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Essays on Labor Market Frictions, Technological Change
and Macroeconomic Fluctuations

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Foreword

Juuso Vanhala's doctoral dissertation is an interesting collection of essays focusing on labor market frictions, technological change and cyclical shocks. The first essay investigates the role of skill mismatch in determining the relation between economic growth and equilibrium unemployment by adding vintage human capital into the standard vintage capital/search model to study the relation between capital-embodied growth and equilibrium unemployment. He shows that faster human capital depreciation reduces unemployment, which leads to more job creation and less job destruction. This effect works in an opposite direction compared with the standard capital obsolescence effect that reduces job duration. The second essay examines a search model with heterogeneous workers and jobs by assuming that high-quality workers can undertake both high-quality and low-quality jobs, while low-quality workers can only carry out low-quality jobs. He extends the existing literature as follows: (i) the distribution of skills across workers is endogenized and (ii) the labour force participation is also endogenized. The third essay concentrates on business cycles and taxation in a matching model by studying the effects of labour taxes and reforms thereof on equilibrium unemployment and the sensitivity of an economy to macroeconomic shocks. The policy instruments to be analyzed are (i) the marginal tax rate, (ii) the tax subsidy to produce a tax progression scheme and (iii) the replacement ratio to account for variability in the outside option for workers. He shows, for example, that a higher marginal tax rate and a higher replacement ratio amplify shocks, while tax subsidies dampen them.

This study is part of the research agenda carried out by the Research Unit of Economic Structure and Growth (RUESG). The aim of RUESG is to conduct theoretical and empirical research with respect to important issues in industrial economics, real option theory, game theory, organization theory, theory of financial systems as well as problems in labour markets, natural resources, taxation and time series econometrics.

RUESG was established at the beginning of 1995 and is one of the National Centres of Excellence in research selected by the Academy of Finland. It is financed jointly by the Academy of Finland, the University of Helsinki, the Yrjö Jahnsson Foundation, Bank of Finland and the Nokia Group. This support is gratefully acknowledged.

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The third essay was written while I was visiting the Bank of Finland Research Unit. I thank the Research Unit for its hospitality and for encouraging me to broaden my research interests into macroeconomic issues. Suggestions and comments from Docent Juha Kilponen, Docent Antti Ripatti, Docent Juha Tarkka and other participants of the research unit workshop improved my work considerably. I owe special thanks to Docent Jouko Vilmunen, who has been a mentor to me throughout the different stages of my thesis work. His encouragement and endless enthusiasm have been invaluable for my work. I made the last revisions to the thesis while working at the Bank of Finland Monetary Policy and Research Department. My warmest thanks go to my superiors, Head of department Antti Suvanto, Head of division Mika Pösö and Advisor Veli-Matti Mattila, for their flexibility.

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Chapter 1

Introduction

1 Background

The state of the labor market and the macroeconomy are shaped by continuous restructuring and reallocation of resources. Old production units and jobs are destroyed and new ones are created as the resources of the economy seek more productive uses. This churning is driven by the basic nature of economic activity. Innovations (e.g. computerization) change the production technologies available to firms and workers, changing consumer preferences alter the demand of goods and services, and shocks that change the economic climate hit economies from time to time. Firms and workers reallocate to accommodate these changes and to seek more profitable economic opportunities. Restructuring takes place at all levels of the economy: individual job-worker matches separate and reallocate when the mutual potential for benefits is exhausted, whole sectors of the economy may become obsolete and be replaced by new emerging sectors. Restructuring is also both a long run and a short run phenomenon: long run growth is characterized by continuous adoption to new production technologies, but restructuring takes place also at business cycle frequency. The large scale and continuously ongoing restructuring and reallocation of resources is often referred to by the Schumpeterian notion of 'creative destruction'. Ideally, the destruction of production units frees inefficient economic resources to more productive use, to match in more profitable combinations, making the economy work at its maximum potential. Reality is not as smooth as this ideal, however, as restructuring involves frictions and rigidities when production factors search new partners to match with.

The large labor market flows and frictions involved in reallocation have received considerable attention in recent research. The flows into and out of unemployment are quantitatively huge although the variations in the stocks of employment and unemployment may be small. The empirical literature (e.g. Davis et al. (1996) or

Cahuc and Zylberberg (2004)) documents typical flows both in and out of unemployment to be of an annual rate of about ten percent of the labor force for OECD economies. Matching frictions or problems of mismatch related to job flows have been evidenced by a large body of studies of a relationship between unemployment and vacancies known as the Beveridge curve (Blanchard and Diamond 1989, Nickell et al. 2003). The Beveridge curve (V/U curve) may be considered as an indicator of mismatches of the labor market as it describes a negative relationship between the equilibrium unemployment rate and the vacancy rate. Whereas movements along the curve indicate cyclical fluctuations, outwards shifts in the curve reflect deterioration in the matching process of firms and workers. These studies have observed an outwards shift of the Beveridge curve for most OECD countries from the 1960s (see e.g. European Central Bank 2002 for a summary). An outward shift of the curve reflects a simultaneous increase in both the number of unemployed workers and the number of open vacancies. This simultaneous excess supply suggests increasing problems of labor market mismatches.¹

This dissertation studies labor market mismatches and rigidities, that arise from exogenous changes in the economy; technological change and macroeconomic shocks. The first essay studies skill mismatch related to long run economic growth and technological change. The second essay investigates skill biased technological change, the allocation of worker and job types, skill mismatch and job competition, which have become important phenomena in European economies. The final essay studies the interaction of matching frictions and labor taxation, and investigates the potential role of taxation and labor market frictions in determining the sensitivity of an economy to cyclical fluctuations. This dissertation analyzes reallocation both from the long run and the short run perspective, but with a common perspective on labor markets: idleness of resources is an inevitable ingredient of reallocation. Although resources are reallocated to more productive use, frictions and mismatches imply idleness of resources as a by-product. The focus of this work is in perhaps the most visible type of economic idleness - unemployment. The dissertation contributes to the literature on theoretical matching models, and their applications to labor markets and the macroeconomy.

¹An interesting feature in the shifts of the Beveridge curves is that after the mid- 1980s, countries fall into two groups. For the first group the shift has continued and for the second group there has been a turnaround s.t. the curve has started moving back to the left . The first group consists of Belgium, Finland, France, Germany, Japan, Norway, Spain, Sweden and Switzerland. The second group consists of Canada, Denmark, Netherlands, the UK and the US. Australia, Austria, New Zealand and Portugal are somewhat ambiguous, although they rather belong to the latter group. (Nickell et al. 2003)

Search-matching models have become the workhorse modelling framework of theoretical labor economics and the literature has become extensive in the past decade (Rogerson et al. 2005). These models acknowledge the central role of the flows and frictions that are associated to the continuously ongoing restructuring in economies. Frictions can basically be considered as anything that interferes with the smooth and instantaneous exchange of goods and services. The labor market has attracted perhaps the most theoretical and empirical interest in this area of research, because of the heterogeneities that prominently feature in these markets, but also the availability of data on job and worker flows enable testing the theory (Pissarides 2002).

The current search-matching models of the labor market have their foundations in the early study by Phelps (1970) who showed search theory to be useful in analyzing the natural rate of unemployment and the trade-off between inflation and unemployment.² The contributions of Diamond (1982), Mortensen (1982) and Pissarides (1985) lead to the development of the current equilibrium models which produce more accurate predictions of the behavior of the labor market than the neo-classical framework (Pissarides 2002). Instead of setting the focus on the real wage cost of labor and the stock of labor demand, the Diamond-Mortensen-Pissarides model centers on the dynamic labor market flows and the matching of firms and workers. In equilibrium the flows into and out of unemployment are equal. The centerpiece of these models is the matching process of firms and workers. These models acknowledge that finding a worker for a job, or for a worker to find an open job vacancy, is a time consuming and costly activity that is characterized by search frictions. Petrongolo and Pissarides (2001) provide an extensive discussion on the matching process of unemployed workers and vacancies and on the micro-foundations of the matching function. The Diamond-Mortensen-Pissarides search and matching model has established itself as the standard theory of equilibrium unemployment. Despite the popularity of the model, several criticisms have been made and important research is being made in many areas of this field.³ The issues studied in this dissertation relate to three currently active areas of research. The first two relate economic restructuring and reallocation in the long run, the third is related to macroeconomic shocks and short run restructuring.

A long tradition of works on growth and unemployment has produced a number

²Even earlier contributions to this area were Stigler's (1961) fixed-sample rule, which chooses the optimal number of seller's before buying at the lowest price, and McCall's (1970) idea of an optimal reservation price, upon which to stop search (see e.g. Pissarides 2002).

³For thorough surveys on search and matching models see Rogerson et al. (2005) or Mortensen and Pissarides (1998). Pissarides (2000) presents an excellent overview of matching models.

of perspectives on the growth-unemployment relationship. Indeed, the relation between technical progress and unemployment has proven to be controversial both in popular debate and in the theoretical literature. Studies on the link between technical progress and unemployment have produced conflicting results and the literature has pointed out that the relation between labor markets and technical progress is far from unidimensional (Calmfors and Holmlund 2000).⁴

In models where the rate of technical progress or growth is assumed exogenous, technology can be assumed to be disembodied or embodied. In the former approach new technology benefits all new and existing jobs, without a need to replace the capital stock for the existing firms (Pissarides 2000). In Aghion and Howitt (1994) firms 'capitalize' the benefits of new technology through spillovers, which increase the returns to jobs and leads to the creation of more vacancies and lower equilibrium unemployment. In the second approach technology is embodied in capital and the idea of 'creative destruction' is predominant. In 'vintage' models of this type, new technology benefits only firms that explicitly invest in it, thus raising only the productivity of matches with the new technology. Consequently, firms using older technology vintages are eventually made obsolete by the new generations of technologies.

Studies by Aghion and Howitt (1994, 1998) and Mortensen and Pissarides (1998) have advocated the vintage capital - matching framework, which has become the benchmark for the study of growth and unemployment. In these models the technological frontier and productivity of new jobs increase in time, but the technology of capital and productivity of a worker in a given match are fixed at the time of job creation. The competing new jobs pay higher wages due to their higher productivity. A feature of these models is the implicit assumption of perfect labor mobility between technology vintages.⁵ New more productive jobs raise the outside option of an employed worker and raise the wage in existing matches. This leads to declining and eventually zero match surplus and destruction of the match. Technological acceleration turns existing jobs unprofitable and obsolete faster, reducing the duration of job matches. Equilibrium unemployment is raised directly due to a higher separation rate but also indirectly because shorter job duration and higher wages reduce the profitability of matches and reduce incentives to create new job vacancies.

Aghion and Howitt (1998) extend the vintage framework to study the effects

⁴See Calmfors and Holmlund (2000) for a survey on growth-unemployment studies.

⁵This means that workers are assumed to be able to change jobs to more recent technology vintages, irrespective of the fact that their skills represent an older, maybe already obsolete technology vintage.

of intersectoral complementarities. The value of jobs in a sector of the economy increases with an increase in productivity in a sector producing complementary intermediate inputs. Thus when sectors are highly complementary, growth of productivity may lead to lower equilibrium unemployment even when there is no capitalization effect. The study also discusses the feedback of unemployment back onto the equilibrium growth rate.⁶ Mortensen and Pissarides (1998) present a more general vintage matching model where firms are allowed to update their technology by paying an implementation cost. When the cost is high enough, renovation of matches is not worthwhile so obsolete jobs are destroyed endogenously and the effect of creative destruction dominates. When the cost of implementation is sufficiently small, firms update obsolete matches and the capitalization effect dominates. In this case faster technical progress makes technological updating of jobs more frequent, which keeps the jobs closer to the productivity frontier and makes them more profitable. This increases job creation and reduces equilibrium unemployment.

An intimately related area of research focuses on the interactions of technological change, skills and specificity. Since the seminal work of Becker (1964) the problems related to skill investment and specificity of skills have been acknowledged in the literature. Caballero and Hammour (1997, 1998a, 1998b) study the implications of specificity and appropriability of job specific rents in the context of growth and job reallocation. They argue that capital is less elastic in the short run than in the long run, and that the attempt by workers to appropriate capital will in the long term lead to a substitution away from labor in favor of capital. The fear of substitution constitutes a mechanism that prevents these attempts. Caballero and Hammour (1998) analyze how quasi-rents and appropriation lead to underinvestment in production factors and sclerotic behavior of the economy. Embodiment of technology in capital vintages and job specific human capital gives rise to appropriable quasi rents that distort job reallocation in the process of creative destruction.

A feature that has received attention in recent research is the idea of skill dynamics, i.e. the idea that human capital evolves over time. Ljungqvist and Sargent (1998) study the determination of equilibrium unemployment in a model where workers accumulate skills on the job and lose skills during unemployment. Their focus is on how labor market policy affects the search incentives and skill depreciation. Their model produces duration dependence to unemployment. Skill dynamics are also present in Laing et al. (2003) who incorporate growth of the stock of public knowledge in the economy. They introduce overlapping generations (vintages) of workers who differ in productivities s.t. knowledge growth is embodied in

⁶Creative destruction effects of are also analyzed in Caballero and Hammour (1994, 1996) with a more cyclical perspective.

new entrant workers. In this setup the entry of new (younger) workers renders the knowledge of older workers obsolete. Galor and Moav (2000) build a growth model which is characterized by ability biased technological transition where technological acceleration leads to a rise in wage inequality both between and within skill groups. In their study, technological progress is assumed to reduce the adaptability of existing human capital for the new technological environment. While superior technologies increase productivity, they also erode existing human capital that is adaptable to the new technologies, thus reducing productivity. A characteristic of their model is that able individuals have a comparative advantage in adapting to new technology. Violante (2002) considers workers with two-dimensional skills. An employed worker accumulates skills by learning-by-doing, but when changing jobs to one of a more recent technological vintage, only a fraction of the workers skills are transferable. Technological acceleration reduces workers' capacity to transfer skills from old to new machines, generating cross-sectional variance of skills and therefore wages. Ljungqvist and Sargent (1998) and Violante (2002) do not study the 'creative destruction' -nature of technological change as their models do not embed the vintage capital structure. Carré and Drouot (2004) introduce learning-by-doing into a vintage capital framework but do not consider transferability of skills between technology vintages.

The first essay (Chapter 2) relates to the issues above. Within a vintage-matching model the essay studies skill transferability between jobs and the effect of growth and technological change on job creation, job destruction and equilibrium unemployment.

Another area of intensive research in the past decade has focused on skill mismatch, job competition and crowding in labor markets. Most OECD countries have experienced a the structural change in both the supply and demand for skills over the last three decades. Universally a strong shift in demand for more highly educated labor has occurred, which is to a large degree associated with skill-biased technological change. At the same time the educational level of workers has increased. Changes in the supply of labor have varied more between countries, but a strong trend toward an increased proportion of workers with college training is universal in the OECD. Along with skill-biased technical change and the higher educational levels, labor markets have been subject to increasing mismatch between the skills of workers and the skills required by firms. In EU countries that have experienced an intense upgrading of tertiary education within the last fifteen years, the supply of skilled workers has outstripped the supply of skilled jobs and 'job competition', overeducation and 'crowding out' have been particularly relevant (Dolado et al.

2003). Collecting results from various studies on the incidence of mismatch Hartog (2000) concludes that the incidence of overeducation has increased while the incidence of undereducation has decreased for three countries for which observations are available (Netherlands 1960-1995, Spain 1985-1990 and Portugal 1982-1992). For some countries however, notably the United States and Great Britain, the development of skill demand and supply has been somewhat different. In particular the United States experienced a sharp slowdown in the growth of the supply of skills after the 1980s (Katz 1994).

A number of theoretical studies have investigated these issues. Davis (1995) and Acemoglu (1999) investigate how well the labor market performs in matching workers and firms, both in terms of total vacancy creation and in the allocation of vacancy creation across job types. They study the performance of a frictional labor market with decentralized bargaining compared to the socially optimal job creation outcome and examine the extent to which the market outcome produces mismatch in the demand and supply for skills. Davis (1995) and Acemoglu (1999) show that matching markets with heterogeneous jobs tend to produce too many 'bad' jobs and too few 'good' jobs. They argue that whenever workers have any bargaining power the firm's marginal cost from investing in high productivity jobs exceeds the marginal increase in the firm's surplus as some fraction of the surplus goes to the worker whereas the cost goes solely to the firm. A zero share of match surplus for workers is then required to provide firms incentives to allocate vacancy creation optimally to high skill jobs. Furthermore, Davis (1995) and Ljungqvist and Sargent (2000) show that there is a fundamental conflict between achieving an optimal allocation of vacancies and an optimal total amount of vacancies. The latter requires that the Hosios (1990) condition is satisfied which implies a positive bargaining power for workers to prevent excess total vacancy creation. This fundamental tension in achieving efficiency in both respects, allocative efficiency requiring zero bargaining power and efficiency in total creation requiring a positive bargaining power for workers, makes policy recommendations particularly difficult.

In the above studies heterogeneity is present only on the firms' side of the market as workers are homogeneous and they are all assumed to be suitable for all jobs. As this assumption is somewhat unappealing, models where heterogeneity is present on both sides of the market have emerged subsequently. Mortensen and Pissarides (1999) study skill biased technological change in a model where jobs have skill requirements and suitability of workers to jobs is a key feature. In their model the labor market is fully segmented to sub-markets according to skill. Marimon and Zilibotti (1999) develop a model of two-sided heterogeneity to study skill biased technological change and the allocation of workers to suitable jobs.

Albrecht and Vroman (2002) and Gautier (2002) introduce the possibility of cross-skill matching equilibrium in a matching model, where workers differ in skills and firms differ in skill requirements. These models assume an exogenous skill distribution of workers and firms make the choice of skill requirements (job type) endogenously. The key feature in their models is the assumption of minimum skill requirements for jobs. High-skill workers are suitable for both high-skilled jobs and low-skilled jobs, but low-skilled workers are suitable only for low-skill jobs. High-skill jobs are more productive than low-skill jobs, but high-skill jobs require a high-skill worker and are therefore harder to fill. This implies that high-skilled workers have a more favorable position in the labor market, as they are suitable for both high and low-skill jobs whereas low-skill workers are suitable for low-skill jobs only. It also implies that the relevant pool of unemployed workers to match with is larger for firms that post low skill vacancies than it is for firms posting high skill vacancies. Albrecht and Vroman (2002) study the equilibrium mix of job types and the equilibrium relationship between worker and job characteristics, wages and unemployment. Building on Albrecht and Vroman (2002), a number of studies have extended the model of two-sided heterogeneity in various directions. Blazquez and Jansen (2005) study the efficiency of matching and find contrasting results to those of Davis (1995) and Acemoglu (1999). They show that matching markets with two-sided heterogeneity tend to produce too many 'good' jobs and too few 'bad' jobs. Mismatch manifests itself as job competition for low skill jobs and 'crowding out' of low-skill workers and is a signal of overeducation in the labor market. Dolado et al. (2002, 2003) introduce on the job search into this type of framework.

The incentives for workers to invest in education have been studied since Becker (1964) by numerous authors. Becker (1964) shows that investment in education is efficient from the social viewpoint in a competitive market as both the costs and returns accrue equally to the worker. In the presence of labor market frictions this is not the case however. As match surplus accrues only partially to the worker while the costs, there is underinvestment in skills (see e.g. Acemoglu 1996). Moen (1999) shows that when the job arrival rate for a worker depends on the skill level relative to other workers, there may be overeducation, at least when job competition is high.

Skill mismatch and job competition are reflected in wages and unemployment rates across and within skill groups. However, as Murphy and Topel (1997) claim, the unemployment rate has become less informative about the state of the labor market. For example, in a study evaluating histories of unemployment and non-employment among American men between 1967-1994, the unemployment rate in 1994, 4.7 percent, is only slightly higher than in 1974 or 1978-1979, whereas non-participation more than doubled from the late 1960's to the 1990's (from 4 percent

to 8.1 percent of the potential labor supply in 1994). When considering differences across skill groups, they observe a declining trend in the returns to work and labor market opportunities, especially among the low skilled workers. This is reflected both as higher unemployment and higher nonparticipation rates.

Considering skill mismatch, unemployment and nonparticipation together is essential as they reflect two sides of the same phenomenon. As Sattinger (1995) points out this leads to an important difference with search models with a fixed labor force: a higher unemployment rate is consistent with higher a higher employment rate. Or vice versa lower employment is consistent with a lower unemployment rate, so that joblessness is absorbed to the pool of nonparticipants. Participation and labor market heterogeneity have been studied in the literature (e.g. Sattinger 1995, Pissarides 2000) by considering heterogeneous preferences of leisure or values of non-market time (determined by e.g. wealth). In earlier studies (e.g. McKenna 1987, Sattinger 1995) it has been common to assume that labor market participation depends positively on the value of being unemployed. More recently, Garibaldi and Wasmer (2003, 2005) and Pries and Rogerson (2004) study the importance of frictions in a model of endogenous participation which feature an irreversible entry cost to participating to the labor market. They show that decisions to participate and stop participating differ, and that labor supply is determined by two margins: exit and entry. These margins coincide when the irreversible sunk cost vanishes.

The second essay (Chapter 3) studies skill mismatch and job competition following this tradition of research. The essay considers skill biased technological change and the endogenous determination of workers' skills and labor market participation.

The business-cycle-frequency implications of reallocation in the labor market and the frictions involved with it have recently been subject to intensive research. There are two active areas of research, one relating to the performance of the Mortensen-Pissarides matching model, the other relating to the performance of business cycle models.

There is a rapidly growing literature on a central handicap of the Mortensen-Pissarides matching framework (Mortensen and Pissarides 1994, Pissarides 2000), often referred to as the 'Shimer critique'. With shocks of a plausible magnitude the matching model cannot generate the business-cycle-frequency fluctuations in two of its key elements, unemployment and vacancies (Shimer 2005). This handicap of the model arises because the wage negotiated by Nash bargaining absorbs too much of the shocks of an economy. Consequently, there has been an active debate in the literature about the role and modeling of the wage in the matching model to achieve more rigid wages (see e.g. Shimer 2005, Hall 2005, Mortensen and Nagypál

2005, Hornstein et al. 2005). Popular solutions to this problem are variants of the solution formalized by Hall (2005), where the wage is partly determined by negotiation and partly by a wage norm. The problem in this solution is that the wage norm is somewhat *ad hoc*.

An area which has received attention recently and where the application of matching models is producing important advances is business cycle theory. A salient feature of models of the business cycle has been their unsatisfactory performance in producing shock responses that correspond to empirical evidence. This has been the case for real business cycle (RBC) models in capturing some of the stylized facts of the labor market and the shock persistence in output (den Haan et al. 1996, Merz 1995). For New Keynesian monetary models, capturing the persistence in output and inflation responses to monetary shocks has posed a major challenge (Walsh 2005, Galí and Gertler 1999).⁷ In addition New Keynesian models have been unable to explain why aggregate shocks cause large and persistent fluctuations in equilibrium unemployment (Trigari 2004).⁸

A number of recent papers have been successful in improving the performance of real business cycle models by combining the search-matching framework of the labor market to business cycle models. The first wave of studies in this area (Merz 1995, Andolfatto 1996, den Haan et al. 2000) applied the search-matching model to real business cycle models. These studies showed that introducing matching frictions of the labor market improves the shock responses of RBC models considerably. In the past few years, the matching model has been applied to the New Keynesian monetary model.⁹ Walsh (2003, 2005) and Trigari (2004) find that introducing la-

⁷Monetary shocks generate a large and persistent hump-shape response in output. For example, Christiano et al. (1999) find that in response to a contractionary policy shock, after a delay of two quarters there is a sustained decline in real GDP with the maximal decline taking place roughly after a year to a year and a half after the policy shock. Inflation behaves in a similar way but the responds more slowly. After an initial delay, the policy shock generates a persistent decline in the index of commodity prices s.t. the GDP deflator is flat for roughly a year and a half after which it declines.

⁸For New Keynesian models with moderate degrees of price stickiness both the persistent output and inflation responses have been difficult to capture and a number of extensions have been studied to generate the observed persistence (see e.g. discussion in Galí and Gertler (1999) and Christiano et al. (2001)). Galí and Gertler (1999) modify the Calvo (1983) -type price stickiness and purely forward looking price setting by firms by allowing for a fraction of firms that use a backward looking rule to set prices. Christiano et. al (2001) extend the baseline New Keynesian model by introducing habit persistence in consumption preferences, adjustment costs in capital investment and variable capital utilization. Dotsey and King (2001) introduce labor supply variability through changes in employment, the extensive margin, instead of in the number of hours, the intensive margin.

⁹See Clarida et al. (1999) for an excellent overview of the New Keynesian framework.

bor market frictions and endogenous job destruction into the New Keynesian model improves the performance of the model in generating shock responses that match with U.S. data. A key feature of these models is that they introduce employment adjustment in business cycle models through changes in the number of employed workers (the extensive margin) instead of in the number of hours (the intensive margin). This, combined with search frictions of the labor market, generates involuntary unemployment and sluggish employment adjustment into the business cycle models. The rigidity in the adjustment of labor has proved to be of essence in generating persistence into the business cycle models. Following the idea of Hall (2005) of a wage that is partly determined by negotiation and partly by a wage norm, wage rigidity has been incorporated along with matching frictions to the New Keynesian model in Krause and Lubik (2005). They demonstrate that the model performance improves further, when wages respond less to shocks. Christoffel and Linzert (2005) and Christoffel et al. (2005) come to similar conclusions. The remaining problem of these models is the *ad hoc* nature of the wage norm. Blanchard and Galí (2005) model wage rigidity to the New Keynesian model to study optimal monetary policy.

The third essay (Chapter 4) applies the Mortensen-Pissarides model to the New Keynesian monetary framework. Particular focus in this study is set on the interaction of labor taxation with wage determination, job creation and destruction in determining shock responses.

Below follows a summary of the three essays of this dissertation.

2 Growth, Skill Mismatch and Unemployment

The first essay (Chapter 2) investigates the role of skill mismatch in determining the relation between growth and equilibrium unemployment. The standard result in vintage capital/search models is that a faster rate of technological growth tends to increase equilibrium unemployment (Aghion and Howitt (1994) and Mortensen and Pissarides (1998)). In a vintage capital model, new more productive technology vintages enter the economy, making older generations of capital relatively less productive. As workers operating old technology vintages may potentially change to a job with a more productive and higher paying technology vintage, the jobs with older technology vintages have to pay wages that take account of this. Eventually the increasing wage drives old capital vintages unprofitable and these jobs are destroyed. A faster rate of growth increases the capital obsolescence effect that tends to shorten job durations and increase unemployment incidence. These models assume that workers skills are fully transferable from older technologies to cutting

edge technologies.

In this essay, the standard vintage capital/search model of Aghion and Howitt (1994) and Mortensen and Pissarides (1998) is extended to incorporate vintage human capital. In addition to vintage capital and capital obsolescence present in earlier models, skill obsolescence of workers is introduced into the vintage framework. Workers learn through learning-by-doing while employed, but their learning is specific to the vintage of technology they operate. Upon separation, workers become unemployed and search for a new job. The amount of skills they can transfer on the new jobs (of the leading edge vintage) is proportional to the technological distance between the old and the new capital they work with. While unemployed, skills keep becoming obsolete.

This essay highlights a new channel for the interaction between growth and unemployment: as the rate of productivity growth increases, we not only have a capital obsolescence effect that tends to shorten job durations and increase unemployment incidence, but we also have a skill obsolescence effect that reduces the value of unemployment and tends to offset the former force. Job creation and destruction are analyzed on the assumption of skill obsolescence being independent of the rate of technical change, and on the assumption that the rate of skill obsolescence and the rate of technical change are related so that transferability is decreasing in the productivity growth parameter.

A two-dimensional nature of skills (Violante 2002) is adopted into the vintage matching framework. There are two technological parameters in the model, the speed of technical change which measures the rate of capital obsolescence and skill transferability which measures the rate of skill obsolescence. One can also conjecture that the rate of skill obsolescence and the rate of technical change are related so that transferability is decreasing in the productivity growth parameter. The productivity growth parameter is thought of as capturing the degree to which new features of the technologies are embodied in capital, and therefore the extent to which capital is different across vintages.

First, assuming that skill transferability is independent of the rate of technical change, an increase in the latter increases both unemployment incidence and unemployment duration. Therefore the equilibrium unemployment rate unambiguously increases as in the standard vintage model. An increase in the transferability parameter has the opposite effect, both unemployment incidence and unemployment duration decrease and equilibrium unemployment decreases.

Next, the impact of an increase in the rate of technical change is studied when skill transferability is decreasing in the productivity growth parameter. A condition to distinguish between two cases is derived: when skills depreciate sufficiently little

with the rate of technical change technological so that obsolescence dominates and when skills depreciate fast enough with the rate of technical change so that skill obsolescence dominates. For the former unemployment increases with an increase in the rate of technical change and for the latter unemployment decreases. The conclusion is that the comparative static results of the standard vintage matching model are reversed when skill depreciation is fast enough.

3 Skill Biased Technological Change, Skill Determination and Labor Market Participation

The second essay (Chapter 3) studies skill biased technological change, skill mismatch and the allocation of workers and firms in the labor market. Within a matching model with two-sided heterogeneity (e.g. Albrecht and Vroman (2002), Gautier (2002)), we study how firms allocate vacancy creation between high and low productivity jobs, and how workers allocate between labor market states. The novelty of this study is to introduce an endogenously determined distribution of the workforce between high and low skill workers. This feature of the model is of relevance at least for long run analysis of labor markets, as one expects workers educational choices to respond to market incentives. The model is then extended to allow for a complete choice of labor market states, by introducing endogenous labor market participation to the model.

The essay builds on a matching model of two-sided heterogeneity with high and low skilled workers and minimum skill requirements for high productivity jobs. As Albrecht and Vroman (2002), we take this to mean that low skilled workers are qualified for low productivity jobs only and high skilled workers are qualified for both high and low productivity jobs. The high skilled, however, are not more productive in the low productivity jobs than low skilled workers. This implies that high-skilled workers have a more favorable position in the labor market, as they are suitable for both high and low-skill jobs whereas low-skill workers are suitable for low-skill jobs only. It also implies that the relevant pool of unemployed workers to match with is larger for firms that post low skill vacancies than it is for firms posting high skill vacancies.

By allowing for workers to invest in skills, we extend a matching model with two-sided heterogeneity to incorporate an endogenous distribution of high and low skill workers. Upon entry to the labor market workers may pay a one time education cost to become high skilled instead of remaining low skilled. Their decision to upgrade their skills depends on the cost of acquiring high skills and on the labor market

prospects for both worker types (i.e. the payoff of upgrading skills). The relative values of being high or low skilled depend on the relative productivities between job/worker types and the job competition that the worker faces in the job market from other workers.

