002). All age groups are affected, but unborn and young children as well as elderly are the most vulnerable.

The Clean Air for Europe program ($CAFE$) and WHO have summarized the European research by collecting 629 peer-reviewed time-series studies and 160 individual or panel studies up to February 2003 (WHO 2004b). In the original studies, daily adult mortality in several European cities was regressed against daily changes in air pollution. The summary estimates indicate that there is a statistically significant 0.6% and 0.3% increase in mortality for each

[1] Term particulate matter (PM) refers to airborne solid particles of varying size, chemical composition and origin. For example the particles in PM_{10} have diameter less than $10\mu m$ and they are mainly combustion-derived, either from traffic or from energy production, often of long-distance sources. There is evidence in support that the smaller the particles are, the more deeply into the lung they penetrate (WHO 2004b).

$10\mu g/m^3$ increase in PM and ozone respectively.[2]

The effects of long-term PM exposure in the US has been analyzed by Pope *et al.* (2002) in a large cohort study in which a questionnaire from 1982 provides data on sex, race, smoking, alcohol consumption $etc.$ so that controlling for alternative risk sources has been possible. The mortality data was collected until 1998 and was regressed against local pollution data to derive 4%, 6%, and 8% increase in all-cause, respiratory, and lung cancer mortality for each $10\mu g/m^3$ increase in PM respectively.[3] These estimates have been applied to the European data by $CAFE$ and WHO to calculate that the short-term and long-term exposures were together responsible for 370000 premature deaths in 2000 (WHO 2004b).

The Clean Air for Europe program and WHO have also provided a synthesis on air pollution and child mortality (WHO 2004a). The synthesis is based on several original studies and the conclusion is given at four-level scale from "sufficient" to "no association". It turns out that there is sufficient evidence from increase in child mortality, mainly due to exposure to particulate matter. No exact summary estimate is provided. In California, the infant mortality risk during the 1990s has been estimated by Currie and Neidell (2004). They applied several covariates and controlled for fetal deaths, low birth weight, and prematurity to exclude the fetal selection bias. The pollutants were particulate matter, ozone, carbon monoxide, and nitrogen dioxide. Single pollutant models supplied significant estimates for ozone, carbon monoxide, and nitrogen dioxide, but when all four pollutants were included, only carbon monoxide was significant. Chay and Greenstone (2003) have also shown that the air quality improvement under the Clean Air Act of 1970 in the US saved more than $1,300$ infants annually.

In the studies of the climate change, the mortality estimates are based on simulations. Tanser *et al.* (2003), for example, have applied the Hadley Centre's climate model to estimate that the projected $5-7\%$ (altitudinal) increase in malaria distribution and the prolonged malaria season would lead to 25% increase in risk to die in malaria by 2100, mainly in Africa. Most victims would be children. The waste literature on climate change has been collected and analyzed by UN's Intergovernmental Panel on Climate Change ($IPCC$). Its Third

[2] This type of meta-analysis tends to have a "publication bias" because the publication of positive results is more likely in the original studies. The authors tried to correct the bias with the outcome that the risk estimate for ozone decreased to 0.02% (WHO 2004b).

[3] Cox proportional hazards survival model was used.

Assessment Report suggests that mortality will increase because of weather extremes, because of environmental changes that lead to diseases and water or food shortages, and because of health consequences or conflicts in displaced populations (IPCC 2001). Relying on $IPCC$, WHO has published a summary report on human health and Climate Change (WHO 2003). This report projects the maximum increase in the risk of 83%, 17%, and 32% for the great killers, malaria, diarrhoea, and malnutrition respectively. There is also a great projected risk increase in coastal floods but the number of deaths may be low at the global level.

