

Research Reports, 1/2002
Department of Social Psychology
Helsinki University

**Cognitive Process and Behavior;
A Conceptual Framework and Simulations**

by

Kullervo Rainio
Professor emeritus
Social Psychology
Helsinki University

ISBN 952-10-0469-X
ISSN 0357-4040

Communications:

Copies available from <http://ethesis.helsinki.fi>

The author:

Prof. emer. Kullervo Rainio

mail address: Petaksentie 44, SF-00630 Helsinki, Finland

e-mail: kullervo.rainio@pp.inet.fi

Department of Social Psychology, Helsinki University:

mail address: PL 9, SF-00014 Helsinki University, Finland

e-mail: tupietil@valt.helsinki.fi

Cognitive Process and Behavior; A Conceptual Framework and Simulations. Research Reports 1/2002, Department of Social Psychology, Helsinki University

by

Kullervo Rainio

Professor emeritus of Social Psychology
Helsinki University

Abstract

The aim of the book is to build a conceptual system which represents the psychological forces as probability vectors determining transitions in topological space. The conceptual system presented in the book is – in a way – a correction and a radical modernization of the classical framework by Kurt Lewin. Unlike in the Lewinian system, also the time variable is handled as a discrete one, *i.e.*, as steps in time. The cognitive process is assumed to occur in the actual life space represented as a graph, the vertices of which indicate cognitive states. The cognitive transitions are determined by the probabilities of cognitive trials from state to state and by the probabilities of their success. Particular rules are given for making the decisions to act.

The book examines the cognitive processes from the holistic viewpoint.

The new conceptual system makes possible to derive stochastic models of cognitive processes to estimate the behavior. In Chapters 3 and 4 this has been done by simulating simple game situations. The simulation programs are given in detail, written in Turbo-Basic language. They are also available in files, in .bas or .exe -forms.

In Chapter 6 some philosophical consequences of the theory are discussed, particularly the psycho-physical problem and Eccles' trial to solve it. A formalization of the Eccles' theory is given. In Chapter 6.4., the occurrences in the macro-physical and quantum systems as well as in biosystems and cognitive systems are compared in terms of the given conceptual framework.

Keywords: balance, behavior, cognition, cognitive, Eccles, field, formal, force, games, graph, graph theory, group maze, Heider, holistic, hodological, learning, Lewin, life space, model, Monte Carlo method, probabilistic, psycho-physical, reality, simulation, stochastic, topological, vector.

Communications:

The whole text available from <http://ethesis.helsinki.fi>

The author: e-mail: kullervo.rainio@pp.inet.fi

The simulation programs are available in .bas and/or in .exe files

Contents:

Page:

Abstract	
Chapter 1. Introduction	7
1.1. Historical background	7
1.2. Epistemological basis	8
1.3. Lewinian conceptual system in a nutshell	9
1.4. Corrections to the Lewinian system	13
Chapter 2. Basic concepts of Theory of Cognitive Process (TCP)	19
2.1. An elementary example	19
2.2. Basic concepts	23
2.3. The cognitive process. Basic assumptions	27
2.4. Cognitive process. Flow chart	28
2.5. Stochastic variables. Learning	29
2.6. Life space. Valence. Field. Potency of field	32
2.7. Behavioral and cognitive dependencies	42
2.7.1. Behavior dependencies	42
2.7.2 Cognitive dependencies. Communication. Group problem	46
Chapter 3. Experiment on Problem Solving by Group and Its Simulation	50
3.1. Group Maze. The experimental set-up	50
3.2. Groups studied. Empirical results	54
3.3. Simulation program	60
3.4. Simulation results. Goodness-of-fit	63
3.4.1. Homogeneous groups. Parameter values and results	63
3.4.2. Unhomogeneous groups. Parameter values and results	70
3.4.3. Fictive groups. Variation of parameters	73
3.5. The problem of roundabout route	76
3.6. Discussion	82
3.6.1. Changes of the Actual Life Space	82
3.6.2. Simplifications	83
3.6.3. Further studies	84
Chapter 4. Heider's balance theory in a mathematical form and its simulation	86
4.1. Heider's conceptual system	86
4.2. Experiment by Esch	87
4.3. Tendency to reach cognitive balance	88
4.3.1. Actual Life Spaces in Esch's experiment	88
4.3.2. Assumptions	90
4.4. Simulation of the experiment by Esch	92
4.4.1. Program	92
4.4.2. Results	94
4.5. Discussion	95
Chapter 5. Further analyses	98
5.1. Chains of cognitive trials. Reality testing. Fancying	98
5.2. Some features of thinking. "Anchor states"	100

5.3. Content of cognitive state	104
5.4. Perception. Forming cognitive maps	104
5.5. Communication	108
Chapter 6. Philosophical perspectives	111
6.1. Psycho-physical problem. Eccles' theory	111
6.2. Eccles' theory formalized	115
6.3. Free will. Commitment	116
6.4. Dynamic properties of the world; a comment	118
6.5. Conclusion	121
Bibliography	122
Appendices	123 - 160
Appendix 1. The Monte Carlo method	
Appendix 2. The assumptions concerning the success of real trials when the environmental change is taken into account	
Appendix 3. Learning operators	
Appendix 4. Flow list of Turbo-Basic program for Group Maze -simulation	
Appendix 5. Group Maze -simulation; basic program	
Appendix 6. Roundaboutroute problem. Program block	
Appendix 7. Simulation of Heider's balance theory; experiment by Esch; basic program	

1. Introduction

1.1. Historical background.

In the last decades, when there has been a great progress within sciences near psychology - neurology, cognitive science, technology of artificial intelligence *etc.* - psychology itself has somehow lost its identity. Most clearly this comes out as a belief that the human consciousness could be built up as a combination of some neurological processes, *i.e.*, of its parts. (Notice the serious speculations of some radical cognitive scientist - Daniel C. Dennett *et al.* - about "conscious machines".)

On many specialized fields in psychology, there is lot of ingenious work going on - no one can neglect it - but we are, it seems to me, more and more in the danger of losing the *holistic* conception of human personality. It is, however, just this holistic view that has made psychology a philosophically extremely significant science. The findings of gestalt psychology in the begin of 20th century changed radically the whole epistemological basis of our thinking. No mechanistic thinking about human behavior and cognition was anymore possible. The man is since then seen as *active* in all his relations to the outer world, in his perception, in his imagination, in his thinking *etc.* And, particularly, person was seen as a *whole*, as a being who makes his *choices*, freely, meaningfully, and creatively.

This was the background on which Kurt Lewin was then building his "dynamic psychology". In conceptualizing the "psychological force" he made just the whole person as the "point of application of force", *i.e.*, it was the person as a whole who made the choices between the behavioral alternatives, not some "inner mechanisms" of neural excitation in him (as some radical thinkers in artificial intelligence -study seem to speculate).

The Lewinian psychology flourished over a period of 1935 - 1950 and had a great impact to social psychology ("group dynamics") but disappeared then gradually. Why? - Only some peripheral ideas of Lewin were used as "theorets" for experimental laboratory studies. Kurt Lewin himself, however, had published his theoretical framework in a very strict and systematic way in his book "The conceptual representation and the measurement of psychological forces" (Lewin, 1938). Why did not his work become a basis for creating psychology toward an exact, developed science?

Maybe sociology of science could give some answer. Perhaps Lewin's mathematical orientation was experienced strange in the humanistic culture, a part of which psychology has been seen. Actually, there was in 1960's a lot of use of mathematics in psychology but it was mostly directed to the methods of measurement, to advanced statistical analysis but not so much to analysis of psychological processes in large. In that time, it was a fashion to build "mathematical models" in psychology of learning and in social psychology. (See, *e.g.*, Bush and Mosteller: Stochastic models for learning (1955) and summarizing articles by Seymour Rosenberg and Robert Abelson in The Handbook of social psychology (1968).) However, the applicability of those models - included my own, concerning learning of social contacts (Rainio, 1962) - was fairly narrow. No total "life space", in the Lewinian sense, was taken into account.

Kurt Lewin had a dream. In the Preface of his book mentioned above he wrote (August 3, 1938):

"Psychology at the moment is rich with more or less 'general approaches'. However, more important for psychology today than general approaches is the development of a type of 'Theoretical Psychology' which has the same relation to 'Experimental Psychology' as Theoretical Physics has to Experimental Physics. Theoretical Psychology then cannot be satisfied with

generalities (however correct they might be) but has to supply specific means of solving the concrete problems of the laboratory and the clinic.” (p. 4)

Lewin intended to build his conceptual framework as a core of such a coherent and strictly systematized ”theoretical psychology”. His ideal was a system as exact as in theoretical physics, but what psychology needed most was its *own* conceptual system. This need led Lewin to his ingenious innovation: he used special geometry, the ”hodological space”, in describing ”psychological locomotion”.

But the dream of Lewin did not come true. And we have not fulfilled the need of the general coherent theory in psychology, in spite of the efficient computing instruments we now have in use - just because there is no exact conceptual system for it.

It seems to me that the quick advance of the computer calculation has merely misled us - misled to handle statistically more and more great amount of details with the consequence that the holistic view of human being as a willing, choosing subject has been forgotten.

Psychology is still searching its identity as a science. That means that its impact to philosophy - once so great - may gradually grow weaker.

I started my research work in psychology by building stochastic models for social contact-making and opinion formation as well as testing them in laboratory. (See, *e.g.*, Rainio, 1961, 1962 and Abelson, 1968.)

Encouraged by good experimental results, I gradually enlarged the application of those models and created a stochastic cognitive model for group problem solving (Rainio, 1970, 1972, and 1986).

At this stage of my study I realized that I actually had modernized Lewin’s theory. The modifications were, however, so radical that it took time to see the connection between both frameworks. (Rainio, 1983)

It came clear that for the lack of great theory in psychology we should not put the blame on the weak interest of the psychologists, only. There were also serious weaknesses in Lewin’s conceptual system. Actually, it has characteristics which make impossible to derive any estimates for behavior from it. Those features are corrected in my study, as we shall see soon.

I can not avoid here to cite Albert Pepitone, a famous student of Kurt Lewin. In a congress of the European Association of Experimental Social Psychology in Tilburg, Netherlands, 1980 [[??]], in discussion after my paper concerning stochastic field theory of behavior, he said: ”Something like that Kurt Lewin had in his mind just before his early death.”

1.2. Epistemological basis

Maybe one important reason among the psychologists to avoid the direct holistic study on cognitive processes - and nowadays so often to approach them by using neurological method instead of the purely psychological investigation - is the tendency to see introspection ”old-fashioned” and unscientific. The emphasis in psychology has been long time on the empirical methods and the introspection regarded very weak as such, merely a speculative way of thinking. Phenomenology - where it still had been used - has suffered for the low status of introspection. It has been seen more a part of philosophy than a science.

Naturally, one may not build a theory totally on introspective method but, to test the consequences derived from the theory, it may prove fruitful and necessary. It *is* one way of scientific observation.

But the introspection, surely, is not the only way to study cognition psychologically. Indirectly it can be studied through the observations of behavior, in a more ”objective” way, *i.e.*, in the intersubjective way (”cognitive etology”). This possibility exists seemingly only in very limited,

simple, and strictly controlled laboratory circumstances. It is true, but it is exaggerated verificationism to require that every theoretical variable and assumption need to be defined operationally. It is epistemologically right and enough if the structure of the theory is logically conclusive and there are possibilities to create such empirical set-ups where the theory could be applied - although in somewhat simplified form.

To my mind, it seems that the Theory of Cognitive Process with its empirical consequences fulfills these epistemological requirements.

1.3. Lewinian conceptual system in a nutshell

To understand historically the starting points of this study it may be informative to examine the Lewinian system and, particularly, its weaknesses in detail.

The basic invention of Lewin was to introduce a new geometrical solution, the "hodological space" ("hodos", a Greek word meaning "way") to describe psychological occurrences, instead of the Euclidean geometry, used in Newtonian physics. Lewin realized clearly that what is relevant in describing the behavior is not the perceived physical changes, not at all, but those states of individual which could be separated of each other on the basis of their psychological *meaning*.

"... The purpose of hodological space", writes Lewin, "is to find a type of geometry which permits the use of the concept of direction in a manner which will correspond essentially with the meaning that direction has in psychology." (Lewin, 1938, p. 23)

The hodological space is a *discrete* presentation of space. Instead of Euclidean continuum of points it is constituted of *regions* with boundaries between them, some regions being "neighboring" each other (*i.e.*, having common boundaries) some not. The wholeness of all regions is called "*life space*". (We should emphasize that, thus, the regions have their meanings only as parts of the life space.)

The psychic *locomotion* of an individual (or behavioral change) occurs from one region to another over a boundary. (The physical locomotion or change one may fancy to happen "inside a region" is thus, naturally, irrelevant. It simply does not exist in this representation. Only the transit from one *state* of behavior to another, *i.e.*, from one region to another, is relevant from the psychic point of view.)

Lewin defines the "distinguished path" as the shortest way from a region to another. This means a queue or a series of neighboring regions, *w*, as shown in Fig. 1.1.

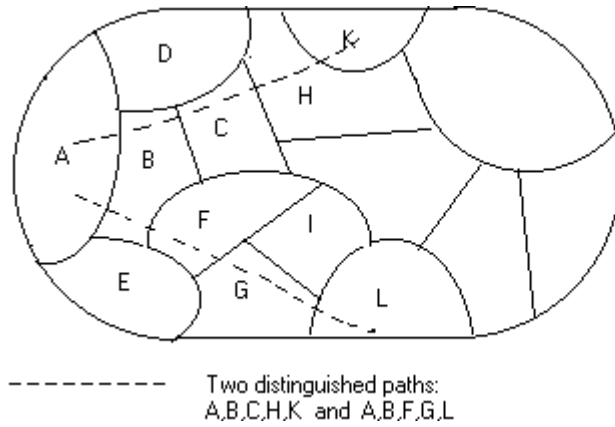
The *direction* from a region to another (from A to K) is then defined as *the first step* (A,B) on the distinguished path. Now, the direction (notated by Lewin as *d* with indices, *e.g.*, $d_{A,K}$) from one region (A) to another (K) is the *same* as the direction toward a third one (L) if the *first step* on the distinguished paths (from A to B in both cases) is the same.

The *psychological force* is now defined effecting in life space as a cause of locomotion of the *point of application* (the person).

A force has: 1) direction, 2) strength, and 3) point of application.