We show that the skill cost wrt. the obtained skill level plays a key role in determining the labor market outcomes of the model. Because workers appropriate surplus from the firms to compensate for the skill investment, a key feature of the model is how large a share of match surplus the workers can appropriate and which type of firms are hit harder by appropriation. The degree to which high skill workers can appropriate match surplus from jobs depends on the costliness of obtaining skills and the distributions of job and worker types. We study the effect of skill biased technological change on job and worker distributions with alternative assumptions on the skill costs. While unemployment increases in most scenarios, the effect on the distribution high and low skill vacancies, high and low skilled workers, both among the total population and the unemployed, vary according to the structure of skill costs.

We extend the model to investigate how job competition and skill biased technological change influence labor market outcomes when also the labor market participation margin is endogenous. In line with several earlier studies (e.g. McKenna 1987, Sattinger 1995) we assume that labor market participation depends positively on the value of being unemployed. The extended model with endogenous labor market participation supports the claim of Murphy and Topel (1997) that the unemployment rate has become less informative of the aggregate state of the labor market. While skill biased technological change has modest effects on unemployment in the model, the participation margin fluctuates much more. In most cases labor market participation decreases with skill biased technological change. We conclude that the unemployment rate gives a misleading picture of the employment effects of skill biased technological change.

4 Labor Taxation, Equilibrium Unemployment, and Macroeconomic Dynamics

The third essay (Chapter 4) investigates the effects of labor taxes and labor tax reform on equilibrium labor market outcomes and on the sensitivity of an economy to macroeconomic shocks. First, the equilibrium labor market effects of the structure of taxes and subsidies are considered. In particular interest is on the effects of changing the degree of tax progression on equilibrium labor market variables and

output. Second, the potential importance of changes in tax structure and compensation schemes for the sensitivity of the economy wrt. exogenous shocks are considered.

For the purpose of analyzing the cyclical behavior of a relatively large set of macroeconomic variables, a search-matching model of the labor market *à la* Mortensen-Pissarides (e.g. Pissarides 2000, Mortensen and Pissarides 1999a) is embedded into a New Keynesian monetary model (e.g. Woodford 2003). This extension also allows the study of e.g. interest rate shocks.¹⁰ This study adopts the view that business cycle fluctuations influence the reorganizational activity in the economy, whereby low productivity job-worker matches are replaced by more profitable ones (see e.g. Hall (2000) and Caballero and Hammour (2005)). This essay contributes to the recent literature that studies the role of matching frictions and the role of factors that produce wage rigidity in improving the performance of business cycle models

Three policy instruments are considered: a marginal tax rate and a tax subsidy to produce tax progression schemes, and a replacement ratio to account for variability in outside options. In equilibrium, the marginal tax rate and replacement ratio dampen economic activity whereas tax subsidies boost the economy. The marginal tax rate and replacement ratio amplify shock responses whereas employment subsidies weaken them. The tax instruments affect the degree to which the wage absorbs shocks. It is shown that the relative effects of the tax instruments and thus the effects of tax progression are sensitive to the initial degree of tax progression in the economy. Increasing tax progression when taxation is initially progressive is harmful for steady state employment and output, and amplifies the sensitivity of macroeconomic variables to shocks. When taxation is initially proportional, increasing progression is beneficial for output and employment and dampens shock responses of macroeconomic variables.

A key issue in both the steady-state equilibrium analysis and the dynamic impulse response analysis is the rigidity of the wage implied by the labor market policy schemes wrt. to exogenous changes in the economy. Progressive taxation essentially renders the wage less sensitive to exogenous shocks, thus shifting adjustments in the labor market to other variables. This outcome relates to the above mentioned 'Shimer' critique of the Mortensen-Pissarides matching model, namely that the model does not produce sufficient fluctuations in vacancies and unemployment because the negotiated wage absorbs too much of the shocks of an economy.

¹⁰In the literature the cyclical properties of the Mortensen-Pissarides type model is often limited to the analysis of productivity shocks, and may thereby miss important mechanisms of the economy.

In the present study tax progression *de facto* makes the wage more rigid, producing a qualitatively similar result to the wage norm solution proposed by Hall (2005) and used by many others. The results are quantitatively not as strong, but no *ad hoc* assumptions on the foundations of wage rigidity are needed.

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Chapter 2

Growth, Skill Mismatch and Unemployment*

Abstract

This paper extends the standard vintage capital/search model of Aghion and Howitt (1994) and Mortensen and Pissarides (1998) to incorporate vintage human capital and studies the impact of capital-embodied growth on equilibrium unemployment. In addition to the capital obsolescence (or creative destruction) effect that tends to raise unemployment, vintage human capital introduces a skill obsolescence effect of faster growth that has the opposite sign. Faster skill obsolescence reduces the value of unemployment, hence wages and leads to more job creation and less job destruction, unambiguously reducing unemployment.

1 Introduction

Economic growth is a process of continuous restructuring and factor reallocation. Technological change, embodied in new generations of capital equipment, renders previous vintages of machines obsolete and eliminates jobs associated to these technologies to be replaced by new ones. Such restructuring is often described by the Schumpeterian notion of 'creative destruction', where new technology vintages enter the economy replacing their predecessors. Vintage economies are characterized by continuous large scale job creation and destruction, where workers from destroyed jobs are reallocated to new jobs at the technological frontier.

The standard result in vintage capital/search models is that a faster rate of technological growth tends to increase equilibrium unemployment (Aghion and Howitt

*This essay is an independent part of an ongoing more extensive research project conducted jointly with Giovanni L. Violante.

(1994) and Mortensen and Pissarides (1998)). In a vintage capital model, new more productive technology vintages enter the economy, making older generations of capital relatively less productive. As workers operating old technology vintages may potentially change to a job with a more productive and higher paying technology vintage, the jobs with older technology vintages have to pay wages that take account of this. Eventually the increasing wage drives old capital vintages unprofitable and these jobs are destroyed. A faster rate of growth increases the capital obsolescence effect that tends to shorten job durations and increase unemployment incidence. These models implicitly assume that workers skills are fully transferable from older technologies to cutting edge technologies. The reallocation of workers implies some unemployment as matching workers to jobs involves frictions, but a factor that has received little attention in vintage models is the evolution of workers' skills and the suitability of workers to the new jobs produced by the economy.

This paper extends the standard vintage capital-search model of Aghion and Howitt (1994) and Mortensen and Pissarides (1998) to incorporate vintage human capital. In addition to vintage capital and capital obsolescence present in earlier models we introduce skill obsolescence of workers into the vintage framework. Workers learn through learning-by-doing while employed, but their learning is specific to the vintage of technology they operate. Upon separation, workers become unemployed and search for a new job. The amount of skills they can transfer on the new jobs (of the leading edge vintage) is proportional to the technological distance between the old and the new capital they work with. While unemployed, skills keep becoming obsolete.

We study the determination of unemployment following the long tradition of works on growth and unemployment. We highlight a new channel for this interaction: as the rate of productivity growth increases, we not only have a capital obsolescence effect that tends to shorten job durations and increase unemployment incidence, but we also have a skill obsolescence effect that reduces the value of unemployment and tends to offset the former force. We analyze job creation and destruction on the assumption of skill obsolescence being independent of the rate of technical change, and on the assumption that the rate of skill obsolescence and the rate of technical change are related so that transferability is decreasing in the productivity growth parameter. Since technological change both eliminates and creates jobs, a search and matching models provide a natural framework for analysis.

The relationship between capital age and unemployment in the U.S. is illustrated in figure 1. The figure draws the rate of unemployment, the age of capital equipment, which also measures job duration in the vintage model, and rate of capital-embodied technical change for capital equipment. According to the stan-

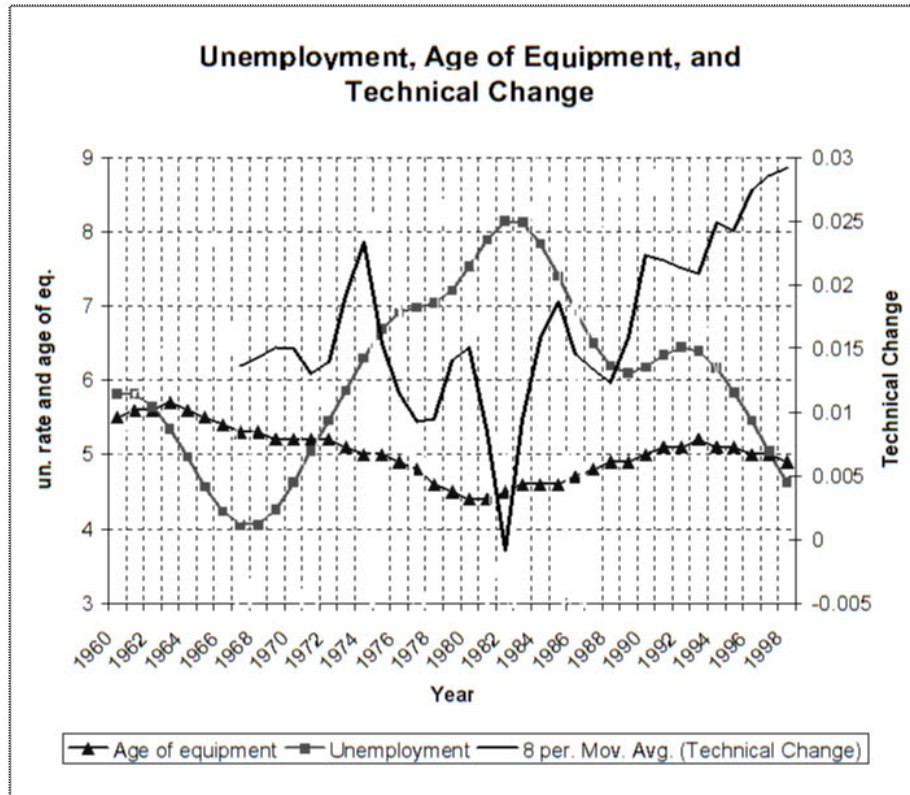


Figure 1: Unemployment, age of equipment, and technical change, U.S. 1960-1998.

dard vintage capital/search model, when the rate of technical change rises, the age of capital falls (job durations fall) and unemployment increases. According to the present model, if skill obsolescence dominates, when the rate of technical change increases, the age of capital rises and unemployment falls. As the figure illustrates, the data of age of capital and unemployment are consistent with both the standard vintage capital model and the model with vintage human capital presented in this paper. However, when the rate of technical change is considered the model of the present paper seems more plausible. Furthermore, it can be noted that wages in the U.S. stagnated over the period 1975-1995 as well, which is consistent with a skill obsolescence story.

In addition to the related studies on growth and unemployment (e.g. Aghion and Howitt (1994) and Mortensen and Pissarides (1998)), a number of papers focus on the interactions of technological change, skills and specificity. Caballero and Hammour (1997, 1998a, 1998b) study the implications of specificity and appropriability

of job specific rents in the context of growth and job reallocation. Embodiment of technology in capital vintages and job specific human capital gives rise to appropriable quasi rents that distort job reallocation in the process of creative destruction. Ljungqvist and Sargent (1998) study the determination of equilibrium unemployment in a model where workers accumulate skills on the job and lose skills during unemployment.¹ Skill dynamics of this type are also present in Laing et al. (2003), but they incorporate growth of the stock of public knowledge in the economy. They introduce overlapping generations (vintages) of workers who differ in productivities s.t. knowledge growth is embodied in new entrant workers. In this setup the entry of new (younger) workers renders the knowledge of older workers obsolete. Galor and Moav (2000) build a growth model which is characterized by ability biased technological transition where technological acceleration leads to a rise in wage inequality both between and within skill groups. Their study is close in spirit to the present paper, as technological progress is assumed to reduce the adaptability of existing human capital for the new technological environment. While superior technologies increase productivity, they also erode existing human capital that is adaptable to the new technologies, thus reducing productivity. A characteristic of their model is that able individuals have a comparative advantage in adapting to new technology. Violante (2002) considers workers with two-dimensional skills. An employed worker accumulates skills by learning-by-doing, but when changing jobs to one of a more recent technological vintage, only a fraction of the workers skills are transferable. Technological acceleration reduces workers' capacity to transfer skills from old to new machines, generating cross-sectional variance of skills and therefore wages. Ljungqvist and Sargent (1998) and Violante (2002) do not study the 'creative destruction' -nature of technological change as their models do not embed the vintage capital structure. Carré and Drouot (2004) introduce learning-by-doing into a vintage capital framework but do not consider transferability of skills between technology vintages.

We adopt the two-dimensional nature of skills (Violante 2002) into the vintage matching framework. There are two technological parameters in the model, the speed of technical change which measures the rate of capital obsolescence and skill transferability which measures the rate of skill obsolescence. One can also conjecture that the rate of skill obsolescence and the rate of technical change are related so that transferability is decreasing in the productivity growth parameter. We think of the productivity growth parameter as capturing the degree to which new features of the technologies are embodied in capital, and therefore the extent to which capital

¹Pissarides (1992) studies unemployment persistence in a model where skill loss during unemployment has an adverse effect on vacancy creation.

is different across vintages.

First, assuming that skill transferability is independent of the rate of technical change, an increase in the latter increases both unemployment incidence (the age of jobs decreases) and unemployment duration (labor market tightness decreases). Therefore the equilibrium unemployment rate unambiguously increases as in the standard vintage model. An increase in the transferability parameter has the opposite effect, both unemployment incidence and unemployment duration decrease (job age and labor market tightness both increase) and equilibrium unemployment decreases.

Next, we study the impact of an increase in the rate of technical change when skill transferability is decreasing in the productivity growth parameter. We derive a condition to distinguish between two cases: when skills depreciate sufficiently little with the rate of technical change so that technological obsolescence dominates and when skills depreciate fast enough with the rate of technical change so that skill obsolescence dominates. For the former unemployment increases (job age and labor market tightness both decrease) with an increase in the rate of technical change and for the latter unemployment decreases (job age and labor market tightness both increase). We conclude that the comparative static results of the standard vintage capital are reversed when skill depreciation is fast enough.

The paper is organized as follows. Section 2 presents the structure of the economy and the evolution of skills and productivity. In section 3 we characterize the balanced growth equilibrium of the model and derive the two key equilibrium conditions: the job creation and destruction conditions. Section 4 describes the behavior of the economy wrt. growth and the key parameters of the model. Section 5 concludes.

2 The Economy

We first describe the structure of the economy and the asset value equations that characterize the flow returns to the firms and workers when participating in a match and when idle. The key element in the structure of the economy is "vintage human capital" which interacts with technological vintage capital present in earlier studies.

Demographics and preferences– Time is continuous. The economy is populated by a measure one of ex-ante equal workers, who are infinitely lived, risk-neutral and discount the future at rate $r > 0$. Every period, workers can be either employed or unemployed. Workers retire exogenously at rate $\rho > 0$ and are replaced by a new inflow of unemployed workers of measure ρ .

Technology– New technologies are embodied in physical capital, and the leading edge technology in the economy advances at an exogenous rate $\gamma > 0$. A firm (or production unit, or machine) created at time t embeds a fixed amount of capital (normalized to unity) of vintage t with productivity $e^{\gamma t}$. Production is decentralized across different firms and takes place when the firm is matched with a worker. Every period, firms can be either vacant or matched. As standard in the literature, we assume that all vacant firms at time t embody the best available technology.

Skills– Human capital is partially vintage specific. New workers entering the labor market at time t have skill level \bar{z}_t specific to the newest technology of vintage t , i.e. the skill level \bar{z}_t measures the productivity of a worker operating capital of vintage t . Over time these skills become obsolete with respect to more recent technology vintages. We will assume that at time t , an unemployed worker with skill level z_τ of vintage τ (s.t. $\tau < t$) can transfer skills $z_t < z_\tau$ on a new job with a machine of the latest vintage t , where z_t is determined by

$$z_t = z_\tau e^{-\phi(t-\tau)}. \quad (1)$$

One may think of the parameter ϕ as a measure of the vintage specificity of the skills of a worker. A worker with a given productivity z_τ in operating capital of vintage τ is less productive in operating capital of a more recent vintage t . The larger is the distance between t and τ , the smaller is the fraction of skills that are relevant or transferable to a more recent vintage t . Given that all vacancies at any time t embody the best vintage of technology, it is convenient to express the skill level of a worker z_τ always in terms of the newest vintage t . This convention and equation (1) imply that for unemployed workers, skills evolve (relative to the best vintage of technology) according to the law of motion $\dot{z} = -\phi z$, where the subscript t is omitted to lighten the notation.

When employed, workers cumulate skills through learning-by-doing at the instantaneous constant learning rate $\lambda > 0$, therefore during employment the law of motion for skills (expressed in terms of the newest vintage t) is $\dot{z} = (\lambda - \phi) z$. The rate of skill obsolescence relative to the leading edge technology is thus lower for employed workers than for unemployed workers, as skills not only depreciate relative to new technology vintages, but also accumulate when employed.² To avoid that the skill space expands infinitely, we make the convenient assumption $\lambda \leq \phi$. With

²Separating the skill evolution parameters into the learning by doing λ and depreciation ϕ parameters is to simplify intuition. Alternatively one could simply assume that skill obsolescence ϕ_i ($i = e, u$) is smaller for employed workers than for unemployed workers, because employed accumulate skills (although their skills also depreciate relative to the most recent vintage).

this assumption, the domain of the skill distribution is bounded between $(0, \bar{z}]$.³

At time t , output produced by a match between a machine of vintage τ and a worker of skills z (always expressed in terms of the newest vintage t) is

$$y(t, \tau, z) = e^{\gamma\tau} z e^{\phi(t-\tau)}, \quad (2)$$

where $e^{\gamma\tau}$ is the fixed productivity of capital of vintage τ , z is the productivity of the worker expressed in terms of the newest vintage t including learning by doing up to date t , and $e^{\phi(t-\tau)}$ converts worker productivity to productivity in operating capital of vintage τ .⁴

Matching—Firms observe workers' skills perfectly and they can direct their search towards any type z , hence the matching markets will be segmented by skill level. Firms pay a search cost

$$c(t, z) = z e^{\gamma t} c \quad (3)$$

to search in market z . The search cost is proportional to the productivity of new jobs in market z , reflecting the realistic feature that job creation and recruiting costs of high productivity jobs are larger than those of low productivity jobs. Each of these matching markets has the same constant returns to scale matching function $m(u(z), v(z))$, where $u(z)$ and $v(z)$ are respectively unemployed workers and vacancy firms in market z . We denote by $\theta(z)$ the vacancy-unemployment ratio (or market tightness) in market z , $p(\theta(z))$ the meeting probability for an unemployed worker and $q(\theta(z))$ the meeting probability for a vacant firm in market z . The meeting rates imply an expected duration of search for firms of $1/q(\theta(z))$ and an expected duration of unemployment of $1/p(\theta(z))$ in market z . Matches dissolve exogenously at rate δ .

Worker's income—The fractions of output going to wages and to profits along the match are determined through Nash bargaining between the firm and the worker. The pair rebargains every instant on the new discounted stream of output. Denote the wage of a match between an employed worker with skill level z on a machine of vintage τ at time t as $w(t, \tau, z)$. Unemployed workers spend all their time endowment searching, hence having no utility from leisure. They receive an

³It is important to make the distinction of *vintage* specificity of skills as opposed to *match* specific skills. In this study, skills are specific to technology vintage, not to an individual match. Therefore, as workers have an initial skill level \bar{z}_t when entering the labor market, even a worker who has never been matched loses skills relative to the leading edge vintage. Match specific skills, acquired in a match, would require a worker to have been matched before any skills could be lost.

⁴If the workers productivity is z on the leading edge capital vintage t , the term $e^{\phi(t-\tau)}$ is included to reverse the skill depreciation over the period $t - \tau$. The workers skills become obsolete wrt. new technology vintages, but they do not depreciate wrt. to the current job of vintage τ .

unemployment income $b(z, t) = ze^{\gamma t}b$ which is proportionate to the growth rate of aggregate income and the evolution of worker's skills. Interestingly, this assumption implies that benefits fall with unemployment duration, as in most of the real-world unemployment insurance systems.

2.1 Value Functions and Match Surplus

The values of participating to the market for the firms and workers are described by a set value equations. At time t , denote the value of vacant firms searching in market z by $V(t, z)$; the value of firms of vintage τ matched with workers of type z by $J(t, \tau, z)$; the value of employed workers in the same type of match by $W(t, \tau, z)$; and the value of unemployed workers with skills z by $U(t, z)$. Then, it is easy to derive that⁵

$$rV(t, z) = \max \left\{ c(t, z) + q(\theta(z)) [J(t, \tau, z) - V(t, z)] + \frac{dV(t, z)}{dt}, 0 \right\} \quad (4)$$

$$rJ(t, \tau, z) = \max \left\{ y(t, \tau, z) - w(t, \tau, z) - (\delta + \rho) \left[J(t, \tau, z) - \max_z V(t, z) \right] + \frac{dJ(t, \tau, z)}{dt}, r \max_z V(t, z) \right\} \quad (5)$$

$$rW(t, \tau, z) = \max \left\{ w(t, \tau, z) - \delta [W(t, \tau, z) - U(t, z)] - \rho W(t, \tau, z) + \frac{dW(t, \tau, z)}{dt}, rU(t, z) \right\} \quad (6)$$

$$rU(t, z) = \max \left\{ b(z, t) + p(\theta(z)) [W(t, \tau, z) - U(t, z)] - \rho U(t, z) + \frac{dU(t, z)}{dt}, 0 \right\} \quad (7)$$

where $\frac{dV(t, z)}{dt} = V_t(t, z)$, $\frac{dJ(t, \tau, z)}{dt} = J_t(t, \tau, z) + \dot{z}J_z(t, \tau, z)$ and $\frac{dW(t, \tau, z)}{dt} = W_t(t, \tau, z) + \dot{z}W_z(t, \tau, z)$ s.t. $\dot{z} = (\lambda - \phi)z$ and $\frac{dU(t, z)}{dt} = U_t(t, z) + \dot{z}U_z(t, z)$ s.t. $\dot{z} = -\phi z$.⁶

⁵A typical derivation of a value equation (as limit of a discrete time model economy) is described in the Appendix.

⁶Observe that whereas z evolves in time for a filled job, an employed worker and an unemployed worker, it is constant in the value equation of a vacancy and $\frac{dV(t, z)}{dt}$ does not involve the effect through z . This is because a firm posts a vacancy with a specific skill requirement z , for example fresh high school graduates, and although the individuals in the pool of unemployed workers with skills z come and go (new graduates with skills z enter and unemployed graduates of an earlier vintage exit the pool as their skills depreciate over time), the pool where the firm searches and therefore z remains the same.

A vacant job in the market for skills z costs $c(t, z)$ per unit time and the firm matches with a worker at rate $q(\theta(z))$. The change of state yields a return of $J(t, t, z) - V(t, z)$ to the firm. An occupied job yields the return $y(t, \tau, z) - w(t, \tau, z)$ per unit time, which is the productive output of the match minus the wage paid to the worker. The job may be terminated due to either an exogenous shock or retirement at the respective rates δ and ρ . The value of the match depends on the skills of the worker, which evolve according to the law of motion $\dot{z} = (\lambda - \phi)z$. The worker accumulates skills at rate λ as long as the firm and worker are matched and the workers skills depreciate relative to the newest technology vintage at rate ϕ .

When employed a worker earns the wage $w(t, \tau, z)$ per unit time. The match may be destroyed due to an exogenous shock at rate δ in which case the worker becomes unemployed and loses the difference between the two labor market states $W(t, \tau, z) - U(t, z)$. In the case of retirement which occurs at rate ρ the worker loses $W(t, \tau, z)$. The evolution of skills for an employed worker follows the law of motion $\dot{z} = (\lambda - \phi)z$. An unemployed worker receives an unemployment income $b(z, t)$ and matches with a firm at rate $p(\theta(z))$ which yields a return of $W(t, t, z) - U(t, z)$. In the case of retirement the worker loses $U(t, z)$. When unemployed, skills depreciate according to the law of motion $\dot{z} = -\phi z$ as no skill accumulation takes place. Note that the value of unemployment $U(t, z)$ for an employed worker in equation (6) evolves according to $\dot{z} = (\lambda - \phi)z$. This is so because as long as a worker is employed, also the value of eventual unemployment is affected by skill accumulation on the job.

3 Balanced Growth Equilibrium

We now characterize the equilibrium of the model, along a balanced growth path where all the values above grow at rate γ . Once we stationarize all values by the growth factor $e^{\gamma t}$ all the relevant equilibrium objects are only a function of the difference $(t - \tau)$ which is “age” and we denote as a . The balanced growth path versions of the value equations are⁷

$$(r - \gamma)V(z) = \max \{-zc + q(\theta(z))[J(0, z) - V(z)], 0\} \quad (8)$$

$$(r - \gamma)J(a, z) = \max \left\{ e^{-\gamma a} z e^{\phi a} - w(a, z) - (\delta + \rho) \left[J(a, z) - \max_z V(z) \right] \right. \\ \left. + J_a(a, z) + (\lambda - \phi)z J_z(a, z), (r - \gamma) \max_z V(z) \right\} \quad (9)$$

⁷See appendix for details.

$$(r - \gamma) W(a, z) = \max \{ w(a, z) - \delta [W(a, z) - U(z)] - \rho W(a, z) + W_a(a, z) + (\lambda - \phi) z W_z(a, z), (r - \gamma) U(z) \} \quad (10)$$

$$(r - \gamma) U(z) = \max \{ bz + p(\theta(z)) [W(0, z) - U(z)] - \rho U(z) - \phi z U_z(z), 0 \} \quad (11)$$

Intuitively, these equations are expressed in terms of age relative to the leading edge technology vintage of age zero. For example, technological productivity $e^{-\gamma a}$ in equation (9) decreases with age relative to the leading edge technology.

The key object for the characterization of the model is the “surplus function”, defined as the value of the match for a worker and a firm, net of their respective outside options. The surplus of a match of age a is given by

$$S(a, z) = J(a, z) + W(a, z) - V(z) - U(z). \quad (12)$$

The worker and the firm divide the match surplus according to the Nash bargaining solution and the first-order condition is

$$\beta [J(a, z; w(a, z)) - V(z)] = (1 - \beta) [W(a, z; w(a, z)) - U(a, z)]. \quad (13)$$

where β represents the worker’s share of match surplus. The division of the match surplus is continuously renegotiated s.t. it reflects the evolution of match surplus over time. Substitution of the value equations into the first-order condition yields the wage equation⁸

$$w(a, z) = \beta z e^{(\phi - \gamma)a} + (1 - \beta) z \left(b + \frac{\beta}{1 - \beta} c \theta(z) \right). \quad (14)$$

The first term is the worker’s share of match output. Relative to productivity in leading edge technology jobs, technological productivity decreases with age a . As z measures the productivity of a worker in a leading edge job, $e^{\phi a}$ reverses skill depreciation, as skill depreciation does not take place as long as the worker continues working with the current technology of age a .⁹ The second term reflects the workers outside option and is expressed in terms of the leading edge vintage. It depends on the unemployment compensation received by unemployed workers of skills z and labor market tightness.

It is useful to formally define a stationary equilibrium for this economy.

Definition: *A stationary (stationarized balanced growth) equilibrium is a list of: (i) values $\{V(z), U(z), J(a, z), W(a, z), S(a, z)\}$, (ii) market tightness function θ , (iii) optimal destruction age \bar{a} , (iv) wage function $w(a, z)$ such that:*

⁸See appendix for detailed derivation.

⁹Recall that z includes learning by doing over the age of the match.

1. The values $\{V(z), J(a, z), W(a, z), U(z)\}$ satisfy equations (8)–(12) above;
2. There is free entry of vacancies in each market z , thus equilibrium market tightness θ satisfies the condition $V(z) = 0$;
3. The optimal destruction age \bar{a} satisfies the condition $S'(\bar{a}, z) = 0$;
4. The wage function $w(a, z)$ solves the Nash bargaining equation (13).

3.1 Characterization

Substituting the value equations (9), (10) and the value of unemployment for an employed worker (with the law of motion $\dot{z} = (\lambda - \phi)z$) into (12) and using $V(z) = 0$ produces¹⁰

$$(r - \gamma + \delta + \rho) S(a, z) = \max \left\{ z e^{(\phi - \gamma)a} - bz - p(\theta(z)) \beta S(0, z) + S_a(a, z) + (\lambda - \phi) z S_z(a, z) \right\}. \quad (15)$$

To see that the surplus (15) is a first order differential equation as a function of t , observe that $S(a, z) = S(a(t), z(t))$ and consequently we can express the derivatives $\frac{dS(a(t), z(t))}{dt} = S_a(a, z) + (\lambda - \phi) z S_z(a, z)$, recall that $\dot{z} = (\lambda - \phi)z$.

From the definition of the surplus and the Nash condition we obtain $S(0, z) = \frac{1}{1 - \beta} J(0, z)$ and using the free entry condition in (8) we get $J(0, z) = \frac{cz}{q(\theta(z))}$. Then (15) reduces to¹¹

$$(r - \gamma + \delta + \rho) S(a, z) - \frac{dS(a, z)}{dt} = \max \left\{ z e^{(\phi - \gamma)a} - bz - \frac{\beta}{1 - \beta} cz \theta(z), 0 \right\}. \quad (16)$$

This is a first order differential equation for the surplus as a function of t . The max operator implies that we have a boundary condition $S(\bar{a}, z) = 0$. The particular solution for this differential equation, once we impose the boundary condition, is

$$S(a, z) = e^{(\phi - \gamma)a} z \int_a^{\bar{a}} [1 - \omega(\theta) e^{-(\phi - \gamma)\tilde{a}}] e^{-(r + \delta + \rho - \lambda)(\tilde{a} - a)} d\tilde{a} \quad (17)$$

where $\omega(\theta) = b + \frac{\beta}{1 - \beta} c\theta$ is the outside option of the worker. From the optimality condition $S'(\bar{a}) = 0$, it follows that the destruction age \bar{a} for the matched pair satisfies the following rule:

$$e^{(\phi - \gamma)\bar{a}} = \omega(\theta) \quad (18)$$

¹⁰See appendix for details.

¹¹Here we have also used the properties of the matching function $\frac{p(\theta(z))}{q(\theta(z))} = \theta$.

This condition is very intuitive and can be explained in two alternative ways. First, let us rewrite the equation above as

$$ze^{\lambda\bar{a}} = e^{\gamma\bar{a}}\omega(\theta)ze^{(\lambda-\phi)\bar{a}}.$$

The left hand side of that expression is the output flow on a machine of age \bar{a} matched with a worker who had skills z upon matching; the right hand side is the flow value of the outside option of the same worker (recall the outside option of the firm V is zero in equilibrium): her skills have increased at the pace λ but, at the same time, became obsolete at rate ϕ . The term $e^{\gamma\bar{a}}$ represents the higher value of job opportunities today compared to \bar{a} periods ago thanks to the growth of the leading edge technology at rate γ .¹²

Second, one can provide an interpretation in terms of the wage $w(a)$. Using (18) in the wage equation (??) it is now immediate to derive that $w(\bar{a}) = e^{(\phi-\gamma)\bar{a}}$. In other words, at age \bar{a} all output is claimed by the worker as wage bill and no more profits can be generated, so the job is destroyed endogenously.