3 The Model

To model the positive check, note that the population growth rate $\dot{L}/L = n$ is the difference between fertility and mortality. We assume that fertility is constant[4] and mortality depends on pollution. Pollution may increase mortality both as emissions E and as stocks S, but it seems appropriate to model in terms of stocks since their mortality effects are more standing. Hence, we assume:

$$n = n(S),\ n(0) > 0,\ n'(S) < 0,\ n\left(\hat{S}\right) = 0, \tag{1}$$

where $\hat{S}$ is the critical stock beyond which population starts to decrease. Normalizing the initial level of population to unity we have at each instant of time

$$L(t) = \exp \int_0^t n[S(\tau)]\, d\tau. \tag{2}$$

Since we want to focus on the population in the absence of production problems we assume the simplest formulation for the rest of the model in line with Foster (1973). The pollution stock accumulates according to

$$\dot{S} = E - \delta(S), \tag{3}$$

where $\delta(S)$ is the abatement function. We assume $\delta(0) = \delta\left(\bar{S}\right) = 0$ and $\delta'(0) > 0$, $\delta'\left(\bar{S}\right) < 0$, $\delta''(S) < 0$ where $\bar{S} > 0$ is the carrying capacity of environment.[5] The abatement function then has the shape of inverted U. We

[4] United Nations has estimated that the currently ongoing fertility transition is mainly over around in 2050 with the global population growth rate less than one percent annually ((United Nations 2005)).

[5] The abatement function is stictly concave. A broad branch of literature deals with the

assume $\bar{S} > \hat{S}$ to allow the possibility of negative population growth in the area of interest.

Consumption C takes place directly at the cost of environment so that $C = E$. Consider an infinitely living central planner (or family head) who derives utility from per capita consumption $C/L = E/L$ and from the number of people.[6] Further, we assume that the instantaneous utility function is of multiplicative form, i.e., at each instant of time, the total utility becomes $u(C/L) \cdot L = u(E/L) \cdot L$, where u satisfies the standard concavity properties and the Inada conditions. In the intertemporal choice, the planner faces the discount factor $\rho > 0$. The planner then chooses emissions $E(t)$ to maximize

$$\begin{aligned} U &= \int_0^\infty u[E(t)/L(t)]\ L(t)\ e^{-\rho t}\, dt \\ &= \int_0^\infty u[E(t)/L(t)]\ e^{-\int_0^t \{\rho - n[S(\tau)]\} d\tau}\, dt, \end{aligned} \tag{4}$$

subject to (3). The mechanism of the model is the following: by the choice of $E(t)$ the planner determines $S(t)$ which in turn gives the population growth rate $n(t)$ and the population $L(t)$. Finally, per capita emissions $E(t)/L(t)$ become determined. The model implies that there is a trade-off between consumption and population because of the positive check. High consumption today necessarily leads to low (possibly negative) population growth rate and small population size in the future.

Unfortunately, the discount factor in (4) is not constant. Therefore, we apply the virtual time technique suggested by Uzawa (1968). Denote

$$\Delta(t) = \int_0^t \{\rho - n[S(\tau)]\}\, d\tau,$$

to get $\frac{d\Delta(t)}{dt} = \rho - n[S(t)]$ and $dt = \frac{d\Delta(t)}{\rho - n[S(t)]}$. The problem can now be rewritten in virtual time as:

$$U = \int_0^\infty \frac{u(E/L)}{\rho - n(S)} \cdot e^{-\Delta} \cdot d\Delta, \tag{5}$$

$$\dot{S} = \frac{dS}{d\Delta} = \frac{dS}{dt}\frac{dt}{d\Delta} = \frac{E - \delta(S)}{\rho - n(S)},$$

problem of non-concavities in the abatement function. For a survey, see Tahvonen and Salo (1996).

[6] The model pays no attention to amenity values provided by unspoiled environment, for a discussion see Krutilla (1967) and Barbier (2003).

where $E = E[\tau(t)]$, $S = S[\tau(t)]$, $L = L[\tau(t)]$. This concave problem with constant discount factor can be solved in virtual time by using standard methods (Benveniste and Scheinkman 1982). The current value Hamiltonian and the necessary conditions become:

$$H(S,E,\lambda) = \frac{1}{\rho - n(S)} \left\{ u(E/L) + \lambda(\tau)\left[E - \delta(S)\right] \right\},$$

$$\frac{\partial H(S,E,\lambda)}{\partial E} = 0 \iff -u'(E/L) = \lambda(\tau) \cdot L, \tag{6}$$

$$\dot{\lambda} = \frac{d\lambda(\tau)}{d\tau} = -\frac{\partial H}{\partial S} + \lambda(\tau), \tag{7}$$

$$\lim_{\tau \to \infty} \lambda(\tau)\, e^{-\tau}\, S = 0. \tag{8}$$

Taking the derivative in (7) and arranging we get

$$\dot{\lambda}/\lambda = -(1/\rho - n)\left\{ n' H/\lambda - (\delta' + \rho - n) \right\}. \tag{9}$$