Thus, using Lewin's notation, the psychological force $f_{A,B}^P$ means "a force in A, which has the direction $d_{A,B}$ and P as point of application" (Lewin, 1938, p. 83). (Fig.1.2)

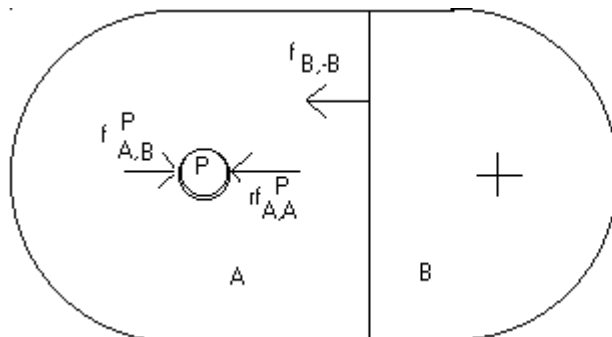
Fig. 1.1. Life space and regions in it.



Lewin uses notation $|f_{A,B}|$ for the *strength* of the force.

(Lewin gives no exact definition for "point of application" of a force. He writes that P, point of application, is "located" in a region A, *i.e.*, $A \supset P$, which means that P is itself a region subsumed to region A. But it is a special region, a region which the psychological forces try to move toward certain direction, *i.e.*, to *restructure* the life space in a certain way. - Usually the meaning of

Fig. 1.2. Life space with regions A and B and forces acting on person P.



the point of application seems clear, it is a person located in a region and according to Lewin the force influencing the behavior of P can be expressed by notation $f_{P,B}$ as well as $f_{A,B}^P$. However, we shall see later that this is a foggy point in Lewin's thinking.)

"A force can influence the behavior of an individual only in regions where the force exists" (Lewin, 1938, p. 75).

There are 3 types of forces (see Fig. 2):

1) Driving force, notated as $df_{A,B}^P$ if needed. It is a force applied to person P, located in the region A, driving him toward the region B.

2) Restraining force, notated as $rf_{A,A}^P$ if needed, or simply $f_{A,A}$. It forces the person P to stay on the same region A where he is already located. Thus, it opposes the driving forces.

3) Boundary force, notated as $f_{B,-B}$, is keeping the person out of the region B when he is in the "boundary region". It "corresponds to the tendency to locomote from B to a region X outside B, for instance to A. The force $f_{B,-B}$ exists, therefore, at least at the common boundary of A and B" (Lewin, 1938, p. 75). When Lewin uses this concept he seems to think that there exists a kind of obstacle between the regions. However, it seems to me strange to see this obstacle as a force, particularly because the boundary is *not* a region. - The concept of boundary force needs clarification.

An essential concept is the *resultant force*, which Lewin denotes by f^* . The resultant force determines the direction of locomotion. Therefore it is extremely important to show how the resultant is derived from the totality of forces applied to the person. Lewin does not succeed in this. He gives a definition of "totality of forces" in a region A:

$$\Sigma f_{A,X} \equiv f_{A,B} + f_{A,C} + \dots + f_{A,N}$$

Lewin gives no clear definition of resultant force. He writes: "The resultant of a group of forces can have the same direction as one force of the group. ... For instance, it might be that $\Sigma f_{A,X} \equiv f_{A,B} + f_{A,C} + \dots + f_{A,N} = f_{A,B}^*$." (Lewin, 1938, p. 85)

In connection to resultant force Lewin introduces a measure for locomotion, namely the concept of *velocity*. If there exists a resultant force $f_{A,B}^*$, *i.e.*, the strength of it is > 0 , formally: $|f_{A,B}^*| > 0$, then the velocity of locomotion is also > 0 , formally $v_{A,B} > 0$.

As far as I can see, Lewin thinks that the resultant force is the biggest of the forces acting in the same time in the same region. According to him, in a certain situation there is one possible resultant force and the behavior could be exactly determined if this could be found out. But Lewin had not such a method. According to him, there exists a resultant force $f_{A,B}^*$ if there appears the locomotion from A to B *afterwards*. He shows no method to *estimate* which direction the locomotion takes. This ruins the usefulness of Lewin's system as an experimentally testable theory. This weakness in the system is understandable only on that ground that Lewin actually tries to create a *deterministic* theory of behavior and, thus, is doomed to failure in his attempt. A radical change at this essential point, as we shall see, is necessary.

The origin of a force is, according to Lewin, a specific characteristic of region, namely the *valence* of it. Lewin gives the definition:

"A region G which has a valence $Va(G)$ is defined as a region within the life space of an individual P which attracts or repulses this individual. In other words:

Definition of positive valence:

If $Va(G) > 0$, then $|f_{P,G}| > 0$."

"Definition of negative valence:

If $Va(G) < 0$ then $|f_{P,-G}| > 0$."

Lewin continues: "The concept of valence ... does not imply any specific statement concerning the origin of the attractiveness or the repulsiveness of the valence. The valence might be

due to a state of hunger, to emotional attachment, or to social constellation... The statement that a certain region of the life space has a positive or negative valence merely indicates that, for whatever reason, at the present time and for this specific individual a tendency exists to act in the direction toward this region or away from it.” (Lewin, 1938, p. 88)

The valence induces a *force field* which reaches more or less all regions in the life space creating in them a force toward the goal region, G , if G has a positive valence, or away from G, if the valence of G is negative. (See fig. 1.3 and 1.4.) ”As in physics, the forces of a force field are only conditional ones; they are those forces which *would* exist in a region if the individual should be located in this region.” (Lewin, 1938, p. 90)

Fig. 1.3. Force field with a positive valence

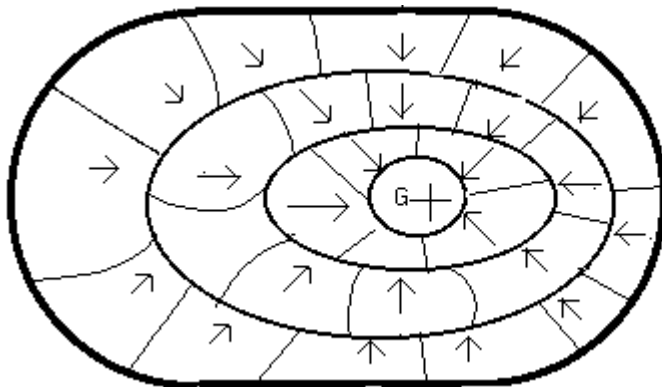
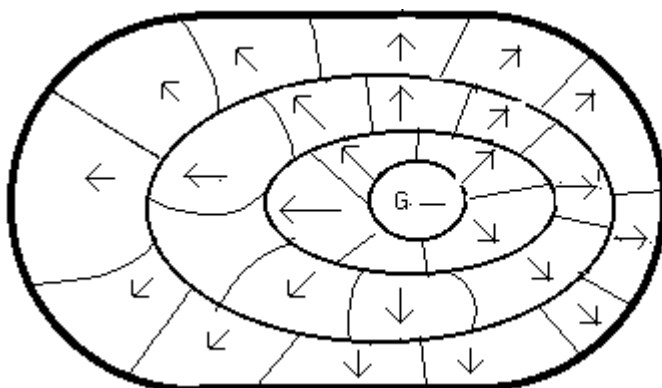


Fig. 1.4. Force field with a negative valence



Lewin makes an additional assumption that the strength of the force in a force field is bigger near the valence region than far from it (measured in number of steps to reach that valence region). (This is illustrated in the figures 1.3 and 1.4 with the length of piles.)

As far as I can see, the Lewin's invention of valence is very suitable just to the description of *intentional* activity, goal-directed behavior.

Lewin does not see the boundaries only as separating regions from each other but gives to them characteristics that are not derived from the formal system of hodological space. They are also obstacles. In spite of the boundary forces mentioned above, the boundary is seen as a *barrier* restraining the locomotion over it. According to Lewin "the *strength of a barrier* is characterized by the maximum driving force which can be applied to the barrier without breaking it" (Lewin, 1938, p. 125). "Breaking it" is a rather curious, even foggy expression in this context. Seemingly Lewin means that the activity a driving force causes is not enough for locomotion over the boundary. It causes only a trial but the trial must also succeed. As far as I can see, the "breaking the barrier" means, thus, succeeding in this trial.

Lewin's attempt to create a conceptual system for holistic psychology has serious weaknesses, as we have seen. Still one should be added to them: Lewin makes no clear distinction between behavioral and cognitive concepts. In the same description he can be dealing with "breakable barriers" or "food" as a goal or "locomotion" in physical sense but also with "driving force" or "valence" which are clearly concepts describing cognition. His expressions are sometimes a kind of mixture of behaviorists and "heterophenomenalists" (see, *e.g.*, Dennett, 1991) use of language. Often the "conceptual representation" by Lewin has an illustrative value only.

Why then notice Kurt Lewin's work at all? - My answer is that Lewin, however, understood very deeply the nature of intentional activity. One can find the points where Lewin's system needs to be corrected (sometimes rather radically, I should say) but, fortunately, also *can* be corrected. That means that we can get of it this way a modern, useful, effective conceptual instrument for theoretical analysis of cognition and behavior. In the following text we shall see the principles how these corrections should be made. In Chapter 3, the concepts and the assumptions of the Theory of Cognitive Process (TCP) will be represented in a more systematic way.

1.4. Corrections to the Lewinian system

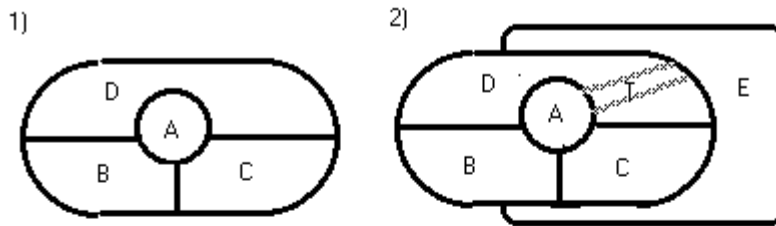
1.4.1. Hodological (topological) representation

In the Lewinian system, the first feature which needs to be changed is the hodological (or topological) representation. The structure of "life space" (*i.e.*, the "neighborhood relations" of regions) is more conveniently represented in a matrix form - and illustrated in graph form if needed. An example of this is shown in Fig. 1.5.A. If we should add to the life space of A,B,C, and D a new region E and it is required that it should be a neighbor of all these regions, the only one solution is that we use a "tunnel" connecting E and A. The matrix form is given in Fig. 1.5.B where the names of rows and columns refer to the "regions" and the elements 1 and 0 to their existing or nonexisting neighborhood.

From the point of view of theory-building this change is, naturally, trivial.

Fig. 1.5. Topological, matrix, and graph representation

A) Topological (hodological) representation



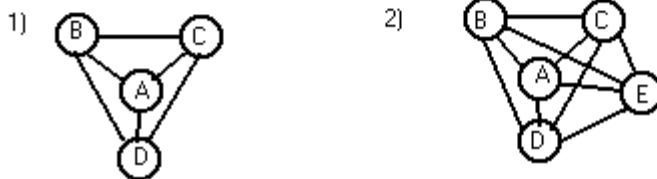
E = neighboring region of A,B,C, and D

T = "tunnel"

B) Matrix representation of 2). 1 = neighboring

	A	B	C	D	E
A		1	1	1	1
B	1		1	1	1
C	1	1		1	1
D	1	1	1		1
E	1	1	1	1	

C) Graph representation



1.4.2. The concept of a resultant force

The concept of psychological force acting in a hodological space was most essential in the Lewinian system - Lewin's central innovation. However, when Lewin, in determining the direction of psychological locomotion, needs the concept of a resultant force, he uses it in a way which makes his system *strictly deterministic*, as mentioned above.

As far as I can see, it is necessary to leave out the concept of a resultant force. *The "psychological forces" should be represented as probabilities.*

To make the use of probabilities possible, we shall handle *also the time variable as discrete, in addition to the discrete space.* The probability of an event will, thus, always mean the probability of it *during a step in time* - the step in time being the unit of time measure.

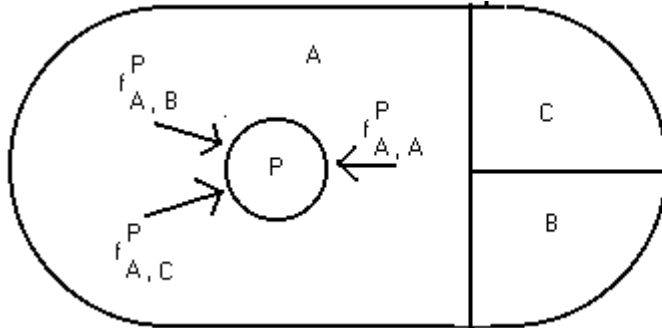
The "psychological force" will be defined as the probability to make a trial during a step in time.

The direction of "locomotion", *i.e.*, the trial alternative, is then determined as an *outcome* from a probabilistic choice using the vector of the trial probabilities in the situation concerned.

An example of this correction is shown in Fig. 1.6.

Fig. 1.6. Resultant force concept

Lewin: $f_{A,C} > f_{A,B} > f_{A,A} \rightarrow f^*_{A,C}$



Correction: Vector $\mathbf{p}_{A,X}$

- Pr[A,B]
- Pr[A,C]
- Pr[A,A]

(No resultant force. Probabilistic choice using vector $\mathbf{p}_{A,X}$ determines the locomotion.)

By this very important revision the Lewinian system loses its deterministic character and we shall be dealing with a stochastic one. This gives an opportunity to use the theory for *estimation* purposes. Not until now the conceptual system becomes a real theory.

1.4.3. "Point of application of forces"

Lewin emphasizes in his work the need to see the individual as a *whole* in analyzing the psychological situation. Thus, "the point of application of forces" in Lewin's representation is the person. Lewin shows this using a special symbol with an index for the formal symbol of force, e.g., P in $f^P_{A,B}$.

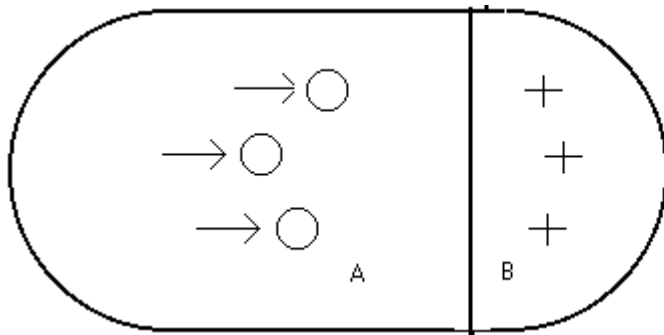
It is, however, remarkable that Lewin forgets all this completely in his discussion of the "inner personality", particularly in his presentation of a "relation between a force for locomotion acting on the person as a whole and the tension in inner systems" (Lewin, 1938, p. 98) He uses in this context, e.g., the expression

$$f_{b_S, S^1}$$

indicating "a force working on the boundary b of S in the direction of the neighboring system S^1 " (Lewin, 1938, p. 99). Which is now the "point of application of forces"? If it is an "inner system" of a person, is it meaningful to discuss the "life space" of such a partial system?