It is important to remark that (18) implies that in order to have a meaningful economic problem, we need to assume throughout that $\gamma > \phi$. If $\gamma \leq \phi$, then jobs are never destroyed endogenously, as the surplus will rise with age (instead of declining as usual in this class of models) because the value of unemployment falls faster than output, due to skill obsolescence.¹³

Now that we have derived a solution for the surplus function, we can show that the model can be reduced to two equations into the pair of unknowns (θ, \bar{a}) . The first equation, the job creation condition, is derived from free entry in equilibrium; the second equation, the job destruction condition, is derived from the optimal separation rule for a match.

3.2 Job Creation condition

The first equation follows directly from the free entry condition $V(z) = 0$, which implies $J(0) = c/q(\theta)$. Using $J(a, z) = (1 - \beta)S(a, z)$ and the definition of the surplus in (17) evaluated at $a = 0$, together with the destruction rule –that we use to eliminate $\omega(\theta)$ from (??)–we arrive at

$$\frac{c}{(1 - \beta)q(\theta)} = \int_0^{\bar{a}} [1 - e^{-(\gamma-\phi)(\bar{a}-\tilde{a})}] e^{-(r+\delta+\rho-\lambda)\tilde{a}} d\tilde{a} \quad (19)$$

¹²Note that since learning by doing increases both the productivity in the current job and the outside option of the employed worker, it cancels out and does not feature in the job destruction condition (18).

¹³Jobs would still be destroyed endogeneously if the worker had a positive return from nonparticipating which is independent of her skill level z . The model would generate direct flows from employment to out-of-the labor force. This is discussed in Vanhala and Violante (2005) in detail.

which characterizes the optimal entry decisions of firms in each of the z markets. Notice that it is an equation in both \bar{a} and θ and that it is positively sloped in the (θ, \bar{a}) space: $q'(\theta) < 0$ and the right-hand-side of the expression above is increasing in \bar{a} . To see that, solve explicitly the right-hand-side to obtain

$$S(0; \bar{a}) = \frac{1}{r + \delta + \rho - \lambda} (1 - e^{-(r+\delta+\rho-\lambda)\bar{a}}) - \frac{1}{r + \delta + \rho - \lambda - \gamma + \phi} [e^{-(\gamma-\phi)\bar{a}} - e^{-(r+\delta+\rho-\lambda)\bar{a}}] \quad (20)$$

and differentiate wrt. \bar{a} , which gives

$$\frac{\partial S(0; \bar{a})}{\partial \bar{a}} = \frac{1}{r + \delta + \rho - \lambda - \gamma + \phi} (\gamma - \phi) e^{-(\gamma-\phi)\bar{a}} [1 - e^{-[r+\delta+\rho-\lambda-(\gamma-\phi)]\bar{a}}] > 0,$$

thus the surplus at age zero is increasing in the destruction threshold \bar{a} . The intuition for why the job creation curve is positively sloped is simple: the surplus function is increasing in \bar{a} thus an increase in \bar{a} makes the marginal job created profitable, and more vacancies will be opened to restore the zero-profit condition (and reduce the meeting probability for the firm), which will increase θ .¹⁴

Let us study the behavior of the curve as $\theta \rightarrow 0$ (i.e. $q(\theta) \rightarrow \infty$): the left-hand-side of (19) goes to zero, and the equation states that $S(0; \bar{a}) = 0$, i.e. $\bar{a}^{\min} = 0$. In other words, as the hiring friction disappears, and the firm instantaneously finds new workers, machines will be constantly updated.¹⁵ To see what happens as $\bar{a} \rightarrow \infty$, use the solution for $S(0; \bar{a})$ in (20) to obtain

$$\frac{c}{(1-\beta)q(\theta^{\max})} = \frac{1}{r + \delta + \rho - \lambda} \Rightarrow \theta^{\max} = q^{-1} \left(\frac{c(r + \delta + \rho - \lambda)}{1 - \beta} \right).$$

In other words, even if the job tenure is infinitely long, the firm needs a minimum meeting rate to recoup the flow vacancy cost c . Figure 2 shows the job creation curve in the (θ, \bar{a}) space.

3.3 Job Destruction condition

The outside option in the optimal separation condition (18) is equal to the value of unemployment for an employed worker and may be expressed in terms of the match surplus. The separation rule may thus be expressed as¹⁶

¹⁴A second reason often cited in the literature, namely that a longer job duration increases the length over which a positive flow surplus accrues to the pair is not true. This is because \bar{a} is chosen optimally by the firm, and by the envelope theorem this effect is zero.

¹⁵Recall that in this model the cost of a new machine is zero, but scrapping an old machine to buy a new one implies separation from the worker. Hence insofar as looking for a new worker takes time, upgrading is costly.

¹⁶See appendix for details.

$$e^{-(\gamma-\phi)\bar{a}} = b + p(\theta) \beta S(0, \bar{a}), \quad (21)$$

Substituting the expression for the surplus (??) evaluated at $a = 0$, and using (18) we obtain the job destruction condition

$$e^{-(\gamma-\phi)\bar{a}} = b + p(\theta) \beta \int_0^{\bar{a}} [1 - e^{-(\gamma-\phi)(\bar{a}-\tilde{a})}] e^{-(r+\delta+\rho-\lambda)\tilde{a}} d\tilde{a}, \quad (22)$$

which traces a negatively sloped curve in the (θ, \bar{a}) space. The intuition for the negative slope is that a rise in \bar{a} reduces the productivity of the marginal job, but it also raises the value of unemployment, as the value of search rises, thus for the destruction rule to be restored, the workers' meeting rate has to fall, so θ has to fall.

Consider the situation where $\theta \rightarrow 0$ (i.e. $p(\theta) \rightarrow 0$), then we have an upper bound for the destruction age

$$\bar{a}^{\max} = -\frac{\ln b}{\gamma - \phi}$$

which is a positive number since $b < 1$.¹⁷ In other words, when the worker's meeting rate is extremely small, the option value of search does not affect the joint destruction decision, only the welfare benefit does.

3.4 Equilibrium Unemployment

In steady state unemployment is constant and the flow of job creation must equal the flow of job destruction. Job creation is given by the flow of new matches

$$JC = m(u, v) = \int_0^{\bar{z}} m(u(z), v(z)) dz.$$

Jobs are destroyed either endogenously or exogenously

$$JD = \delta \left(1 - \int_0^{\bar{z}} u(z) dz \right) + e^{-\delta\bar{a}} \int_0^{\bar{z}} m(u(z), v(z)) dz$$

where we have normalized the labor force to one. A fraction δ of employed workers become unemployed due to exogenous job destruction. A fraction $e^{-\delta\bar{a}}$ of matches survive until the age of obsolescence and are then destroyed endogenously.

To derive the aggregate unemployment rate denote $u = u(z)$, $v = v(z)$ and $m(u, v) = \int_0^{\bar{z}} m(u(z), v(z)) dz$. In steady state $JC = JD$ so we have

$$m(u, v) = \delta(1 - u) + e^{-\delta\bar{a}} m(u, v) \quad (23)$$

¹⁷This has to be true in a viable labor market where the normalized productivity of the best vintage is 1.

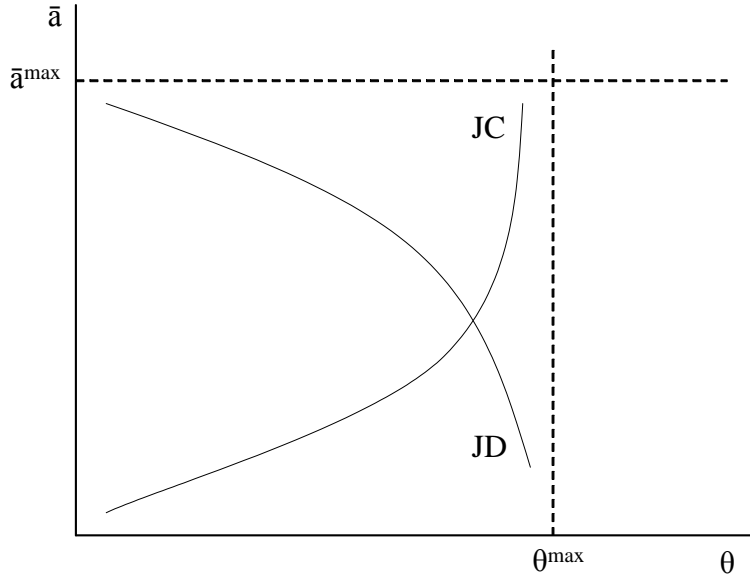


Figure 2: Job creation and destruction

where $1 - u = e$. Rearranging produces

$$p(\theta) u (1 - e^{-\delta \bar{a}}) = \delta (1 - u) \quad (24)$$

where $p(\theta) u = m(u, v)$ by the properties of the matching function. Rearranging gives the equilibrium unemployment rate

$$u = \frac{\delta}{\delta + p(\theta) (1 - e^{-\delta \bar{a}})} \quad (25)$$

The equilibrium unemployment rate is decreasing in age \bar{a} and labor market tightness θ . This is intuitive as higher \bar{a} implies lower unemployment incidence and higher θ implies shorter unemployment duration.

4 Equilibrium Comparative Statics

There are two key technological parameters in the model, the speed of technical change γ which measures the rate of capital obsolescence and the rate of skill obsolescence ϕ . Recall that the parameter ϕ should be interpreted as a measure of specificity of skills: a large value of ϕ implies vintage specific skills, whereas a low value of ϕ means general skills that are largely transferable to other job vintages.

We start by the simplest case, whereby ϕ is independent of γ . Alternatively, one can reasonably conjecture that ϕ and γ are related by the function $\phi(\gamma)$, with $\phi' > 0$. In other words, transferability is decreasing in the productivity growth parameter γ as, for example, in Galor and Moav (2000), Violante (2002), and Gould, Moav and Weinberg (2001).¹⁸ We proceed by first examining the effect of the technological parameters on market tightness and the destruction age, then we consider the effects on equilibrium unemployment.

4.1 The Effects on (θ, \bar{a})

The impact of γ — Let's start with the effect of the rate of technical change γ on the surplus function $S(0, \bar{a})$. Differentiating (20), we obtain

$$\frac{\partial S(0, \bar{a})}{\partial \gamma} = \frac{e^{-(\gamma-\phi)\bar{a}} \{ [r + \delta + \rho - \lambda - (\gamma - \phi)] \bar{a} + e^{-(r+\delta+\rho-\lambda-\gamma+\phi)\bar{a}} - 1 \}}{(r + \delta + \rho - \lambda - \gamma + \phi)^2} > 0$$

where the inequality follows from the fact that

$$\frac{1 - e^{-(r+\delta+\rho-\lambda-\gamma+\phi)\bar{a}}}{r + \delta + \rho - \lambda - \gamma + \phi} = \int_0^{\bar{a}} e^{-(r+\delta+\rho-\lambda-\gamma+\phi)x} dx < \bar{a}$$

as the argument of the integral is strictly less than 1 everywhere on the domain $[0, \bar{a}]$. Therefore, the job creation curve (19) shifts downward: for a given contact rate θ , the faster technical change increases the value of the surplus, thus \bar{a} has to fall to rebalance the job creation condition.

It is easy to see that the job destruction curve (22) shifts downwards as well: the rise in the surplus raises the value of unemployment, for given θ . At the same time, the marginal value of a job $e^{-\gamma\bar{a}}$ falls, thus the meeting probability has to decline as well to rebalance this condition. In conclusion, we have an unambiguous rise in unemployment incidence (\bar{a} decreases) but ambiguous effects on unemployment duration (expected unemployment duration is given by $1/p(\theta)$).

The impact of ϕ — Consider now a change in the transferability parameter ϕ . From the surplus function

$$\frac{\partial S(0, \bar{a})}{\partial \phi} = \frac{-e^{-(\gamma-\phi)\bar{a}} \{ [r + \delta + \rho - \lambda - \gamma + \phi] \bar{a} + e^{-(r+\delta+\rho-\lambda-\gamma+\phi)\bar{a}} - 1 \}}{(r + \delta + \rho - \lambda - \gamma + \phi)^2} < 0,$$

¹⁸We think of γ as capturing the degree to which new features of the technologies are embodied in capital, and therefore the extent to which capital is different across vintages. Note that γ is often measured through quality-adjusted relative prices, consistently with this view. This effect is called in the literature the “human capital erosion effect” due to faster growth.

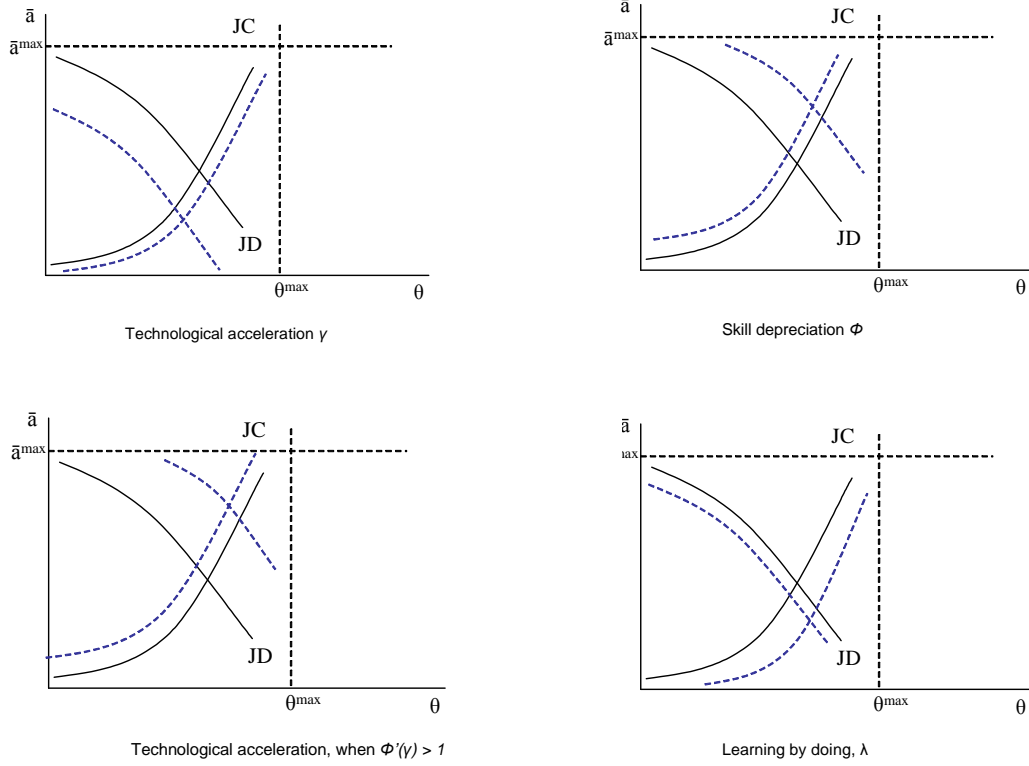


Figure 3: Equilibrium comparative statics.

thus following the same logic we have used above, it is easy to show that both curves shift upward, leading to a rise in \bar{a} (and a fall in unemployment incidence) and ambiguous effects on unemployment duration.

The impact of γ when ϕ depends on γ – The derivative of the surplus becomes

$$\begin{aligned}
 & \frac{\partial S(0, \bar{a})}{\partial \gamma} \\
 = & \frac{[1 - \phi'(\gamma)] e^{-(\gamma - \phi)\bar{a}} \{ [r + \delta + \rho - \lambda - (\gamma - \phi(\gamma))] \bar{a} + e^{-[r + \delta + \rho - \lambda - (\gamma - \phi(\gamma))\bar{a}] - 1} \}}{[r + \delta + \rho - \lambda - \gamma + \phi(\gamma)]^2}
 \end{aligned}$$

which is positive when $\phi'(\gamma) < 1$ and negative when $\phi'(\gamma) > 1$. For example, suppose that $\phi(\gamma) = \phi\gamma$. Then the assumption $\phi(\gamma) < \gamma$ implies that $\phi < 1$ and $\phi'(\gamma) < 1$. Thus, when ϕ is linear, the comparative statics are like in the case where ϕ is independent of γ . However, when nonlinearities are present and $\phi'(\gamma) > 1$, we can have a situation where $\frac{\partial S(0, \bar{a})}{\partial \gamma} < 0$ and a rise in γ has the same comparative statics as a rise in ϕ . In this case the specificity of skills increases and consequently

the transferability of skills is reduced with the rate of technical change.

The impact of λ – Finally consider a change in the learning rate λ . The derivative of the surplus function is

$$\begin{aligned} \frac{\partial S(0; \bar{a})}{\partial \lambda} &= \frac{-\bar{a}e^{-(r+\delta-\lambda)\bar{a}}(r+\delta+\rho-\lambda)+1-e^{-(r+\delta+\rho-\lambda)\bar{a}}}{(r+\delta+\rho-\lambda)^2} \\ &\quad -e^{-(\gamma-\phi)\bar{a}} \times \\ &\quad \frac{1-e^{-[r+\delta+\rho-\lambda-(\gamma-\phi)]\bar{a}}-\bar{a}e^{-[r+\delta+\rho-\lambda-(\gamma-\phi)]\bar{a}}[r+\delta+\rho-\lambda-(\gamma-\phi)]}{[r+\delta+\rho-\lambda-(\gamma-\phi)]^2} \end{aligned}$$

which is easily seen to be positive. Again, following the same logic we have used above, we can show that both the job creation and destruction curves shift downwards. Unemployment incidence increases (\bar{a} decreases) and the effect on unemployment duration is ambiguous.

4.2 The Effects on Equilibrium Unemployment

The job creation and destruction conditions derived above produce unambiguous comparative statics for \bar{a} and unemployment incidence. However the comparative statics with respect to θ and unemployment duration remain ambiguous. To determine the effects of (γ, ϕ, λ) on θ express the equilibrium job creation condition $c/q(\theta) = (1-\beta)S(0)$ as

$$\frac{c}{q(\theta)} = (1-\beta) \int_0^{\bar{a}} \left[1 - e^{(\gamma-\phi)\bar{a}} \left(b + \frac{\beta}{1-\beta} c\theta \right) \right] e^{-(r+\delta+\rho-\lambda)\bar{a}} d\bar{a}. \quad (26)$$

This equation allows us to perform simply the comparative statics on θ since it is (locally) independent of \bar{a} due to the envelope theorem. It is therefore easy to show that

$$\frac{d\theta}{d\gamma} < 0, \quad \frac{d\theta}{d\phi} > 0, \quad \frac{d\theta}{d\lambda} > 0.$$

Thus we conclude that an increase in the rate of technical change γ raises both unemployment incidence (\bar{a} decreases) and unemployment duration (θ decreases), and the equilibrium unemployment rate unambiguously increases. An increase in the transferability parameter ϕ has the opposite effect: both unemployment incidence and unemployment duration decrease (\bar{a} and θ both increase), and equilibrium unemployment decreases. Finally, an increase in the learning rate λ increases unemployment incidence by reducing \bar{a} , and reduces unemployment duration by increasing θ with ambiguous effect on the equilibrium unemployment rate.

To see the impact of γ when ϕ depends on γ , it is enough to recognize from (26) that $\frac{d\theta^*}{d\gamma}$ and $\frac{d\theta^*}{d\phi}$ are equal in absolute value. When $\phi'(\gamma) > 1$, the indirect

effect of γ through skill obsolescence dominates and a technological acceleration reduces unemployment. When $\phi'(\gamma) < 1$ the direct effect of γ through technological obsolescence dominates and we have the standard result that a technological acceleration increases unemployment.

These results have a very intuitive interpretation. A higher rate of technical change increases the value of frontier jobs relative to older vintages. In the standard vintage capital-search model this increases the worker's outside option. The consequent higher wage demands of workers and less profitable firms leads to less job creation and more job destruction, leading ultimately to higher equilibrium unemployment. The skill obsolescence effect counteracts this mechanism, by reducing the outside value of the worker relative to the current job. This is because skills are not fully transferable to new jobs. When the skill obsolescence effect dominates, workers settle for lower wages, which implies more profitable firm's and ultimately higher equilibrium unemployment.

5 Concluding Remarks

Models on growth and unemployment (eg. Aghion and Howitt 1994, Mortensen and Pissarides 1998) have focused on the 'creative destruction'-nature of technical change that takes place as new capital vintages replace old ones. The capital obsolescence effect present in these models governs job creation and destruction, as jobs are destroyed along with the obsolete capital they are matched with and new jobs are created at the technological frontier where new capital is introduced. This paper extends the standard vintage capital/search model to incorporate vintage human capital by introducing skill obsolescence. The novel feature of the present model is that workers skills are two-dimensional: skill accumulation and depreciation take place simultaneously. During the lifetime of a job workers accumulate skills that are relevant to the capital they currently operate, but these skills are only partially transferable to jobs of more recent capital vintages. Therefore, as job tenure increases so does the distance to the technological frontier where new jobs are created.

In addition to the capital obsolescence (or creative destruction) effect that tends to raise unemployment, vintage human capital introduces a skill obsolescence effect of faster growth that has the opposite sign. Faster skill obsolescence reduces the value of unemployment, hence wages and leads to more job creation and less job destruction, unambiguously reducing unemployment.

The search model with vintage human capital presented in this study provides

a framework to study questions pertaining to the impact of capital-embodied technological change on important labor market equilibrium outcomes such as the skill distribution and inequality in the wage distribution and labor market participation. An important extension to the presented model is to allow for the updating of capital in existing matches. Issues related to the determination of reservation wages and accepting jobs as well as search intensity provide interesting topics for extensions of the model.

An interesting extension to the matching model with vintage human capital is to study the labor force participation decision of workers when nonemployment involves an income unconditional on search. In the presence of an income from home-production, or a welfare payment unconditional on search, the model implies that some workers will choose optimally to exit the labor force because their skills have depreciated so much that their value of searching is below what is offered as non-participants. Murphy and Topel (1998) show that in synchrony with the rise of inequality, a larger fraction among the low-skilled agents have left the labor force for good. They have argued that the same labor demand shift is responsible for both. In the present model, a rise in the productivity growth of new capital increases the rate of obsolescence of skill, so it might lead in equilibrium to a larger fraction of agents whose skills end up commanding a wage lower than the value of nonparticipation. The participation decision depends on the threat point of the employed worker, whether it is unemployment or nonparticipation. The various cases of vintage human capital and nonparticipation are considered in ongoing research (Vanhala and Violante 2005).

A Appendix

A.1 Derivation of a typical value equation

A typical derivation of the value equations goes as follows. Consider the value of a vacant job. The strategy is to write the functions in discrete time, with interval length dt and take the limit as dt approaches zero.

$$V(t, z) = \max \left\{ \frac{1}{1 + rdt} \left[\int_t^{t+dt} -y(x, z) c dx + q(\theta) dt J(t + dt, t + dt, z) \right. \right. \\ \left. \left. + (1 - q(\theta) dt) V(t + dt, z) \right], 0 \right\}$$

$$(1 + rdt) V(t, z) = \max \left\{ \left[\int_t^{t+dt} -y(x, z) c dx + q(\theta) dt J(t + dt, t + dt, z) \right. \right. \\ \left. \left. + (1 - q(\theta) dt) V(t + dt, z) \right], 0 \right\}$$

Multiplying both sides by $(1 + rdt)$, subtracting $V(t, z)$ from both sides and dividing by dt , we obtain

$$rV(t, z) = \max \left\{ \left[\frac{\int_t^{t+dt} -y(x, z) c dx}{dt} + q(\theta) [J(t + dt, t + dt, z) - V(t + dt, z)] + \frac{V(t + dt, z) - V(t, z)}{dt} \right], 0 \right\} \quad (27)$$

where

$$\frac{\int_t^{t+dt} -y(x, z) c dx}{dt} = \frac{-\int_t^{t+dt} e^{\gamma x} z c dx}{dt} = \frac{-e^{\gamma t} z c}{\gamma} \left(\frac{e^{\gamma dt} - 1}{dt} \right).$$

We now take limits as $dt \rightarrow 0$ for each component of (27). To begin with,

$$\lim_{dt \rightarrow 0} \left(\frac{-e^{\gamma t} z c}{\gamma} \right) \left(\frac{e^{\gamma dt} - 1}{dt} \right) = \frac{-e^{\gamma t} z c}{\gamma} \gamma = -e^{\gamma t} z c = -y(t, z),$$

where we have made use of De L'Hopital rule for the term $\left(\frac{e^{\gamma dt} - 1}{dt} \right)$. Moreover,

$$\lim_{dt \rightarrow 0} [J(t + dt, t + dt, z) - V(t + dt, z)] = J(t, t, z) - V(t, z).$$

Furthermore, we have

$$V_1(t, z) = \lim_{dt \rightarrow 0} \frac{V(t + dt, z) - V(t, z)}{dt}.$$

Therefore, $rV(t, z)$ in (27) simplifies to

$$rV(t, z) = \max \{ [-y(t, z) + q(\theta) [J(t, t, z) - V(t, z)] + V_1(t, z)], 0 \}.$$

A.2 Wage equation

The wage is given by the Nash bargaining solution and maximizes

$$w(a, z) = \arg \max [J(a, z) - V(z)]^{1-\beta} [W(a, z) - U(z)]^\beta. \quad (28)$$

Observe that in this condition $U(z)$ is the value of unemployment for an employed worker and is thus affected by both learning by doing λ and skill depreciation ϕ . Therefore the law of motion for $U(z)$ in (28) is $\dot{z} = (\lambda - \phi) z$.

The first order condition, given $V(z) = 0$, is

$$\begin{aligned} & (1 - \beta) \frac{\partial J(a, z)}{\partial w(a, z)} [J(a, z) - V(z)]^{1-\beta} [W(a, z) - U(z)]^\beta \\ & + \beta \frac{\partial W(a, z)}{\partial w(a, z)} [J(a, z) - V(z)]^{1-\beta} [W(a, z) - U(z)]^{\beta-1} \\ & = 0 \end{aligned} \quad (29)$$

Divide both sides by $[J(a, z) - V(z)]^{-\beta} [W(a, z) - U(z)]^{\beta-1}$ and use $\frac{\partial W(a, z)}{\partial w(a, z)} = -\frac{\partial J(a, z)}{\partial w(a, z)}$ to get

$$\beta J(a, z) = (1 - \beta) [W(a, z) - U(z)]. \quad (30)$$

By substitution from equations (9), where $y(a, z) = ze^{(\phi-\gamma)a}$, and (10) and using the free-entry condition $V(z) = 0$, we obtain

$$\begin{aligned} & \beta \left[\frac{ze^{(\phi-\gamma)a} - w(a, z) + J_t(a, z) + \dot{z}J_z(a, z)}{r + \delta + \rho} \right] \\ &= (1 - \beta) \left[\frac{w(a, z) + \delta U(z) + W_t(a, z) + \dot{z}W_z(a, z)}{r + \delta + \rho} - U(z) \right], \end{aligned} \quad (31)$$

which by cancelling terms and rearranging gives

$$\begin{aligned} w(a, z) &= \beta ze^{(\phi-\gamma)a} + (1 - \beta)(r + \rho)U(z) \\ &+ \beta [J_t(a, z) + \dot{z}J_z(a, z)] - (1 - \beta) [W_t(a, z) + \dot{z}W_z(a, z)] \end{aligned} \quad (32)$$

Substitute the value equation for unemployment for an employed worker to get

$$\begin{aligned} w(a, z) &= \beta e^{\gamma\tau} ze^{\phi(t-\tau)} + (1 - \beta) \{bz e^{\gamma t} + p(\theta(z)) [W(0, z) - U(z)]\} \\ &+ \beta [J_t(a, z) + \dot{z}J_z(a, z)] \\ &- (1 - \beta) [W_t(a, z) + \dot{z}W_z(a, z) - U_t(z) - \dot{z}U_z(z)] \end{aligned} \quad (33)$$

As the value of unemployment in the wage bargain is that of an employed worker, observe that all the values $J(a, z)$, $W(a, z)$ and $U(z)$ have the law of motion $\dot{z} = (\lambda - \phi)z$.

Using the first order condition (30) to cancel the derivative terms we obtain

$$w(a, z) = \beta ze^{(\phi-\gamma)a} + (1 - \beta) \{bz + p(\theta(z)) [W(0, z) - U(z)]\} \quad (34)$$

By the first order condition $W(0, z) - U(z) = \frac{\beta}{1-\beta} J(0, z)$, and $V(z) = 0$ in (8) implies $J(0, z) = \frac{zc}{q(\theta)}$. Substituting into (34) and using the properties of the matching function the wage equation reduces to

$$w(a, z) = \beta ze^{(\phi-\gamma)a} + (1 - \beta) z \left(b + \frac{\beta}{1 - \beta} c\theta \right). \quad (35)$$

A.3 Match Surplus

To derive the equations (8) – (11) given in the text, substitute (2), (3) and (14) into the value equations (4) – (7) and stationarize the equations. Along the balanced growth path all the values above grow at rate γ . Let $a = t - \tau$ denote the age of the

production unit. Hence, it follows that $V(t, z) = e^{\gamma t} V(z)$, with $V_t(t, z) = \gamma e^{\gamma t} V(z)$; $J(t, \tau, z) = e^{\gamma t} J(a, z)$, with $J_t(t, \tau, z) = \gamma e^{\gamma t} J(a, z) + e^{\gamma t} J_a(a, z)$; $W(t, \tau, z) = e^{\gamma t} W(a, z)$, with $W_t(t, \tau, z) = \gamma e^{\gamma t} W(a, z) + e^{\gamma t} W_a(a, z)$; $U(t, z) = e^{\gamma t} U(z)$, with $U_t(t, z) = \gamma e^{\gamma t} U(z)$. Finally, we have $w(t, \tau, z) = e^{\gamma t} w(a, z) = e^{\gamma t} z w(a)$.