To eliminate λ, we follow the usual procedure by taking the derivative of (6) in terms of time. In this case, the virtual time is relevant. The derivatives are denoted by $\dot{E} = dE/d\tau$ and $\dot{L} = dL/dT$. To simplify the analysis, we also adopt the $CIES$ utility function $u(E/L) = \left[(E/L)^{1-\theta}\right]/(1-\theta)$, $\theta \neq 1$ with $u'' \cdot (E/L)/u' = -\theta$. We get

$$\dot{\lambda}/\lambda = -\theta \dot{E}/E + (\theta - 1)\dot{L}/L, \tag{10}$$

which together with (9) gives $\dot{E}/E = [1/\theta(\rho - n)]\left\{ -n' H/\lambda - \left(\delta' + \rho - \theta n\right) \right\}$, where $\dot{L}/L = n/(\rho - n)$ is applied. Substituting $-n' H/\lambda = [n'/(\rho - n)][\theta E/(\theta - 1) - \delta]$ and noting $\dot{E} = E/(\rho - n)$ we finally derive

$$\frac{\dot{E}}{E} = \frac{1}{\theta}\left\{ \frac{n'}{\rho - n}\left[\frac{\theta E}{\theta - 1} - \delta\right] - \left(\delta' + \rho - \theta n\right) \right\}. \tag{11}$$

The non-linear equations (3) and (11) give the solution of the model. The phase lines become:

$$\frac{\dot E}{E} = 0 \iff E = \frac{\theta-1}{\theta}\left\{\delta + \frac{\rho-n}{n'}\left[\delta' + \rho - \theta n\right]\right\}, \tag{12a}$$

$$\dot S = 0 \iff E = \delta. \tag{12b}$$

In the $(S,E)-$space, the shape of the $\dot S = 0-$line is that of δ, i.e., inverted U with $\delta(0) = \delta(\bar S) = 0$. The shape of the $\dot E = 0-$line depends on the value of θ. Hall has argued that empirical elasticities tend to be large (Hall 1988). Therefore, we assume $\theta > 1$ but nothing essential is changed if $\theta < 1$ is assumed. The following is the sufficient condition for the existence of at least one interior steady state:

Proposition 1 *If $\delta'(0) + \theta n(0) > \rho$ and $\delta'(\bar S) + \theta n(\bar S) < \rho$ then the problem (5) has a steady state $S^* \in (0, \bar S)$.*

Proof. In the $(S,E)-$space the $\dot S = 0-$line hits the $S-$axis at $S = 0$ and at $S = \bar S$. For $S = 0$ and $S = \bar S$ (12a) then becomes $\dot E = 0 \iff E = \frac{\theta-1}{\theta}\left(\frac{\rho-n}{n'}\left[\delta' + \rho - \theta n\right]\right)$. By assumption, $\theta - 1 > 0$, $\rho - n > 0$ and $n' < 0$. Thus, if $\delta'(0) + \theta n(0) > \rho$ and $\delta'(\bar S) + \theta n(\bar S) < \rho$, the $\dot E = 0-$line lies below the $\dot S = 0-$line for $S = 0$ and above it for $S = \bar S$. By continuity, the $\dot E = 0-$line intersects the $\dot S = 0-$line at least once. ■

To comprehend, consider a marginal emission E. If consumed tomorrow, it is discounted by ρ. If consumed today, it adds to pollution stock S and produces a change in abatement $\delta'(S)$ and population $n(S)$ (both as denominator and multiplicator in (4)). If the sum of latter two is larger, consumption today pays. Especially, the first unit of emission is consumed if $\delta'(0) + \theta n(0) > \rho$. On the other hand, if $\delta'(\bar S) + \theta n(\bar S) < \rho$ it never pays to pollute until the carrying capacity $\bar S$.