It seems to me that, in forgetting the problem of point of application of forces, Lewin commits a fatal error. Some "group dynamists" are guilty of the same kind of an "inventory error" when they describe forces at the same time both on the group and on the members of that group (e.g., J. P. French Jr.)

Fig. 1.7. A group in a "common" Life Space; an erroneous description



Correction:

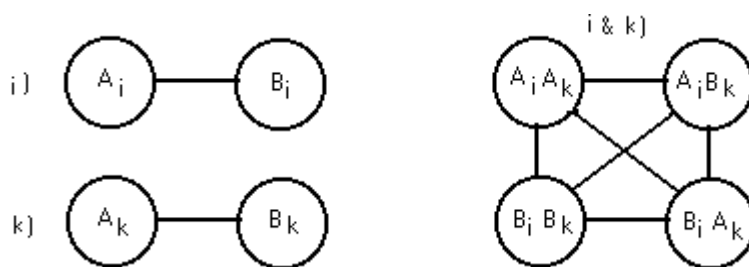
Matrix form:

	i)			k)	
	A_i	B_i			A_k
B_k					
A_i	1	1		A_k	1
1					
B_i	1	1		B_k	1
1					

i and k)

$A_i A_k$	$A_i B_k$	$B_i A_k$	$B_i B_k$	
$A_i A_k$	1	1	1	1
$A_i B_k$	1	1	1	1
$B_i A_k$	1	1	1	1
$B_i B_k$	1	1	1	1

Graph form:



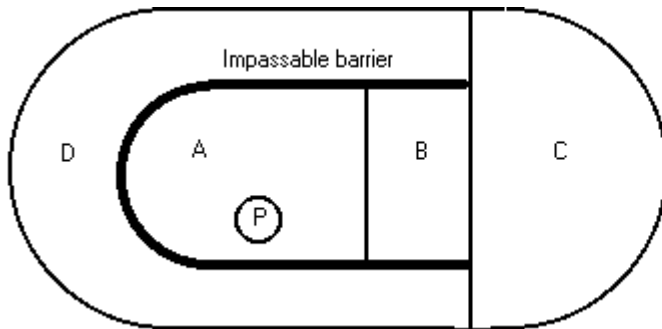
A simple example in Fig. 1.7 shows the point. The only way I see to solve the formal problem and avoid the confusion is to keep the "life spaces" of different subjects totally separate from each other and to form the space where the behavior of the aggregate is represented by defining it as a graph which includes all possible combinations of the vertices (regions) of the separate spaces as the vertices in it. (A matrix, analogous to the graph, is also shown in Fig. 1.7.)

1.4.4. Barrier strength

The concept "strength of a barrier" by Lewin has nothing to do with topological (or hodological) representation. (See Fig. 1.8) It has an illustrative character only.

Fig. 1.8. Barrier strength

Lewin:



Correction:

Matrix form:

	A	B	C	D
A	1	.9	0	0
B	.8	1	.6	0
C	0	.5	1	.3
D	0	0	.6	1

Lewin seems to have touched here, intuitively, on an important topic, the resistance of the psychic locomotion. But one can ask why he has not applied the idea more systematically to *all* boundaries between the regions.

It seems convenient to define - in analogy to the strength of a barrier - a particular concept "probability of the success of a trial" and apply it to *all* the relations between the graph vertices (regions). Thus, the matrix showing the relations between the regions in the life space (in graph representation: the matrix of the weights of edges) will have a new meaning: the probability values show the "strength of the barrier" between them. (We shall call them "success probabilities".)

1.4.5. Cognitive and "real" world

One of the most confusing aspect in Lewin's representation is his tendency to sometimes describe cognitive phenomena and sometimes the so-called "reality" by using the same terms and without formally making any clear distinction between them. In order to create an *exact* theory in the modern sense, *i.e.*, a programmable one, it is, however, quite necessary to make this distinction, as clearly as possible.

In the theory of cognitive process (TCP) as I should call the system described in Chapter 4, the psychic processes will be presented as occurring, naturally, in the cognitive space. The "regions" (the vertices, "points", in our graph representation) stand for *cognitive states* of the subject. The psychic forces act always and only in this cognitive field - activating cognitive trials.

The graph is also used for the "real" world (*i.e.*, for the world seen by an observer and interpreted according to the psychological - behavioral - relevance). Thus the objective states of an individual - relevant to us from the psychological point of view - are represented as vertices of a graph but this representation has no *a priori* linkage to the cognitive world of the person in question. The linkage is given by a particular concept of "correspondence", *i.e.*, in the form of a correspondence probability matrix where the rows indicate cognitive states, the columns real world states, and the elements the probabilities of these states corresponding to each other (see the Table 1.1).

Table 1.1. Cognitive and "real" world

Matrix of correspondence probabilities:

Cognitive states	States in "real" world				
	r ₁	r ₂	r ₃	...	r _n
c ₁	Pr _{1,1}	Pr _{1,2}	Pr _{1,3}	...	Pr _{1,n}
c ₂	Pr _{2,1}	Pr _{2,2}	Pr _{2,3}	...	Pr _{2,n}
...
c _m	Pr _{m,1}	Pr _{m,2}	Pr _{m,3}	...	Pr _{m,n}

It needs to be emphasized that the content of the correspondence matrix is left totally open. In some cases, *e.g.*, in game situations with clearly defined rules, the probabilities might equal 1 or 0. The correspondence is then complete. An opposite case to that should be a matrix with totally homogeneous probability vectors where the cognitive and real states correspond to each other randomly.

Actually we shall take in our use two kind of correspondence matrices:

1) RCC - (reality-cognition) -correspondences, probabilities indicating which "cognitive map" and state in it correspond to a certain real situation. ("Cognitive map", see Chapter 4.)

2) CRC - (cognition-reality) -correspondence, probabilities indicating which is the real state to which the subject's certain cognitive state refers, *e.g.*, which behavioral movement follows a certain decision to move.

Above we have seen how, in principle, the *psycho-physical problem* can be solved formally in our system. Thus, our theory is not based on any neurophysiological hypotheses. In mathematical analysis of cognition and behavior on our abstract level it is not needed. We shall see that it is not needed in applications either. (In Chapter 6 we shall, however, discuss briefly neurophysiological solution of the psycho-physical problem. There will be given a new rather interesting interpretation of CRCor -probabilities.)

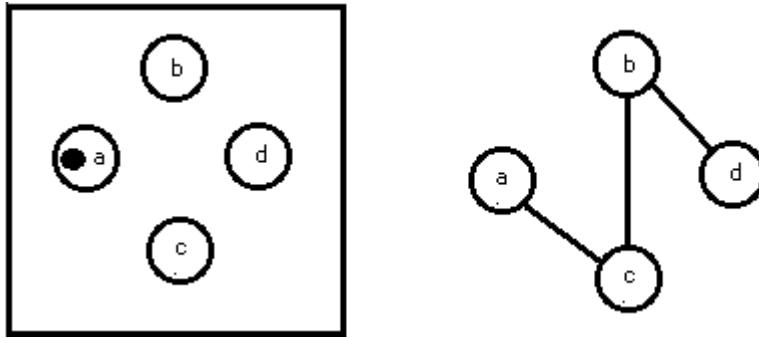
2. Basic concepts of Cognitive Process

2.1. An elementary example

Before systematic and abstract presentation of our conceptual framework, it may be illustrating to describe individual's behavior in a very simple maze and his cognitive process corresponding to it.

Let us observe the game represented in Fig. 2.1.A - C.

Fig. 2.1. The elementary example. Four Position Game



A) The game board.

B) The problem in graph form

	a	b	c	d
a	0	0	1	0
b	0	0	1	1
c	1	1	0	0
d	0	0	0	1

C) The problem in a matrix form

The graph in A presents the game board and shows 4 positions and one piece on the position **a**. In B a graph is shown which is used only by the experimenter and hidden to the players. The lines in that graph mean the allowed ways to move the piece. Thus, moves from **a** to **c**, from **c** to **b**, and from **b** to **d** are possible, only.

The instruction to the player:

"There are 4 different positions in the game: **a**, **b**, **c**, and **d** (Fig. 2.1. A). There is a piece at the position **a**. You should move your piece to some of the other positions, *i.e.*, to **b**, **c**, or **d**. You do not know yet which of the moves are allowed. After you have placed your piece on some position you are informed whether the move was allowed or not. When you move to **d** you get 1 point for that move and for all moves left. Namely, there are 10 moves altogether available."

After the first move the experimenter uses the graph representing the problem (Fig. 2.1 B) and tells whether the move was allowed or not. The lines in the graph mean the allowed transitions.

The experimenter continues the instruction:

(In the case of a failed move:) "The move did not succeed. You have to continue from the position **a**."

(In the case of a succeeded move:) "This move succeeds. You are now on the position **c** and may continue from that."

When the move toward **d** is made the experimenter instructs:

"Now you have reached the position which gives you 1 and" (in case this was the 6th move) "4 points for the moves left. Because you can keep the piece in **d**, the game is now in its end and we simply sum the points over all moves. You got 5 points. - Now we start again from the position **a** and you may make your move."

The game is repeated then several times and points scored for each game.

Actually, as far as I know, this (boring) game has not been experimented. It is used here only as an example simple enough to illustrate the main structure of the conceptual framework which is later given more systematically and in detail.

In behavioristically oriented learning psychology some groups of individuals were tested and the errors summed up for the first game, second game etc. A mathematical law for the means of the decreasing of the errors would be found and - that's enough.

In our conceptual system, the behavioristic description would use the following concepts:

A real state **a** of an individual **i** at time point **t**. Formally we write that:

$Be(i,a,t)$

A trial of the individual from a certain real state, say **a**, toward another, say **b**, at time point **t**.

Formally this written:

$Tr(i,a,b,t)$

Such a trial succeeds – formally: $Succ(Tr(i,a,b,t))$ – or fails – formally $\neg Succ(Tr(i,a,b,t))$.

If the trial succeeds, the individual **i** will behave at the next point in time by the way **b**.

Formally: $Be(i,b,t+1)$.

If the trial does not succeed, the individual **i** will behave at the next point in time by the way **a**, *i.e.*, stay in the state **a**. Formally: $Be(i,a,t+1)$.

We need only these 3 concepts – state of behavior, trial, and the success of trial – to describe the behavioral process, the "locomotion" in psychological sense.

The success and the failure of a trial is determined by the problem matrix (Fig. 2.1 C) where the numbers just indicate the probabilities of the successes of trials, but how to determine which trial occurs at each step in time?

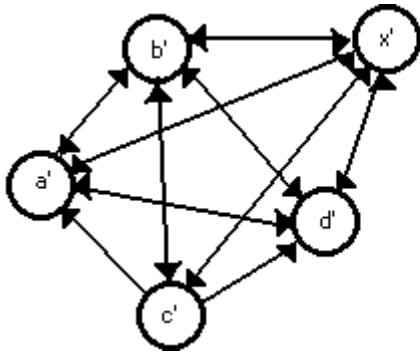
There is a pure behavioristic way to compute from a large group of individuals the relative frequencies of trials from each position toward the others, from **a** to **b**, **c**, and **d** (in the beginning), *e.g.*:

	a	b	c	d
a	0	.1	.1	.8

It is natural to assume now that these relative frequencies correspond to the trial probabilities of an individual from the state **a** toward the other states. We can, namely, use the means of the relative frequencies of moves as estimates of these probabilities. (From game to game these probabilities then change according to some mathematical learning law. Those are easily derived from the empirical results, too.)

There is in our simple example no big difference between the behavioral and cognitive description of occurrences. We shall, however, analyze the cognition in this trivial case, just to show how some basic concepts of our theory are applied.

Fig. 2.2. The cognitive states and the trials, the cognitive map



A) The cognitive map in graph form. The cognitive states and transition possibilities.

	a'	b'	c'	d'	x'
a'	1	1	1	1	1
b'	1	1	1	1	1
c'	1	1	1	1	1
d'	1	1	1	1	1
x'	0	0	0	0	1

B) The cognitive map in the matrix form

	a'	b'	c'	d'	x'	Σ
a'	0	.099	.099	.8	.002	1
b'	.05	0	.15	.8	0	1
c'	.05	.15	0	.8	0	1
d'	0	0	0	1	0	1
x'	0	0	0	0	1	1

C) The probabilities of the cognitive trials, in the beginning

It is assumed in this simple case that the cognitive representation of the problem situation is in the one-to-one correspondence to the real situation ("real" meaning, as mentioned in the Introduction, psychologically adequate description of the situation). The individual *i* sees himself in this structure situating in a state **a'** which corresponds to the initial state in the game, **a**. This assumed representation of the problem situation in *i*'s cognition we shall call a "cognitive map". This is shown in Fig. 2.2.

There is, however, an additional state **x'** in the cognitive map. Why? – It is assumed in our theory that a person, when accepting the rules of the game, limits his cognitive action taking into account only the states relevant to the problem situation. But he does not do that totally because this decision is made by his (free) *will*. That means that he is also able in any time to "step out from the field", *i.e.*, to refuse to behave or act cognitively according to the rules. To keep this freedom in our framework we need the state **x'** (and the probability > 0 of the cognitive trial to transit into it, as

well as a probability > 0 of succeeding in this trial). Practically this assumption has not much effect because the people use to follow the rules very regularly in simple game and test situations.

The cognitive locomotion, *i.e.*, transiting from one state to another, is assumed to occur in the beginning without any limiting factor, because its natural to think that **i**, knowing nothing of the possible obstacles, do not see any reason why a certain locomotion should not be possible. (If there were such reasons we should take it into account by giving to the probabilities values other than 1.)

The success possibility which is shown by lines in the directed graph representation in Fig. 2.2. A is described by probabilities 1 in the matrix form in Fig. 2.2. B.

In Fig. 2.2.C the probabilities of making cognitive trials, in the beginning, are given. The numeric values of them are estimated intuitively (because we do not have any empirical results on which to base them). Those values are changing after each trial according to learning.

The vector **a'** (the row **a'**) determines probabilistically the first cognitive trial. (In simulation that means that we should use the so-called Monte Carlo -method to select the move by drawing lots. (See the Appendix 1 to get information about MC-method in detail.)

Because, according to instruction, **d** is the state which is rewarding with the 1 point score, it is natural to think that the probability for making a trial to this direction is high (assumed to be .8). The probability of trial toward **x'** is assumed be very small (.002). This is in accordance to the experience that normal individuals break off the test very seldom. The probabilities of trials toward **b'** and **c'** are assumed to be equal to each other, .099, so that the sum equals to 1.