The balanced growth path versions of the value equations are (after some manipulation)

$$(r - \gamma) V(z) = \max \{-zc + q(\theta(z)) [J(0, z) - V(z)], 0\} \quad (36)$$

$$(r - \gamma) J(a, z) = \max \left\{ e^{-\gamma a} z e^{\phi a} - w(a) z - (\delta + \rho) \left[J(a, z) - \max_z V(z) \right] \right. \\ \left. + J_a(a, z) + (\lambda - \phi) z J_z(a, z), (r - \gamma) \max_z V(z) \right\} \quad (37)$$

$$(r - \gamma) W(a, z) = \max \{ w(a) z - \delta [W(a, z) - U(z)] - \rho W(a, z) \\ + W_a(a, z) + (\lambda - \phi) z W_z(a, z), (r - \gamma) U(z) \} \quad (38)$$

$$(r - \gamma) U(z) = \max \{ bz + p(\theta(z)) [W(0, z) - U(z)] - \rho U(z) \\ - \phi z U_z(z), 0 \} \quad (39)$$

Substituting the value equations (37), (38) and the value of unemployment for an *employed* worker (with the law of motion $\dot{z} = (\lambda - \phi) z$) into (12), using $V(z) = 0$ and the definition of surplus produces

$$(r - \gamma + \delta + \rho) S(a, z) = \max \left\{ e^{(\phi - \gamma)a} z - bz - p(\theta(z)) [W(0, z) - U(z)] \right. \\ \left. + J_a(a, z) + (\lambda - \phi) z J_z(a, z) \right. \\ \left. + W_a(a, z) + (\lambda - \phi) z W_z(a, z) \right. \\ \left. - (\lambda - \phi) z U_z(z), 0 \right\} . \quad (40)$$

Use the the Nash first order condition to substitute $[W(0, z) - U(z)] = \beta S(0, z)$ and definition of surplus to combine the derivative terms to obtain

$$(r - \gamma + \delta + \rho) S(a, z) = \max \left\{ e^{(\phi - \gamma)a} z - bz - p(\theta(z)) \beta S(0, z) \right. \\ \left. + S_a(a, z) + (\lambda - \phi) z S_z(a, z) \right\} . \quad (41)$$

To see that (41) is a first order differential equation as a function of t , observe that $S(a, z) = S(a(t), z(t))$ and consequently we can express the derivatives $S_a(a, z) + (\lambda - \phi) z S_z(a, z) = \frac{dS(a(t), z(t))}{dt}$, recall that $\dot{z} = (\lambda - \phi) z$. Furthermore, use the Nash condition $S(0, z) = \frac{1}{1 - \beta} J(0, z)$ and the free entry condition $V(z) = 0$ in the value equation for $J(0, z)$ to get $J(0, z) = \frac{cz}{q(\theta(z))}$ to obtain

$$(r - \gamma + \delta + \rho) S(a, z) - \frac{dS(a, z)}{dt} = \max \left\{ e^{(\phi - \gamma)a} z - bz - \frac{\beta}{1 - \beta} cz \theta(z), 0 \right\} \quad (42)$$

where we have used the properties of the matching function to produce $\frac{p(\theta(z))}{q(\theta(z))} = \theta(z)$. This equation is a first order differential equation for the surplus as a function of t .

Note that z represents the skills of a worker on the current leading edge technology, in terms of which the unemployment compensation b and search cost c are expressed. However, as the worker in a match of age a operates capital of that age, z is augmented by the factor $e^{\phi a}$ in the first right-hand-side term.

The max operator implies that we have a boundary condition $S(\bar{a}, z) = 0$. The general solution for the differential equation, using $\omega(\theta)z = bz + \frac{\beta}{1-\beta}cz\theta(z)$, for the surplus is

$$S(a, z) = Ae^{-(r-\gamma+\delta+\rho)a} \times \\ + \int_a^{\bar{a}} [e^{-\gamma\tilde{a}}e^{\phi a}ze^{\lambda(\tilde{a}-a)} - \omega(\theta)ze^{(\lambda-\phi)(\tilde{a}-a)}] e^{-(r-\gamma+\delta+\rho)(\tilde{a}-a)} d\tilde{a}$$

Note that in the first term in the integral, worker productivity is constant over time ($e^{\phi a}z$), except for learning by doing from age a onwards ($e^{\lambda(\tilde{a}-a)}$). In the second term in the integral, $e^{(\lambda-\phi)(\tilde{a}-a)}$ accounts for the workers skills depreciating at rate ϕ relative to the newest technology and accumulating by learning by doing at rate λ from age a onwards, from the initial level z . Divide and multiply by $e^{(\lambda-\phi)(\tilde{a}-a)}$ to get

$$S(a, z) = Ae^{-(r-\gamma+\delta+\rho)a} \times \\ + \int_a^{\bar{a}} [e^{-\gamma\tilde{a}}e^{\phi a}ze^{\lambda(\tilde{a}-a)}e^{-(\lambda-\phi)(\tilde{a}-a)} - \omega(\theta)z] e^{-(r-\gamma+\delta+\rho+\phi-\lambda)(\tilde{a}-a)} d\tilde{a}$$

and cancel terms

$$S(a, z) = Ae^{-(r-\gamma+\delta+\rho)a} \times \\ + \int_a^{\bar{a}} [e^{(\phi-\gamma)\tilde{a}}z - \omega(\theta)z] e^{-(r-\gamma+\delta+\rho+\phi-\lambda)(\tilde{a}-a)} d\tilde{a}$$

Using the boundary condition $S(\bar{a}, z) = 0$ implies that we must have $A = 0$ as the integral term in the differential equation is equal to zero at \bar{a} . Therefore after rearranging, we obtain,

$$S(a, z) = e^{(\phi-\gamma)a}z \int_a^{\bar{a}} [e^{(\phi-\gamma)(\tilde{a}-a)} - \omega(\theta)e^{-(\phi-\gamma)a}] e^{-(r-\gamma+\delta+\rho+\phi-\lambda)(\tilde{a}-a)} d\tilde{a}$$

Divide and multiply by $e^{(\phi-\gamma)(\tilde{a}-a)}$ to get

$$S(a, z) = e^{(\phi-\gamma)a}z \int_a^{\bar{a}} [1 - \omega(\theta)e^{-(\phi-\gamma)\tilde{a}}] e^{-(r+\delta+\rho-\lambda)(\tilde{a}-a)} d\tilde{a}. \quad (43)$$

A.3.1 Job Creation

Using the Nash first order condition and the free entry condition we obtain the job creation condition from (36)

$$\frac{c}{q(\theta)} = e^{(\phi-\gamma)a} \int_a^{\bar{a}} [1 - \omega(\theta) e^{-(\phi-\gamma)\bar{a}}] e^{-(r+\delta+\rho-\lambda)(\bar{a}-a)} d\bar{a}$$

where z cancels out from both sides which is independent of z . This equation implicitly solves θ , and is a function of the destruction age \bar{a} and the parameters of the model. z does not feature in this equation.

Furthermore, using $\omega(\theta) = b + \frac{\beta}{1-\beta}c\theta$ the job destruction condition is given by

$$e^{(\phi-\gamma)\bar{a}} = b + \frac{\beta}{1-\beta}c\theta$$

or

$$\bar{a} = -\frac{1}{\gamma-\phi} \ln \left(b + \frac{\beta}{1-\beta}c\theta \right) \quad (44)$$

Substitute this for \bar{a} in the job creation condition. The resulting equation implicitly solves θ as a function of the parameters of the model and this equation is independent of z .

A.3.2 Job Destruction

The outside option in the optimal separation condition (18) is equal to the value of unemployment for an employed worker

$$\omega(\theta) = b + \frac{\beta}{1-\beta}c\theta = b + \frac{\beta}{1-\beta}p(\theta) \frac{c}{q(\theta)}$$

where the last equality follows the properties of the matching function. Use $J(0, \bar{a}) = \frac{c}{q(\theta)}$ and the Nash first-order condition to get

$$\omega(\theta) = b + p(\theta) [W(0, z) - U(z)] \quad (45)$$

or

$$\omega(\theta) = b + p(\theta) \beta S(0, \bar{a}). \quad (46)$$

The value equation for an employed is given by

$$(r - \gamma + \rho + \phi - \lambda)U(z) = b + p(\theta) \beta S(0), \quad (47)$$

hence the outside option is equal to the value of unemployment for an employed worker.¹⁹

¹⁹The value of unemployment for an employed worker includes learning by doing as discussed above, hence the term λ .

Substituting (46) into (18) gives

$$e^{-(\gamma-\phi)\bar{a}} = b + p(\theta) \beta S(0, \bar{a}). \quad (48)$$

Substituting the expression for the surplus (17) evaluated at $a = 0$, and using (18) we obtain the job destruction condition

$$e^{-(\gamma-\phi)\bar{a}} = b + p(\theta) \beta \int_0^{\bar{a}} [1 - e^{-(\gamma-\phi)(\bar{a}-\tilde{a})}] e^{-(r+\delta+\rho-\lambda)\tilde{a}} d\tilde{a}, \quad (49)$$

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Chapter 3

Skill Biased Technological Change, Skill Determination and Labor Market Participation

Abstract

This paper studies the effect of skill biased technological change on skill mismatch and the allocation of workers and firms in the labor market. By allowing workers to invest in education, we extend a matching model with two-sided heterogeneity to incorporate an endogenous distribution of high and low skill workers. We consider various possibilities for the cost of acquiring skills and show that the skill cost is a key element in determining the effects of skill biased technological change on labor market variables. While unemployment increases in most scenarios, the effect on the distribution of vacancy and worker types varies according to the structure of skill costs. When the model is extended to incorporate endogenous labor market participation, we show that the unemployment rate becomes less informative of the state of the labor market as the participation margin absorbs employment effects of skill biased technological change.

1 Introduction

Most OECD countries have experienced a structural change in both the supply and demand for skills over the last three decades. Universally a strong shift in demand for more highly educated labor has occurred, which is to a large degree associated with skill-biased technological change. At the same time the educational level of workers has increased. Changes in the supply of labor have varied more between countries, but a strong trend toward an increased proportion of workers with college training is universal in the OECD (Katz 1994). Along with skill-biased technical

change and higher educational levels, labor markets have been subject to increasing mismatch between the skills of workers and the skills required by firms. In EU countries that have experienced an intense upgrading of tertiary education within the last fifteen years, the supply of skilled workers has outstripped the supply of skilled jobs. In these countries job competition, overeducation and crowding out have been particularly relevant (Dolado et al. 2003).¹

Skill mismatch and job competition are reflected in wages and unemployment rates across and within skill groups. However, as Murphy and Topel (1997) claim, the unemployment rate has become less informative about the state of the labor market. For example, in a study evaluating histories of unemployment and non-employment among American men between 1967-1994, the unemployment rate in 1994, 4.7 percent, is only slightly higher than in 1974 or 1978-1979, whereas non-participation more than doubled from the late 1960's to the 1990's (from 4 percent to 8.1 percent of the potential labor supply in 1994). When considering differences across skill groups, they observe a declining trend in the returns to work and labor market opportunities, especially among the low skilled workers. This is reflected both as higher unemployment and higher nonparticipation rates.

This paper studies skill biased technological change, skill investment and the allocation of workers and firms to jobs of different productivities. Within a matching model with two-sided heterogeneity, we study how firms allocate vacancy creation between high and low productivity jobs, and how workers allocate between labor market states. The novelty of this study is to introduce an endogenously determined skill distribution of the workers by allowing the possibility for workers to invest in skills when entering the labor market. This feature of the model is of relevance at least for long run analysis of labor markets, as one expects workers educational choices to respond to market incentives. From a policy perspective, to affect allocational mismatches of the labor market, the structure of the costs of skills is an obvious variable that may be influenced by policy. The model is then extended to allow for a complete choice of labor market states, by introducing endogenous labor market participation to the model. Thus, workers may choose whether to participate to the labor market or not, and when choosing participating the workers decide on their skill level endogenously.

¹Collecting results from various studies on the incidence of mismatch Hartog (2000) concludes that the incidence of overeducation has increased while the incidence of undereducation has decreased for three countries for which observations are available (Netherlands 1960-1995, Spain 1985-1990 and Portugal 1982-1992). For some countries however, notably the United States and Great Britain, the development of skill demand and supply has been somewhat different. In particular the United States experienced a sharp slowdown in the growth of the supply of skills occurred after the 1980s (Katz 1994).

We consider a matching model of two-sided heterogeneity with high and low skilled workers and minimum skill requirements for high productivity jobs. As Albrecht and Vroman (2002) or Gautier (2002) we take this to mean that low skilled workers are qualified for low productivity jobs only and high skilled workers are qualified for both high and low productivity jobs. The high skilled, however, are not more productive in the low productivity jobs than low skilled workers. This implies that high-skilled workers have a more favorable position in the labor market, as they are suitable for both high and low-skill jobs whereas low-skill workers are suitable for low-skill jobs only. It also implies that the relevant pool of unemployed workers to match with is larger for firms that post low skill vacancies than it is for firms posting high skill vacancies. We extend the model of Albrecht and Vroman (2002) by incorporating an endogenously determined skill distribution. Upon entry to the labor market workers may pay a one time education cost to become high skilled instead of remaining low skilled. For workers to invest in skills, the improvement in labor market prospects must compensate the cost of acquiring skills. The payoff of upgrading depends on the relative productivities of job types and the job competition that the worker faces in the job market from other workers.

Skill biased technological change is characterized by an increase in the productivity of high skill jobs relative to low skill jobs. The driving force of the model is the way skill biased technological change affects the expected values of high and low skill jobs, and how the skill cost features in these values. First, output in high skill jobs increases but also the outside option of high skill workers increases. The latter effect moderates the increase in the value of high skill jobs. For low skill firms, the cost of cross-skilled high skill workers increases with an increase in their outside option, but there is no increase in output. Thus, for low skill firms there is only a negative effect. In a standard setup, the expected value of high skill vacancies increases relative to low skill vacancies and the fraction of low skill vacancies tends to fall. In the present model, skill biased technological change increases the cost of acquiring skills and alters the value of the outside option of high skill workers. Workers must also consider the change in the degree of job competition that arises from changes in the distributions of both vacancies and workers.

We show that the skill cost wrt. the obtained skill level plays a key role in determining the labor market outcomes of the model. Because workers appropriate surplus from the firms to compensate for the skill investment, a key feature of the model is how large a share of match surplus the workers will appropriate and which type of firms are hit harder by appropriation. Typically, the higher is the marginal cost of skills, the more the value of high skill vacancies increases relative to low skill vacancies. Additionally, the general equilibrium effects through changes

in the distributions of worker and firm types will interact with this effect. We study the effect of skill biased technological change on job and worker distributions with alternative assumptions on the skill costs. While unemployment increases in most scenarios, the effect on the distribution high and low skill vacancies, high and low skilled workers, both among the total population and the unemployed, vary according to the structure of skill costs.

When the marginal cost of acquiring higher skills is relatively high, the model produces an expected result. Skill biased technological change increases firms' incentives to create high skill vacancies relative to low skill vacancies, because of higher productivity in high skill jobs and because the appropriation of surplus by workers alters the values of job types in favor of high skill jobs. Consequently the share of low skill unemployed increases. Although the increase in the skill cost is relatively high, it is compensated for in the unemployment value by the enhanced labor market prospects for high skill workers. Consequently more workers acquire skills and the fraction of high skill workers in the population increases. When the marginal cost of skills wrt. the attained skill level falls, the shift towards high skill vacancies becomes smaller. We show that the share of high skill vacancies may actually fall, leading also to a fall in the share of high skilled workers. Despite a smaller skill cost, workers do not have incentives to invest in skills anymore, because labor market prospects for high skill workers are not promising enough. The share of high skill workers in the population falls and low skill unemployment increases. Although total output increases, unemployment tends to rise in all cases.

We then extend the model further to investigate how job competition and skill biased technological change influence labor market outcomes when also the labor market participation margin is endogenous. Considering unemployment and non-participation together is essential as they reflect two sides of the same phenomenon. Participation and labor market heterogeneity have been studied in the literature (e.g. Sattinger 1995, Pissarides 2000) by considering heterogeneous preferences of leisure or values of non-market time (determined by e.g. wealth). In line with several earlier studies (e.g. McKenna 1987, Sattinger 1995) we assume that labor market participation depends positively on the value of being unemployed.² As Sattinger (1995) points out this leads to an important difference with search models

²More recently Garibaldi and Wasmer (2003, 2005) study the importance of frictions in a model of endogenous participation. These studies feature an irreversible entry cost to participating to the labor market. They show that decisions to participate and stop participating differ, and that labor supply is determined by two margins: exit and entry. These margins coincide when the irreversible sunk cost vanishes. See also Pries and Rogerson(2004) for a dynamic model with participation costs.

with a fixed labor force: a higher unemployment rate is consistent with higher a higher employment rate. Or vice versa lower employment is consistent with a lower unemployment rate, so that joblessness is absorbed to the pool of nonparticipants. The novelty of this study is to consider participation in a matching model where the markets of heterogenous workers and firms interact. In addition to participation effects, the participation margin also interacts with job competition and crowding out.

The extended model with endogenous labor market participation supports the claim of Murphy and Topel (1997) that the unemployment rate has become less informative of the aggregate state of the labor market. While skill biased technological change has modest effects on unemployment in the model, the participation margin fluctuates much more. In most cases labor market participation decreases with skill biased technological change. We conclude that the unemployment rate gives a misleading picture of the employment effects of skill biased technological change.

The structure of the paper is the following. Section 2 constructs a matching model with two-sided heterogeneity with an endogenously determined skill distribution of workers. In section 3 we derive and characterize the steady state equilibrium of the model. Section 4 analyses the effect of skill biased technological change on the labor market variables. In section 5 we extend the model to incorporate endogenous labor market participation. Section 6 concludes.

2 Model

We consider a continuous time matching model with two types of workers and two types of firms in the spirit of Albrecht and Vroman (2002).

Population and skills– In the basic model the labor force (participants) L equals the working age population P and is normalized to one. Workers can then be either unemployed u or employed $1 - u$. Workers endogenously decide whether or not to invest in the higher of two possible skill levels at a cost $K(\tilde{s})$. The skill investment cost depends on the difference between the two skill levels $\tilde{s} = s_2 - s_1$ and those who do not invest in skills remain at the lower level s_1 . We denote the fraction of workers with skills s_1 with p and the fraction of workers with skills s_2 with $1 - p$.

Skill requirements and job productivity– Firms' jobs are either filled or vacant. Firms can post vacancies with different productivity requirements for the worker. We assume that high skill workers are suitable for both high skilled jobs and low skilled jobs, but low skilled workers are suitable only for low skill jobs. We assume

that even though high skilled workers may work in a low skilled job, they are not more productive than low skilled workers in these jobs. Throughout the study we will assume that the parameter values of the model are such that high skill workers are willing to accept low skilled jobs instead of waiting for a high skill job to come along. We call this type of equilibrium a cross-skill matching equilibrium following previous studies.³ Denote by s the skill level of a worker and y is the skill requirement of the firm. The productivity of the job is then

$$x(s, y) = \begin{cases} y & \text{if } s \geq y \\ 0 & \text{if } s < y \end{cases}$$

This way of modeling skills and productivity has the idea of a minimum skill requirement for jobs. The firm pays the worker a wage $w(s, y)$ which depends both on the job type and the worker's skills and is negotiated by Nash bargaining. The wage will be discussed in more detail below. Jobs break up at an exogenous destruction rate δ . Firms that post vacancies pay a (flow) vacancy cost c during search. The cost is equal to all firms.

Search— We assume that search is undirected.⁴ The matching rate is governed by a matching function $m(u, v)$ which has constant returns to scale and the usual properties.⁵ The hazard for an unemployed worker to meet a vacant job is $\frac{m(u, v)}{u} = \theta q(\theta)$, where $\theta = \frac{v}{u}$ characterizes labor market tightness. The hazard for a vacant firm to meet an unemployed worker is $\frac{m(u, v)}{v} = q(\theta)$. The rate $\theta q(\theta)$ is increasing and $q(\theta)$ is decreasing in labor market tightness θ . The matching function is the same for all types, so the hazards do not depend on type. The fraction of low skill vacancies is ϕ and the fraction of high skilled vacancies is $1 - \phi$. Low skilled workers do not qualify for high skill jobs so the effective probability to meet a relevant vacancy for low skilled workers is $\phi \theta q(\theta)$. The fraction of low skilled unemployed workers is γ and that of high skilled unemployed workers is $1 - \gamma$. Low skill firms accept all workers but high skill firms accept only high skill workers. High skill firms have thus the effective probability $(1 - \gamma) q(\theta)$ of meeting a suitable worker.

The equilibrium solves for five endogenous variables: labor market tightness θ , the fraction p of low skilled workers in the population, the share of low skilled workers among the unemployed γ , the share of low skilled vacancies ϕ and the equilibrium unemployment rate u . In equilibrium the surplus for all matched agents

³Eg. Albrecht and Vroman (2002) and Dolado et al. (2002).

⁴Directed search has received considerable attention in recent research, but we abstract from these issues in this study.

⁵The matching function is increasing and concave in both of its arguments with $m(0, v) = m(u, 0) = 0$.

must be positive and there is free entry for firms in both markets. In steady state the flows into and out of unemployment must be equal. Accordingly we have a free entry condition and a steady state labor market flow condition for each market. Also it must be that $\phi, \gamma, u, p \in [0, 1]$ and $\theta > 0$.

2.1 Match surplus and value equations

Matches between vacant firms and unemployed workers are formed whenever the surplus of the match is nonnegative. We denote the value of unemployment for a worker of skills s is $U(s)$ and the value for a firm of holding a vacant job with skill requirement y is $V(y)$, the value of being employed in a job with skill requirement y for a worker with skills s is $W(s, y)$ and the value of a filled job with skill requirement y and a worker with skills s is $J(s, y)$. A match between a firm with skill requirement y and a worker of skills s produces a surplus of $S(s, y)$ and for the match to be formed it must be that

$$S(s, y) = J(s, y) + W(s, y) - V(y) - U(s) \geq 0. \quad (1)$$

2.1.1 Firms

The flow value of a filled job with skill requirement y and a worker of skills s is

$$rJ(s, y) = y - w(s, y) - \delta [J(s, y) - V(y)]. \quad (2)$$

The flow value equals the productivity y of the match minus the wage $w(s, y)$ paid to the worker and the loss of the firm's surplus in case of exogenous job destruction which takes place at rate δ .

The value equations for low and high skill vacancies are

$$rV(s_1) = -c + q(\theta) \{ \gamma [J(s_1, s_1) - V(s_1)] + (1 - \gamma) \max [J(s_2, s_1) - V(s_1), 0] \} \quad (3)$$

$$rV(s_2) = -c + q(\theta) (1 - \gamma) [J(s_2, s_2) - V(s_2)] \quad (4)$$

The flow value equals the search cost c plus the payoff of matching. The low skill firm has the probability $q(\theta) \gamma$ of matching with a low skill worker and the probability $q(\theta) (1 - \gamma)$ of matching with a high skilled worker. The last term in the value equation for a low skill vacancy takes account of the requirement that a cross-skill match must yield positive surplus, $J(s_2, s_1) - V(s_1) \geq 0$. The value equation for the high skill vacancy takes account of only the high skill workers being suitable for high skill jobs. The high skill firm meets a high skill worker with probability $q(\theta) (1 - \gamma)$.

Firms enter into both markets until the value of posting a vacancy goes to zero $V(s_i) = 0$. Substituting these into the asset value equations for vacancies in each market gives the job creation conditions for both vacancy types

$$\gamma J(s_1, s_1) + (1 - \gamma) J(s_2, s_1) = \frac{c}{q(\theta)} \quad (5)$$

$$(1 - \gamma) J(s_2, s_2) = \frac{c}{q(\theta)} \quad (6)$$

These conditions state that the expected surplus for the firm of each job type equals the expected job creation cost.

2.1.2 Workers

The flow value for an employed worker of skills s in a job with skill requirement y is

$$rW(s, y) = w(s, y) - \delta [W(s, y) - U(s)], \quad (7)$$

and has an analogous interpretation to the value equation for a filled job. The worker receives a wage $w(s, y)$ and faces the risk of loss of surplus at the destruction rate δ .

The flow values for low and high skilled unemployed workers are given by separate value equations.

$$rU(s_1) = b + \theta q(\theta) \phi [W(s_1, s_1) - U(s_1)] \quad (8)$$

$$\begin{aligned} rU(s_2) = & b + \theta q(\theta) \{ \phi \max [W(s_2, s_1) - U(s_2), 0] \\ & + (1 - \phi) [W(s_2, s_2) - U(s_2)] \} \end{aligned} \quad (9)$$

For low skilled unemployed workers the flow value of unemployment equals an unemployment benefit b plus the expected payoff of search. For the low skilled worker the probability of being matched is $\theta q(\theta) \phi$ as only low skill jobs can be accepted. For the high skilled worker the probability of being matched is $\theta q(\theta)$. For the high skilled workers, the payoff depends on the shares of each type of vacancies in the market. With probability ϕ a low skill vacancy is met and with probability $(1 - \phi)$ a high skill vacancy is met. Cross skill matching requires that the surplus must be positive for the match to be formed, $W(s_2, s_1) - U(s_2) \geq 0$.

Initially all workers are endowed with skills s_1 . Before entering the labor market workers may upgrade their skills from s_1 to s_2 by incurring a lump sum education cost $K(\tilde{s})$. The education cost depends on the productivity difference between the two job types $\tilde{s} = s_2 - s_1$, and $K(\tilde{s}) > 0$ and $K'(\tilde{s}) \geq 0$. The education cost is paid

upon entry to the labor market, after which the skill type of the worker does not change.⁶

The form of the education cost function is specified as

$$K(\tilde{s}) = \frac{\kappa}{\alpha} (s_2 - s_1)^\alpha \quad (10)$$

where $\kappa > 0$ and $\alpha > 0$. The cost for a worker of skills s_1 of acquiring the higher skill level s_2 may depend on a variety of features. One can think of various plausible scenarios:

Convex cost function– We first assume increasing marginal cost of skills s.t. $\alpha > 1$. In this case acquiring higher skills becomes more costly with as s_2 increases. This standard cost function has the interpretation that learning new skills becomes harder as more effort is needed to acquire the higher skill level.

Linear cost function– We then assume that $\alpha = 1$ s.t. the cost function is linear in the productivity difference. The costliness of acquiring higher skills is proportional to the skill difference between the two skill levels.

Concave cost function– In this case $0 < \alpha < 1$ so that the marginal cost of skills is decreasing. Here learning may be considered to have economies of scale. For example, learning new to use new software may initially be relatively hard, but the more programmes a person has learned, the easier it becomes to learn new ones. This is a typical case of learning by doing.

The distribution of worker types is determined by the difference in the unemployment values for each skill type and the lump sum education cost

$$U(s_2) - U(s_1) = K(\tilde{s}). \quad (11)$$

This condition equates the gain in unemployment values to the education cost.⁷

2.2 Wages

Match surplus is divided between the firm and the worker by the Nash bargaining solution. The first order condition is

$$\beta [J(s, y) - V(y)] = (1 - \beta) [W(s, y) - U(s)] \quad (12)$$

⁶This assumption is made to avoid complications related to the flows between skill types.

⁷In this setup we do not consider education as time consuming (see e.g. Becker (2005)), as this would require the introduction of an additional labor market state to the model. Implicitly we assume that the time required to obtain higher skills is consumed before entering search in the labor market.

where $0 \leq \beta \leq 1$ is the bargaining power of the worker. In equilibrium $V(y) = 0$ and by substituting from the value equations we obtain

$$w(s, y) = \beta y + (1 - \beta) rU(s). \quad (13)$$

The wage consists of two components. The first part βy is determined by match productivity y of which the worker gets a share according to the bargaining parameter β . The second part is determined by the workers outside option i.e. the value of unemployment for the worker. There are three possible wages

$$\begin{aligned} w(s_1, s_1) &= \beta s_1 + (1 - \beta) rU(s_1) \\ w(s_2, s_1) &= \beta s_1 + (1 - \beta) rU(s_2) \\ w(s_2, s_2) &= \beta s_2 + (1 - \beta) rU(s_2) \end{aligned}$$

The wage for a low skilled worker is $w(s_1, s_1)$. It depends on the match productivity s_1 and on the value of unemployment for a low skilled worker $rU(s_1)$. The low skilled worker would be equally productive in any other potential job so the value of unemployment depends on s_1 .

For a high skilled worker there are two possible wages. If the high skilled worker is in a low skilled job, her productivity equals that of a low skilled worker. However, the outside option of a high skilled worker is higher and therefore the wage depends on $rU(s_2)$. The high skilled worker has potential high productivity jobs included in the outside option. In a high skilled job the high skilled worker has a higher wage than in a low skill job because the workers productivity is higher.

3 Equilibrium

The model produces two different types of equilibria, depending on the parameter values of the model. In 'cross skill matching equilibrium' high skill workers accept both high and low skill jobs. In 'ex post segmentation equilibrium' high skill workers do not find it beneficial to accept low skill vacancies and therefore there are no mismatched workers (i.e. high skill workers in low skill jobs). We will limit the analysis to a parameter range such that high skill workers accept both types of jobs so that we have an equilibrium with cross skill matching. This requires that $s_1 \geq rU(s_2)$. Generally, a sufficiently high share of low skilled workers p or a sufficiently small spread between different productivity types produces this case. The conditions for cross skill matching are presented below.

The equilibrium is solved by deriving the equilibrium values for five endogenous variables: labor market tightness θ , the fraction p of low skilled workers in the

population, the share of low skilled workers among the unemployed γ , the share of low skilled vacancies ϕ and the equilibrium unemployment rate. In equilibrium the surplus for all matched agents must be positive and there is free entry for firms in both markets. The latter implies that $V(s_i) = 0$ in both markets. With the free-entry conditions we determine labor market tightness θ (reflecting total vacancy creation) and the allocation ϕ of vacancies between job types from the firms' optimizing behavior. In addition there is a steady-state labor market flow condition for each market from which we obtain the equilibrium unemployment rate and the fractions of worker types in the labor force. Additionally we require that $\phi, \gamma, u, p \in [0, 1]$ and $\theta > 0$.

3.1 Steady-state flows

In steady state the flow out of and into unemployment must be equal for both worker types. For low skilled workers the steady state condition is

$$\phi\theta q(\theta)\gamma u = \delta(p - \gamma u). \quad (14)$$

The probability of meeting a low skilled vacancy for the worker is $\phi\theta q(\theta)$ and γu is the measure of low skilled unemployed workers. The flow out of unemployment for low skilled workers is thus given by $\phi\theta q(\theta)\gamma u$. The flow into unemployment is given by $\delta(p - \gamma u)$ where δ is the rate of exogenous job destruction. p is the share of low skilled workers in the workforce and $p - \gamma u$ is thus the measure of employed low skilled workers.

An analogous condition for the high skilled workers is

$$\theta q(\theta)(1 - \gamma)u = \delta[1 - p - (1 - \gamma)u]. \quad (15)$$

As high skilled workers are suitable for all jobs and given our assumptions they will also accept all jobs, the probability of meeting an acceptable vacancy is simply $\theta q(\theta)$. With $(1 - \gamma)u$ being the measure of unemployed high skilled workers the outflow of unemployment is given by $\theta q(\theta)(1 - \gamma)u$. The inflow to unemployment $\delta[1 - p - (1 - \gamma)u]$ is the destruction rate multiplied by the share of employed high skilled workers.