In what follows, we assume that the number of the steady states is one. Standard local analysis shows that this steady state is a saddle with stable manifolds running from the North-West and South-East (see Appendix A). Figure 1 illustrates. The following lemma characterizes the steady state:

Lemma *Inefficient underaccumulation of the pollutant is not possible.*

Proof. Equations (12a) and (12b) imply that in a steady state

$$\frac{\theta-1}{\theta}\left\{\delta + \frac{\rho-n}{n'}\left[\delta' + \rho - \theta n\right]\right\} = \delta \tag{13}$$

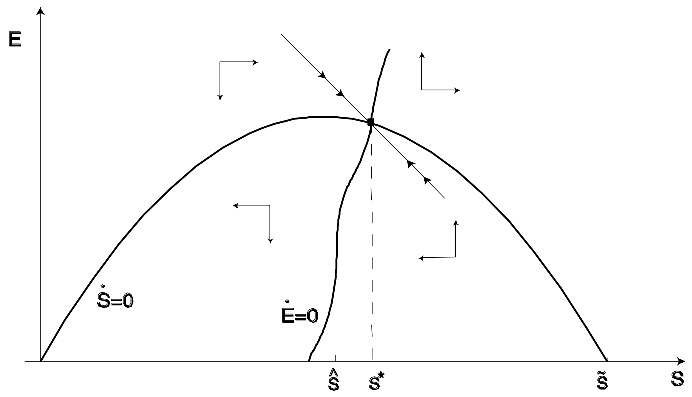

Figure 1: The phase diagram of the model. Note that since $\hat{S} < S^*$, the implied steady state population growth rate $n(S^*)$ is negative.

The transversality condition is $\lim_{t\to\infty}\left\{\lambda(t)\, e^{-t}\, S(t)\right\} = 0$. Because the model tends to the steady state, $S(t)$ goes to a positive constant S^*. In a steady state, $\dot{E} = 0$ so that (10) implies $\dot{\lambda}/\lambda = (\theta - 1)\dot{L}/L$ which is a constant in the steady state. The transversality condition then requires $(\theta - 1)\dot{L}/L - 1 < 0$. Because $\dot{L}/L = n/(\rho - n)$ we get $(\theta - 1)n(S^*)/(\rho - n(S^*)) - 1 < 0$ and further

$$\rho - \theta n(S^*) > 0. \tag{14}$$

Arranging and using (13) we get $\rho - \theta n = \frac{n'}{(\rho - n)(\theta - 1)}\delta - \delta' > 0$. Because $\frac{n'}{(\rho - n)(\theta - 1)}\delta < 0$ it must be $\delta'(S^*) < 0$. Therefore, the steady state is located on the downwards sloping part of the $\dot{S} = 0$ - line. ■

4 Demographic Sustainability

The Brundtland Comission 1987 defines sustainable development as a development that "that meets the needs of the present without compromising the ability of future generations to meet their own needs" (WCED 1987). This definition refers to non-decreasing consumption or non-decreasing utility and these concepts are also applied by most economists (for a review, see Pezzey 1992). From the demographic point of view, a supplement is needed since one can hardly think as sustainable a situation where consumption increases at the cost of human lives. The following alternative definition is given:

Definition *An optimal path is demographically sustainable if it provides non-decreasing consumption for a non-decreasing population.*

Consider first the steady state. Recall that $E = C$ saying that total emission equals total consumption. The growth rate of the per capita emission is $\gamma_{E/L} = \dot{E}/E - \dot{L}/L$. In the steady state, E is constant so that $\gamma_{E/L} = -\dot{L}/L = -n(S^*)$. Three alternatives are possible. For $n(S^*) > 0$ an ever increasing population enjoys ever decreasing per capita consumption. For $n(S^*) < 0$ the contrary is true. For $n(S^*) = 0$ both the population and per capita consumption are constants. This is the consumption-population trade-off in plain. Given the one and lonely planet, each additional inhabitant on it is to dilute the per capita consumption of others. Therefore, the following holds:

Proposition 2 *The only demographically sustainable steady state is that of constant population and constant per capita consumption.*