There are 3 alternative positions available in the beginning: **b'**, **c'**, and **d'**. Although **d'** is the most probable, let us suppose that **c'** has been chosen *i.e.*, $\text{Ctr}(i, a', c', t', t)$ occurs and **i**'s cognition is:

$C(i, c', t'+1)$. We can make now a simplified assumption that there follows a real trial as soon as a succeeded cognitive trial is made. (Later we shall abandon this simplification and study the decision making in detail.) Thus, **i** makes a real trial

$\text{Tr}(i, a, c, t)$

According to the matrix in Fig.2.1.B, this trial succeeds because the probability of success of that real trial is 1. Thus, **i** behaves in the way **c** at the time point $t+1$, formally:

$\text{Be}(i, c, t+1)$

The second cognitive trial will be then determined by Monte Carlo method using the vector **c'** of the matrix in Fig. 2.2.C. Suppose that the outcome of it will be **d'**. Thus, the cognitive trial $\text{Ctr}(i, c', d', t', t+1)$ occurs and **i**'s cognition is now $C(i, d', t'+1)$. It is followed by the real trial $\text{Tr}(i, c, d, t+1)$. According to the success probability matrix of the real trials (Fig. 2.1 C), this succeeds and **i** behaves in the way **d** at the time point $t+2$, formally:

$\text{Be}(i, d, t+2)$

We have assumed above only succeeding trials, both cognitive and real. What happens when a trial fails?

Let's suppose that the first cognitive trial is made toward **b'**, *i.e.*, formally $\text{Ctr}(i, a', b', t', t)$. This leads to the real trial toward **b**, *i.e.*, $\text{Tr}(i, a, b, t)$, which fails – according to the matrix in Fig. 2.1. C. That means that **i** will stay in the state **a**:

$\text{Be}(i, a, t+1)$

One of basic assumptions will be that there occurs a *learning* process after each real trial, a reward after a success and a punishment after a failure. This means that the probabilities used in the cognitive trial process (the success probabilities included) will be changed – according to some mathematical learning law; we shall apply the Bush and Mosteller's two operator model which will be described later. Usually we shall take the *transfer* in learning also into account.

Thus, without showing the computations, the two **a'** vectors may after a failed cognitive trial toward **d'** look out as follows:

The probabilities of success of cognitive trials:

	a'	b'	c'	d'	x'
a'	1	1	1	.2	1

The probabilities of cognitive trials:

	a'	b'	c'	d'	x'	Σ
a'	0	.299	.299	.4	.002	1

Now it may occur that a cognitive trial from **a'** to **d'** fails. That means that **i** stays cognitively in **a'**, formally: $C(i, a', t'+1)$. Notice the cognitive time point $t'+1$. The real time continues to be t and will have the value $t+1$ not until the real trial occurs.

This may be enough for illustrating the main features of our framework. The purpose of this elementary example was to define tentatively the basic concepts, in words and formally, as well as show the course of the cognitive and behavior processes and the formal connection between them. We have left out many important questions (how to estimate cognitive trial probabilities, what rules follow the decision making, which exact models to use for getting the learning phenomenon computable, *etc.*)

The simplicity of our elementary example may mislead some readers. It is necessary to emphasize that the report of the Group Maze experiment and its simulation will show the usefulness and the estimation power of our theory, as well as the Chapters 4 and 5 the flexibility of our conceptual system in forming applications.

In the next Chapter we shall present the basic concepts and assumptions systematically. We can not avoid some complicated formal expressions, because the exactness (and programmability) requires that, but the full interpretation using normal language is always given.

2.2. Basic concepts

In our theory, the state of the subject is described as his position at a certain vertex (point) of a graph at certain point in discrete time, the locomotion of the subject being determined by probabilities of trials and their success from one position to another in one step in time. Unlike in the Lewinian system, *both space and time appear as discrete variables.*

The basic concepts in our theory are the following:

Symbol:	Verbal description:
$Be(i, a, t)$	Behavioral state: i behaves in the way a at the moment t – in other words: i is in the <i>real</i> state a in a space of action states at t . Note 1: By "real state" we mean psychologically relevant situation of an individual, available for observation. It may be a state of eating, as well as seeking food. It may be a situation in a game – and then to be in a situation of waiting the opponent's move is psychologically a state totally different from the state of being in turn to move, although the situation on the board is physically the same in both case. – Writing is not a real state but "being writing" is! The pure occurrence of writing as a more or less motoric process is not relevant from our point of view, but "writing a letter" ("being writing") is, because it <i>means</i> some action seemingly corresponding to some meaningfully experienced cognitive situation by the person. The occurrence of not-writing <i>per se</i> is not relevant. But "not yet writing" – if one "should write" – is relevant, as well as "not writing anymore" –

if one has already ended the writing action. – If a child is writing letter A, this may be relevant (if we are interested in his canning and uncanning).

Just taking one more example: "Walking" *per se* is not relevant but "to be on walking" or "walking toward some place" may be (if we are analyzing some psychologically meaningful process where those actions are essential parts.)

When dealing with "real states", the term "relevant" means, thus, being a meaningful part of the process to be analyzed - meaningful in the way an observer sees it. (There arises a philosophical question: may the observer be the subject himself – not only an outsider? Why not? Naturally the "real state" seen by an outsider may be different than the "real state" as observed by the acting subject himself. That difference may in some cases be essential and had to be taken into account in theoretical modeling. If such difference exists, it is – according to our theory – merely the subject himself who should be seen as the observer. We shall, however, not handle this problem in our short presentation of the theory.)

C(i,a',t')
point Cognitive state: **i** imagines in the way **a'** at a cognitive time point **t'** – in other words: **i** is in the state **a'** in the space of cognitive states at the cognitive time **t'**.

Note 2:

A "cognitive state" may be "feeling hunger", "listening a violin concert", imagining a dinosaur, recognition of being in a situation were chess mate will come in to moves, *etc.*

Cognitive state is always a relevant part of the whole of the actualized consciousness of the subject. Certainly, consciousness is tremendously rich of details in every moment (*e.g.*, in perception of the environment). A totally free fantasy process may be so diffuse that in that it would be impossible to assume theoretically any separable "states" of cognition. (Through introspection that might be possible, in some cases. Notice, *e.g.*, the way one tells his dreams: The dream "moves" from one situation to another, the subject being somehow capable to see what is relevant in it and to separate such relevant parts – states – from each other.) – The free fantasy is too unique to be an object of theoretical analysis.

There is, however, choice and decision-making processes, general enough to be analyzed in theoretical way. In such cases, namely, the subject may *focus* his consciousness and eliminate all such content of his cognition that is not relevant to the choice process. With a quite great confidence we can then assume the relevant part of the cognition (the actual life space) to have a certain structure we can interpret as separable cognitive states and their relations. In some simple game situations, for instance, this is rather easily done: one can roughly assume that the real game situations and the cognitive states correspond to each other in one-to-one relations.

Tr(i,a,b,t) Behavioral trial: **i**, being in the state **a**, tries to behave in the way **b** at the time point **t**.

To be exact, we should use a separate expression for the trial aspect of **i**'s behavior, namely $tr(i,b,t)$. Thus, $Tr(i,a,b,t)$ means the conjunction $Be(i,a,t) \wedge tr(i,b,t)$. We shall use the shortened expression $Tr(i,a,b,t)$.

Ctr(i,a',b',t') Cognitive trial: **i** tries to make a cognitive choice **b'** at the cognitive time point **t'**.

This is also a shortened expression of: $\text{Ctr}(i, a, a', b', t', t)$ which is a shortened expression itself of conjunction: $\text{Be}(i, a, t) \wedge \text{C}(i, a', t') \wedge \text{tr}'(i, b', t') \wedge t \leq t' < t+1$. We shall use the shortened expression and leave the real state and real time out, if they are obvious.

$\text{Succ}(i, a, b, t)$ Success of a trial, a shortened expression of $\text{Succ}(\text{Tr}(i, a, b, t))$. Correspondingly, $\neg\text{Succ}(i, a, b, t)$ means failure of the trial $\text{Tr}(i, a, b, t)$.

$\text{Succ}'(i, a', b', t')$ Success of a cognitive trial $\text{Ctr}(i, a', b', t')$. $\neg\text{Succ}'$ means failure.

$\text{ECh}(a, b, t)$ Environmental change: the behavior alternative **b** is substituted for **a** at the time point **t** and at any other following time point.

Thus, if **i** behaves in the way **a** at the time point **t** and then the environmental change occurs at the same time point then the behavior of **i** is changed to **b** *without* any trial. – Because we are dealing in the applications of the theory primarily with controlled laboratory situations we shall use *constant environment hypothesis*. Therefore the environment change is not analyzed in this report in details. It is mentioned here only for reason of completeness of the theory.

$\text{NS}'(i, a_0', b', t)$ Cognitive success sum: number of consecutive successful cognitive trials by **i** toward **b'** from the state a_0' at real time point **t**.

The use of this variable is based on the hypothesis that the cognitive process leading to the decision to make a real trial is dependent on the success experienced in cognitive trials. The success sum (NS') is one way to measure that dependence, but certainly there might be others. We shall discuss this essential feature of our theory later.

$\text{AT}(i)$ Action threshold: an integer indicating the number of consecutive successful cognitive trials (NS') needed to start a choice action (a real trial).

This formal description implies that the threshold value for the individual concerned is constant over time (a "character feature" by **i**). High value of this constant means seemingly a cautious character, even a tendency to hesitate in decision-making, the low value of AT being a sign of a "laissez-faire" character. – We may emphasize here the flexibility of our system: If the constant hypothesis of AT comes out to be too rough, there is no hinder to handle AT as variable, *e.g.*, assume it dependent of time. In that case AT should be written as $\text{AT}(i, t)$.

Cognition and reality:

$\text{RCCor}(i, a, M', t)$ Reality-cognition correspondence: **i**, behaving in the way **a** at the time point **t**, chooses the *cognitive map* M' as the graph where **i**'s cognitive processes will occur.

The cognitive map resembles, for its structure, the graph of the real states but need not to be identical with it. In the case of free fantasy (starting when **i** is in the state **a**), we may assume it totally independent of the real environment. On the other hand, in a problem-solving situation, where **i** commits himself to the rules of the problem, the cognitive states of M' can be in one-to-one relation to the real states. Those are the extremities of the correspondence.

In a matrix form of M' , the rows and columns represent the cognitive states and the elements the success probabilities of the cognitive trials from one state (row) to another (column).

It is also assumed that i has some idea of his position among the real states, *i.e.*, some cognitive state, say a' , corresponding to the real state a , will be the starting state of the cognitive process.

How to determine which cognitive map will be taken in the use in certain situation, that will be presented in the next chapter.

CRCor(i, a', a, t) Cognition-reality correspondence: in i 's cognition, the cognitive alternative a' corresponds to the behavior alternative a at the moment t .

The correspondence between individual's cognition and the behavioral space (the totality of the relevant real states) will be given by a matrix of probabilities $p(\text{CRCor}(i, x', x, t))$, x' meaning the cognitive alternative and x the behavior alternative. How these probabilities determine i 's real trial, that will be presented in the next chapter.

(Taking the correspondence concept into use is our technical solution to the *psychophysical problem*. Our solution is only mathematical. It says nothing about that *how* in real sense this correspondence is understood. There will be a short philosophical discussion about this problem in Chapter 6.)

Cognitive and real time:

Real time points are denoted by $t, t+1, t+2$ etc. Analogously, $t', t'+1, t'+2$ etc. indicate points in cognitive time.

The relation between cognitive and real time is given by the following basic assumption:

Cognitive time is measured by taking a point in real time as the starting point and the sum of all cognitive time steps during the cognitive process leading to the next action equals one step in real time.

When dealing with one individual alone, we do not need to "measure" how many cognitive steps there might be in one step in real time. This question has no relevance in that case.

However, in group problem solving, for instance, because the common real time exists, we need to take into account a kind of "cognitive speed" – *i.e.*, we have to define how many cognitive trials there are available for each individual in one step in real time. For that we shall use formal expression:

$nt'(i)$.

Thus, we may, in principle, assume different nt' values ("cognitive speed") for different individuals. (Instead of treating nt' as a constant "character feature" of individual, we may, if necessary, assume that it is a function of time, too – $nt'(i, t)$. The "cognitive speed" may then, *e.g.*, "slow down" during the problem solving.)

The actual sum of the steps taken in the i 's cognitive time after the time point t in real time we shall denote formally by:

$\Sigma t'(i, t)$.

2.3. The cognitive process. Basic assumptions

The choice process of the individual is - in a stochastic way - determined by the following basic assumptions:

(In the following text, **Outc** means the outcome of a probabilistic choice by the Monte Carlo - method, using the given probability vector.)

(1) Individual **i** behaves in some way at the first moment of the real time considered. Formally:

$$(\exists a_0)Be(i, a_0, t_0)$$

Thus, the starting state is assumed to be given. The following states of behavior are determined by the rules of the process.

Note: A "passive" state is also assumed to be a way of behavior in this general sense.

(2) There exists for every state of behavior a cognitive map M' determined as an outcome by a probabilistic choice. Formally:

$$(\forall a)(\exists M') \text{Outc}(\mathbf{RCCor}(i, a, t)) = M'$$

The cognitive map is given in the form of two matrices: 1) The cognitive trial probability matrix $\mathbf{p}(\text{Ctr}(x', y'))$ where the elements are of the form $\text{Pr}[\text{Ctr}(i, a, x', y', t', t)]$. – 2) The success of the cognitive trial probability matrix $\mathbf{p}(\text{Succ}'(x', y'))$ where the elements are of the form:

$\text{Pr}[\text{Succ}'(i, a, x', y', t', t)]$. – It is assumed that M' gives also the information which cognitive state will be the starting point of the cognitive process (\mathbf{a}_0').