The two steady-state conditions can be solved for the share of low skill workers in the population p and the equilibrium unemployment rate u respectively, which produce

$$p = \frac{\gamma[\delta + \theta q(\theta)\phi]}{\delta + \theta q(\theta)[1 - \gamma(1 - \phi)]} \quad (16)$$

and

$$u = \frac{\delta(1 - p)}{(1 - \gamma)[\delta + \theta q(\theta)]}. \quad (17)$$

We see that p is a function of θ, γ and ϕ and the parameter δ and that u is a function of θ, γ and the parameter δ . The share of low skilled workers p in the population is decreasing in the variable θ and increasing in the variables γ and ϕ . The equilibrium unemployment rate is decreasing in θ and p and increasing in γ .

3.2 Job creation

The total amount of vacancies and their allocation across markets are determined by the free-entry conditions given above. Substituting from the value equations for $J(s, y)$ and the wage equation the job creation conditions yield

$$\gamma [s_1 - rU(s_1)] + (1 - \gamma) [s_1 - rU(s_2)] = \frac{(r + \delta) c}{(1 - \beta) q(\theta)} \quad (18)$$

$$(1 - \gamma) [s_2 - rU(s_2)] = \frac{(r + \delta) c}{(1 - \beta) q(\theta)} \quad (19)$$

where the flow values of unemployment $rU(s_1)$ and $rU(s_2)$ are⁸

$$rU(s_1) = \frac{(r + \delta) b + \beta \theta q(\theta) \phi s_1}{r + \delta + \beta \theta q(\theta) \phi} \quad (20)$$

$$rU(s_2) = \frac{(r + \delta) b + \beta \theta q(\theta) [\phi s_1 + (1 - \phi) s_2]}{r + \delta + \beta \theta q(\theta)} \quad (21)$$

Instead of working with the above job creation conditions (18) and (19), we follow Albrecht and Vroman (2002) and derive two alternative conditions to simplify later analysis. Free entry for firms implies that the value of posting a vacancy in either market must be equal and vacancies enter until zero profits prevail in both markets. Therefore $V(s_1) = V(s_2) = 0$. Subtracting (18) from (19) produces an 'equal value' condition for the allocation of vacancies

$$(1 - \gamma) (s_2 - s_1) = \gamma [s_1 - rU(s_1)]. \quad (22)$$

Substituting $rU(s_1)$ into the equation gives the form

$$(1 - \gamma) (s_2 - s_1) = \gamma \frac{(r + \delta) (s_1 - b)}{r + \delta + \beta \theta q(\theta) \phi}. \quad (23)$$

Substituting the equal value condition (23) into (19) we obtain the second job creation condition

$$s_1 - b = \frac{[r + \delta + \beta \theta q(\theta)] c}{(1 - \beta) q(\theta)}. \quad (24)$$

This condition determines labor market tightness θ which characterizes total vacancy creation. Conditions (23) and (24) can be used instead of (18) and (19) in determining the equilibrium of the model.

⁸See appendix for derivations.

3.3 Skill choice condition

To obtain the final equilibrium equation, we rewrite the 'workers equal value' condition (11) as

$$rU(s_2) - rU(s_1) = rK(\tilde{s}). \quad (25)$$

This may be interpreted as a flow version of the 'workers equal value' condition. Substituting the expressions (20) and (21) into this equation we obtain

$$\frac{(r + \delta)b + \beta\theta q(\theta)[\phi s_1 + (1 - \phi)s_2]}{r + \delta + \beta\theta q(\theta)} - \frac{(r + \delta)b + \beta\theta q(\theta)\phi s_1}{r + \delta + \beta\theta q(\theta)\phi} = rK(\tilde{s}) \quad (26)$$

which is a function of labor market tightness θ , the share of high skill vacancies ϕ and parameters. As it is assumed that $K'(\tilde{s}) \geq 0$, both sides of the equation are increasing in the skill level s_2 . The distribution of vacancy types plays an important role in determining the payoff of skill upgrading. The higher is the share $1 - \phi$ of high skill vacancies, the better are the labor market prospects for high skill workers.

A key feature in the workers decision to acquire skills is the possibility for high skill workers to extract surplus from firms. A high skilled worker benefits from the fact that high skill firms cannot employ low skill workers. This implies that workers can appropriate surplus from firms to compensate for the skill investment. But low skill firms must also pay high skill workers according to their outside option as they recognize that otherwise these workers will choose to search for a high skilled job. Thus high skill worker's wages are affected in both job types. The higher unemployment value $rU(s_2)$ for high skilled workers will be a key element in determining the firms incentives to allocate vacancies between the two job types. In general equilibrium ϕ and θ will adjust to balance changes of s_2 in equation (26).

3.4 Equilibrium conditions

For easier reference we collect the equilibrium equations for the five endogenous variables u, p, γ, ϕ and θ . The skill upgrading condition (26) of workers has been replaced by an alternative expression which is more convenient in the analysis that will follow.⁹

- Equilibrium unemployment rate

$$u = \frac{\delta(1 - p)}{(1 - \gamma)[\delta + \theta q(\theta)]} \quad (27)$$

⁹See appendix for derivation.

- Fraction of low skilled in the population

$$p = \frac{\gamma [\delta + \theta q(\theta) \phi]}{\delta + \theta q(\theta) [1 - \gamma(1 - \phi)]} \quad (28)$$

- Skill upgrading condition

$$\frac{1}{\gamma} (s_2 - s_1) - \frac{(r + \delta)(s_1 - b)}{(1 - \gamma)[r + \delta + \beta\theta q(\theta)]} = rK(\tilde{s}) \quad (29)$$

- Equal value condition

$$(1 - \gamma)(s_2 - s_1) = \gamma \frac{(r + \delta)(s_1 - b)}{r + \delta + \beta\theta q(\theta) \phi} \quad (30)$$

- Total vacancy creation, labor market tightness θ

$$s_1 - b = \frac{[r + \delta + \beta\theta q(\theta)]c}{(1 - \beta)q(\theta)} \quad (31)$$

The model can be solved recursively. Due to the properties of the matching function, equation (31) gives a unique solution for θ . Equation (29) is monotonously increasing in (θ, γ) – space so when there is a cross skill matching equilibrium, equations (31) and (29) produce a unique solution for the pair (θ, γ) . With knowledge of θ and γ equation (30) solves ϕ . Finally, equations (27) and (28) solve for u and p after substituting θ, γ and ϕ .

3.5 Parameter restrictions for cross-skill matching

As discussed previously cross skill equilibrium requires that $s_1 \geq rU(s_2)$. Generally, a sufficiently high share of low skilled workers p or a sufficiently small difference between the two productivity types $s_2 - s_1$ produces this case. Reformulating the requirement for cross-skill matching gives

$$(r + \delta)(s_1 - b) \geq \beta\theta q(\theta)(1 - \phi)(s_2 - s_1) \quad (32)$$

In the case of cross-skill matching there is also the possibility of a corner solution in which firms post only low skill vacancies, so that $\phi = 1$. The corner solution occurs when the value of posting a low skill vacancy is positive at $\phi = 1$. We can show however, that the present model rules out this possibility. A sufficient condition to rule out the corner solution is

$$(1 - p)(s_2 - s_1) > p \frac{(r + \delta)(s_1 - b)}{r + \delta + \beta\theta q(\theta)} \quad (33)$$

It is easy to show that when there is cross-skill matching, $\gamma > p$.¹⁰ Consequently, whenever $K > 0$ and the education investment condition holds it is the case that (33) is valid.

4 Skill-biased technological change

We now proceed to analyze the effects of skill biased technological change on the labor market variables. Skill-biased technological change is characterized by an increase in the spread between the productivities of high and low skilled jobs, so that s_2 increases while keeping s_1 fixed.

The driving force of the model is the way skill biased technological change affects the expected values of high and low skill jobs, and how the skill cost features in these values. First, when s_2 increases for given values of the endogenous variables, output in high skill jobs increases but also the outside option of high skill workers increases. The latter effect moderates the output effect and the incentives to shift vacancy allocation to high skill jobs. For low skill firms, the cost of cross-skilled high skill workers increases with an increase in their outside option, but there is no increase in output. Thus, for low skill firms there is only a negative effect. *Ceteris paribus*, vacancies would typically be allocated towards high skill jobs. Second, recall from (25) that $rU(s_2) = rU(s_1) + rK(\tilde{s})$. A rise in s_2 implies a rise in the skill cost, and for (25) to be restored the value of unemployment for a high skill worker must rise, the more so the larger the rise in the skill cost. By substituting (25) for $rU(s_2)$ into (18) and (19) one may see the effect of the skill cost on job creation. The higher the marginal cost of skills, the smaller is the increase in surplus induced by a rise in s_2 for high skill jobs, and the larger is the negative effect on low skill surplus, *ceteris paribus*. Thus the marginal cost of skills is of key importance in affecting the relative values of the two job types. Typically, a higher marginal cost alters job values in favor of high skill jobs. Additionally, the effects through the endogenous variables also affect the changes in the relative values of the job types.

We first derive the general conditions for the comparative statics and then discuss alternative cases for the skill cost function, illustrating them by numerical examples. We set standard values for the parameters of the model. The matching

¹⁰This follows from the requirement that $1 - p$ is a fraction i.e.

$$1 - p = \frac{(1 - \gamma) [\delta + \theta q(\theta)]}{\delta + \theta q(\theta) [1 - \gamma(1 - \phi)]}$$

function is assumed to take the Cobb-Douglas specification $m(u, v) = u^\sigma v^{1-\sigma}$ and the worker's matching elasticity parameter is set to $\sigma = 0.4$. The job destruction rate is $\delta = 0.1$. These values are consistent with the empirical literature. We use the standard assumptions that firms and workers receive an equal share of the match surplus s.t. $\beta = 0.5$ and the flow job creation cost is set at $c = 0.3$. The unemployment compensation is $b = 0.2$. and the interest rate is $r = 0.05$. Productivity in low skill jobs is $s_1 = 1$ and productivity in high skill jobs $s_2 = 1.2$ in the initial cross-skill matching equilibrium. The cost function for skills has the form $K(\tilde{s}) = \frac{\kappa}{\alpha} (s_2 - s_1)^\alpha$ and we set the value of κ so that in the initial state the fraction of low skilled workers is $p = 2/3$ as in Albrecht and Vroman (2002). This parametrization produces an equilibrium where $u = 0.063, p = 0.667, \gamma = 0.695, \phi = 0.868$ and $\theta = 2.252$. Table 1 presents the numerical examples.

4.1 Equilibrium comparative statics $(\theta, \gamma, \phi, p, u)$

From equation (31) it is immediate that labor market tightness is not affected by the increase in s_2 so we have

$$\frac{d\theta}{ds_2} = 0$$

regardless of the functional form of the education cost function.

As labor market tightness is unaffected by skill biased technological change, the key block of equations are (29) and (30), which determine the workers incentives to acquire higher skills or remain low skilled and the firms incentives to allocate vacancies between the two types. Differentiating the equations produces the comparative statics for the share of unemployed workers γ and low skill vacancies ϕ

$$\frac{d\gamma}{ds_2} = \frac{\gamma(1-\gamma) \left[\frac{1}{\gamma}(s_2 - s_1) - \alpha r K(\tilde{s}) \right]}{(s_2 - s_1) \left[\frac{1}{\gamma}(s_2 - s_1) - \gamma r K(\tilde{s}) \right]} \quad (34)$$

$$\frac{d\phi}{ds_2} = \frac{rK(\tilde{s})(\gamma - \alpha)}{\left[\frac{1}{\gamma}(s_2 - s_1) - \gamma r K(\tilde{s}) \right] \frac{\beta\theta q(\theta)(s_2 - s_1)}{r + \delta + \beta\theta q(\theta)\phi}} \quad (35)$$

where $\alpha = \frac{K'(\tilde{s})\tilde{s}}{K(\tilde{s})}$ is the elasticity of the education cost wrt. to the productivity differential $\tilde{s} = s_2 - s_1$.¹¹ Note that the denominator is positive in both equations. It is immediate to see that the form of the skill cost function will play a central role for the comparative statics of γ and ϕ .

To determine $\frac{d\gamma}{ds_2}$, use $\alpha = 1$ in the numerator of (34) and substitute for $rK(\tilde{s})$

¹¹See appendix for detailed derivations.

from (29) to produce

$$\text{sign} \left(\frac{d\gamma}{ds_2} \right) = \text{sign} \left[\frac{(r + \delta)(s_1 - b)}{(1 - \gamma)[r + \delta + \beta\theta q(\theta)]} \right] > 0.$$

From this it follows that $\frac{d\gamma}{ds_2} > 0$ when the cost function is concave, i.e. when $\alpha < 1$. As α increases the effect becomes smaller and turns negative at a threshold level. The threshold level can be expressed as¹²

$$\text{sign} \left(\frac{d\gamma}{ds_2} \right) = \text{sign} \left[\frac{\frac{1}{\gamma}(s_2 - s_1)}{rK(\tilde{s})} - \alpha \right]. \quad (36)$$

Because γ is a fraction it is immediate from (35) that for convex and linear cost functions $\frac{d\phi}{ds_2} < 0$. At the threshold $\alpha = \gamma$ the effect on ϕ turns positive.

Ultimately the comparative statics of γ and ϕ determine the outcomes of the remaining variables. As θ is unaffected by s_2 , the effect of skill biased technological change on p and u are given by

$$\text{sign} \left(\frac{dp}{ds_2} \right) = \text{sign} \left[\frac{1}{(1 - \gamma)\gamma} \frac{d\gamma}{ds_2} + \frac{\theta q(\theta)}{\delta + \theta q(\theta)\phi} \frac{d\phi}{ds_2} \right] \quad (37)$$

$$\text{sign} \left(\frac{du}{ds_2} \right) = \text{sign} \left[(1 - \phi) \frac{d\gamma}{ds_2} - \gamma \frac{d\phi}{ds_2} \right]. \quad (38)$$

With the general conditions for the comparative statics we now proceed to examine the effect of skill biased technological change in the various cases of the skill cost function.

4.2 Skill biased technological change and the structure of skill costs

We begin with a convex skill cost function ($\alpha = 4$ in Table 1) as it provides perhaps the most intuitive results. An increase in s_2 raises productivity in high skill jobs. The outside option of high skilled workers increases as well, moderating the positive effect on the surplus of high skill jobs and reducing expected surplus of low skilled jobs. Vacancy creation changes towards high skill jobs ($\partial\phi/\partial s_2 < 0$) because the relative values of job types change in favor of high skill jobs. A relatively high marginal cost of skills (and thus in the unemployment value $U(s_2)$) moderates the incentives for workers to invest in skills, but because of the improvement in the prospects for high skilled workers more workers invest in skills ($\partial p/\partial s_2 < 0$). Low skilled unemployment increases ($\partial\gamma/\partial s_2 > 0$) somewhat due to the reduction in the

¹²See appendix.

share of low skill jobs. Total output increases, despite an increase in unemployment. These results are qualitatively similar to those in earlier studies.¹³

When the convexity of the cost function decreases ($\alpha = 2$ to $\alpha = 0.9$ in Table 1), skill biased technological change has a smaller effect on the marginal cost of skills. The relative values of the job types are still altered in favor of high skill jobs and the share of high skill jobs does increase ($\partial\phi/\partial s_2 < 0$), but less than above. Even though the increase in the skill cost is lower than above, less workers invest in skills ($\partial p/\partial s_2 > 0$). This is because the labor market prospects for high skilled workers are not improved enough. The share of low skilled unemployed increases ($\partial\gamma/\partial s_2 > 0$) more than above, because now there are more low skilled workers in the labor force in addition to the smaller increase in the share of high skilled vacancies. Unemployment and output increase.

Reducing further the slope of the cost curve ($\alpha = 0.5$ to $\alpha = 0.1$ in Table 1), the effect of skill biased technological change on the marginal cost of skills decreases further. Despite this, the share of low skilled workers increases ($\partial p/\partial s_2 > 0$), because the relative values of the two job types changes in favor of low skilled jobs. Vacancy creation is altered towards low skill jobs ($\partial\phi/\partial s_2 < 0$) and the labor market prospects for low skilled workers are improved. However, despite the increase in the fraction of low skill jobs, low skill unemployment increases due to the higher share of low skilled workers in the labor force.

The effects on the skill distribution of the labor force and among the unemployed as well as the allocation of vacancies depends on the marginal cost of skills. The driving force behind these effects is the effect of the skill cost on the appropriability of match surplus by workers. Although output increases in all scenarios, so does unemployment. The effect on the distribution high and low skill vacancies, high and low skilled workers, both among the total population and the unemployed, vary according to the structure of skill costs. The model does not give a complete picture of the employment effects of skill biased technological change however. This is discussed in the following section: when the model is extended to incorporate endogenous labor market participation, we show that the unemployment rate becomes less informative of the state of the labor market as the participation margin absorbs employment effects of skill biased technological change.

¹³Increasing the convexity of the cost function enough produces a range where γ may decrease. However, this area does not necessarily feature in the cross-skill equilibrium range.

	s_2	θ	ϕ	p	γ	u	Y
Initial equilibrium	1.20	2.252	0.868	0.667	0.695	0.063	0.999
Skill biased technological change High margin cost							
$\alpha = 4$	1.25	2.252	0.733	0.651	0.713	0.071	1.012
Skill biased technological change Medium margin cost							
$\alpha = 2$	1.25	2.252	0.823	0.695	0.732	0.066	1.006
$\alpha = 1$	1.25	2.252	0.857	0.710	0.738	0.064	1.004
$\alpha = 0.9$	1.25	2.252	0.860	0.711	0.739	0.064	1.004
Skill biased technological change Low margin cost							
$\alpha = 0.5$	1.25	2.252	0.871	0.715	0.741	0.064	1.003
$\alpha = 0.1$	1.25	2.252	0.882	0.720	0.743	0.063	1.003

Table 1. The effect of skill biased technological change on equilibrium labor market variables.

5 Endogenous labor market participation

To complete the analysis of workers' labor market choices we extend the model by incorporating a third labor market status for workers, non-participation. Workers who are without a job may choose either to search actively for a new job or they may decide not to participate in search and thus remain outside the labor force. Matching depends on the measure of open vacancies and the pool of actively searching workers. We make a distinction between the workers without a job who actively search for one and those who do not search and are considered as out of the labor force. In this extended setup there are thus three labor market states for workers: employed, unemployed who actively search and non-participants. We abstract from the possibility of workers moving directly from out of the labor force to employment. As discussed in Petrongolo and Pissarides (2001) transitions directly from out of the labor force into employment may be considered as a result of inadequate measurement. For example, in data with monthly or quarterly survey periods, an inactive worker who starts search and finds a job within a week is considered as moving from out of the labor force to employment. However, this worker was an actively searching worker when finding a job although the search period was short. Therefore, when the period of analysis is sufficiently short we can assume all hires to come from the ranks of the unemployed.¹⁴

¹⁴Blanchard and Diamond (1989) consider various definitions of the relevant pool of searching workers, but they find that inactive workers do not enter the matching function with a significant

5.1 Matching and skill distribution

We continue to assume that the size of the working age population normalized to one. Now some workers opt for nonparticipation, so the working age population $P = 1$ and the labor force $LP = L$ are no longer equal. Unemployment u now denotes the fraction of unemployed workers as a fraction of the labor force L . Vacancies v are also referred to as the number of vacant jobs as a fraction of the labor force. We express the matching function as

$$mL = m(uL, vL).$$

By the homogeneity of the matching function, the hazard rates for the firm and worker are, respectively,

$$\begin{aligned} \frac{m(uL, vL)}{vL} &= m\left(\frac{u}{v}, 1\right) = q(\theta) \\ \frac{m(uL, vL)}{uL} &= m\left(1, \frac{v}{u}\right) = \theta q(\theta). \end{aligned}$$

The hazard rates are functions of the ratio of vacancies to unemployment (labor market tightness) $\theta = \frac{v}{u}$ only. The effective hazard rates with different types are also unchanged. The fractions of low and high skill vacancies are ϕ and $(1 - \phi)$ and the hazards for a worker of meeting a low and high skill vacancy are, respectively $\phi\theta q(\theta)$ and $(1 - \phi)\theta q(\theta)$. The fraction of unemployed low skilled unemployed workers is γ and that of high skilled unemployed workers is $(1 - \gamma)$. Here it is just to bear in mind that these are fractions of uL , the share of unemployed of the labor force which does not include the nonparticipants. High and low skill firms have thus the effective probability $(1 - \gamma)q(\theta)$ and $q(\theta)$, respectively, of meeting a suitable worker.

Although the overall hazard rates are independent of the participation rate, the fractions of different worker types among the nonemployed require caution. We will show below that if any low skill workers participate, all nonparticipants will be low skilled. Consequently the share of high and low skill workers will not be the same in the total labor force and among the participants.

The workers enter the economy with exogenously given low skills and decide on their labor market state at the moment of entry. They face three alternatives: participate with the given low skill level, pay the education cost and participate as high skilled, or choose nonparticipation. The low and high skilled shares of the population are, respectively,

$$\chi = (1 - L) + pL \tag{39}$$

$$1 - \chi = (1 - p)L. \tag{40}$$

coefficient.

The share of low skill workers in the population is given by the fraction $1 - L$ of workers endogenously choosing nonparticipation (and consequently staying low skilled) and the fraction p of the participants who endogenously decide to remain low skilled. The share of high skilled of the population are the fraction $(1 - p)$ of the participants L who acquire skills.

The equilibrium unemployment rate (of the labor force participants L) is given by equation (27) as above and the nonemployment rate is given by

$$n = 1 - L + uL. \quad (41)$$

The nonemployed consist of the nonparticipants $1 - L$ and the share of the unemployed in the labor force uL .

5.2 Value equations

We continue to assume free entry of firms, firms may post high or low skill vacancies and cross skill matching takes place in the above described parameter range. The firms' value equations are thus the same as above, (3) and (4).

Next consider the asset value equations of the workers. As above, the low skilled workers may be matched with a low skill job and the high skilled workers may be matched either with a high skill job or the may be mismatched (matched with a low skill job). The value equations for being employed obey (7) for skills s and output (job type) y as in the basic setup. In addition workers without a job have the possibility of choosing not to participate to the labor market and producing through a home-production technology (or getting a social assistance payment) which is not proportional to their skill level s_i .

The asset-value equations for the low-skilled and high skilled unemployed workers are respectively

$$rU(s_1) = \max \{b + \theta q(\theta) \phi [W(s_1, s_1) - U(s_1)], rN\} \quad (42)$$

$$rU(s_2) = b + \theta q(\theta) \{ \phi [W(s_2, s_1) - U(s_2)] + (1 - \phi) [W(s_2, s_2) - U(s_2)] \} \quad (43)$$

The asset value equation for the low skilled unemployed workers involves the term rN which is the flow value of non-participation. The low skill unemployed workers maximize between participating to the labor market or leaving the labor force. The first alternative involves the value of being unemployed b plus the expected payoff of search whereas nonparticipation involves the flow value of home production rN . Because $rU(s_2) > rU(s_1)$ for all feasible parameter values, the participation choice concerns only the low skilled workers.¹⁵ Those that choose participation face the same skill investment choice as in the previous section, given by equation (26).

¹⁵We show in the appendix that $rU(s_2) > rU(s_1)$ for all parameter values.

5.3 Participation rule

In modeling the participation decision of workers we follow McKenna (1987), by assuming that workers make the labor market participation decision when entering the economy. We thus abstract from flows between participation and nonparticipation, as they complicate the analysis considerably in the current setup.

The opportunity cost of participation is forgone leisure or home production l , and we assume that individuals differ according to their valuation of l . The valuation of leisure is distributed according to $F(l)$ which is assumed to be continuous and stationary over time. There will be a reservation valuation l_0 for participation and a fraction $F(l_0)$ of workers have a value of leisure equal to or less than l_0 and participate to the market. The reservation rule for participation is given by

$$rN = l_0 = rU(s_1) \quad (44)$$

so we have

$$l_0 = \frac{(r + \delta)b + \beta\theta q(\theta)\phi s_1}{r + \delta + \beta\theta q(\theta)\phi}. \quad (45)$$

The reservation value of leisure l_0 is increasing in the labor market prospects for low skill workers (recall that the participation decision is relevant only to the low skill workers). Therefore l_0 increases in everything that increases the value of unemployment for low skilled workers $rU(s_1)$, so l_0 increases in labor market tightness θ , the share of low skill vacancies ϕ , low skill productivity s_1 and the unemployment compensation b .

As $P = 1$, the labor force participation rate is given by

$$L = F(l_0) = F\left(\frac{(r + \delta)b + \beta\theta q(\theta)\phi s_1}{r + \delta + \beta\theta q(\theta)\phi}\right). \quad (46)$$

5.4 Equilibrium with endogenous labor market participation

The equilibrium equations for the endogenous variables $n, L, l_0, u, \chi, p, \gamma, \phi$ and θ are

- Nonemployment

$$n = 1 - L + uL \quad (47)$$

- Labor force

$$L = F(l_0) \quad (48)$$

- Participation rule

$$l_0 = \frac{(r + \delta) b + \beta \theta q(\theta) \phi s_1}{r + \delta + \beta \theta q(\theta) \phi} \quad (49)$$

- Equilibrium unemployment rate

$$u = \frac{\delta(1-p)}{(1-\gamma)[\delta + \theta q(\theta)]} \quad (50)$$

- The share of low skilled workers in the population

$$\chi = 1 - L(1-p) \quad (51)$$

- Fraction of low skilled workers in the labor force

$$p = \frac{\gamma[\delta + \theta q(\theta)\phi]}{\delta + \theta q(\theta)[1 - \gamma(1 - \phi)]} \quad (52)$$

- Skill upgrading condition

$$\frac{1}{\gamma}(s_2 - s_1) - \frac{(r + \delta)(s_1 - b)}{(1 - \gamma)[r + \delta + \beta \theta q(\theta)]} = rK(s_2) \quad (53)$$

- Equal value condition

$$(1 - \gamma)(s_2 - s_1) = \gamma \frac{(r + \delta)(s_1 - b)}{r + \delta + \beta \theta q(\theta)\phi} \quad (54)$$

Total vacancy creation

$$s_1 - b = \frac{[r + \delta + \beta \theta q(\theta)]c}{(1 - \beta)q(\theta)} \quad (55)$$

The solution is recursive as above. Equation (55) provides a unique solution for θ , which can be substituted into (53) to obtain the solution for γ . Having θ and γ equation (54) solves for ϕ . With knowledge of θ, γ and ϕ we can solve p using (52). Using these equilibrium values we can solve (49) for l_0 , then (48) for L and finally (51) for χ and (47) for n .

5.5 Skill biased technological change

We now proceed to study the effects of skill biased technological change when in addition to an endogenous skill distribution the participation rate is endogenous. The comparative statics of the variables discussed in the benchmark model u, p, γ, ϕ and θ are the same as discussed above. One should bear in mind two features of the extended model. Now p represents the share of low skilled workers among the

participants L , not the total population P . Also remember that γ is the fraction of low skilled of the unemployed that participate uL , not among all the nonemployed $n = 1 - L(1 - u)$. In this section we thus focus on l_0, L, χ and n . The comparative static effects are summarized in table 2.

In the linear and convex skill cost cases (see $\alpha = 1$ to $\alpha = 4$ in Table 1), the labor market prospects of low skill workers deteriorate with skill biased technological change, leading to a decrease in the threshold participation level l_0 and a fall in the labor force participation rate L . Nonemployment n unambiguously increases in these two cases, reflecting the claim of Murphy and Topel (1997) that among the low skilled, deteriorating labor market prospects are reflected as both higher unemployment and higher nonparticipation rates. In this case the equilibrium unemployment rate understates the negative employment effect that takes place. In the ranges where the marginal cost of skills is not extremely high, the fraction of low skilled in the total working age population increases, as both the nonparticipation and the share of low skilled of the labor force participants increases.¹⁶

The threshold participation level l_0 increases with skill biased technological change when the marginal cost of skills is sufficiently low (see $\alpha = 0.5, \alpha = 0.1$ in Table 1). This is because more vacancy creation is allocated to low skill jobs, as discussed above. The reservation value of leisure l_0 is increasing in the labor market prospects for low skill workers, so more low skill vacancies raises l_0 and consequently the fraction of workers who participate in the labor force L increases. Nonparticipation falls.

The extended model with endogenous labor market participation supports the claim of Murphy and Topel (1997) that the unemployment rate is less indicative of the aggregate state of the labor market. Changes in the economic environment that affect equilibrium unemployment and the allocation of unemployment between high and low skilled labor also affect the labor market at the participation margin.

In the range where the marginal cost of skills is high, the participation rate falls simultaneously with the rise in the unemployment rate. The participation margin thus amplifies the negative employment effect, so the unemployment rate understates the effect on employment. On the other hand, when the marginal cost of skills is relatively low, the rise in the unemployment rate is accompanied by an increase in the participation rate. In this case the unemployment rate gives a qualitatively misleading picture of the employment effect of skill biased technological change. Summarizing, both the unemployment and participation rate should be considered when evaluating employment effects in the economy.

¹⁶When the marginal cost of skills is very high, the effect on χ turns negative, but this case may not feature in the cross-skill matching range.

	s_2	θ	ϕ	p	γ	u	l_0	L	χ	n
Initial equilibrium	1.20	2.252	0.868	0.667	0.695	0.063	0.860	0.720	0.760	0.326
Skill biased technological change High margin at cost										
$\alpha = 4$	1.25	2.252	0.733	0.651	0.713	0.071	0.840	0.679	0.763	0.369
Skill biased technological change Medium margin at cost										
$\alpha = 2$	1.25	2.252	0.823	0.695	0.732	0.066	0.854	0.707	0.784	0.339
$\alpha = 1$	1.25	2.252	0.857	0.710	0.738	0.064	0.858	0.717	0.792	0.329
$\alpha = 0.9$	1.25	2.252	0.860	0.711	0.739	0.064	0.859	0.718	0.793	0.328
Skill biased technological change Low margin at cost										
$\alpha = 0.5$	1.25	2.252	0.871	0.715	0.741	0.064	0.860	0.721	0.795	0.325
$\alpha = 0.1$	1.25	2.252	0.882	0.720	0.743	0.063	0.862	0.723	0.797	0.322

Table 2. Skill biased technological change and endogenous participation.

6 Concluding remarks

The shift in demand towards more highly educated labor has been a widespread phenomenon in OECD countries, but the labor market experiences wrt. the incidence of unemployment between high and low skilled workers have varied. Job competition and crowding out of low skilled workers by high skilled workers vary between countries. In addition to the unemployment consequences, participation rates reflect the consequences of skill biased technological change and job competition.

In this study, the costliness of acquiring additional skills plays a key role in determining labor market outcomes in response to skill biased technological change. The responses of workers to the shift in demand towards more educated labor depend on the prospects of the labor market relative to the marginal cost of attaining higher skills. In terms of policy, this could imply that by shaping the structure of the costs of education, one may influence labor market outcomes in desired ways.