To stipulate $\gamma_{E/L} = \dot{E}/E - n$ during the off-steady-state transitional period, assume that the economy starts with zero initial pollution stock $S = 0$ and then moves towards the steady state along the North-Western saddle path (see Figure 1). Consider first $\dot{E}/E$. Equation (11) can be rewritten as

$$\begin{aligned}\frac{\dot{E}}{E} &= \frac{1}{\theta}\left\{\frac{n'}{\rho - n}\left(\frac{\theta E - \theta\delta}{\theta - 1}\right) + \frac{n'}{\rho - n}\frac{1}{\theta - 1}\delta - \left[\delta' + (\rho - \theta n)\right]\right\} \\ &= \frac{1}{\theta}\left\{\frac{n'}{(\rho - n)(\theta - 1)}\left(\theta \dot{S} + \delta - \frac{(\rho - n)(\theta - 1)}{n'}\left[\delta' + (\rho - \theta n)\right]\right)\right\}.\end{aligned}$$

Note that $(1/\theta)\left\{\delta - \left[(\rho - n)(\theta - 1)/n'\right]\left[\delta' + (\rho - \theta n)\right]\right\}$ is the difference of the $\dot{S} = 0$ and $\dot{E} = 0$ - lines which it is positive for $S \in (0, S^*)$ (see Figure 1). Because $\dot{S} > 0$ along the North-Western branch and $n'(S) < 0$ by assumption, we have $\frac{\dot{E}}{E} < 0$ for all $S \in (0, S^*)$. Next consider $\gamma_{E/L} = \dot{E}/E - n$. By assumption, $n(0) > 0$ so that $\gamma_{E/L} < 0$ for $S = 0$. Because $\lim_{S \to S^*} \dot{E}/E = 0$ we have $\lim_{S \to S^*} \gamma_{E/L} = -n(S^*)$. The above three cases now appear. If $n(S^*) > 0$ we have $\gamma_{E/L} < 0$ for all $S \in (0, S^*)$ and $\lim_{S \to S^*} \gamma_{E/L} < 0$. If $n(S^*) = 0$ we have $\gamma_{E/L} < 0$ for all $S \in (0, S^*)$ and $\lim_{S \to S^*} \gamma_{E/L} = 0$. Since $\dot{S} > 0$ along the North-Western branch, the two cases imply that the time path for per capita consumption E/L is decreasing and approaches the steady state limit E^*/L^* which is zero for $n(S^*) > 0$ and positive for $n(S^*) = 0$. These time paths are depicted in the leftmost panel of Figure 2. Finally, if $n(S^*) < 0$ we have $\lim_{S \to S^*} \gamma_{E/L} > 0$. Since $\gamma_{E/L} < 0$ for $S = 0$ there exists at least one S such that $\gamma_{E/L}$ = 0 by continuity of $\gamma_{E/L}$. Therefore, the implied E/L first decreases and then increases as is shown in the rightmost panel of Figure 2.

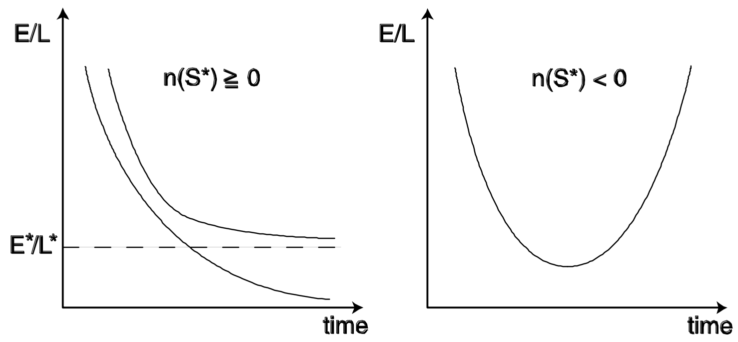

Figure 2: The time paths for per capita emission (consumption) E/L.

Which of these three cases is to be realized? First note that the *a priori* assumptions are $\rho > 0$ and $\rho - n(S) > 0$ for all S and that they pose no explicit limit to $sign\ n(S^*)$. An other candidate to limit $sign\ n(S^*)$ is the transversality condition in (14) but for suitable values of ρ, θ the transversality condition can hold for positive and negative values of $n(S^*)$. We summarize this discussion as follows:

Proposition 3 *In the steady state S^*, the population may be constant, increasing or decreasing.*

The steady state population growth rate $n^* = n(S^*)$ is endogenously determined in the model and the utilitarian objective functional $\int_0^\infty u(E/L)\, Le^{-\rho t}\, dt$ may take its maximum both at high E/L and low L or vice versa. It may well be optimal to choose increasing consumption at the cost of population. Therefore, each of the transitional time paths in Figure 2 is possible. Or, to put it differently, it possible that the (asymptotic) extinction of the species *Homo Sapiens* is optimal.