(3) The cognitive map determined, the cognitive process by **i** starts at the cognitive moment t_0' from the starting cognitive state \mathbf{a}_0' . Formally:

$$\text{Outc}(\mathbf{RCCor}(i, a, t)) = M' \rightarrow C(i, \mathbf{a}_0', t_0')$$

(4) There exists at every cognitive moment for every cognitive state, an outcome of probabilistic choice where the probability vector of making a cognitive trial is used. Formally:

$$(\forall t')(\forall x')(\exists y') \text{Outc}(\mathbf{Ctr}(i, x', t')) = y'$$

(5) If **i** makes a cognitive trial at the moment t' , an outcome of its success is determined by a probabilistic choice. Formally:

$$\text{Ctr}(i, a, a', b', t', t) \rightarrow \text{Outc}(\mathbf{Succ}'(i, a', b', t')) = x; x \text{ has two alternatives } \text{Succ}' \text{ and } \neg\text{Succ}'.$$

(6) *Simplified version of transition to real trial:*

If **i**'s cognitive trial from the cognitive starting state **a'** toward another cognitive state **b'** succeeds, **i** makes a real trial toward the corresponding state **b**. Formally:

$$\text{Succ}'(i, a_0', b', t') \rightarrow \text{Tr}(i, a, b, t)$$

Thus, our simplified version indicates that only one successful cognitive trial from starting state \mathbf{a}_0' is needed, *i.e.*, the action threshold $\text{AT}(i) = 1$, and the correspondence probability $\text{Pr}[\mathbf{CRCor}(i, b', b, t)] = 1$.

If the simplified version of transition to real trial is not adequate, the more *general version* shall be taken in the use. Thus, instead of the assumption 6, we shall use in this general version the assumptions 6A - 6D:

(6A) If **i**'s cognitive trial toward **b'** from the cognitive state \mathbf{a}_0' at the cognitive moment t' succeeds then 1 is added to the cognitive success'-sum of b' . Formally:

$$\text{Succ}'(i, a_0', b', t') \rightarrow \text{NS}'(i, a_0', b', t'+1, t) = \text{NS}'(i, a_0', b', t', t) + 1$$

(6B) If i 's cognitive trial from state a_0' fails then all the cognitive success-sums assume the value 0. Formally:

$$\neg \text{Succ}'(i, a_0', b', t') \rightarrow (\forall x') \text{NS}'(i, a_0', x', t'+1, t) = 0$$

(6C) If any of the cognitive success-sums reaches the action threshold, in steps in cognitive time less or equal than the maximum amount of the steps available, then an outcome by probabilistic choice using the correspondence matrix is determined and a real trial is made according to the outcome, at the real time point t . Formally:

$$\text{NS}'(i, a_0', b', t', t) = \text{AT}(i) \wedge \Sigma t'(i, t) \leq nt'(i, t) \rightarrow (\exists x) (\text{Outc}(\mathbf{RCRCor}(i, b', t)) = x \wedge \text{Tr}(i, a, x, t))$$

(6D) If no cognitive success-sum has reached the value of action threshold when the sum of cognitive time steps equals the maximum amount of the steps available, the real behavior of i continues at the next real time point $t+1$. Formally:

$$\text{Be}(i, a, t) \wedge (\forall x') \text{NS}'(i, a_0', x', t', t) < \text{AT}(i) \wedge \Sigma t'(i, t) = nt'(i, t) \rightarrow \text{Be}(i, a, t+1)$$

(7) If i makes a real trial at the moment t , a probabilistic choice is made at the same moment to determine whether the trial succeeds or not. Formally:

$$\text{Tr}(i, a, b, t) \rightarrow \text{Outc}(\mathbf{Succ}(\text{Tr}(i, a, b, t))) = x; x \text{ has two alternatives succ and } \neg \text{succ}'.$$

The following assumptions are based on the *constant environment hypothesis*. In a more general form – where that hypothesis is *not* made – they are presented in the Appendix 2.

(8a) If i who behaves in the way a at the time point t , succeeds in his trial to behave in the way b then i behaves in the way b at the next moment $t+1$. Formally:

$$\text{Succ}(\text{Tr}(i, a, b, t)) \rightarrow \text{Be}(i, b, t+1)$$

(8b) If i who behaves in the way a at the time point t , fails in his trial to behave in the way b then i continues to behave in the way a at the next moment $t+1$. Formally:

$$\neg \text{Succ}(\text{Tr}(i, a, b, t)) \rightarrow \text{Be}(i, a, t+1)$$

The cognitive process continues then as described above, beginning from the point 2.

2.4. Cognitive process. Flow chart

As a summary the following list of the steps in cognitive process is represented, as well as the corresponding graphic scheme in Fig. 2.3.

- 1) Start. (Starting real state = a_0 , and the starting real time point = t_0)
- 2) Set $x = a_0$ and $t = t_0$ in $\text{Be}(i, x, t)$
- 3) Determine the cognitive map M' according to the RCCor-probabilities
- 4) Give to the cognitive time point t' the value t_0'
- 5) Set x' in the cognitive state $C(i, x', t_0')$
- 6) Determine the cognitive trial $\text{Ctr}(i, x', y', t')$, outcome denoted by b'
- 7) Add 1 to t' . Test t' . If $t' = nt'(i)$ then add 1 to t and goto 4
- 8) Determine the success of the cognitive trial, outcome being succ' or $\neg \text{succ}'$
- 9) Test the outcome of success of the cognitive trial

- 10) If \neg succ' then goto 6
- 11) If succ' then set $y=b$ in the real trial $Tr(i,x,y,t)$
- 12) Determine the success of real trial $Tr(i,x,b,t)$, outcome being succ or \neg succ. Add 1 to t
 - 13) If \neg succ then goto 6
 - 14) If succ then set $x = b$ in the real behavior $Be(i,x,t)$
- 15) Let t_n indicate the end of time available. Test t. If $t < t_n$ then goto 4
- 16) End

2.5. Changes of probabilities. Learning

According to our assumptions, there are five instances where a probabilistic choice was made to determine the outcomes:

- 1) the choice of a cognitive map
- 2) the choice of a cognitive alternative
- 3) the determination of a success of a cognitive trial
- 4) the determination of an action (real trial) according to the CRCor probabilities
- 5) the determination of the success of the real trial

We have already made an assumption that in each behavioral state the individual chooses the relevant cognitive map according to a probability vector showing the probabilities for several possible maps in that situation. The ability to select the "right" map have to be a consequence from a long learning process but we can well assume that, in the situation in question, those probabilities do not change essentially. Thus, we shall leave out from the analysis the point 1. – The same seems to be true considering the point 4: the CRCor probabilities are results of experiences of individual and may be assumed constant in the short analysis period – usually equaling 1 and 0 in our examples.

The probabilities of success of real trials (point 5 above) describe the problem situation. They belong to the environment and follow special rules given from outside. They can not be the object of the cognitive analysis.

There are points 2 and 3 on the list left. The probabilities of the choice of cognitive trial alternative and of the success of cognitive trial we assume to change during the cognitive and behavioral process. What rules these changes may follow?

We shall take as a starting point the assumption that those probabilities follow the *learning laws*.

When we try to develop our theory to make it applicable, we are forced to make some assumptions about the mathematical laws which the changes of these probabilities follow, *i.e.*, to select some mathematical model for our use. As far as I can see, the problem of which model to select is a question of secondary importance compared with the decisions we have to make in developing assumptions about the reward and punishment effects of the outcomes. These decisions are necessary in using any mathematical learning model.

In behavioristically oriented learning research, such procedures as feeding or electric shocks are used as rewards and punishment. We shall have a different point of view. In cognition, it is the *knowledge* that some valuable (rewarding) thing is available, if the goal is reached, that has an effect on the cognition and decisions to act. But this knowledge means a totally different thing than reward. We shall handle it later, as a source of *valences* in the life space (see the next chapter).

We shall see the *success* of a trial – not any concrete thing – as the source of reinforcement.

Thus, we set the following assumption concerning the reward and punishment:

The success of a trial, cognitive or real, produces reward, and the failure punishment.

After a *real* trial, both the probabilities of cognitive success and the probabilities of cognitive trials are changed according to mathematical learning law, success having a rewarding and failure a punishing effect.

The success and failure of *cognitive* trials produce, in principle, the same kind of effect. We shall assume this, however, weaker and usually take it not into account in our applications.

What kind of mathematical learning law we shall apply? There are several statistical models of learning available (*i.e.*, W.K. Estes, R.R. Bush and F. Mosteller). We shall use Bush & Mosteller's Two Operator Learning Model (Bush & Mosteller, 1955).

Let us apply the model to the elementary example.

Suppose that a cognitive trial from state **a'** to **d'** was made, $\text{Ctr}(i, a', d', t')$. The probability of the success of this trial, $\text{Succ}'(i, a', b', t')$, was 1 (see the Fig.2.2.B). But the real trial toward **d** failed. The failure produces a *punishing* learning effect. Thus, the probability

$\text{Pr}[\text{Succ}'(i, a', d', t', t)]$ will decrease.

The amount of the decrease is determined by the model according to the mathematical law:

$$p_{t+1} = p_t - \beta \cdot p_t,$$

where p_t is the shortened expression of the $\text{Pr}[\]$, above, and p_{t+1} the same probability but in the next time point, $t+1$, β being the learning coefficient used in case of punishment; $0 \leq \beta \leq 1$.

Suppose that $\beta = .8$. The formula gives to the p_{t+1} the value .2, because $1 - .8 \cdot 1 = .2$.

Let's then look the first vector of *cognitive trial* probabilities (see Fig. 2.2.C). (For convenience, we shall leave out the state x' .)

		$\mathbf{p}(\text{Ctr}(i, a', x', t', t))$				
		a'	b'	c'	d'	Σ
a'		0	.1	.1	.8	1

Let p_t be a shortened expression of $\text{Pr}[\text{Ctr}(i, a', d', t', t)]$ which has the value .8. The punishment means a decrease of this probability. If β is now supposed to be, *e.g.*, .5, then according to the formula above the new value will be:

$$.8 - .5 \cdot .8 = .4.$$

We have computed one probability in the vector. The question arises how the others have to be changed so that the sum on the all elements would be 1. Seemingly we have to increase them. If we share the increase *proportionally to the size* of elements, we shall get the following vector at the next time point $t+1$:

		$\mathbf{p}(\text{Ctr}(i, a', x', t', t+1))$				
		a'	b'	c'	d'	Σ
a'		0	.3	.3	.4	1

There is another reasonable assumption, too: We can share the increase *equally* to the elements. In that case the vector has the following form:

		$\mathbf{p}(\text{Ctr}(i, a', x', t', t+1))$				
		a'	b'	c'	d'	Σ
a'		.133	.233	.233	.4	1

This latter assumption has been used in some earlier studies (Rainio, 1986, p. 50). It seems also well argued from some psychological point of view, because it is in

accordance with the frustration type of behavior. - In this study the first assumption will be used.

Let's suppose that the first trial in our example were **b'**, instead of **d'**. Also in that case punishment will occur. $\beta = .5$. According to the model the **p**-vector is transformed as follows:

$$\begin{array}{rcccccc}
 & & \mathbf{p}(\text{Ctr}(i,a',x',t',t+1)) & & & \\
 & & a' & b' & c' & d' & \Sigma \\
 a' & & 0 & .05 & .1056 & .8448 & 1
 \end{array}$$

Notice that $\text{Pr}[\text{Ctr}(i,a',b',t',t)]$, that was .1, is now decreased to .05, namely: $.1 - .5 \cdot .1 = .05$.

Now, let's examine the *rewarding* learning effect. According to the Bush-Mosteller model the probability is increased after reward applying the following formula:

$$p_{t+1} = p_t + \alpha(1 - p_t)$$

In our elementary example we may suppose that the first trial was **c**. That trial succeeds and, thus, is rewarding to the cognitive trial and cognitive success probabilities. Because the latter is already 1 (see Fig. 2.2.B), it does not change. But let's look at the cognitive trial probabilities:

$$\begin{array}{rcccccc}
 & & \mathbf{p}(\text{Ctr}(i,a',x',t',t)) & & & \\
 & & a' & b' & c' & d' & \Sigma \\
 a' & & 0 & .1 & .1 & .8 & 1
 \end{array}$$

Now the probability of cognitive trial toward **c'** is increased according to the formula above when α is supposed to be .4 :

$$.1 + .4(1 - .1) = .1 + .36 = .46$$

The increase .36 has to be taken from the other probabilities in proportion to their size. That gives to the vector the new form:

$$\begin{array}{rcccccc}
 & & \mathbf{p}(\text{Ctr}(i,a',x',t',t+1)) & & & \\
 & & a' & b' & c' & d' & \Sigma \\
 a' & & 0 & .06 & .46 & .48 & 1
 \end{array}$$

For calculations needed, there is a handy matrix-algebraic way to get the whole vector transformed: The vector has to be multiplied by a matrix called *learning operator*. (See also Appendix 3.)

In the case of reward, the learning operator matrix \mathbf{Q}_{rew} has – in our example, **c** being the trial – the form:

$$\begin{array}{rcccc}
 & & & & \mathbf{Q}_{\text{rew}} \\
 & & a' & b' & c' & d' \\
 a' & & 1-\alpha & 0 & 0 & 0 \\
 b' & & 0 & 1-\alpha & 0 & 0 \\
 c' & & \alpha & \alpha & 1 & \alpha \\
 d' & & 0 & 0 & 0 & 1-\alpha
 \end{array}$$

The \mathbf{p} vector has to be multiplied by this matrix. In the following we use the numerical values in our example, α being = .4 :

	\mathbf{Q}_{rew}				\mathbf{p}_t	\mathbf{p}_{t+1}
.6	0	0	0	0	$p_1 = 0$	
0	.6	0	0	.1	$p_2 = .06$	
.4	.4	1	.4	.1	$p_3 = .46$	
0	0	0	.6	.8	$p_4 = .48$	

The calculations in this case are:

$$\begin{aligned}
 p_1 &= .6 \cdot 0 + 0 \cdot .1 + 0 \cdot .1 + 0 \cdot .8 = 0 \\
 p_2 &= 0 \cdot 0 + .6 \cdot .1 + 0 \cdot .1 + 0 \cdot .8 = .06 \\
 p_3 &= .4 \cdot 0 + .4 \cdot .1 + 1 \cdot .1 + .4 \cdot .8 = 0 + .04 + .1 + .32 = .46 \\
 p_4 &= 0 \cdot 0 + 0 \cdot .1 + 0 \cdot .1 + .6 \cdot .8 = .48
 \end{aligned}$$

The general form of operators and the multiplication of a vector by a matrix in details are given in Appendix 3.

The use of Bush and Mosteller's learning model is no basic assumption of our theory. Other models may well be used if they are more relevant in certain cases. The matrix form of learning operators is flexible to modeling other types of learning, too.