When the model is extended to incorporate an endogenous labor market participation margin, it supports the claim of Murphy and Topel (1997) that the unemployment rate is less indicative of the aggregate state of the labor market. Changes in the economic environment that affect equilibrium unemployment and the allocation of unemployment between high and low skilled labor also affect the labor market at the participation margin. The equilibrium unemployment rate does not fully characterize the state of the labor market and therefore the focus should rather be on employment and joblessness, the latter including both the unemployed and nonparticipants. This applies also to policy measures intended to reduce unemploy-

ment. Focusing solely on the unemployment rate may thus be misleading for policy recommendations.

This study has been limited to study skill biased technological change in a model of two sided heterogeneity with cross skill matching. An obvious extension would be to consider ex post segmentation equilibria. Another perhaps interesting avenue for future research would be to consider education as a time consuming activity, as in eg. Becker (2005). This would incorporate an additional labor market state into the model, but it could provide interesting aspects of education choice decisions of workers.

A Appendix

A.1 Derivation of unemployment values

The flow value of unemployment for a low skilled worker is

$$rU(s_1) = b + \theta q(\theta) \phi [W(s_1, s_1) - U(s_1)].$$

Substitute $W(s_1, s_1) = \frac{w(s_1, s_1) + \delta U(s_1)}{r + \delta}$ and $w(s_1, s_1) = \beta s_1 + (1 - \beta) rU(s_1)$ to get

$$rU(s_1) = b + \theta q(\theta) \phi \left[\frac{\beta s_1 + (1 - \beta) rU(s_1) + \delta U(s_1) - (r + \delta) U(s_1)}{r + \delta} \right]$$

cancel terms and rearrange to obtain

$$(r + \delta) rU(s_1) = (r + \delta) b + \beta \theta q(\theta) \phi [s_1 - rU(s_1)]$$

and thus

$$rU(s_1) = \frac{(r + \delta) b + \beta \theta q(\theta) \phi s_1}{r + \delta + \beta \theta q(\theta) \phi} \quad (56)$$

The flow value of unemployment for a high skilled worker is derived in an analogous manner and produces

$$rU(s_2) = \frac{(r + \delta) b + \beta \theta q(\theta) [\phi s_1 + (1 - \phi) s_2]}{r + \delta + \beta \theta q(\theta)} \quad (57)$$

A.2 Derivation of equilibrium equations

In the derivation of the equilibrium equations we use an approach that is analogous to Albrecht and Vroman (2002). Instead of using (18),(19) and (26), we use equivalent

conditions $V(s_1) = V(s_2)$, $V(s_2) = 0$ and an alternative expression for (26). The first condition is derived by subtracting (18) from (19)

$$(1 - \gamma)(s_2 - s_1) = \gamma[s_1 - rU(s_1)] \quad (58)$$

and substituting (20) to get the 'equal value' condition

$$(1 - \gamma)(s_2 - s_1) = \gamma \frac{(r + \delta)(s_1 - b)}{r + \delta + \beta\theta q(\theta)\phi}. \quad (59)$$

Multiply this expression by $r + \delta + \beta\theta q(\theta)\phi$ and add $(1 - \gamma)(r + \delta)(s_1 - b)$ to both sides to obtain

$$(1 - \gamma)[(r + \delta)(s_2 - b) + \beta\theta q(\theta)\phi(s_2 - s_1)] = (r + \delta)(s_1 - b) \quad (60)$$

which will prove useful in what follows.

To derive the second equilibrium condition, substitute (21) into (19) to get

$$(1 - \gamma) \frac{(r + \delta)(s_2 - b) + \beta\theta q(\theta)\phi(s_2 - s_1)}{r + \delta + \beta\theta q(\theta)} = \frac{(r + \delta)c}{(1 - \beta)q(\theta)}. \quad (61)$$

Substitute (60) to the left hand side, cancel and rearrange to obtain

$$s_1 - b = \frac{[r + \delta + \beta\theta q(\theta)]c}{(1 - \beta)q(\theta)}. \quad (62)$$

Due to the properties of the matching function this equation produces a unique solution for θ .

The third equilibrium condition is derived by adding and subtracting $s_2 - s_1$ on the left hand side (25) to get

$$s_2 - s_1 - [s_2 - rU(s_2)] + [s_1 - rU(s_1)] = rK(s_2). \quad (63)$$

Use (58) to substitute the last left hand side term, substitute the numerator of the second term using (60) and combine terms to obtain

$$\frac{1}{\gamma}(s_2 - s_1) - \frac{(r + \delta)(s_1 - b)}{(1 - \gamma)[r + \delta + \beta\theta q(\theta)]} = rK(s_2) \quad (64)$$

A.3 Skill biased technological change

A.3.1 Low skill unemployment

For the comparative static effects of s_2 recall from (31) that labor market tightness is not affected. To derive the comparative statics for γ and ϕ we use equation (30) and (29). By the implicit function rule we obtain

$$\frac{d\gamma}{ds_2} = \frac{\frac{1}{\gamma} - rK'(\tilde{s})}{\frac{1}{\gamma^2}(s_2 - s_1) + \frac{(r + \delta)(s_1 - b)}{(1 - \gamma)^2[r + \delta + \beta\theta q(\theta)]}}$$

and substitution of (29) into the denominator gives

$$\frac{d\gamma}{ds_2} = \frac{(1-\gamma) \left[\frac{1}{\gamma} - rK'(\tilde{s}) \right]}{\frac{1}{\gamma^2} (s_2 - s_1) - rK(\tilde{s})}. \quad (65)$$

Multiply and divide by $\frac{1}{\gamma} (s_2 - s_1)$ to get

$$\frac{d\gamma}{ds_2} = \frac{\gamma(1-\gamma) \left[\frac{1}{\gamma} (s_2 - s_1) - \alpha rK(\tilde{s}) \right]}{(s_2 - s_1) \left[\frac{1}{\gamma} (s_2 - s_1) - \gamma rK(\tilde{s}) \right]}. \quad (66)$$

As the denominator and $\gamma(1-\gamma)$ is positive we have

$$\text{sign} \left(\frac{d\gamma}{ds_2} \right) = \text{sign} \left[\frac{1}{\gamma} (s_2 - s_1) - \alpha rK(\tilde{s}) \right] \quad (67)$$

Then substitute from (29) for $rK(\tilde{s})$ to obtain

$$\text{sign} \left(\frac{d\gamma}{ds_2} \right) = \text{sign} \left\{ (1-\alpha) \frac{1}{\gamma} (s_2 - s_1) + \alpha \frac{(r+\delta)(s_1-b)}{(1-\gamma)[r+\delta+\beta\theta q(\theta)]} \right\}. \quad (68)$$

For the linear cost function $\alpha = 1$ so

$$\text{sign} \left(\frac{d\gamma}{ds_2} \right) = \text{sign} \left[\frac{(r+\delta)(s_1-b)}{(1-\gamma)[r+\delta+\beta\theta q(\theta)]} \right] > 0. \quad (69)$$

For the general case use (30) to substitute $\frac{1}{\gamma} (s_2 - s_1)$ to get

$$\text{sign} \left(\frac{d\gamma}{ds_2} \right) = \text{sign} \left\{ (1-\alpha) \frac{(r+\delta)(s_1-b)}{(1-\gamma)[r+\delta+\beta\theta q(\theta)\phi]} + \alpha \frac{(r+\delta)(s_1-b)}{(1-\gamma)[r+\delta+\beta\theta q(\theta)]} \right\}. \quad (70)$$

A.3.2 Vacancy allocation

From (30) we obtain

$$\frac{d\phi}{ds_2} = \frac{\left[s_2 - s_1 + \frac{(r+\delta)(s_1-b)}{r+\delta+\beta\theta q(\theta)\phi} \right] \frac{d\gamma}{ds_2} - (1-\gamma)}{\gamma \frac{(r+\delta)(s_1-b)\beta\theta q(\theta)}{[r+\delta+\beta\theta q(\theta)\phi]^2}}.$$

From (30) we have

$$\frac{1-\gamma}{\gamma} (s_2 - s_1) = \frac{(r+\delta)(s_1-b)}{r+\delta+\beta\theta q(\theta)\phi}$$

so we obtain

$$\frac{d\phi}{ds_2} = \frac{\frac{1}{\gamma} (s_2 - s_1) \frac{d\gamma}{ds_2} - (1-\gamma)}{(1-\gamma) \frac{\beta\theta q(\theta)(s_2-s_1)}{r+\delta+\beta\theta q(\theta)\phi}}. \quad (71)$$

Substitute (66) to get

$$\frac{d\phi}{ds_2} = \frac{(1-\gamma) \left[\frac{\frac{1}{\gamma}(s_2-s_1) - \alpha r K(\tilde{s})}{\left[\frac{1}{\gamma}(s_2-s_1) - \gamma r K(\tilde{s}) \right]} \right] - (1-\gamma)}{(1-\gamma) \frac{\beta \theta q(\theta)(s_2-s_1)}{r+\delta+\beta \theta q(\theta)\phi}}. \quad (72)$$

Cancel and rearrange terms to get

$$\frac{d\phi}{ds_2} = \frac{(\gamma-\alpha) r K(\tilde{s})}{\left[\frac{1}{\gamma}(s_2-s_1) - \gamma r K(\tilde{s}) \right] \frac{\beta \theta q(\theta)(s_2-s_1)}{r+\delta+\beta \theta q(\theta)\phi}} \quad (73)$$

or

$$\frac{d\phi}{ds_2} = \frac{(\gamma-\alpha) r K(\tilde{s})}{\left[\frac{1}{\gamma}(s_2-s_1) - \gamma r K(\tilde{s}) \right] \frac{1-\gamma}{\gamma} \frac{\beta \theta q(\theta)(s_2-s_1)^2}{(r+\delta)(s_1-b)}}. \quad (74)$$

A.3.3 Share of low skill workers

The share of low skill workers is given by

$$p = \frac{\gamma [\delta + \theta q(\theta) \phi]}{\delta + \theta q(\theta) [1 - \gamma (1 - \phi)]}. \quad (75)$$

As θ is unaffected by s_2 , the effect of skill biased technological change is given by

$$\frac{dp}{ds_2} = \frac{\partial p}{\partial \gamma} \frac{d\gamma}{ds_2} + \frac{\partial p}{\partial \phi} \frac{d\phi}{ds_2} \quad (76)$$

which produces

$$\frac{dp}{ds_2} = \frac{[\delta + \theta q(\theta) \phi] [\delta + \theta q(\theta)]}{\{\delta + \theta q(\theta) [1 - \gamma (1 - \phi)]\}^2} \frac{d\gamma}{ds_2} + \frac{\gamma (1 - \gamma) \theta q(\theta) [\delta + \theta q(\theta)]}{\{\delta + \theta q(\theta) [1 - \gamma (1 - \phi)]\}^2} \frac{d\phi}{ds_2}. \quad (77)$$

This expression reduces to

$$\text{sign} \left(\frac{dp}{ds_2} \right) = \text{sign} \left[\frac{1}{\gamma (1 - \gamma)} \frac{d\gamma}{ds_2} + \frac{\theta q(\theta)}{\delta + \theta q(\theta) \phi} \frac{d\phi}{ds_2} \right] \quad (78)$$

or

$$\text{sign} \left(\frac{dp}{ds_2} \right) = \text{sign} \left[\frac{p}{\gamma} \frac{d\gamma}{ds_2} + \frac{\gamma (1 - \gamma) \theta q(\theta)}{\delta + \theta q(\theta) [1 - \gamma (1 - \phi)]} \frac{d\phi}{ds_2} \right] \quad (79)$$

A.3.4 Equilibrium unemployment

To derive the effects of skill biased technological change on u we first use (28) to derive

$$1 - p = \frac{(1-\gamma) [\delta + \theta q(\theta)]}{\delta + \theta q(\theta) [1 - \gamma (1 - \phi)]} \quad (80)$$

which we substitute into (27) and cancel terms to obtain

$$u = \frac{\delta}{\delta + \theta q(\theta) [1 - \gamma (1 - \phi)]} \quad (81)$$

which gives the equilibrium unemployment rate as a function of γ and ϕ .

$$\frac{du}{ds_2} = \frac{\partial u}{\partial \gamma} \frac{d\gamma}{ds_2} + \frac{\partial u}{\partial \phi} \frac{d\phi}{ds_2} \quad (82)$$

which reduces to

$$\frac{du}{ds_2} = \frac{\delta \theta q(\theta) \left[(1 - \phi) \frac{d\gamma}{ds_2} - \gamma \frac{d\phi}{ds_2} \right]}{\{\delta + \theta q(\theta) [1 - \gamma(1 - \phi)]\}^2} \quad (83)$$

$$\frac{du}{ds_2} = u \frac{\theta q(\theta) \left[(1 - \phi) \frac{d\gamma}{ds_2} - \gamma \frac{d\phi}{ds_2} \right]}{\delta + \theta q(\theta) [1 - \gamma(1 - \phi)]} \quad (84)$$

It is immediate that

$$\text{sign} \left(\frac{du}{ds_2} \right) = \text{sign} \left[\frac{1}{\gamma} \frac{d\gamma}{ds_2} - \frac{1}{1 - \phi} \frac{d\phi}{ds_2} \right]. \quad (85)$$

A.4 Unemployment values: $rU(s_2) > rU(s_1)$

We can formally show that $rU(s_2) > rU(s_1)$ for all parameter values of the model.

We have

$$\begin{aligned} rU(s_2) &> rU(s_1) \\ \frac{(r + \delta)b + \beta \theta q(\theta) [\phi s_1 + (1 - \phi)s_2]}{r + \delta + \beta \theta q(\theta)} &> \frac{(r + \delta)b + \beta \theta q(\theta) \phi s_1}{r + \delta + \beta \theta q(\theta) \phi} \end{aligned}$$

Multiply both sides by $r + \delta + \beta \theta q(\theta)$ and rearrange to get

$$(1 - \phi) \beta \theta q(\theta) s_2 > [(r + \delta)b + \beta \theta q(\theta) \phi s_1] \left[\frac{r + \delta + \beta \theta q(\theta)}{r + \delta + \beta \theta q(\theta) \phi} - 1 \right]$$

or

$$(1 - \phi) \beta \theta q(\theta) s_2 > [(r + \delta)b + \beta \theta q(\theta) \phi s_1] \left[\frac{\beta \theta q(\theta) (1 - \phi)}{r + \delta + \beta \theta q(\theta) \phi} \right]$$

Cancel $(1 - \phi) \beta \theta q(\theta)$ from both sides, multiply and divide the left hand side by $[r + \delta + \beta \theta q(\theta) \phi]$ and rearrange to get

$$\frac{(r + \delta)(s_2 - b) + \beta \theta q(\theta) \phi (s_2 - s_1)}{r + \delta + \beta \theta q(\theta) \phi} > 0.$$

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Chapter 4

Labor Taxation, Equilibrium Unemployment and Macroeconomic Dynamics

Abstract

This paper studies the effects of labor taxes and labor tax reform on equilibrium labor market outcomes and macroeconomic dynamics in a New Keynesian model with matching frictions. Three policy instruments are considered: a marginal tax rate and a tax subsidy to produce tax progression schemes, and a replacement ratio to account for variability in outside options. In equilibrium, the marginal tax rate and replacement ratio dampen economic activity whereas tax subsidies boost the economy. The marginal tax rate and replacement ratio amplify shock responses whereas employment subsidies weaken them. The tax instruments affect the degree to which the wage absorbs shocks. We show that the relative effects of the tax instruments and thus the effects of tax progression are sensitive to the initial degree of tax progression in the economy. Increasing tax progression when taxation is initially progressive is harmful for steady state employment and output, and amplifies the sensitivity of macroeconomic variables to shocks. When taxation is initially proportional, increasing progression is beneficial for output and employment and dampens shock responses of macroeconomic variables.

1 Introduction

The relatively poor performance of European labor markets is frequently blamed on high and distorting labor taxes as well as generous unemployment compensation schemes. Consequently, the restructuring of employment taxes and other labor market institutions feature prominently on the European policy agenda. According to the European Commission Guidelines for Employment Policies of the

Member States (2005) policies should: *"Ensure employment-friendly labour cost developments and wage-setting mechanisms by... reviewing the impact on employment of non-wage labour costs and where appropriate adjust their structure and level" ... "Ensure that wage developments contribute to macroeconomic stability and growth"*. Two concerns are immediate in the policy guidelines. First, wage determination and non-wage labor costs are pointed out as key determinants of equilibrium employment. Second, nominal wages and labour cost developments should be consistent with price stability and the trend in productivity over the medium term. Accordingly, labor market policies should reinforce the macroeconomic framework by increasing flexibility and adjustment capacity in response to e.g. technological advances and cyclical changes. Although extensive academic research effort has been devoted to labor taxation and compensation schemes, policy conclusions do not seem conclusive.

This paper investigates the role of income taxation and unemployment compensation schemes in an economy from a dual perspective. First, we study the equilibrium labor market effects of the structure of taxes and subsidies. In particular we are interested of the effects of changing the degree of tax progression on equilibrium labor market variables and output. Second, we investigate whether changes in tax structure and compensation schemes matter for the sensitivity of the economy wrt. exogenous shocks. Because changes in taxation alter the steady state of the economy, the new equilibrium levels and especially relative values of the variables generate shock responses different to those in the initial equilibrium. For the purpose of analyzing the cyclical behavior of a relatively large set of macroeconomic variables, we embed a search-matching model of the labor market *à la* Mortensen-Pissarides (e.g. Pissarides 2000, Mortensen and Pissarides 1999a) into a New Keynesian monetary model (e.g. Woodford 2003). This extension also allows the study of e.g. interest rate shocks.¹ The present model incorporates three labor market policy instruments: a marginal income tax, a tax subsidy for employed workers and a replacement ratio for unemployed workers. The marginal income tax and employment subsidy jointly determine the degree of progression in income taxation and the replacement ratio determines the income when unemployed.

The present study contributes to the all but conclusive theoretical literature on tax structure and labor market outcomes. Koskela and Vilmunen show that under plausible assumptions increased tax progression lowers wages and unemployment in three trade union models of the labor market; the monopoly union, the 'right-to-

¹In the literature the cyclical properties of the Mortensen-Pissarides type model is often limited to the analysis of productivity shocks, and may thereby miss important mechanisms of the economy.

manage' and the efficient bargain model. They conclude that the effects of taxation appear to be very sensitive to the structure of labor markets. Indeed, Pissarides (1998) studies the effects of employment tax cuts on unemployment and wages in four different equilibrium models of the labor market: competitive, union bargaining, search and efficiency wages. He points out that there is no definitive model of the European labor market and shows that effects of changes in the structure and level of taxation sometimes depends on the underlying model of the labor market. He finds that when wages are determined by bargaining, a revenue neutral increase in tax progression reduces unemployment in steady state. In a more general setting with endogenous job destruction Sinko (2005) obtains qualitatively similar results. Mortensen and Pissarides (2003) consider various tax and subsidy effects on wages and unemployment. They study policies that drive the labor market closer to 'efficiency' in terms of search frictions but they do not explicitly address tax progression schemes. Their calibrations show that the tightness to which the labor market is calibrated matters for the steady state outcomes. The interaction between shocks and institutions in a matching model is studied in Mortensen and Pissarides (1999b), but their focus is in unemployment compensation and employment protection policies.

In this paper we study labor tax reforms when taxation is initially proportional and when it is initially progressive. Our analysis shows that when income taxation is initially proportional as in Pissarides (1998) and Sinko (2005), a tax revenue neutral increase in tax progression boosts employment and output as in these earlier studies. But when income taxation is initially (sufficiently) progressive, the opposite is true: further progression is detrimental for employment and output. This new result contrasts the earlier findings of Pissarides (1998) and Sinko (2005). When implementing tax revenue neutral changes in tax progression, the relative weights of changes in the dampening effect of the marginal tax and the stimulating effect of the tax subsidy depend on the initial degree of tax progression. In an initially proportional tax system, the stimulating effect of the tax subsidy dominates the dampening effect of the marginal tax. In an initially (sufficiently) progressive tax system the opposite takes place.

We then proceed to comparing the dynamic behavior of pre and post tax reform economies in a full scale macro model. A recent body of literature has explored the role of real rigidities of the labor market in business cycle models by combining the search-matching framework of the labor market to real business cycle models (Merz 1995, Andolfatto 1996, den Haan et al. 2000) and the New Keynesian monetary model (Walsh 2003, 2005, Trigari 2004, Krause and Lubik 2005). These studies have been successful in improving the performance of business cycle models in generating shock persistence in macroeconomic variables observed in the data. A key

feature of these models is that they introduce employment adjustment in business cycle models through changes in the number of employed workers (the extensive margin) instead of in the number of hours (the intensive margin). This, combined with search frictions of the labor market, generates involuntary unemployment and sluggish employment adjustment into the business cycle models. The rigidity in the adjustment of labor has proved to be of essence in generating persistence into the business cycle models.

We show that taxes alter the rigidity of labor market adjustment and thus changes in individual tax policy instruments produce well-defined effects on the dynamics of the economy. Higher marginal tax rates and replacement ratios amplify shock responses both in terms of peak effects and persistence whereas higher tax subsidies dampen the impulse responses. These clear cut results abstract from any tax revenue questions, so we proceed to study the effects of tax revenue neutral changes in tax progression with alternative assumptions on the initial tax scheme of the economy. We show that the effects of tax revenue neutral changes in tax progression depend crucially on the initial degree of tax progression in the labor market. When taxation is progressive in the initial state, the effect of the marginal tax on labor market variables dominates the tax subsidy effect. This implies that the dynamic responses to exogenous shocks are amplified by tax progression. When taxation is initially proportional, increasing progression dampens shock responses of macroeconomic variables. This is because the relative strengths of the two tax effects are reversed when the tax subsidy is sufficiently small. Thus we find that a government tax revenue neutral change in tax progression has opposite effects on the steady state and shock responses depending on the degree of tax progression in the initial steady state.

A key issue in both the steady-state equilibrium analysis and the dynamic impulse response analysis is the rigidity of the wage implied by the labor market policy schemes wrt. to exogenous changes in the economy. Progressive taxation essentially renders the wage less sensitive to exogenous shocks, thus shifting adjustments in the labor market to other variables. This outcome relates to an active area of current research. A critique of the Mortensen-Pissarides matching model that has been strongly advocated by Shimer (2005) is that it does not produce sufficient fluctuations in vacancies and unemployment because the negotiated wage absorbs too much of the shocks of an economy. This critique has led to an active debate in the literature about the modeling of the wage in the matching model to achieve more rigid wages (see e.g. Shimer 2005, Hall 2005, Mortensen and Nagypál 2005, Hornstein et al. 2005). The solution of Hall (2005) is a wage that is partly determined by negotiation and partly by a wage norm. This solution has been adopted in a

number of studies, but the wage norm still remains *ad hoc*. In business cycle models wage rigidity has been incorporated along with matching frictions to the New Keynesian model in e.g. Krause and Lubik (2005). They demonstrate that the model performance improves further, when wages respond less to shocks. Christoffel and Linzert (2005) and Christoffel et al. (2005) come to similar conclusions. We show that tax progression *de facto* makes the wage more rigid, producing a qualitatively similar result to the wage norm solution. Our results are quantitatively less strong, but no *ad hoc* assumptions on the foundations of wage rigidity are needed.

The structure of this study is as follows. In section 2 we construct a New Keynesian model which incorporates matching frictions of the labor market and the tax policy instruments. Section 3 characterizes and solves the steady state of the model and presents the linearized system of equations. The model calibration is discussed in section 4. In section 5 we first analyze the effects of labor market policy on the steady state of the model at some length, as this reveals intuition and the mechanisms that drive the dynamics of the model. Thereafter we consider the dynamic responses to shocks for various tax policy regimes. Section 6 summarizes.

2 Model

The model economy follows the structure of Trigari (2004) and Walsh (2003, 2005) by incorporating a Mortensen-Pissarides type of labor market with matching frictions into a New Keynesian monetary model. The full-scale macro model allows us to extend the dynamic analysis to a larger range of macroeconomic variables, rather than labor market variables only. The two main driving forces of the model's dynamics are nominal rigidities in price setting and matching frictions. A characteristic feature of the model is the separation of firms into two types, each type taking account of one type of rigidity. This separation is made to separate the nominal rigidities from the real rigidities, thus making the model more tractable. The economy consists of the following:

Households– Households supply labor, purchase goods for consumption and hold bonds. Labor is supplied at the extensive margin, so adjustment in the labor market takes place through additional employed workers rather than varying the hours of work. We consider the households as extended families who pool consumption. This assumption is conventional and is made to avoid distributional issues. Households own the firms in the economy.

Firms– There are two types of profit maximizing firms: wholesale and retail firms. Production takes place in the wholesale firms who use labor as the sole

factor of production. Matching workers and wholesale firms is a time consuming and costly process which generates real rigidity into the economy. Wholesale firms sell all their output to the retail firms at a competitive price. Retail firms transform the intermediate goods purchased from the wholesale firms into differentiated final goods and sell them in a monopolistically competitive market with staggered pricing which generates the nominal rigidity of the model.

Government– The government raises tax revenue by levying an income tax from employed workers. The tax revenue is used to finance unemployment benefits, tax subsidies paid to workers and exogenous government expenditures.

Central bank– The central bank controls the nominal interest rate according to an exogenous policy rule.

2.1 Households

There is a continuum of households on the unit interval in a discrete-time economy. The representative household maximizes the expected present discounted utility

$$E_t \sum_{i=0}^{\infty} \beta^i u(\mathcal{C}_{t+i}, \mathcal{C}_{t+i-1}) \quad (1)$$

where $\mathcal{C}_t = C_t + \psi h$, and C_t is the consumption of a market purchased composite good which will be defined below. The composite good consists of the differentiated goods produced by the retail firms. h is nontradable home production and ψ is an indicator function taking the value of zero when an individual is employed and one otherwise. The utility function allows for habit persistence. As monetary policy is represented by an interest rate rule and our focus is not on the stock of money, we consider a limit economy where the weight of the utility of the household's holdings of real money balances approaches zero in the utility function.

The households budget constraint is

$$P_t D_t + (1 + i_{t-1}) B_{t-1} = P_t C_t + B_t \quad (2)$$

where D_t is the family income which consists of wage income, unemployment income and family share of firms profits. B_t is the household's nominal holdings of bonds and P_t is the retail price index. Using (1) and (2) we can derive the first-order condition

$$\lambda_t = \beta (1 + i_t) E_t \left(\frac{P_t}{P_{t+1}} \right) \lambda_{t+1} \quad (3)$$

which is the household's Euler condition, where

$$\lambda_t \equiv u_1(\mathcal{C}_t, \mathcal{C}_{t-1}) + \beta E_t u_2(\mathcal{C}_{t+1}, \mathcal{C}_t).$$

2.2 Wholesale firms and labor market search

Production takes place in the wholesale (intermediate product) firms. Labor is the sole input in production and the matching of workers and firms involves search frictions. A job-worker match generates a surplus, which is the sum of the gain for a firm and worker of being matched relative to not being matched. The surplus is divided by Nash bargaining as is common in the literature. Consequently the wage in the intermediate sector does not equal the marginal productivity of a worker, as it would in a Walrasian setup. In addition to match productivity, the wage depends on the value of being idle for the firm and worker and the ease with which each side can find an alternative match. Unemployed workers receive an unemployment benefit and enjoy a value of nontradable home production (or leisure). The match surplus and labor market tightness influence the wage rate and govern job creation and destruction.

2.2.1 Match productivity and job flows

To keep the model simple we assume that labor is the only input in the production of intermediate goods. Match productivity is given by

$$y_{it} = a_{it}z_t$$

where a_{it} is match specific productivity and z_t is a common aggregate productivity measure. Each period a_{it} is drawn from a time-invariant distribution with c.d.f. $F(a)$ and density $f(a)$. Denote the price at which wholesale firms sell output to monopolistically competitive retail firms by P_t^w , the retail price index is P_t and $\mu_t = \frac{P_t}{P_t^w}$ is the markup of retail over wholesale prices. The real value of output in terms of time t consumption is $\mu_t^{-1}a_{it}z_t$.

Production takes place once a firm and worker are matched. Matching of firms and workers in the intermediate sector is characterized by a constant returns to scale matching function

$$m(u_t, v_t) = Au_t^\alpha v_t^{1-\alpha}$$

where u_t and v_t are unemployed workers and open vacancies at time t respectively, $0 < \alpha < 1$ and $A > 0$ is a shift parameter.² The hazard rates for a firm of meeting a worker and a worker of meeting a firm are respectively

$$q_t^f = \frac{m(u_t, v_t)}{v_t} = A\theta_t^{-\alpha} \quad (4)$$

$$q_t^w = \frac{m(u_t, v_t)}{u_t} = A\theta_t^{1-\alpha} \quad (5)$$

²The Cobb-Douglas matching function is supported by a number of empirical studies. For a survey on the matching function see Petrongolo and Pissarides (2001).

where $\theta_t = \frac{v_t}{u_t}$ is labor market tightness. The tighter the labor market, the easier it is for the worker to find a firm and harder for a firm to find a worker. Thus q_t^f is decreasing and q_t^w is increasing in θ_t .

Jobs are destroyed due to exogenous shocks and endogenous separation decisions of firms and workers. Exogenous shocks arrive at rate ρ^x at the beginning of each period. For the matches that survive, the firm and worker jointly observe the realization of match productivity and decide whether to continue or destroy the match. Jobs with a productivity realization that is below an endogenous reservation productivity \tilde{a}_t are destroyed. Endogenous job destruction is then

$$\rho_t^n = \Pr[a_t \leq \tilde{a}_t] = F(\tilde{a}_t) \quad (6)$$

and the aggregate separation rate is

$$\rho_t = \rho^x + (1 - \rho^x) \rho_t^n. \quad (7)$$

With job creation and destruction characterized as above, the number of matches that enter period $t + 1$ is

$$n_{t+1} = (1 - \rho_t) n_t + m(u_t, v_t) \quad (8)$$

where n_t is period t employment. The measure of searching workers is

$$u_t = 1 - n_t + \rho_t n_t = 1 - (1 - \rho_t) n_t. \quad (9)$$

The number of searching workers in period t differs from the number of unemployed workers, $1 - n_t$, in the beginning of period t as some of the employed workers separate from their matches and start searching for a new job within the same period.

Furthermore, we determine the net job creation and destruction rates. Each period $q_t^f v_t$ vacancies are filled. A fraction ρ^x of the new and previously existing matches are destroyed exogenously immediately at the beginning of the period. The rate of turnover is then $q_t^f \rho^x n_t$ and the net job creation rate can be expressed as

$$jc_t = \frac{q_t^f v_t}{n_t} - q_t^f \rho^x. \quad (10)$$

The net job destruction rate is

$$jd_t = \rho_t - q_t^f \rho^x \quad (11)$$

where ρ_t is the aggregate job destruction rate and $q_t^f \rho^x$ are the exogenously destroyed matches that rematch within the same period.