4.1 A Parametric Example

Chapter 2 reports some recent evidence of the positive Malthusian check but there is only little evidence on the functional formula by which this check steps in. However, in the Report of Rome (Meadows *et al.* 1972) some alternatives, repeated in Figure 3, are suggested. In A population growth decreases linearly, in C the negative effect is exponential, and in B mortality increases as pollution bypasses a threshold level. We concentrate on case B since the existence of such a threshold is often discussed (see already Meadows *et al.* 1972).

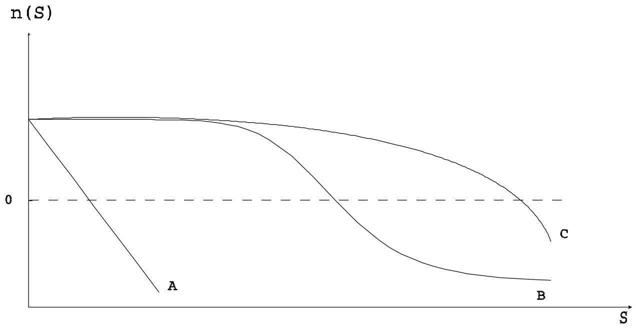

Figure 3: Possible functional formulas for the positive check. Meadows *et al.* 1972.

One of the simplest algebraic expressions to produce B is

$$n(S) = \beta - \frac{\alpha}{1 + (\mu S)^{-\gamma}},$$

in which β gives the population growth for $S = 0$, $\beta - \alpha$ gives the lowest population growth reached, $\mu > 0$ multiplies the effect of S such that large values of μ lead to negative population growth at low concentrations. Finally, $\gamma > 0$ gives the curvature of the function with high values referring to curved shape and severity of the crisis after the threshold. Further, we assume that the abatement function takes the logistic formula

$$\delta(S) = rS\left(1 - \frac{S}{\bar{S}}\right), \tag{15}$$

in which $r > 0$ is the intrinsic rate of decay.

Let conventional values $\theta = 3$, $\rho = 0.04$ describe the preferences (compare for example Barro and Sala-i-Martin 1995). Further, let $r = 0.15$ and $\bar{S} = 1000$ describe the abatement function. Let the demographic parameters be $\beta = 0.005$ referring to 0.5% population growth rate for $S = 0$, and $\alpha = 0.01$ saying that the lowest population growth is -0.5%, a value that seems to be modest rather than inordinate. Further, let $\mu = 0.002$ and $\gamma = 6$. These parameters imply that $\hat{S} = \frac{1}{2}\bar{S} = 500$, i.e., population starts to decrease as the concentration reaches half of the carrying capacity limit. The model has a steady state at $S^* = 683$ and the steady state population growth rate is $n(S^*) = -0.0037$ so that in the steady state the population halves in every 187 years whereas per capita consumption doubles in the same time. The depicted off-steady state path for population shows that the critical value $\hat{S} = 500$ is reached in only about 20 years because the positive population growth increases the pace of pollution initially. From this on, population decreases, pollution accumulates much slowlier and per capita emission (consumption) starts to increase after some 50 years. After 270 the population is only 40% of its original size.

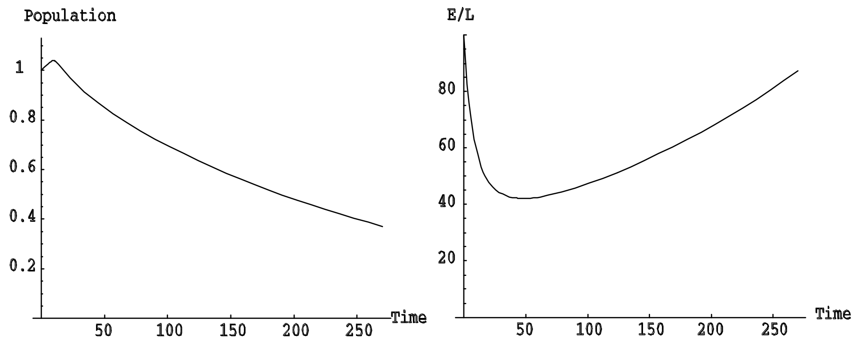

Figure 4: The parametric time paths for population and per capita consumption in the case $n(S^*) = -0.0037$.