2.6. Life space. Field. Valence. Potency of field

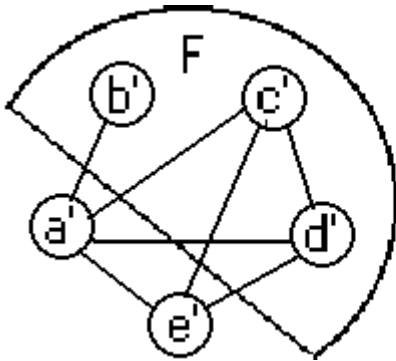
2.6.1. Field and Life Space

Thus far we have treated the positions – in real behavior as well as in cognition – as undivided. In the Lewinian system it is often illustrative and fruitful to describe a position of person simultaneously in a region and its subregion, or in an intersection of two or more overlapping regions. In our study this seems necessary, too. We have to keep in mind our *holistic view* which is fundamental: The consciousness is considered as a whole which is differentiated to parts and subparts. This means, apparently, that we have to bind together several positions, *i.e.*, to form groups of states which correspond relevantly to the categorization of things in cognition. Those groups of states may, of course, be included to each other or be overlapping as well.

Let us take an example of the grouping of states. See the graph of states $\mathbf{a}' - \mathbf{e}'$ in Fig.2.3.

We suppose that states \mathbf{b}' , \mathbf{c}' , and \mathbf{d}' are elements of group (cognitive category) F. Then, to be in the state \mathbf{b}' or \mathbf{c}' or \mathbf{d}' means as well to be *inside* F, while to be in \mathbf{a}' or in \mathbf{e}' means being *outside* it. One can think that to stay inside or outside of F would be important to a person while it does not matter so much to him in which states he stays. (Why then to handle separate states at all? – Considering the further *transitions*, it may be relevant – *e.g.*, from \mathbf{b}' there exists no further transition possibility while from \mathbf{c}' and \mathbf{d}' there exists.)

Fig. 2.3. A field



Groups of cognitive states we shall call **fields** and denote them by F . (Because we handle only cognitive fields, we leave the sign ' out. For sake of convenience, we avoid indices and name the different fields like F , F_1 , F_2 , F_x , F_n etc.)

There may be one or more states as elements of a field.

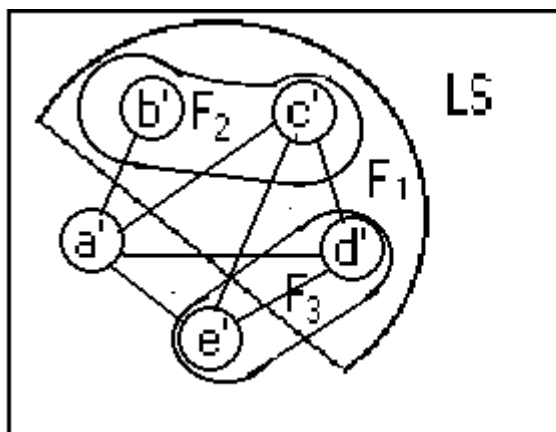
A state may be an element in many fields.

The fields may be overlapping.

One field may also include another: $F_x \subset F_y$, if all elements of the former field are also elements of the latter one.

In Fig. 2.4, three fields are shown, one, F_2 , being included to another, F_1 , and two overlapping, F_1 and F_3 , the state d' being located in the *intersection* of them.

Fig. 2.4. Life space and three fields in it



Formally we can define all possible groups of states as fields, but psychologically relevant may be only some of them. In our analysis we set for field the requirement that it needs to be also psychologically relevant. – As before, we consider here that the relevancy depends on the actual case and describing it is one of the main problems of the theoretical analysis of the situation.

It seems reasonable to think that the "life space" itself is a field. This field includes, then, all the cognitive states as its elements. As such a field it means the *whole consciousness* of person. However, we obviously can not see it relevant in any casual situation. We may describe some main characteristics of it, in principle, but our theory in its exact form does not, practically, reach it. Therefore, we shall use the term "actual life space" and denote that by LS. (Lewin seemingly uses this term in the same way.) The *actual life space*, LS, has now a restricted meaning: It is the *focused* part of consciousness – *actualized* in a certain situation.

Thus, *the actual life space, LS, is the field of all situationally relevant cognitive states.*
Formally:

$$(\forall x') x' \in LS \wedge (\forall F) F \subset LS$$

It is fair to emphasize that a person's choice to follow the rules in an application of our theory or to follow not, is a question of *free will*. We assume that he forms (selects) a cognitive map and then acts in his cognition according to that map by his free will, in principle. As long as he stays inside the actual life space, he can be taken an object of our study, but he has at every moment the possibility to leave it (by free will, too) – *i.e.*, he tries and succeeds to go from the field LS to the field \neg LS outside it, from actual life space to unfocused part of the total consciousness. We can not know even the probability of doing that. This has the consequence that we can, practically, study only those cases where the probability of using the free will for leaving the actual life space is reasonably low, *i.e.*, experiments where the subjects are well "motivated" to follow the instructions.

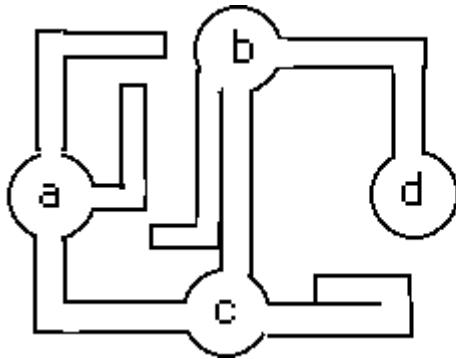
2.6.2. Valence

Before going to the formal description of fields, let us consider our elementary example again (see Fig. 2.1). If the "points" to be given in **d'** are there is not mentioned in the instruction, then the whole game should be seen as *meaningless*. Nobody could see it reasonable to move a piece from one position to another, only. – (Maybe somebody could play the game in that simple form for the curiosity sake, waiting some surprise. But that is an event of "higher order", not relevant in this context.)

From a purely *behavioristic* point of view, we shall see our elementary example as follows:

We may well see the game as a maze (see Fig. 2.5) and, *e.g.*, put a rat at the position **a**. When the rat starts to run in that maze, the running is understood as *instinctive behavior*, not relevant to our analysis of cognition.

Fig. 2.5. The game of the elementary example as a maze.



The rat finds the reward, e.g., food, in **d** after the first run. Before that there was no goal. It is reasonable to think that, in the begin, behavioral trial probabilities by the rat are homogeneous (equal for each alternative in the vector), as shown in the Table 2.1.

Table 2.1. The behavioral trial probabilities in the begin, in case of rat.

	$p(\text{Tr}(\text{rat}, x, y, t_0))$				
	a	b	c	d	Σ
a	.25	.25	.25	.25	1
b	.25	.25	.25	.25	1
c	.25	.25	.25	.25	1
d	.25	.25	.25	.25	1

From the *cognitive* point of view, the example is seen very different:

To make the game meaningful requires some "goal", a *symbolic representation* of reinforcement, located somewhere in the cognitive map. The *knowledge* of the possibility to "get points" in **d** – and the belief that it will happen – creates an idea of "goal" as an essential feature of the cognitive map.

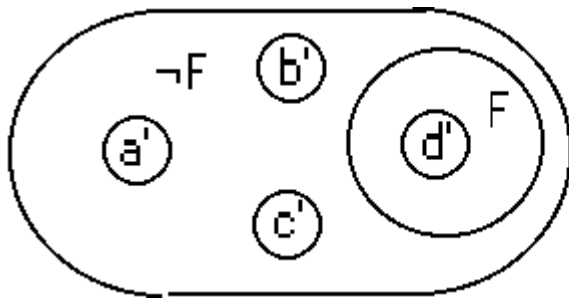
Lewin introduced in his conceptual framework the concept *valence* to indicate that character of a region which creates, induces, in the life space driving forces toward that region (positive valence) or out and further out of it (negative valence). (See Lewin, 1938, pp. 87 - 92.)

We emphasize that *the valences give to the cognition its meaningful character* arousing *intentional*, goal-seeking activity. And *vice versa*, the meaningfulness of cognition, intentional activity, requires the description of valences in the life space.

Let us examine our elementary experiment again. We can describe the cognitive map using the concept of field (see Fig. 2.6). The Life Space is now divided to two subregions, F and $\neg F$, the states being elements of the groups F or $\neg F$:

$$F = \{d'\} \text{ and } \neg F = \{a', b', c'\}$$

Fig. 2.6. Field F in the elementary example



To give a *positive valence* to the field F means psychologically that it attracts person, *i.e.*, keeps him inside or induces moving from outside to inside. In our mathematical language:

The probability of keeping the subject inside F , *i.e.*, $\Pr[F, F]$ needs to be higher than the probability of letting the subject go to outside it, $\Pr[F, \neg F]$. The attraction means that $\Pr[\neg F, F]$ needs to be greater than $\Pr[\neg F, \neg F]$. This is summarized in the following matrix as a numerical example:

	\mathbf{p}_F	
	F	$\neg F$
F	.8	.2
$\neg F$.7	.3

Lewin did not have any quantitative measure of valence in his conceptual framework (see Lewin, 1938, pp. 87 - 92). In accordance to our way of thinking, we can easily form a *measure of valence* of a field:

$$Va(F) = (\Pr[F, F] - .5) + (\Pr[\neg F, F] - .5)$$

It can be seen without difficulty that the formula is the measure of both the positive and of the negative valence.

The negative valence means that $\Pr[F, F] < .5$ and $\Pr[\neg F, F] < .5$ so that the valence $Va(F)$ gets a negative value.

What about the case where $\Pr[F, F] < .5$ but $\Pr[\neg F, F] > .5$ or *vice versa*? That case means psychologically an *ambivalence* of F and we have a measure for it, too:

$$Ambiv(F) = (\Pr[F, \neg F] - .5) + (\Pr[\neg F, F] - .5)$$

The maximum values of positive and negative valences are shown Table 2.2.

Table 2.2. The maximum values of valences

	p_F			p_F	
	F	$\neg F$		F	$\neg F$
F	1	0	F	0	1
$\neg F$	1	0	$\neg F$	0	1
Maximum positive valence, $Va(F) = 1$			Maximum negative valence, $Va(F) = -1$		

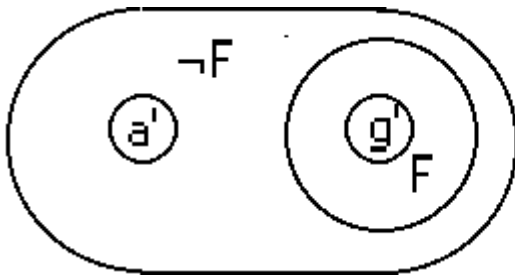
2.6.3. Valences and cognitive trial probabilities

Lewin's assumption that valence induces psychological forces (driving forces) has to be translated to the terms of our theory:

The cognitive probabilities are derived from the field probabilities. In what way, that will be shown as follows:

Let us analyze first a simple case with one field and two states as shown in Fig. 2.7.

Fig. 2.7. One field and two states



There are two groups: $F = \{ g' \}$ and $\neg F = \{ a' \}$. It is easily seen that the cognitive trial probabilities from the state to state correspond to the field probabilities, because there is always only one direction to which each of the field probability effects. Using abbreviation

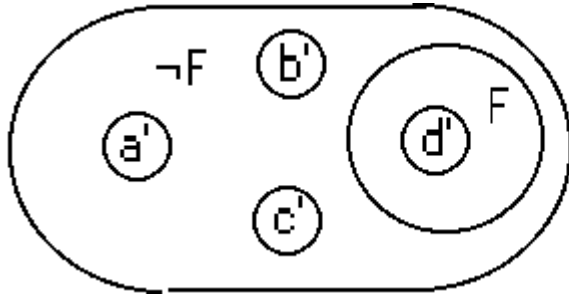
$$p(x,y) = \Pr[\text{Ctr}(i,x',y',t',t)]$$

we can, thus, write

$$\begin{aligned} p(a,a) &= \Pr[\neg F, \neg F] \\ p(a,g) &= \Pr[\neg F, F] \\ p(g,a) &= \Pr[F, \neg F] \\ p(g,g) &= \Pr[F, F] \end{aligned}$$

What about the case where there are more elements in some field? Let us examine again our elementary example (see Chapter 2.1) from the point of view of the fields and cognitive trial probabilities. (See Fig. 2.8)

Fig. 2.8. The elementary example. One field



We have one state in F and three states in $\neg F$. It is quite reasonable to think that the field probability should be divided equally to the states in the same region – to **a'**, **b'**, and **c'**, because they are located in same region $\neg F$. If the field probabilities are, *e.g.*,

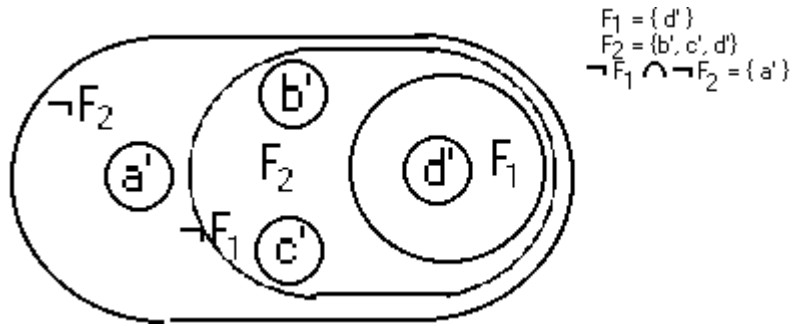
	F	$\neg F$
F	.7	.3
$\neg F$.82	.18

then the cognitive trial probabilities should be, according to that principle:

	a'	b'	c'	d'
a'	.06	.06	.06	.82
b'	.06	.06	.06	.82
c'	.06	.06	.06	.82
d'	.1	.1	.1	.7

How do the field probabilities determine the cognitive trial probabilities in the case when there are more than one field? Suppose that this was the case in our elementary example. (This is also psychologically reasonable, because we may well assume that the subject sees the states **b'** and **c'** somehow as "nearer **d'**" and, valuable to reach.) (See Fig. 2.9)

Fig. 2.9. The elementary example. Two fields



F_1		F_2	
	F_1	$\neg F_1$	
F_1	.8	.2	
$\neg F_1$.7	.3	
↓			
	$\neg F_1$	F_1	
	a	b	c
$\neg F_1$.1	.1	.1
b	.1	.1	.1
c	.1	.1	.1
F_1	.07	.07	.07
	.8		

F_2		$\neg F_2$	
	F_2	$\neg F_2$	
F_2	.6	.4	
$\neg F_2$.9	.1	
↓			
	$\neg F_2$	F_2	
	a	b	c
$\neg F_2$.1	.3	.3
b	.4	.2	.2
c	.4	.2	.2
F_2	.4	.2	.2

Means

	a	b	c	d
a	.1	.2	.2	.5
b	.25	.15	.15	.45
c	.25	.15	.15	.45
d	.23	.13	.13	.50

In this case there is one state, d' , inside the field F_1 and three outside it, while three states, b' , c' , and d' are inside the field F_2 and one, a' , outside that.