2.2.2 Match surplus and value functions

Match surplus is a key object in determining job creation and destruction. The surplus is the difference of the values of being matched and the outside values and is given by

$$S_t(a_{it}) = J_t(a_{it}) + W_t(a_{it}) - V_t - U_t \quad (12)$$

where $J_t(a_{it})$ and $W_t(a_{it})$ are the values for a firm and worker of being matched and V_t and U_t are the values of idleness for the worker and firm, that is having an open vacancy for the firm and being unemployed for the worker.

Firm The value for a firm of a filled job $J_t(a_{it})$ and a vacancy V_t are given by

$$J_t(a_{it}) = \frac{a_{it}z_t}{\mu_t} - w_t(a_{it}z_t) + \max E_t \beta_{t+1} \left[(1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}) + \rho^x V_{t+1} \right] \quad (13)$$

$$V_t = -\kappa + E_t \beta_{t+1} \left[q_t^f (1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}) + (1 - q_t^f) V_{t+1} \right] \quad (14)$$

The value of a filled job is determined by the real value of match output $\frac{a_{it}z_t}{\mu_t}$ (in terms of time t consumption goods) minus the wage $w_t(a_{it}z_t)$ paid to the worker, and the expected future value of the job, which is discounted according to the discount factor $\beta_{t+i} = \frac{\beta^i \lambda_{t+i}}{\lambda_t}$. The wage paid by the firm includes income taxes paid by the worker to the government. The expected value of the job takes into account that the job may be destroyed due to an exogenous shock with probability ρ^x and that jobs with a productivity realization $a_{it+1} < \tilde{a}_{t+1}$ will be destroyed endogenously.

The value of having an open vacancy involves a periodical cost κ of having an open vacancy and the expected surplus of a filled job. The latter depends on the endogenous rate q_t^f of finding an appropriate worker, and that the job is not destroyed due to an exogenous shock or endogenously due to a low realization of match specific productivity. There is free-entry of firms to the market so firms enter until $V_t = 0$. Substituting this into (14) produces the job creation condition

$$\frac{\kappa}{q_t^f} = (1 - \rho^x) E_t \beta_{t+1} \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}). \quad (15)$$

The job creation equation states that the expected surplus for the firm must equal the cost of posting a vacancy. The right hand side of the equation gives the expected surplus that accrues to the firm from a filled job. The left hand side is the

expected cost of filling the vacancy, where q_t^f is the probability of the firm finding a worker so $\frac{1}{q_t^f}$ is the expected duration of search.

Worker The values for the worker of employment $W_t(a_{it})$ and unemployment U_t are respectively

$$W_t(a_{it}) = w_t(a_{it}z_t) - T(w_t(a_{it}z_t)) + E_t\beta_{t+1} \left[(1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} W_{t+1}(a_{it+1}) dF(a_{it+1}) + \rho^x U_{t+1} \right] \quad (16)$$

$$U_t = h + b_t(H(\tilde{a}_t)z_t) + E_t\beta_{t+1} \left[q_t^w (1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} W_{t+1}(a_{it+1}) dF(a_{it+1}) + (1 - q_t^w (1 - \rho^x)) U_{t+1} \right] \quad (17)$$

An employed worker earns a wage of $w_t(a_{it}z_t)$ and makes a transfer $T(w_t(a_{it}z_t))$ to the tax authorities which will be discussed in more detail below. The expected value of employment depends on the probability of not being destroyed by an exogenous shock and that the match specific productivity realization satisfies $a_{it+1} \geq \tilde{a}_t$. In the case of destruction the worker enjoys the value of unemployment U_{t+1} . An unemployed worker enjoys the value of leisure (or home production) h and an unemployment compensation $b_t(w_t^e(H(\tilde{a}_t)z_t))$, which will be defined below. The probabilities and values of being employed or unemployed next period affect the value of unemployment in the current period.

2.2.3 Employment taxes and unemployment income

From the variety of possible tax policy schemes we will focus on income taxation and unemployment benefits.³ Taxes on labor income and unemployment earnings are modeled in a simple manner by using three policy instruments: a marginal tax on total labor earnings, a tax subsidy for employed workers and unemployment compensation. We assume that wage taxes are linear and smooth functions of income. In our benchmark case employed workers receive a tax subsidy v and are subsequently taxed for their total earnings, the subsidy included, at proportional rate τ (*s.t.* $0 \leq \tau \leq 1$).⁴ The net income of a worker with match specific productivity a_{it} is then $(1 - \tau)[w_{it}(a_{it}z_t) + v]$, where $w_{it}(a_{it}z_t)$ is the wage of a worker with

³We abstract from other policy aspects such as employment protection or promotion through firing costs and hiring subsidies respectively, or the role of payroll taxes.

⁴The benchmark labor market policy setup follows Pissarides (2000), but in the analysis that follows we will consider departures from these assumptions.

match-specific productivity a_{it} . The transfer from the worker to the tax authorities is

$$T_{it}(w_{it}(a_{it}z_t)) = \tau w_{it}(a_{it}z_t) - (1 - \tau)v \quad (18)$$

The marginal tax rate is $T'(w_{it}(a_{it}z_t)) = \tau$. A marginal increase in the wage of a worker increases the tax transfer by τ . When the tax subsidy v is positive, taxation is progressive s.t. the average tax rate increases with the wage. When $v = 0$ taxation is proportional.

Unemployment compensation is modeled to be a policy determined replacement ratio of net income.⁵ As there is a distribution of wages, one possibility would be to set the unemployment compensation proportional to the average net wage. To simplify the model we use instead average productivity and assume that the unemployment compensation is proportional to the sum of the average productivity and the tax subsidy. The unemployment compensation is then

$$b_t = \rho^\tau (1 - \tau) (H(\tilde{a}_t) z_t + v). \quad (19)$$

where ρ^τ is the replacement rate and $H(\tilde{a}_t)$ is the conditional expectation $E[a | a \geq \tilde{a}_t]$. This setup effectively implies that the unemployment benefit is subject to the marginal tax rate.⁶

2.2.4 Bargaining and the wage

The wage is determined by Nash bargaining and satisfies⁷

$$w_t(a_{it}z_t) = \arg \max [J_t(a_{it}) - V_t]^{1-\eta} [W_t(a_{it}) - U_t]^\eta \quad (22)$$

⁵This is not the case in all European countries.

⁶An alternative way to model taxes and the unemployment compensation scheme would be to follow e.g. Pissarides (1998) by assuming that the net income of a worker with match specific productivity a_{it} is $(1 - \tau)w_{it}(a_{it}z_t) + v$ i.e. the tax subsidy is not subject to the marginal tax (in the benchmark case we assumed that employed workers receive a tax subsidy v and are subsequently taxed for their total earnings, the subsidy included). The transfer from the worker to the tax authorities in the alternative setup is then

$$T_{it}(w_{it}(a_{it}z_t)) = \tau w_{it}(a_{it}z_t) - v. \quad (20)$$

Unemployment compensation can be assumed to be either fixed or proportional to the average productivity (without the tax subsidy)

$$b_t = \rho^\tau H(\tilde{a}_t) z_t. \quad (21)$$

The results presented above are qualitatively unambiguous and general and are not sensitive to the calibration of the model or specific policy setup. However, the particular policy setup does influence the quantitative effects of the policy instruments.

Another possibility to model tax progression is to use a tax exemption and a marginal tax.

⁷See appendix for detailed derivation of the wage.

where η is the workers share of the match surplus. The first order condition is

$$\eta(1 - \tau) J_t(a_{it}) = (1 - \eta) [W_t(a_{it}) - U_t] \quad (23)$$

and implies the following relations

$$J_t(a_{it}) - V_t = \frac{1 - \eta}{1 - \eta\tau} S_t(a_{it}) \quad (24)$$

$$W_t(a_{it}) - U_t = \frac{\eta(1 - \tau)}{1 - \eta\tau} S_t(a_{it}). \quad (25)$$

The share parameter η increases the worker's share of match surplus. Increasing the marginal tax rate τ has qualitatively similar effects to the division of surplus as a decrease in the share parameter: the higher the marginal tax rate, the lower is the worker's share of surplus relative to the firm's. Substituting the value equations into (23) and rearranging yields the wage equation⁸

$$w_t(a_{it}z_t) = \eta \left(\frac{a_{it}z_t}{\mu_t} + \kappa\theta_t \right) + (1 - \eta) \left(\frac{h}{(1 - \tau)} + \rho^\tau H(\tilde{a}_t) z_t - (1 - \rho^\tau) v \right). \quad (26)$$

In addition to the real value of the marginal product $\frac{a_{it}z_t}{\mu_t}$ of the match, the wage depends on the cost related to search in case of separation, as well as the outside value of the worker. The wage is increasing in labor market tightness θ_t which reflects the ease with which a worker can find an alternative job in the case of separation. The higher the value of home production h , the higher is the required wage for the worker to agree to work. The wage is increasing in the bargaining share η of the worker.

The partial comparative statics of the wage wrt. the policy parameters are

$$\frac{\partial w_t}{\partial \rho^\tau} > 0, \quad \frac{\partial w_t}{\partial v} < 0, \quad \frac{\partial w_t}{\partial \tau} > 0.$$

A higher replacement rate ρ^τ raises the worker's unemployment income and threat point in the wage bargain, thus raising the wage. The tax subsidy v paid to an employed worker reduces the negotiated wage. This is because the cost of labor to the firm is reduced as the worker's employment is partly compensated by the tax subsidy. As the wage is bargained for, the firm and worker share the subsidy in the same way as they share the surplus of the job. The net gain from the subsidy received upon job formation is $(1 - \rho^\tau) v$: employed workers receive the full subsidy v , but as the unemployment benefit is proportional to net income (including the subsidy), they already received a fraction ρ^τ of it in their unemployment benefit.

⁸Note that this is the gross wage that the firm pays to a worker while the worker's after tax net wage is $w_t(a_{it}z_t) - T(w_t(a_{it}z_t)) = (1 - \tau) [w_t(a_{it}z_t) + v]$.

The marginal tax τ reduces the worker's share of match surplus. From any increase in the wage conceded by the firm, the worker receives only a fraction $1 - \tau$, so there is a joint loss to the firm and worker from the marginal tax. As the value of unemployment includes home production which is not taxed, the marginal tax reduces the value of working relative to being unemployed. Therefore the bargained wage has to increase with the marginal tax to restore the value of working.

Note that the wage consists of market and nonmarket components. The first term in (26) consists of variables that reflect market conditions, match productivity and labor market tightness. The wage responds to changes and volatility in the labor market through this term. The second term consists of non market or fixed parameters. The larger is this part of the wage relative to the market part, the more rigid is the wage. The relative importance of these two terms determines how much of exogenous shocks are absorbed by the wage. The more rigid the wage, the more the shocks are transferred onwards to the profitability of jobs and thus to the job creation and destruction margins.

To illustrate this, consider two extreme cases of the wage negotiation outcome, namely approaching solutions where one of the partners has all of the bargaining power. When the worker's bargaining power approaches unity ($\eta \rightarrow 1$) the second term in the wage equation approaches zero and the wage equation becomes

$$w_t(a_{it}z_t) = \frac{a_{it}z_t}{\mu_t} + \kappa\theta_t.$$

Now there are no fixed components and the whole wage consists only of 'market terms' making it more sensitive to market disturbances. The whole of the real value of the marginal product $\frac{a_{it}z_t}{\mu_t}$ of the match accrues to the worker and the value of unemployment becomes irrelevant. The worker can appropriate all of the match surplus. The policy parameters have no influence in this extreme case.

In the other extreme the firm has all bargaining power ($\eta \rightarrow 0$) and the wage equation reduces to

$$w_t = \frac{h}{(1 - \tau)} + \rho^\tau H(\tilde{a}_t) z_t - (1 - \rho^\tau) v.$$

In this case the wage is immune to labor market tightness, and productivity affects only through the replacement ratio. However now the policy parameters have a key influence on the wage. In this case the match surplus goes entirely to the firm and the wage paid to workers will be only as high as the value of leisure and unemployment compensation. Here the policy parameters have qualitatively similar, but more important, effects on the wage as in the basic case.

2.2.5 Job creation and destruction

To derive expressions for job creation and destruction we first manipulate the value equation for a filled job following Pissarides (2000, ch. 2). Substitute the wage equation (26) and the free-entry condition $V_t = 0$ into the value equation for a filled job (13) to get

$$J_t(a_{it}) = (1 - \eta) \left(\frac{a_{it}z_t}{\mu_t} - \frac{h}{(1 - \tau)} - \rho^\tau H(\tilde{a}_t) z_t + v(1 - \rho^\tau) \right) - \eta\kappa\theta_t \quad (27)$$

$$+ E_t \beta_{t+1} (1 - \rho^x) \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}).$$

Evaluate this expression at $a_{it} = \tilde{a}_t$ and subtract the resulting equation from (27) after noting that $J_t(\tilde{a}_t) = 0$ by the definition of reservation productivity (jobs are destroyed when match surplus goes to zero).⁹ We obtain

$$J_t(a_{it}) = (1 - \eta) \frac{z_t}{\mu_t} (a_{it} - \tilde{a}_t). \quad (28)$$

Substituting this into the job creation condition (15) we get

$$E_t \beta_{t+1} (1 - \rho^x) (1 - \eta) \frac{z_{t+1}}{\mu_{t+1}} \int_{\tilde{a}_{t+1}}^{\bar{a}_{t+1}} (a_{it+1} - \tilde{a}_{t+1}) dF(a_{it+1}) = \frac{\kappa}{q_t^f}. \quad (29)$$

The left-hand side of the equation is decreasing in the reservation value \tilde{a}_{t+1} for match specific productivity and the right-hand side is increasing in labor market tightness θ_t (through q_t^f), so (29) traces a negatively sloped curve in the (θ, \tilde{a}) -space.

Jobs are destroyed when match surplus is zero, $J_t(\tilde{a}_{it}) = 0$. Setting (27) to equal zero and substituting the job creation condition for the second row we obtain

$$\frac{\tilde{a}_t z_t}{\mu_t} - \frac{h}{(1 - \tau)} - \rho^\tau H(\tilde{a}_t) z_t + v(1 - \rho^\tau) - \frac{\eta}{1 - \eta} \kappa \theta_t + \frac{1}{1 - \eta} \frac{\kappa}{q_t^f} = 0. \quad (30)$$

The left-hand side of the equation is increasing in the reservation value \tilde{a}_t and decreasing in labor market tightness, so (29) traces a positively sloped curve in the (θ, \tilde{a}) -space. With these formulations of the job creation and destruction conditions the policy instruments are present only in the latter.

2.3 Aggregate output and consumption

The aggregate output of the economy produced by all firm-worker matches is given by

$$Q_t = (1 - \rho_t) n_t z_t \int_{\tilde{a}_t}^{\bar{a}_t} \frac{a_{it} f(a_{it}) da_{it}}{1 - F(\tilde{a}_t)} = (1 - \rho_t) n_t z_t H(\tilde{a}_t) \quad (31)$$

⁹The firm and worker agree when to separate as $J_t(a_{it}) = 0$ implies $W_t(a_{it}) - U_t = 0$ by the Nash bargaining rule. Therefore we may consider job destruction from either the firm's or worker's perspective.

where $H(\tilde{a}_t)$ as the conditional expectation $E[a \mid a \geq \tilde{a}_t]$. Finally, we also require that consumption C_t equals aggregate household income Y_t which equals production net of vacancy costs

$$C_t = Y_t = (1 - \rho_t) n_t z_t H(\tilde{a}_t) - \kappa v_t. \quad (32)$$

2.4 Retail firms and price rigidity

There is a continuum of monopolistically competitive retail firms on the unit interval. Retail firms buy output of wholesale firms at price P_t^W , differentiate the good and sell it to households. No other inputs or costs are used in the production of final goods, thus retail firm's marginal cost is P_t^W and real marginal cost is $\frac{P_t^W}{P_t}$.

Output sold by retail firm j is y_{jt} at price p_{jt} . Final goods y_t are a composite of individual retail goods

$$y_t = \left[\int_0^1 y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $\varepsilon > 1$ is the the elasticity of substitution across the differentiated retail goods. If resources are used efficiently output of good j equals the demand (consumption) of good j , $y_{jt} = c_{jt}$ so we have

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The demand for good j can be written as

$$c_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\varepsilon} C_t \quad (33)$$

where the price elasticity of good j is ε . As $\varepsilon \rightarrow \infty$, the goods become closer substitutes and firms have less market power.

Following Walsh (2005) and Christiano et al. (2001) a fraction $1 - \omega$ of randomly chosen firms adjusts its price optimally each period and a fraction ω adjusts according to a rule of thumb.¹⁰ Optimally adjusting firms set their price to maximize the expected discounted value of current and future profits and all adjusting firms choose the same price p^* . Profits at a future date $t+i$ are affected by the price chosen at date t if the firm has not had the possibility to update its price optimally after t . The probability of this is ω^i . Firms choose p_{jt} to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\frac{p_{jt}}{P_{t+i}} c_{jt+i} - \frac{P_{t+i}^w}{P_{t+i}} c_{jt+i} \right]. \quad (34)$$

¹⁰This is a variant of Calvo (1983).

Using the demand curve (33) faced by the firm to eliminate c_{jt} from the objective function and substituting $\mu_{t+i}^{-1} = \frac{P_{t+i}^w}{P_{t+i}}$ we obtain

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\varepsilon} - \mu_{t+i}^{-1} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\varepsilon} \right] C_{t+i}. \quad (35)$$

The first order condition is after some manipulation¹¹

$$\frac{p_t^*}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\mu_{t+i}^{-1} \left(\frac{P_{t+i}}{P_t} \right)^{\varepsilon} C_{t+i} \right]}{E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\left(\frac{P_{t+i}}{P_t} \right)^{\varepsilon-1} C_{t+i} \right]}. \quad (36)$$

This equation gives the price chosen by the firms that adjust their price optimally.

The retail price index P_t is given by

$$P_t = \left[\int_0^1 p_{jt}^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \quad (37)$$

from which we get

$$P_t^{1-\varepsilon} = (1 - \omega) (p_t^*)^{1-\varepsilon} + \omega p_{jt-1}^{1-\varepsilon} \quad (38)$$

where a fraction $(1 - \omega)$ adjusts price optimally and a fraction ω adjusts according to rule of thumb. We assume that firm j uses a rule of thumb based on the most recently observed rate of inflation and the most recently observed price level P_{t-1} ,

$$p_{jt} = \pi_{t-1} P_{t-1}. \quad (39)$$

To obtain an expression for aggregate inflation, (36) and (38) can be approximated around a zero average inflation steady state equilibrium and combined to yield the New Keynesian Philips curve. We obtain

$$\pi_t = \frac{\beta}{1 + \beta} E_t \pi_{t+1} + \frac{1}{1 + \beta} \pi_{t-1} - \frac{\zeta}{1 + \beta} \hat{\mu}_t. \quad (40)$$

where $\zeta = \frac{(1-\omega)(1-\omega\beta)}{\omega}$ and $\hat{\mu}_t$ is the deviation of the price markup from the steady state value.¹²

2.5 Monetary authority

The central bank controls the nominal rate of interest according to a modified Taylor rule. The short-term nominal interest rate follows the process

$$R_t = R_{t-1}^{\rho_R} \left(\frac{P_t}{P_{t-1}} \right)^{\phi_{\pi}(1-\rho_R)} e^{\phi_t} \quad (41)$$

¹¹See appendix for detailed derivation.

¹²See e.g. Walsh (2003) for a textbook derivation.

where ρ_R is the degree of interest rate smoothing, $\phi_\pi > 1$ is the response coefficient for inflation and ϕ_t is a serially uncorrelated, mean zero stochastic process. With this policy rule for the nominal rate of interest, the nominal quantity of money adjusts endogenously to satisfy the demand for money.

2.6 Government tax revenue

The government levies income taxes from workers to finance unemployment benefits and tax subsidies paid to workers. The government tax revenues are given by

$$G_t = (1 - \rho^x) n_t T [w_t^e (H(\tilde{a}_t) z_t)] - (1 - n_t) b_t (H(\tilde{a}_t) z_t) \quad (42)$$

where $T [w_t^e (H(\tilde{a}_t) z_t)]$ and $b_t (H(\tilde{a}_t) z_t)$ are given by (18) and (19) respectively. The government receives tax payments (marginal tax on gross income net of the tax subsidy) from all employed workers whose jobs are not destroyed in the current period. The unemployed workers receive an unemployment compensation from the government.

3 Steady state solution and dynamics

In steady state we have $\pi_t = 0$ and $p_t^* = P_t = P$ and $z_t = z = 1$. This implies that the household's Euler condition reduces to $R = \frac{1}{\beta}$ and the steady state values of $n, \rho, u, q^f, q^w, j_c, \theta, w, \tilde{a}, C$ and the policy variables TR, T and b are given by the steady state versions of equations (4), (5), (7), (8), (9), (10), (29), (26), (30), (32), (42), (18) and (19)

- Firm's hazard rate

$$q^f = \frac{m(u, v)}{v} \quad (43)$$

- Worker's hazard rate

$$q^w = \frac{m(u, v)}{u} \quad (44)$$

- Destruction rate

$$\rho = \rho^x + (1 - \rho^x) F(\tilde{a}) \quad (45)$$

- Employment

$$\rho n = m(u, v) \quad (46)$$

- Unemployed job seekers

$$u = 1 - (1 - \rho) n \quad (47)$$

- Net job creation

$$jc = \frac{m(u, v)}{n} - q^f \rho^x \quad (48)$$

- Government tax revenue

$$G = (1 - \rho^x) nT(w) - (1 - n)b \quad (49)$$

- Worker's tax transfer to government

$$T = \tau w - (1 - \tau)v \quad (50)$$

- Unemployment compensation

$$b = \rho^\tau (1 - \tau)(w + v). \quad (51)$$

- Wage

$$w = \eta \left(\frac{H(\tilde{a})}{\mu} + \kappa\theta \right) + (1 - \eta) \left(\frac{h}{1 - \tau} + \rho^\tau H(\tilde{a}) - (1 - \rho^\tau)v \right) \quad (52)$$

- Free-entry

$$\frac{\kappa}{q^f} = \beta (1 - \rho^x) (1 - \eta) \frac{1}{\mu} \int_{\tilde{a}}^{\bar{a}} (a_i - \tilde{a}) dF(a_i) \quad (53)$$

- Job destruction treshold \tilde{a}

$$\frac{\tilde{a}}{\mu} - \frac{h}{(1 - \tau)} - \rho^\tau H(\tilde{a}) + v(1 - \rho^\tau) - \frac{\eta}{1 - \eta} \kappa \frac{q^w}{q^f} + \frac{1}{1 - \eta} \frac{\kappa}{q^f} = 0 \quad (54)$$

- Aggregate income and consumption

$$Y = C = (1 - \rho) nH(\tilde{a}) - \kappa v \quad (55)$$

- The steady-state price markup

$$\mu = \frac{\varepsilon}{\varepsilon - 1}. \quad (56)$$

The steady state solution can be solved analytically. Equations (53) and (54) produce a unique solution in the two endogenous variables \tilde{a} and θ (through q^f and q^w). We can then substitute these equilibrium values into (52) and solve the rest of the equations recursively. The linearized equations used in the dynamic analysis are presented in the appendix.

4 Calibration

The baseline parameter values are calibrated to a stylized U.S. economy and to be in line with previous literature.¹³ As information on all parameters is not available, we calibrate these values indirectly as residual parameters from the steady state equations. The model's parameters can be separated into six groups: labor market parameters, labor market policy parameters, household preferences, parameters characterizing the degree of price rigidity, interest rate parameters and the parameters of exogenous shocks.

Labor market– Job flows are determined by the matching and separation probabilities of firms and workers. We set the time period to one quarter and the job finding rate of workers and the rate of filling vacancies at $q^w = 0.6$ and $q^f = 0.7$ respectively. The matching function parameters are set to $\alpha = 0.4$ for the worker's elasticity parameter and $1 - \alpha = 0.6$ for the firm's elasticity parameter. These are in accordance with empirical studies of the matching function.¹⁴ The shift parameter of the matching function is $A = 0.65$, a value similar to that in e.g. Walsh (2004). The size of the labor force is normalized to one and the employment rate is set to $n = 0.94$, which implies an unemployment rate of 6 percent. The steady-state number of workers searching for a job is then $u = 0.154$, as u also includes the total ρn of workers who move to the matching market because their matches dissolve before production is started. The total job destruction rate is set to $\rho = 0.1$ which is roughly consistent with a large body of empirical studies.¹⁵ These values and the matching function also imply $v = 0.134$. For the exogenous job destruction rate we use the value calibrated by den Haan et al. (2000) $\rho^x = 0.068$ implying the endogenous job destruction rate $\rho^n = F(\tilde{a}) = 0.034$. The reservation productivity \tilde{a} can be derived from $\tilde{a} = F(\rho^n)^{-1}$. Following eg. Mortensen and Pissarides (2003) we assume that $F(a)$ is the uniform c.d.f. with support $[\gamma, 1]$. In the linearized model we need the elasticity of the c.d.f. at the reservation productivity level \tilde{a} , which is given by $e_{F,a} = \frac{\partial F(\tilde{a})}{\partial \tilde{a}} \frac{\tilde{a}}{F(\tilde{a})} = \frac{\tilde{a}f(\tilde{a})}{F(\tilde{a})}$. For the conditional expectation of a given the reservation productivity \tilde{a} we have $H(\tilde{a}) = \int_{\tilde{a}}^{\bar{a}} a \frac{f(a)}{1-F(\tilde{a})} da$ and the elasticity $e_{H,a} = \frac{\partial H(\tilde{a})}{\partial \tilde{a}} \frac{\tilde{a}}{H(\tilde{a})}$. The worker and firm are assumed to get an equal share of the match surplus in the wage bargaining so we set $\eta = 0.5$. The value of leisure h and the lower support of the productivity distribution γ are calibrated s.t. the model is consistent with the values for ρ^n and n above. Finally q and κ are calibrated as

¹³See eg. Walsh (2003, 2004), Trigari (2004), Krause and Lubik (2003) and den Haan et al. (2000).

¹⁴See e.g. Petrongolo and Pissarides (2001) and Blanchard and Diamond (1989).

¹⁵See e.g. Davis et al. (1998).

residual parameters from the steady state equations.

Labor market policy– We calibrate the policy parameters together with the value of home production in such a way that we obtain steady state values that are roughly consistent with Walsh (2003, 2005) for reasonable tax parameter values. Our strategy is to first set the policy parameters to benchmark values s.t. taxation is initially progressive and the replacement ratio similar to examples of the U.S. used in the literature. We then reverse calibrate the value of home production h s.t. the model produces steady state values that are consistent with eg. Walsh (2003, 2005) calibrations. For the baseline calibration we set the marginal tax rate to $\tau = 0.25$ and the tax subsidy $v = 0.03$. The positive tax subsidy implies that income taxation is progressive. Finally, the replacement rate is set to $\rho^\tau = 0.2$ and reverse calibration of the value of home production produces $h = 0.53$.

Household preferences– We follow Walsh (2005) for the utility function $u(C_{t+i}) = \frac{(C_{t+i} - \chi C_{t+i-1})^{1-\sigma}}{1-\sigma}$ where χ is a parameter of habit persistence, and choose values for the parameters of household preferences that are standard in the literature. We set $\chi = 0.5$, $\beta = 0.989$ and the coefficient of relative risk aversion is chosen to be $\sigma = 2$. The steady state price markup for retail firms is set to equal $\mu = 1.1$ which implies $\varepsilon = 11$, which is the parameter that determines the elasticity of demand of differentiated retail goods.

Price rigidity– The degree of price rigidity is determined by the share of firms who do not optimally adjust their price. We follow Walsh (2003) and set this fraction to equal $\omega = 0.67$.

Monetary policy– We set the parameters of the interest rate rule to equal $\phi_\pi = 1.10$, which gives a 110 basis points long-run nominal response to a 100 basis point increase in inflation, and $\rho_R = 0.9$ which is roughly consistent with the empirical evidence on high inertia displayed by central bank policy rules (Walsh 2005). The standard deviation of the monetary policy shock is set to $\phi_t = 0.002$.

Productivity shock– We assume that the log aggregate productivity shock to follow an AR(1) process $\log z_t = \rho_z \log z_{t-1} + \epsilon_t$ with the serial coefficient $\rho_z = 0.95$ and the standard deviation of the productivity shock shock ϵ_t , to be $\sigma_\epsilon = 0.01$.

5 Model analysis

We proceed by first analyzing the steady state of the model and the comparative statics of the labor market policy parameters. Then we move to the impulse response analysis to study the effects of taxes on the dynamic behavior of the model.

	<i>Benchmark</i>	$d\tau$	dv	$d\rho^r$
y	0.66	0.62	0.69	0.62
q	0.67	0.63	0.70	0.63
n	0.94	0.91	0.96	0.90
θ	0.88	0.76	1.02	0.74
qf	0.70	0.73	0.65	0.74
qw	0.60	0.56	0.67	0.55
a	0.58	0.59	0.56	0.59
jc	0.031	0.033	0.253	0.034

Figure 1: Percentage point changes in policy parameters.

5.1 Steady state labor market policy analysis

5.1.1 Employment taxes and unemployment income

First we consider the effects of changes in policy parameters on the steady state of the economy independently of tax revenue considerations (figure 1). We then investigate compensating policy changes to study the impact of changes in the tax structure (figure 2). With tax revenue neutral changes we fix the government tax revenue and consequently the tax subsidy solves as an endogenous variable of the model which depends on the marginal tax rate. The effects of policy work through the wage on the job destruction condition, which jointly with the job creation condition determines the equilibrium destruction productivity and labor market tightness.

Marginal tax rate– Consider a marginal increase in the income tax rate τ . As home production (or leisure) is not taxed it's value relative to working increases making the latter less attractive. To restore the attractiveness of working the wage must be increased. Higher wages imply lower job creation and lower labor market tightness; less vacancies and more unemployed workers. Output falls as less people are employed and jobs are fewer.

Tax subsidy– Increasing the tax subsidy v has opposite effects to the marginal tax rate. The tax subsidy paid to an employed worker reduces the negotiated wage as the worker's employment is partly compensated by the tax subsidy. Bargaining implies that the firm and worker share the subsidy. The reduction in the negotiated wage raises job creation, vacancies and labor market tightness. Unemployment falls as the job finding probability for workers increases. Output increases.

Replacement ratio– A higher replacement rate increases the worker's unemployment income and threat point in the wage bargain. The wage increases with effects

similar to those of the marginal tax.

5.1.2 Tax progression

We next examine the importance of the structure of taxes for the labor market equilibrium. Keeping the government tax revenue fixed, we increase tax progression by increasing the marginal tax rate and then increase the tax subsidy so much that the change in tax revenue implied by the marginal tax raise is exhausted. Given the comparative statics of the marginal tax and tax subsidy described above, the effects of increasing tax progression are ambiguous *a priori*, and depend on the relative magnitude of the effects of the tax instruments.