5 Technical progress

The optimists argue that technical progress will warrant the sustainability (Neumayer 1999). To see if this optimism is supported by the model let $A(t)$ be the available technology at time t and assume that technical progress is exogenously running at rate x so that $A(t) = e^{xt}$ for $A(0) = 1$. Further, let technical progress be "consumption augmenting" in the meaning that at every instant of time t, we have $C = e^{xt}E$, i.e., for given emissions it is possible to consume more than earlier (Krautkraermer 1985). Per capita consumption then becomes

$$C/L = e^{xt}E \,/\, L. \tag{16}$$

Per capita consumption C/L grows at rate $\gamma_{C/L} = E/E + x - n$. In a steady state, $E/E = 0$ so that $\gamma_{C/L}$ is positive if $x > n(S^*)$. Therefore, in the presence of technical progress it is *possible* to have growing per capita consumption and growing population together.[7] However, the positive population growth is by no means warranted. To see why, apply (16) to (5)–(10) to derive[8]

$$\frac{\dot{E}}{E} = 0 \Leftrightarrow E = \left\{ \frac{\theta - 1}{\theta}\delta + \left[\frac{\rho - n}{n'}\delta' + (\theta - 1)x + (\rho - \theta n)\right] \right\}, \quad (17)$$

The derivative of (17) in terms of x is:

$$\frac{\partial E}{\partial x}\bigg|_{\dot{E}=0} = \frac{(\theta - 1)^2 (\rho - n)}{\theta n'} < 0.$$

Therefore, the $\dot{E} = 0$ – line shifts down as the pace of technical progress increases (Figure 5).

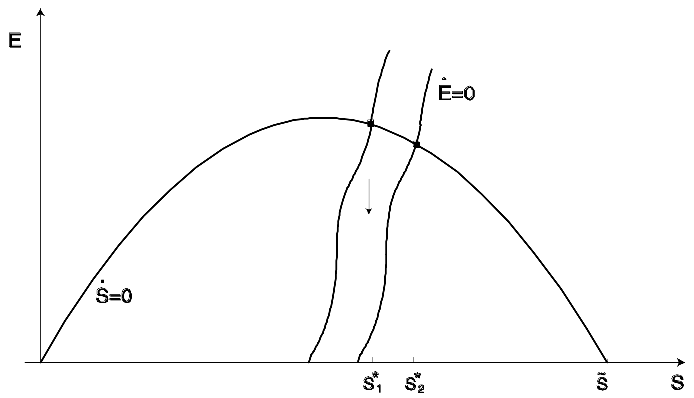

Figure 5: Technical progress shifts the $\dot{E} = 0$ – line down.

To comprehend, note that in the equilibrium, the *negative* utility effect of a marginal emission through the increase in S and decrease in population growth, and its *positive* utility effect through the increase in consumption just cancel and the marginal emission is rejected. Technical progress increases the positive consumption effect and the marginal emission is accepted with the result that the steady state pollution stock S^* increases. Given the biologically determined $\hat{S}$, it is then more likely that $\hat{S} < S^*$ i.e., $n(S^*)$ is negative. On the contrary to common wisdom, we thus find that technical progress does not necessarily save

[7] To determine the off-steady state path is slightly more complicated than above. Most importantly, however, it is possible to have increasing per capita consumption during the transition if $x > n(S^*) > 0$.

[8] Technical progress has no effect on the $\dot{S} = 0$ – line.

us. Quite the opposite, it may shorten the expected future of humans because it makes extra consumption and emission to pay. This result naturally depends on the chosen type of technical progress. If we argue that technical progress takes place in medicine and eliminates the link from S to n, then the conclusion may be reversed. In the real world, both types are present, and the race between them determines the net demographic e¤ect of technical progress.