The effects of both fields are shown separately, following the same principle as above, namely, that the field probability is divided equally to the states being located in the same region.

But what is now the common effect of the field upon the cognitive trial probabilities? - One solution is simply to compute the means of them, as shown in the last matrix, "Means".

The solution seems to be too simple. Instead of that we shall compute *the weighted sum of separate field effects to get the final cognitive trial probabilities*. This leads us to the concept of the *potency of field*. (Following Lewins terminology we use the term "potency" instead of "potential", although the latter may sound suitable, too. (Lewin, 1938.))

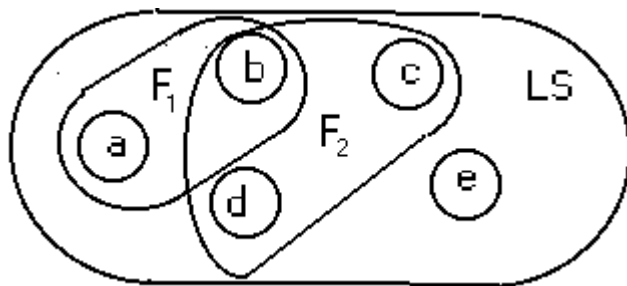
2.6.4. Potency of field

In summing up the cognitive trial probabilities in the earlier chapter, we actually used equal weights, .5 and .5, for getting the means of the effects of two fields.

We use the term *potency of a field* to indicate *the weights used in combining (summing up) the effects of the field in calculation of the cognitive trial probabilities*.

Let us take an example. (See Fig. 10)

Fig. 2.10. Two overlapping fields



There are two fields F_1 and F_2 in the Life Space, LS, formally:

$$F_1 = \{a, b\}$$

$$F_2 = \{b, c, d\}$$

and, of course, there exists the $LS = \{a, b, c, d, e\}$.

Let the *field probabilities* be:

	F1	-F1		F2	-F2
F1	1	0	F2	1	0
-F1	1	0	-F2	1	0

and the *potencies* of the fields:

$$\text{Pot}(F_1) = .6$$

$$\text{Pot}(F_2) = .4$$

Because the cognitive trial probabilities are the weighted sums of field probabilities divided equally to the states being located in the same regions (F or -F), we can compute one vector of the cognitive trial probabilities in this numerical example as follows:

$$p(a, b) = \text{Pot}_1 \cdot \text{Pr}[F_1, F_1] / N_{F_1} + \text{Pot}_2 \cdot \text{Pr}\{-F_2, F_2\} / N_{F_2}$$

$$\begin{aligned}
&= .6 \cdot 1/2 + .4 \cdot 1/3 = .433 \\
p(a,c) &= .6 \cdot \Pr[F1, \neg F1] / NF_1 + .4 \cdot \Pr[\neg F2, F2] / NF_2 \\
&= .6 \cdot 0/3 + .4 \cdot 1/3 = .133 \\
p(a,d) &= p(a,c) = .133 \\
p(a,e) &= .6 \cdot 0/3 + .4 \cdot 0/2 = 0 \\
p(a,a) &= .6 \cdot 1/2 + .4 \cdot 0/2 = .3 \\
\Sigma &= 1
\end{aligned}$$

The other vectors, $\mathbf{p}(b,x)$, $\mathbf{p}(c,x)$, *etc.* can be computed according to the same principle.)

Now we are ready to write generally the calculation rule of the cognitive trial probabilities as a formula:

We use the abbreviation

$$\Pr[x,y] = \Pr[\text{Ctr}(i,x',y',t',t)]$$

and make the following notation:

$R_k(x)$ = that region in LS where x is located in relation to field F_k .

$$x \in F_k \rightarrow R_k(x) = F_k$$

$$x \in \neg F_k \rightarrow R_k(x) = \neg F_k$$

Let NR_k indicate the number of elements (states) being located in the region $R_k(x)$.

Pot_k = potency of the field F_k .

The cognitive trial probabilities are now calculated by the formula:

$$\begin{aligned}
\Pr[x,y] &= Pot_1 \cdot \Pr[R_1(x), R_1(y)] / NR_1(y) + Pot_2 \cdot \Pr[R_2(x), R_2(y)] / NR_2(y) + \\
&+ \dots + Pot_m \cdot \Pr[R_m(x), R_m(y)] / NR_m(y)
\end{aligned}$$

The same formula can be written also in an abbreviated form as follows:

$$\Pr[x,y] = \sum_{j=1}^m Pot_j \cdot \Pr[R_j(x), R_j(y)] / NR_j(y)$$

where j varies from 1 to m , m being the total number of fields.

Note: The Life Space, LS, should be understood as a field, too. In the case we assume the potency of Life Space to be 1, there can not exist any other fields. Using the formula above we get the result that in that case all the vectors of cognitive trial probabilities need to be homogeneous, *i.e.*, equal to $1/n$ if n indicates the total number of states in LS. That is just the matrix we have to use to represent cognition in pure trial and error situation - as was described above (see Chapter 2.1). – In the next Chapter we shall discuss about the question of the relation between Life Space and the other fields in detail.

2.6.5. Field effect. Changes of potencies of fields

We make here an important assumption that *the cognitive trial probabilities are a mixture of the component where Life Space effects alone ($Pot_{LS} = 1$) and of the other component computed according to the field probabilities when the potency of Life Space is assumed to have value 0, ($Pot_{LS} = 0$).*

Thus, we take two aspects of the behavior into account, the behavioristic trial and error type of behavior and the cognition where differentiated cognitive map is used. Valences are not effective

in the former type of behavior, we can simply say that they do not exist. It has been already mentioned that the existence of valences require symbol formation (the so called symbol function).

It is psychologically reasonable to think that, at least in every problem-solving situation, both aspects of activity are present, with varying emphasis.

The result, the final cognitive trial probabilities, is, thus, the weighted sum of these two components. The weight used for the latter component (the fields) we shall call a *field effect*.

Formally the two components are:

- 1) $\mathbf{p}_1(x,y) = \text{PotLS} \cdot 1 / n = 1/n$; where n is the total number of elements in the LS.
- 2) $\mathbf{p}_2(x,y) = \sum \text{Pot}_j \cdot \text{Pr}[R_j(x), R_j(y)] / \text{NR}_j(y)$; where $j \neq \text{LS}$, *i.e.*, $\text{PotLS} = 0$.

Let us notate the field effect by w_{FE} . Then the resulting matrix, the cognitive trial probabilities, is:

$$\mathbf{p}(x,y) = (1 - w_{\text{FE}}) \cdot \mathbf{p}_1(x,y) + w_{\text{FE}} \cdot \mathbf{p}_2(x,y)$$

We assume – and this is a very essential assumption – that the field effect follows also the learning laws in such a way that the failure decreases the field effect, while the success increases it. The Bush and Mosteller model will be used also in this context. The psychological argument of this procedure is that a failure somehow distracts (or destroys) the structure of cognitive map, while the success supports the belief that the structure is suitable. The consequence of a failure, thus, causes a kind of *regression* of the cognition.

These assumptions concerning the changes of field effect we shall apply to the simulation of Group Maze (see the Chapter 3). We shall see that those are well supported by empirical results.

The analysis of fields, their valences and potencies, enlighten many interesting psychological situations Kurt Lewin described, qualitatively, in his studies (see Lewin, 1938, pp. 203 - 209). The theory of cognitive process and behavior gives an opportunity to create exact stochastic hypotheses for testing them.

2.7. Behavioral and cognitive dependencies

We have already met conditionality when we defined the trial:

$\text{Tr}(i,a,b,t)$ was an abbreviation of the expression of $\text{tr}(i,b,t)$ and $\text{Be}(i,a,t)$. *i.e.*, trial by \mathbf{i} to the direction \mathbf{b} at the time point \mathbf{t} and the behavior of \mathbf{i} in the way \mathbf{a} at the same point in time. Formally this can be written

$$\text{tr}(i,b,t) \mid \text{Be}(i,a,t)$$

This means that a trial can occur only by the condition that the subject behaves in certain way at the same time point (is in a certain real state), which is obvious.

In this chapter we shall describe some dependencies in the large and complicated world of the conditionalities of psychological occurrences.

2.7.1. Behavior dependencies

1) *Environmental change*:

In an early stage of our analysis we made the *constant environment assumption* which means that we have made all our examinations with the condition $\neg\text{ECh}$.

All the probabilities we have used have been, then, conditional in the form:

$$\text{Pr}[x \mid \neg\text{ECh}]$$

If the constant environment assumption is rejected, the environment change needs to be taken into account and the process follows different conditional probabilities, *e.g.*:

$$\Pr[\text{Be}(i,b,t+1) \mid \text{ECh}(a,b,t) \ \& \ \text{Be}(i,a,t)] = \Pr[\text{Be}(i,a,t)]$$

It is the trial to stay in **a** – or a failed trial from **a** to some **x** – which lead to behavior **b** at the next time point $t+1$, because at that time point **a** is changed to **b**.

We do not handle here more cases where ECh has taken into account. (See the Appendix 2 which gives more information about that.) The environmental changes are directly observable and should be handled as characteristics of the situation, described together with the other observables defining that. They are not *per se* objects of psychological analysis.

2) Success of real trial dependencies:

There may be events which form conditions for the success of real trial, so that the probabilities get the form:

$$\Pr[\text{Succ}(\text{Tr}(i,a,b,t)) \mid E] \text{ where } E \text{ means a conditional event.}$$

Because we are dealing with real trials, their conditions have to be some kind of physical events.

Let us take a simple example:

a = "to be at home 8.00 a.m"

b = "to be in laboratory 8.30 a.m."

E = "bus 131 is going ordinarily"

The real trial by **i** to move from **a** to **b** starting at time point **t** (= 8.00 a.m.) is now conditional, the condition being event **E**. The probability of the success of it is:

$$\Pr[\text{Succ}(\text{Tr}(i,a,b,t)) \mid E]$$

Suppose that the probability of success were .9 if the condition, event **E**, is fulfilled. If the bus is going ordinarily with the probability, say, .8 then the success will occur with the probability $.9 \cdot .8 = .72$. If the bus is going maximally ordinarily (probability = 1), then the success probability have to be .9.

3) Social behavioral dependencies:

There are cases more interesting than that above – namely such situations where the conditions of one subject's behavior is the behavior of another or others.

Formally these conditional probabilities are expressed as follows:

$$(\exists j) \Pr[\text{Succ}(\text{Tr}(i,a,b,t)) \mid \text{Be}(j, x_j, t)] \text{ or}$$

$$(\exists j) \Pr[\text{Succ}(\text{Tr}(i,a,b,t)) \mid \text{Tr}(j, x_j, y_j, t)]$$

There may well be more than one other person, **j**, whose behavior or trial at the same point of time as **i**'s trial form the conditions.

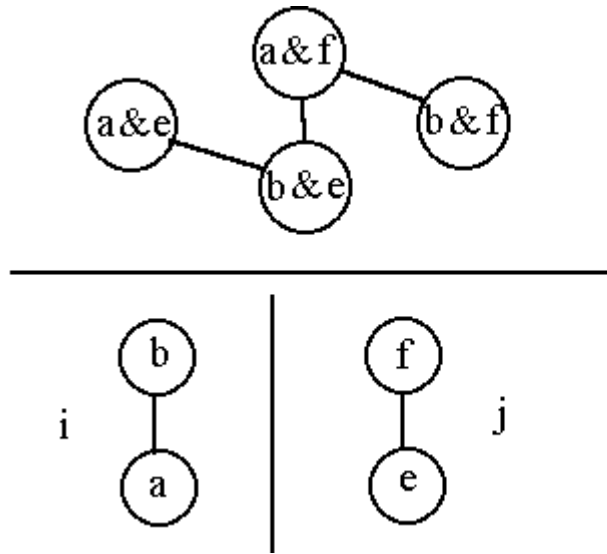
We shall call the dependencies described above as *social behavioral dependencies* and they are essential for description of any kind of *group problems*.

Let us come back to our elementary example.

The example could be seen as an experiment of a group problem if we suppose that the four states, **a** - **d**, are actually combinations of two states of two individuals. Thus, we describe the game graph as seen in Fig. 2.11.A where the lines of the graph indicate the allowed moves.

Fig. 2.11. Two-Person Game. Graphs

A)



B)

There are only two possible states, **a** and **b**, for the first subject, **i**, to choose, and correspondingly two states, **e** and **f**, for second subject, **j** (see Fig. 2.11.B).

In the game, the starting position is supposed to be **a & e** and the rules are:

1. The subjects are asked make the choice between the two alternatives available, **a** or **b** for **i**, and **e** or **f** for **j**, but *separately, without knowing the other's choice*, and – naturally – not knowing the solution graph represented in Fig. 2.11.A.

2. After the choices, the subjects are told whether their choices succeeded or failed and asked to make the next choices.

3. When the experimenter (game leader who knows the solution graph, the graph in Fig. 2.11.A, and is able to observe the moves of both subjects) sees that, after the subjects have reached the state **a & f**, they are choosing **b & f**, he declares one round of game ended, one point given, and the new round of game starting.

It should be emphasized that the players may not even know that there are two playing the game and that the moves are mutually dependent. Thus, they do not form a group in social-psychological meaning. We shall call this kind of combination of individuals an *aggregate*.

The success probabilities of real trials are in this case conditional in that way that the trial of the other subject, made at same point in time than the trial of the subject concerned, is the condition.

We call this kind of conditionality *behavioral social dependence* – in contrast to the *cognitive social dependence*, which we shall analyze later.

Although there exists no group, the subjects are, paradoxically, in a situation of solving group problem – without knowing that! It may seem that the solution will be found only randomly and no learning will appear. No, the case needs not to be so trivial. Although no experimentation with this game have been made, as far as I know, we can argue theoretically that the solution may be learned.

We have to look the cognition of the subject in this situation.

What kind of cognitive map is taken in the use by the subject? – If he is only vaguely committed himself to the game (experiencing it, maybe, too boring), it is well assumed that his cognitive map consists of two cognitive states, **a'** and **b'** (subject **i**), only – without any valences. It is difficult to see that here could be any possibility to learn the path to the solution.