Progressive taxes in initial equilibrium– Figure 2 shows how a revenue neutral increase in tax progression affects the steady state of the economy in the benchmark calibration. The wage rate increases, inducing more job destruction, reducing labor market tightness and thus raising unemployment. Output decreases. The effect of a tax revenue neutral increase in tax progression is qualitatively similar to an increase in the marginal tax. The effect on the steady state values of the change in the marginal tax dominates the effect of the tax subsidy. However the effect of the marginal tax is moderated by the opposite effect of the tax subsidy. A more progressive tax scheme thus shifts the economy to a lower output and higher unemployment equilibrium. To our knowledge this result is new to the literature and opposite to results in earlier studies using a similar modelling framework. This tax structure involves a trade-off between income equality considerations and equilibrium unemployment and output.

Proportional taxes in initial equilibrium– The above result is in contrast with the results of Pissarides (1998) and Sinko (2005). In their studies increasing tax progression has a positive employment effect, whereas we find a negative one. The key issue between these opposite results is the initial degree of tax progression. Pissarides (1998) and Sinko (2005) consider the effects of a tax revenue neutral increase in tax progression when taxation is initially proportional (the tax subsidy is zero), whereas we start from an initially progressive tax scheme. Experimenting with the policy instruments reveals that our model also produces qualitatively similar results to the above studies when taxation is proportional in the initial state. The smaller is the tax subsidy in the initial state, the smaller is the negative effect of the marginal tax increase on employment relative to the positive effect of the tax subsidy increase. For a sufficiently small tax subsidy the relative effects are reversed and the employment effect turns positive. The wage rate decreases with tax progression, inducing more job creation and vacancies, higher labor market tightness and

	$\tau = 0.25, \nu = 0.03$	$\tau = 0.26, \nu = 0.038$	$\tau = 0.30, \nu = 0.071$
y	0.659	0.655	0.641
q	0.667	0.663	0.650
n	0.94	0.937	0.951
θ	0.880	0.865	0.819
qf	0.70	0.70	0.71
qw	0.60	0.60	0.58
a	0.577	0.578	0.583
jc	0.030	0.031	0.032

Figure 2: Tax revenue neutral increase in tax progression.

lower unemployment. Output increases. In this case promoting income equality is consistent with lower equilibrium unemployment and higher output.

Our opposite results to those of Pissarides (1998) and Sinko (2005) show that the effects of increasing tax progression depend on the initial degree of tax progression in the economy. Our results are not in conflict with these studies, but completes them by emphasizing the mechanism by which progression works. Our simulations show that, starting from a proportional tax scheme, the relative strength of the two tax policy instruments is reversed as progression increases. Initially the effect of the tax subsidy dominates, but once the initial tax scheme is sufficiently progressive, the effect of the marginal tax dominates. This implies that for economies with an initially low degree of tax progression, increasing it is beneficial in terms of employment and output. But for economies with a sufficiently progressive tax scheme initially, increasing progression further is harmful in terms of employment and output.

5.2 Tax reform and shock propagation

Now we investigate how changes in labor market policy instruments affect shock propagation. As in the previous section, our strategy is to first look at the effect of policy parameters separately without government tax revenue considerations and then examine tax revenue neutral changes in the tax structure.

5.2.1 Employment taxes and unemployment income

The effects of the tax instruments on the impulse response functions to productivity and interest rate shocks are plotted by the solid lines in figure 3 and 4 respectively.

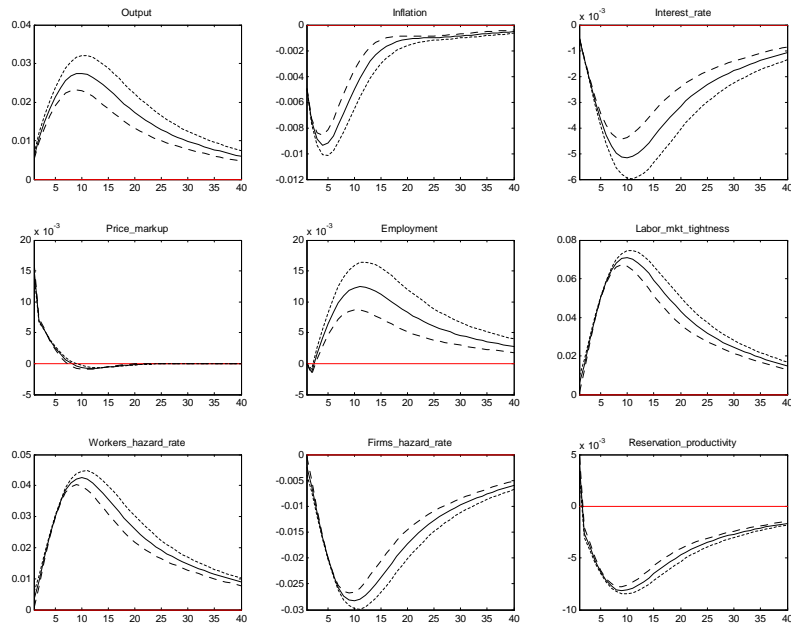


Figure 3: Impulse response functions to output shock. The baseline case is plotted by the solid lines, the dotted lines plot the impulse responses for $\tau = 0.26$ and the dashed lines plot the impulse response functions for $v = 0.04$.

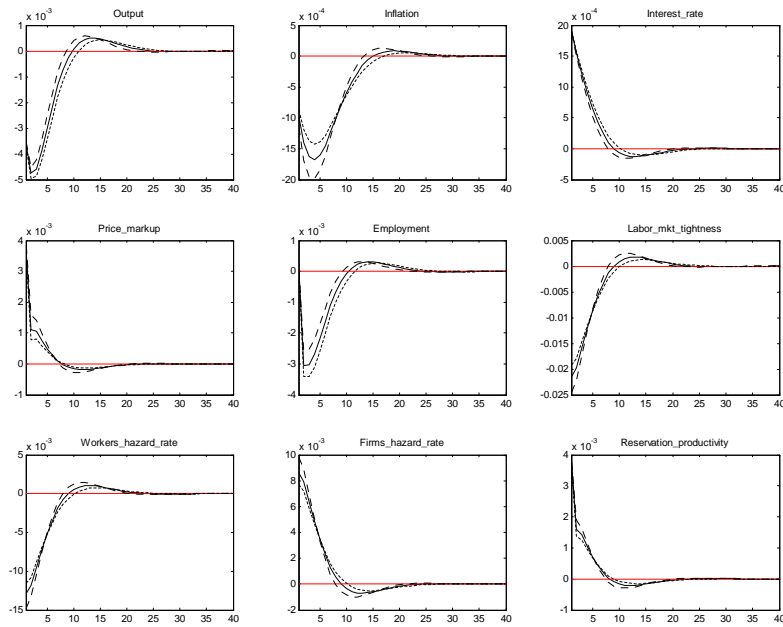


Figure 4: Impulse response functions to policy shock. The baseline case is plotted by the solid lines, the dotted lines plot the impulse responses for $\tau = 0.26$ and the dashed lines plot the impulse response functions for $v = 0.04$.

The dotted lines plot the impulse responses for a percentage point increase in the income tax rate. The impulse response functions for a percentage point increase in the tax subsidy are produced by the dashed lines. For the sake of clarity in the figure, the impulse response functions for the replacement rate is not plotted as the plots overlap closely those of the marginal tax.

Marginal tax rate– The impulse response functions of a productivity shock are generally amplified by the marginal tax increase, but the shapes of the functions remain qualitatively the same. Both peak effects are larger and the shocks are more persistent. In fact, this effect is similar to the effect of reducing the bargaining power of workers described in Walsh (2003). This should not be surprising, considering the discussion in section 2.2.4 on the way the marginal tax affects the division of match surplus. A higher marginal tax increases the weight of the 'non market' component in the wage equation relative to the market sensitive part. This implies that the wage becomes more rigid and absorbs less of shocks, transmitting them on to the rest of the economy through job creation and destruction. The marginal tax affects the impulse responses to an interest rate shock in a more diverse way. The impulse responses of output and employment are amplified both in the peak effect and persistence. The peak effect of inflation is moderated but the impulse response is more persistent. This applies to the labor market tightness and to the firms and workers hazard rates as well.

Tax subsidy– The tax subsidy has the opposite effect to the marginal tax rate. The impulse responses to a productivity shock are smoothed: both peak effects and persistence are reduced by the tax subsidy. The tax subsidy increase has an opposite effect to the marginal tax in the wage equation. An increase in the tax subsidy increases relative size of the market sensitive component of the wage. This implies that the wage absorbs more of the shocks and less of them transmit to the rest of the economy. The tax subsidy smooths the impulse responses of output and employment wrt. an interest rate shock. The peak effects of inflation, labor market tightness and the hazard rates are amplified but the impulse responses are more persistent.

Replacement ratio– The replacement rate has qualitatively similar effects to the marginal tax for similar reasons.

5.2.2 Tax progression

We now proceed to investigate the importance of the structure of taxation for the dynamics of the economy wrt. shocks. As in the steady state analysis we consider increasing the marginal tax rate and making the necessary increase in the

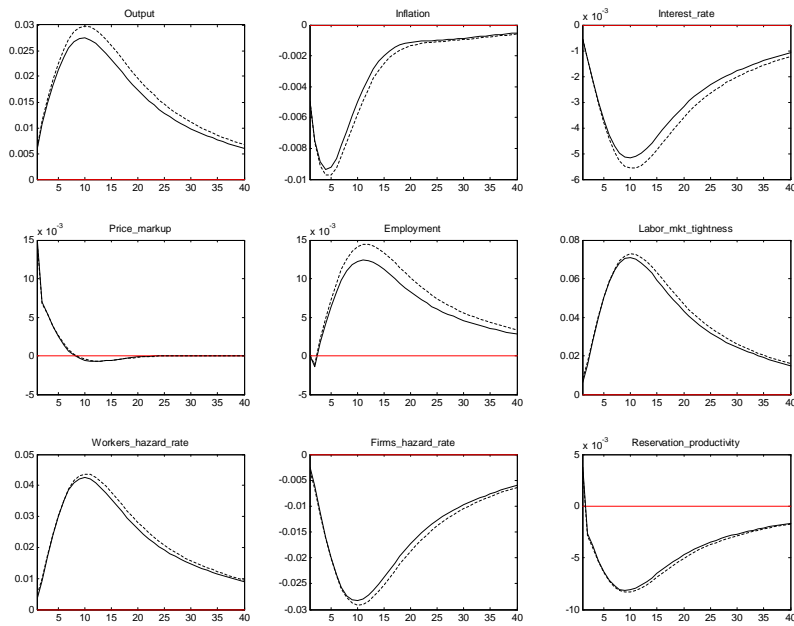


Figure 5: Impulse responses to productivity shock and initially progressive taxation. The baseline case is poltted by solid lines and increased progression is plotted by the dotted lines.

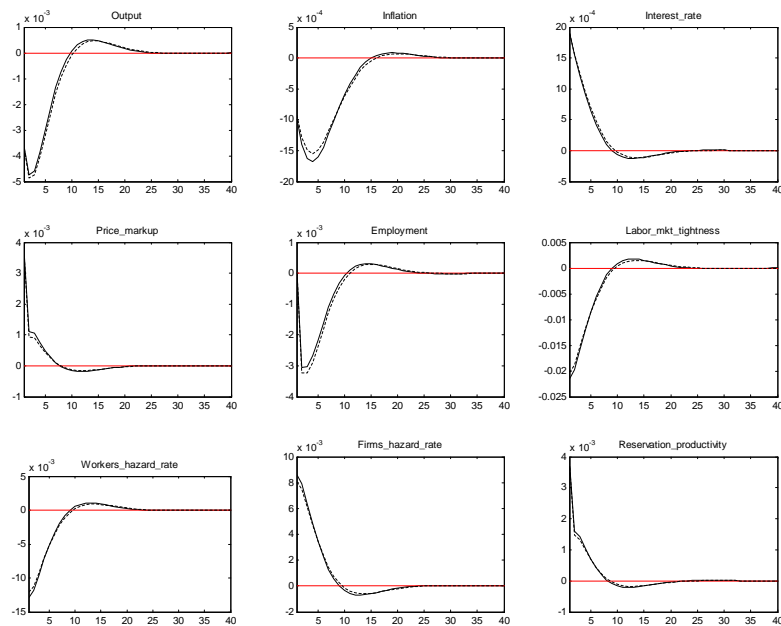


Figure 6: Impulse responses to policy shock and initially progressive taxation. The baseline case is poltted by solid lines and increased progression is plotted by the dotted lines.

tax subsidy to keep government tax revenues neutral. The same forces are at work here as in the steady state analysis: the initial degree of tax progression determines the relative effects of the two tax parameters and thus the effects of increasing tax progression.

Progressive taxes in initial equilibrium— Figures 5 and 6 plot the impulse responses of the benchmark setup to productivity and interest rate shocks respectively. With an increase in tax progression the impulse responses are amplified, both in terms of peak effects as well as persistence. The reasoning is analogous to that of the previous section where the steady state effects were analyzed. The amplifying effect on the impulse responses of the marginal tax dominates that of the tax subsidy. For interest rate shocks the impulse responses are affected by tax progression qualitatively in the same way as by the marginal tax. Overall, tax progression implies a more volatile economy in the benchmark calibration.

Proportional taxes in initial equilibrium— The impulse responses to productivity and interest rate shocks for a calibration with the tax subsidy being zero in the initial state are plotted in figures 7 and 8. As in the steady state analysis the results of the alternative setup are opposite to the benchmark case. Now the impulse responses wrt. to a productivity shock are smoother and less persistent, both in terms of peak effects as well as persistence. In this alternative setup tax progression, or promoting income equality is consistent with a less volatile economy.

The implications of labor tax reform depend crucially on the initial tax scheme. Summarizing, increasing tax progression has desirable output and employment effects when initially progression is low and the tax subsidy effect dominates. But the higher is the initial level of progression, further increases in tax progression are less and less desirable because the effect of the marginal tax strengthens relative to the tax subsidy effect. Thus, when tax progression is initially high, the marginal tax effect dominates the tax subsidy effect and the output and employment effects become detrimental.

6 Concluding remarks

The paper illustrates the importance of the initial degree of tax progression in determining both the steady state and dynamic effects of labor tax reforms. The main conclusion is that the macroeconomic outcomes of tax reforms depend on the initial degree of tax progression which determines the relative effects of the tax instruments. In an economy with initially proportional labor taxation, increasing progression has desirable equilibrium employment and output effects and stabilizing

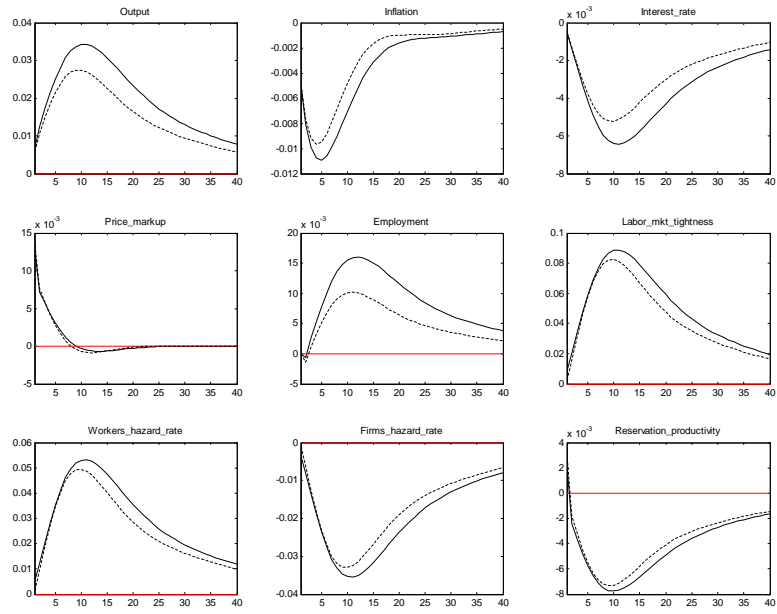


Figure 7: Impulse responses to productivity shock and initially proportional taxation. The initial value responses are plotted by solid lines and increased progression is plotted by the dotted lines.

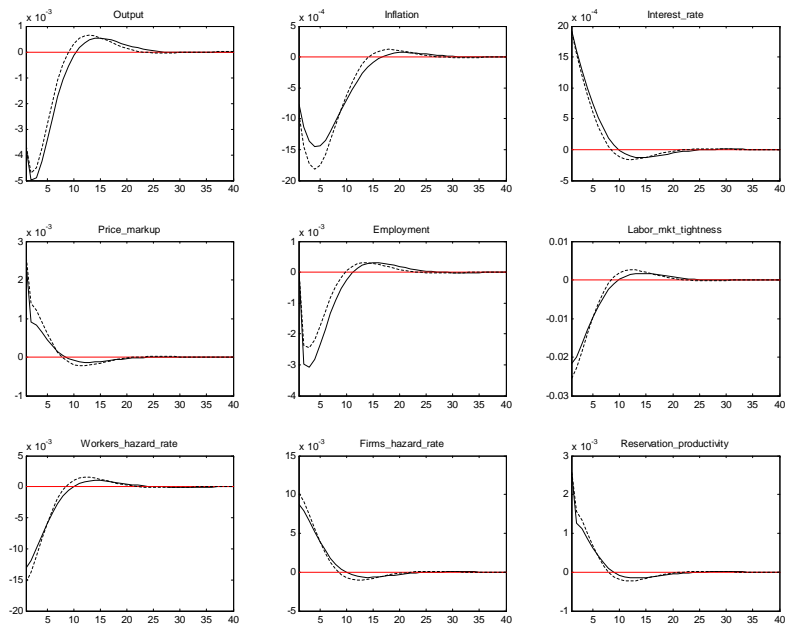


Figure 8: Impulse responses to policy shock and initially proportional taxes. The initial value responses are plotted by solid lines and increased progression is plotted by the dotted lines.

dynamic effects. However, if the tax scheme is initially sufficiently progressive, increasing progression has opposite effects: the equilibrium employment and output effects are negative and the sensitivity to shocks is amplified.

Our simulations show that interactions of policy tools differ depending on the state of the labor market. As very different policy schemes are implemented in European countries and these countries have large variation in labor market outcomes, it would be of interest to study the implications of tax reforms in these different setups. Also, as a large set of policy instruments is available to the policy maker, a more comprehensive study including tools such as payroll taxes, hiring subsidies and firing costs would offer more insight into the effects of tax reforms and the alternatives available and trade-offs involved when designing tax reforms.

There are several issues that deserve attention in future research. We have investigated the effects of taxation on macroeconomic outcomes in a framework which incorporates the search-matching model of the labor market to a New Keynesian business cycle model. A word of caution regarding the results may be in order. Pissarides (1998) points out that there is no definitive model of the European labor market and shows that effects of changes in the structure and level of taxation sometimes depends on the underlying model of the labor market. One avenue for future research would be to consider the implications of the choice of the labor market model nested in the New Keynesian framework.

Finally, an important issue in matching models is the inefficiency typically produced by matching frictions and decentralized bargaining. An alternative approach to labor market policy is to design taxation so as to internalize search externalities and improve the efficiency of resource allocation. This question is also left for future work.

A Appendix

A.1 Bargaining and wage

The match surplus is shared between the firm and the worker according to the parameter η which represents the workers share of the match surplus (bargaining power). The wage rate thus satisfies

$$w_t = \arg \max [W_t(a_{it}) - U_t]^\eta [J_t(a_{it}) - V_t]^{1-\eta}. \quad (57)$$

The first order condition is given by

$$\begin{aligned} & \eta \frac{\partial W_t(a_{it})}{\partial w_t} (W_t(a_{it}) - U_t)^{\eta-1} (J_t(a_{it}) - V_t)^\eta \\ & + (1 - \eta) \frac{\partial J_t(a_{it})}{\partial w_t} (W_t(a_{it}) - U_t)^\eta (J_t(a_{it}) - V_t)^{-\eta} \\ & = 0 \end{aligned}$$

Divide both sides by $[J_t(a_{it}) - V_t]^{\eta-1} [W_t(a_{it}) - U_t]^{-\eta}$ to get

$$\eta \frac{\partial W_t(a_{it})}{\partial w_t} (J_t(a_{it}) - V_t) + (1 - \eta) \frac{\partial J_t(a_{it})}{\partial w_t} (W_t(a_{it}) - U_t) = 0$$

where $\frac{\partial J_t(a_{it})}{\partial w_t} = -1$ and $\frac{\partial W_t(a_{it})}{\partial w_t} = 1 - T'(w_t) = 1 - \tau$ so the first order condition becomes

$$\eta(1 - \tau) J_t(a_{it}) = (1 - \eta) (W_t(a_{it}) - U_t). \quad (58)$$

Substituting the value equations (13), (16), (17) and $V_{t+1} = 0$ into the first order condition and cancelling terms produces

$$\begin{aligned} & (1 - \eta\tau) w_t(a_{it}z_t) \quad (59) \\ & = \eta(1 - \tau) \frac{a_{it}z_t}{\mu_t} + \eta(1 - \tau) E_t \beta_{t+1} \left[(1 - \rho^x) q_{t+1}^w \int_{\bar{a}_{t+1}}^{\bar{a}_{t+1}} J_{t+1}(a_{it+1}) dF(a_{it+1}) \right] \\ & + (1 - \eta) [A + h + b_t + T(w_t(a_{it}z_t))] \end{aligned}$$

where we have used the first order condition to obtain the last term of the second row. Substituting equations (18), (19) and

$$\begin{aligned} E_t \beta_{t+1} (1 - \rho^x) [W_{t+1}(a_{it}) - U_{t+1}] & = \frac{\eta(1 - \tau)}{(1 - \eta)} E_t \beta_{t+1} (1 - \rho^x) J_{t+1}(a_{it}) \\ & = \frac{\eta(1 - \tau)}{(1 - \eta)} \frac{\kappa}{q_t^f} \end{aligned}$$

into (59) and dividing both sides of the resulting equation by $(1 - \tau)$ produces

$$w_t(a_{it}z_t) = \eta \left(\frac{a_{it}z_t}{\mu_t} + \kappa\theta \right) + (1 - \eta) \left(\frac{A + h}{(1 - \tau)} + \rho^\tau a_t^e z_t - (1 - \rho^\tau) v \right) \quad (60)$$

where $\mu_t = \frac{P_t}{P_t^W}$.

A.2 Price rigidity and Phillips curve

Firms choose p_{jt} to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\frac{p_{jt}}{P_{t+i}} c_{jt+i} - \frac{P_{t+i}^w}{P_{t+i}} c_{jt+i} \right]. \quad (61)$$

where $\beta_{t+i} = \frac{\beta^i \lambda_{t+i}}{\lambda_t}$. Using the demand curve (33) faced by the firm we can eliminate c_{jt} to get the objective function and substitute and $\mu_{t+i}^{-1} = \frac{P_{t+i}^w}{P_{t+i}}$ to get

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\varepsilon} - \mu_{t+i}^{-1} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\varepsilon} \right] C_{t+i}. \quad (62)$$

The first order condition is

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[(1-\varepsilon) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\varepsilon} + \varepsilon \mu_{t+i}^{-1} \left(\frac{p_t^*}{P_{t+i}} \right)^{-\varepsilon-1} \right] \frac{1}{P_{t+i}} C_{t+i} = 0.$$

Re-express this as

$$E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[(1-\varepsilon) \left(\frac{1}{P_{t+i}} \right) + \varepsilon \mu_{t+i}^{-1} \frac{1}{p_t^*} \right] \left[\left(\frac{p_t^*}{P_t} \right) \left(\frac{P_t}{P_{t+i}} \right) \right]^{-\varepsilon} C_{t+i} = 0$$

Divide by $\frac{p_t^*}{P_t}$ and rearrange

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\left(\frac{1}{P_{t+i}} \right) \left(\frac{P_t}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \\ &= \frac{\varepsilon}{(\varepsilon-1)} E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \mu_{t+i}^{-1} \left[\frac{1}{p_t^*} \left(\frac{P_t}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \end{aligned}$$

Multiply and divide the left side by P_t

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\frac{1}{P_t} \left(\frac{P_t}{P_{t+i}} \right) \left(\frac{P_t}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \\ &= \frac{\varepsilon}{(\varepsilon-1)} E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \mu_{t+i}^{-1} \left[\frac{1}{p_t^*} \left(\frac{P_t}{P_{t+i}} \right)^{-\varepsilon} C_{t+i} \right] \end{aligned}$$

Then multiply both sides by p_t^* and rearrange to obtain

$$\frac{p_t^*}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1} \right) \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\mu_{t+i}^{-1} \left(\frac{P_{t+i}}{P_t} \right)^{\varepsilon} C_{t+i} \right]}{E_t \sum_{i=0}^{\infty} \omega^i \beta_{t+i} \left[\left(\frac{P_{t+i}}{P_t} \right)^{\varepsilon-1} C_{t+i} \right]} \quad (63)$$

The retail price index P_t is given by

$$P_t = \left[\int_0^1 p_{jt}^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \quad (64)$$

from which we get

$$P_t^{1-\varepsilon} = (1-\omega) (p_t^*)^{1-\varepsilon} + \omega p_{jt-1}^{1-\varepsilon} \quad (65)$$

where a fraction $(1 - \omega)$ adjusts price optimally and a fraction ω adjusts according to rule of thumb. We assume that firm j uses a rule of thumb based on the most recently observed rate of inflation and the most recently observed price level P_{t-1} ,

$$p_{jt} = \pi_{t-1} P_{t-1}. \quad (66)$$

To obtain an expression for aggregate inflation, equations (63) and (65) can be approximated around a zero average inflation steady state equilibrium.

A.3 Linearized equations

Linearizing the model around the non-stochastic zero-inflation steady state produces the equations below. The variables are expressed in terms of percentage deviations around the steady state.

- The Euler condition from household's problem

$$0 = E_t \hat{y}_{t+1} - \hat{y}_t - \frac{1}{\sigma} (\hat{r}_t - E_t \hat{\pi}_{t+1}) \quad (67)$$

- Survival rate of matches $\varphi_t = 1 - \rho_t$

$$\hat{\varphi}_t = - \left(\frac{\rho^n}{1 - \rho^n} \right) e_{F,a} \hat{a}_t \quad (68)$$

where $e_{F,a} = \frac{\partial F(\bar{a})}{\partial \bar{a}} \frac{\bar{a}}{F(\bar{a})}$.

- Employment (evolution of number of matches) n_{t+1}

$$\hat{n}_{t+1} = \bar{\varphi} \hat{n}_t + \bar{\varphi} \hat{\varphi}_t + \left(\frac{\bar{v} \bar{q}^f}{\bar{n}} \right) \hat{v}_t + \left(\frac{\bar{v} \bar{q}^f}{\bar{n}} \right) \hat{q}^f \quad (69)$$

- Unemployment (number of unemployed job seekers u_t)

$$\hat{u}_t = - \frac{\bar{\varphi} \bar{n}}{\bar{u}} \hat{n}_t - \frac{\bar{n} \bar{\varphi}}{\bar{u}} \hat{\varphi}_t \quad (70)$$

- Probability of filling vacancy for firm q^f

$$\hat{q}_t^f = \alpha (\hat{u}_t - \hat{v}_t) \quad (71)$$

- Equality of firms filling vacancies and workers finding jobs

$$\hat{v}_t + \hat{q}_t^f = \hat{u}_t + \hat{q}_t^w \quad (72)$$

- The nominal interest rate rule

$$\hat{r}_t = \rho_R \hat{r}_{t-1} + \phi_\pi (1 - \rho_R) \pi_t + \phi_t \quad (73)$$

- Inflation

$$\hat{\pi}_t = \frac{\beta}{1 + \beta} E_t \hat{\pi}_{t+1} + \frac{1}{1 + \beta} \hat{\pi}_{t-1} - \frac{\zeta}{1 + \beta} E_{t-1} \hat{\mu}_t \quad (74)$$

where $\zeta = \frac{(1-\omega)(1-\omega\beta)}{\omega}$.

- Output equation

$$\hat{y}_t = \frac{\bar{Q}}{\bar{y}} (\hat{z}_t + e_{H,a} \hat{a}_t + \hat{\varphi}_t + \hat{n}_t) - \frac{\kappa \bar{v}}{\bar{y}} \hat{v}_t \quad (75)$$

where $e_{H,a} = \frac{\partial H(\bar{a})}{\partial \bar{a}} \frac{\bar{a}}{H(\bar{a})}$.

- Wage at destruction margin

$$\hat{w}_t = \eta \left(\frac{\bar{a}}{\bar{w} \bar{\mu}} (\hat{a}_t + \hat{z}_t - \hat{\mu}_t) + \frac{\kappa \bar{\theta}}{\bar{w} \bar{\lambda}} (\hat{\theta}_t - \hat{\lambda}_t) \right) + \frac{(1 - \eta)}{(1 - \tau)} \left(\frac{\bar{b}}{\bar{w}} \hat{b}_t - \frac{h}{\bar{w} \bar{\lambda}} \hat{\lambda}_t \right) \quad (76)$$

- Unemployment compensation

$$\hat{b}_t = \rho^\tau (1 - \tau) \frac{\overline{H(\bar{a})}}{\bar{b}} (e_{H(a), \bar{a}} \hat{a}_t + \hat{z}_t) \quad (77)$$

- Endogenous job creation

$$\begin{aligned} -\hat{q}_t^f &= \hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{\varphi}_{t+1} + \hat{z}_{t+1} - \hat{\mu}_{t+1} \\ &+ E_t \beta (1 - \eta) \frac{\bar{\varphi} \bar{q}^f}{\bar{\mu} \kappa} \left[\overline{H(\bar{a})} e_{H,a} - \bar{a} \right] \hat{a}_{t+1} \end{aligned} \quad (78)$$

- Endogenous job destruction

$$\frac{\bar{a}}{\bar{\mu}} (\hat{a}_t + \hat{z}_t - \hat{\mu}_t) - \bar{w} \hat{w}_t - \frac{\kappa}{\bar{\lambda} \bar{q}^f} (\hat{\lambda}_t + \hat{q}_t^f) = 0 \quad (79)$$

or, substituting the wage and unemployment benefit equations into the job destruction equation yields

$$\begin{aligned} &\frac{\bar{a}}{\bar{\mu}} \hat{a}_t + \left[\frac{\bar{a}}{\bar{\mu}} - \rho^\tau \frac{H(\bar{a}_t)}{\bar{\mu}} \right] (\hat{z}_t - \hat{\mu}_t) - \rho^\tau \frac{H(\bar{a}_t) e_{H,a}}{\bar{\mu}} \hat{a}_t \\ &- \frac{\eta \kappa \bar{q}^w}{(1 - \eta) \bar{q}^f} \hat{q}_t^w - \frac{(1 - \eta \bar{q}^w) \kappa}{(1 - \eta) \bar{q}^f} \hat{q}_t^f \\ &= 0. \end{aligned}$$

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