6 Discussion

The model of optimal pollution above introduces population that is endogenous to pollution by assuming the Malthusian positive check. The model shows that a con‡ict between optimality and sustainability may appear. It may well be optimal to consume at the cost of human lives. Solow has suggested that "The theory of optimal growth ... is thoroughly utilitarian in conception. It is also utilitarian in the narrow sense that social welfare is (usually) de…ned as the sum of the utilities of di¤erent individuals or generations" (Solow 1974). In the case of endogenous population, the utilitarism takes an extreme expression: a path, which ultimately leads into extinction, may still be optimal. Naturally, di¤erent results would have been derived if positive population were posed as *a priori* constraint to the optimization. Perhaps one argues that this should have been done, but looking at the actual behavior around one may conclude that the utilitarian approach as a description of the current state of a¤airs is not so distorted after all.

An article of sustainable growth is, more or less, a wake-up call. Broadly speaking, one wants to tell, what may happen, if environmental issues are not considered seriously. Therefore, it is important to give the right signal. With population exogenous, the warning signal is that per capita consumption is decreasing. On the contrary, with population endogenous and with the positive check present, increasing consumption signals that sustainability fails. It tells that even if some people su¤er and die, as long as the waste majority goes happily consuming the path is really "optimal" in the meaning that total utility is maximized. Naturally, projecting these results to the real world must be done with care. In the model, there is perfect foresight and a once-for-all choice of the optimal path but in the real world, each generation is free to makes its own choices and the future is less transparent. However, if consumption keeps increasing and if pollution-related mortality keeps tolerable, the annoying conclusion is that — in the real world as well as in the model — the incentives for a change in economic behavior are not sufficient.

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A Appendix: Local Stability of the steady states

Lets write $\dot S = \varphi(S, E)$ and $\dot E = \phi(S, E)$. The Jacobian of the model is

$$J = \begin{bmatrix} \varphi_S & \varphi_E \\ \phi_S & \phi_E \end{bmatrix}.$$

As evaluated around the steady state, its elements become

$$\begin{aligned}
\varphi_S &= -\delta', \\
\varphi_E &= 1, \\
\phi_S &= \frac{E}{\theta}\left\{ \frac{-n''(\rho - n) - (n')^2}{(\rho - n)^2}\left(\frac{\theta E}{1-\theta}+\delta\right) - \frac{n'}{\rho - n}\left[\delta'\right] - \left[\delta'' - \theta n'\right]\right\}, \\
\phi_E &= \frac{1}{\theta}\left\{\frac{n'}{\rho - n}\left(\frac{\theta E}{\theta - 1} + \delta\right) - \left(\delta' + \rho - \theta n\right)\right\} \\
&\quad + \frac{E}{\theta}\left\{\frac{n'}{\rho - n}\frac{\theta}{\theta - 1}\right\} \\
&= \frac{n' E}{(\rho - n)(\theta - 1)},
\end{aligned}$$

in which the last row is derived by using (13) and (12b). Because ϕ_S contains the undefined second derivative of $n(S)$, we write

$$\begin{aligned}
DET\ J &= \varphi_S \cdot \phi_E - \phi_S \cdot \varphi_E \\
&= \left(-\frac{\varphi_S}{\varphi_E} - \left(-\frac{\phi_S}{\phi_E}\right)\right)(-\varphi_E)\cdot\phi_E.
\end{aligned}$$

The expression $(-\varphi_E)\cdot\phi_E = -\frac{n'E}{(\rho - n)(\theta - 1)}$ is positive. The expression in the square brackets is the difference in the slopes of the phase lines $\dot S = 0$ and $\dot E = 0$. In the steady state, the $\dot E = 0-$line hits the $\dot S = 0-$line from below and this expression is negative implying $DET\ J < 0$. Therefore, the steady state is a saddle. Because $\varphi_E > 0$, we have $\dot S > 0$ $(\dot S < 0)$ above (below) the $\dot S = 0-$line. Because $\phi_E < 0$, we have $\dot E > 0$ $(\dot E < 0)$ below (above) the $\dot E = 0-$line as is depicted in Figure 1.