But this needs not be the case. We can well argue that the cognitive map could be based on the cognitive states where the conditions are taken into account, so that, instead of the cognitive states **a** and **b**, only, there are, *e.g.*, 30 different cognitive states. (We shall call them "conditional states".) The matrix of the cognitive trial probabilities between these conditional states, in the begin, is given in Table 2.3:

Table 2.3. Two-person game. Conditional states

Matrix $\mathbf{p}(\text{Ctr}(i,x',y',t',t))$

	y'							
x'	$a' a$	$b' a$	$a' aa$	$b' aa$	$a' ab$	$b' ab$	<i>etc.</i>	
a	.5	.5	0	0	0	0	...	
$a' a$	0	0	.5	.5	0	0	...	
$b' a$	0	0	0	0	.5	.5	...	
$a' aa$	<i>etc.</i>							
$b' aa$								
$a' ab$								
$b' ab$								
$a' aaa$								
$b' aaa$								
$a' aab$								
$b' aab$								
$a' aba$								
$b' aba$								
$a' abb$								
$b' abb$								
$a' aaaa$								
$b' aaaa$								
$a' aaab$								
$b' aaab$								
<i>etc.</i>								
...								
$b' abbb$								

(*Mutatis mutandis*, $\mathbf{p}(\text{Ctr}(j,x',y',t',t))$ with states e and f.)

(The matrices of the probabilities of success of the cognitive trials are the same as above if 1 is substituted for .5.)

If the subject remembers the "history" of the actual game process so that, *e.g.*, subject **i** learns to know that after succeeded trials **a** and **b** it is **a** which succeeds more probably, and after the chain **a,b,a** he should choose **b**, then he may gradually learn to repeat this chain of choices. If the other

subject, **j**, learns, correspondingly, gradually the chain **e,e,f,f**, then both subjects can reach their maximal success by repeating those chains.

(What has been said here about the cognitive trial probabilities suits as well to the cognitive success probabilities.)

The learning process were much easier for **i**, if the other subject, **j**, repeats his chain with high probability. If the choices of the other subject, **j**, are totally deterministic, forming the chain **eeff**, the learning process by **i** is, seemingly, maximally fast.

Because there is no communication between the subjects in this case, we may well call the "other subject" – who behaves in the deterministic way – the *nature* (macro-physical nature, to be exact).

In relation to the nature the human being is forced to adjust his trials to the occurrences on which he is dependent, without any communication (in the usual sense). The nature "communicates" only by its occurrences at the same time point they happen – which is "too late" from the real communications point of view. (Fortunately, in the case of deterministic process, the occurrences themselves tell the future. Who can read such messages can trust them better than the information got from human beings, concerning their plans.)

2.7.2. Cognitive dependence. Communication. Group problem

From the psychological point of view, our game example is maybe more interesting if we let the subjects be in communicative connection with each other. Only in this case we can speak about "group" in socialpsychological meaning.

We leave here out the question what are the minimum requirements of "communicative connection". Should it be enough that there is the knowledge of the existence of one or more other subjects and of the mutual dependence between them? If so, then the *expectation* of the other's behavior is seen as communication in a large sense. We shall argue in this way, because in the case of expectation there exists cognitive dependence, *i.e.*, the person makes his choices conditionally according to what he expects the other will do.

Communication is handled here as an aspect of behavior which produces *cognitive dependence*. Thus, the communication acts are included in the behavioral states, formally in **a** of $Be(j,a,t)$, and they effect on the cognitive maps of other individuals, making the cognitive states conditional.

To illustrate the cognitive dependence of the group members on each other, let us examine our elementary example, assuming a restricted form of communication possible.

In the Two-Person Game, the cognitive states need now to be named in the new way. There is a reason to assume that the player imagines now *common states* instead of the place of his own piece, only. Thus, the cognitive states are named as in Fig. 2.12.

The communication can have meaning only if the subjects know that the states are now common and that one have to take the other's choice into account. Thus, we need to assume that the matrices of the probabilities of the cognitive trials have to take the form shown in Table 2.4. (It is assumed also that the goal, the point-giving, has been told.)

Fig.2.12. Two-Person Game. Cognitive states

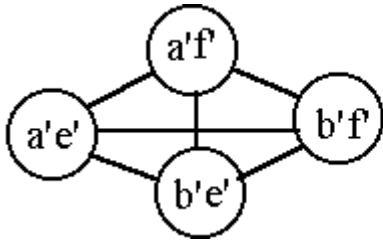


Table 2.4. Matrices of cognitive trial probabilities

$$p(\text{Ctr}(i,x',y',t',t))$$

	ae	af	be	bf
ae	.1	.1	.1	.7
af	.1	.1	.1	.7
be	.1	.1	.1	.7
bf	.1	.1	.1	.7

(Mutatis mutandis, changing **i** to **j**, we get the matrix applied to **j**.)

The situation from **i**'s point of view is now following:

As long as **j** has not sent any message, **i** can choose "freely", *i.e.*, there are two possibilities: 1) **i** chooses, *e.g.*, **b'** without thinking **j**'s intentions at all, or 2) **i** chooses, *e.g.*, **b'** expecting that **j** will make his choice according to that alternative **i** is imagining as **j**'s choice.

The latter case is interesting. The probability on which the choice was based is formally:

$$\Pr[\text{Ctr}(i,a,a',b',t',t) \mid \text{Expec}'(i,j,f,t+1)]$$

which can be read: "probability that **i** chooses in his cognition the move toward **b'** if he expects **j** to move toward **f** on his trial".

Expectation can be defined formally as follows, to be exact:

$$\text{Expec}'(i,j,x,t+\tau) = C(i,Be'(j,x,t+\tau),t',t)$$

The expectation makes the probability conditional. When we say, that **i** makes his choice "freely", we assume that his expectation is certain, *i.e.*, $\Pr[\text{Expec}'(i,j,t+1)] = 1$

In the Table 2.4., the abbreviations actually mean:

$$ae = a' \supset C(i,Be'(j,e,t),t',t)$$

The cognitive state **ae** is an abbreviation meaning that **i**'s cognitive state **a'** includes that he imagines **j** to behave in the way **e** at present.

$$bf = b' \supset C(i,Be'(j,f,t+1),t',t)$$

The state **bf** as the goal of a cognitive trial means that **i**'s cognitive state **b'** includes that **i** imagines **j** to behave in the way **f** at the next point in time, *i.e.*, to make a trial toward **f** at the same time as **i**.

What about the case where **j** has already sent a message that he will make the choice **f** ?

If **i** makes the cognitive choice **bf**, then we can argue that the message simply makes the expectation **f** more accurate. Thus, it means no difference from the **i**'s point of view.

But, suppose that **j** has already sent a message to choose **e** and **i** makes his cognitive choice **bf**. There exists now a clear contradiction: **i** wishes that the state **bf** could be chosen but he can control his own move only, and the fact that **j** intends to choose otherwise is in conflict with **j**'s intention. There seems to be now three possibilities (before the decision to choose the real trial):

- 1) **i** uses the possibility to send a message to **j** suggesting him to change his choice.
- 2) **i** makes a new cognitive trial himself.
- 3) **i** simply ignores the message and keeps on his "wishful thinking", *i.e.*, decides to choose cognitively **bf** and makes the move **b** according to that.

The message sent in the case 1 we shall call a *change suggestion*.

It should be noted that also **i** was able to send a suggestion to **j** already before he had got any information from **j**. Such a message we shall call a *dominance suggestion*.

When **j** has sent a suggestion to **i** to behave in a certain way, **i** has, seemingly, three possibilities:

- 1) **i** may follow the suggestion and make his cognitive choice according to that without thinking what will be the combined state. This case we shall call *submissive behavior*.
- 2) **i** may take the suggestion into account and make his cognitive trials according to that, *i.e.*, imaging to choose in the suggested way, but testing the success of his cognitive trials.
- 3) **i** may make his cognitive trials quite independently, as if there were no suggestion at all.

This brief description of some features of communication in Two-Person Game may be enough for an illustration of the dependencies produced by the message sending. It serves also as an introduction to the Group Maze simulation – to be presented in Chapter 3.

To show the way in which the communication process can be built into the conceptual system of our theory, we define the basic concepts of the process of sending and receiving information as follows:

1) *Content of a message* is a meaningful piece of information included to a cognitive state:

$$x' = \text{Cont}' \rightarrow C(j, y' \supset x', t', t), \text{Cont}' \text{ meaning the content of message}$$

2) *Expression of a message* is a component of a real state (way of behavior by the message sender) determined as an outcome of CRCor-probability choice:

$$x = \text{Expr}(j, t) \rightarrow \text{Outc}(\mathbf{p}(\text{CRCor}(j, \text{Cont}', t', t))) = x$$

The expression of a message is, thus, an observable *physical* thing, a symbolic description or a signal, which corresponds more or less to the meaning the sender of message wanted to indicate.

3) *Received message*, RecMess, is that form of an expression that is included to a real state of behavior of the subject called the receiver:

$$x = \text{RecMess} \rightarrow (\exists y) \text{Be}(i, y \supset x, t)$$

The received message is a physical, observable thing, too. It should be emphasized that receiving a message does not mean that it is already interpreted by the receiving person, not at all. It simply means that there exists in a person's situation a psychologically relevant physical phenomenon which *could* be interpreted as a message having some meaningful content. (Because the expression may be changed physically so that the form in which it will be received can be different, we use the different notation Mess for this received form.)

4) *Interpreting* (or understanding) *a message* occurs when, as an outcome of determination the cognitive map of the receiver, a piece of meaningful information (corresponding to the received message – to a certain amount) becomes included to a cognitive state as a *content of message by the receiver*. This concept is formally defined as follows:

$$x' = \text{Cont}'(i, t') \rightarrow \text{Be}(i, y \supset \text{RecMess}, t) \ \& \ (\exists M') \text{Outc}(\mathbf{p}(\text{RCR}(i, y, t))) = M' \ \& \ M' \supset x'$$

Apparently, the content of message, interpreted by the receiver, needs not to be the "same" as the content of the message by the sender. An essential problem in the communication is just how "far" they are from each other. Applying our theory we are able, in principle, to answer to the question, what is the probability that a certain content of the message by the receiver comes out if the content of the message by the sender is given. This measure can be estimated if the conditional probabilities in the following chain are known:

$$\begin{aligned} & \Pr[\text{CRCor}(j, \text{Expr}(j, t), t) \mid \text{Cont}'_j] \\ & \Pr[\text{RecMess}(i, t) = \text{Expr}(j, t)] \\ & \Pr[M' \supset \text{Cont}'_i \mid \text{Be}(i, x' \supset \text{RecMess}, t+1)] \end{aligned}$$

5) An important aspect of communication has to be added: the *believing* of the truth of the content of the message (by the receiver). Believing the content to be true of false has, apparently, a great effect upon the cognitive process and decisions to choose the behavior alternatives.

Note: Not-believe, $\neg\text{Bel}$, should be understood differently from the believing that the content is false, $\text{Bel}(\text{Cont}_i = \text{false})$. Not-believing means, clearly, the act to ignore the message.

There exist, obviously, complicated philosophical problems concerning the psychological communication process, problems which can not be handled here. The purpose of this analysis was only to show that our basic conceptual system is flexible enough for an exact representation of some essential and psychologically relevant aspects of communication. The great advance of our theoretical framework, as compared with the pure verbal theorizing, is that it gives the possibility for mathematically testable estimation.

Chapter 3. Experiment on Problem Solving by Group and Its Simulation

3.1. Group Maze. The experimental set-up

From the point of view of the operationalization of the problem solving behavior, the traditional tasks used in the individual level are not very fruitful, because too many details remain hidden in the individual cognitive process. Because the subjects in group problem solving have to communicate between each other, much more observable data – the messages sent between the group trials – are produced.

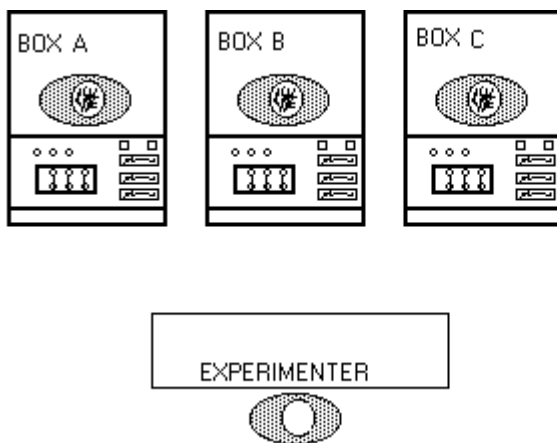
Every group task can be represented in a form of a graph on which all possible combinations of the individual behavioral states appear as vertices. Further, a graph is only another name for a maze. Thus, to study group problem solving, a particular "Group Maze" experiment was set up.

For an illustrative purpose, we shall first give a detailed picture of the Group Maze experiment and later in presenting the simulation procedure, show the links between it and our stochastic theory of cognitive process.

For the experiments reported in the following, groups of only *three individuals* were studied in a game in which each person was allowed *two alternatives*. To eliminate distractions, the game was made extremely abstract - so abstract that the problem arose of how to motivate the subjects enough to play it.

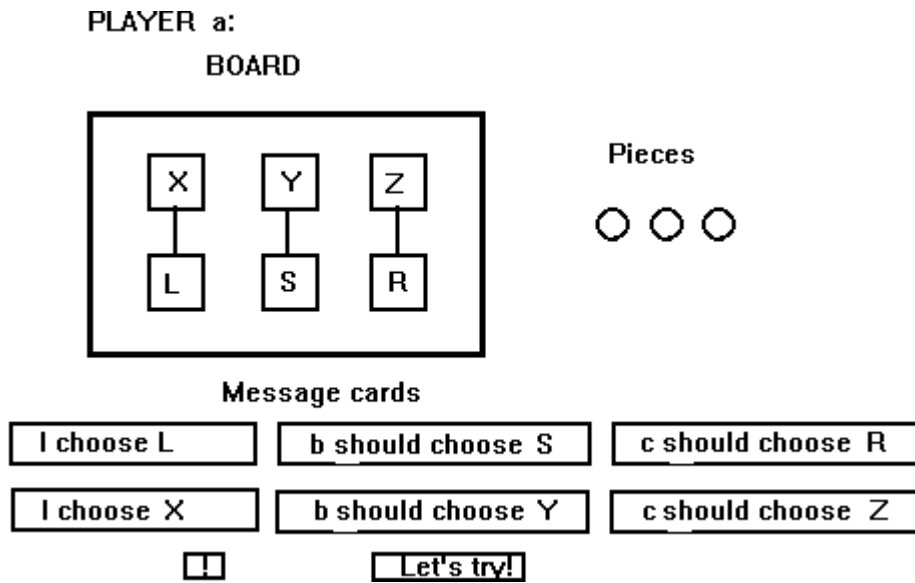
As it was absolutely necessary to eliminate all immediate communication between the subjects, they were made to sit in separate boxes with no face-to-face contact, and in total silence (see Fig.3.1.). They could communicate only through the experimenter, using special message cards.

Fig.3.1. Group Maze experiment. Subjects and experimenter



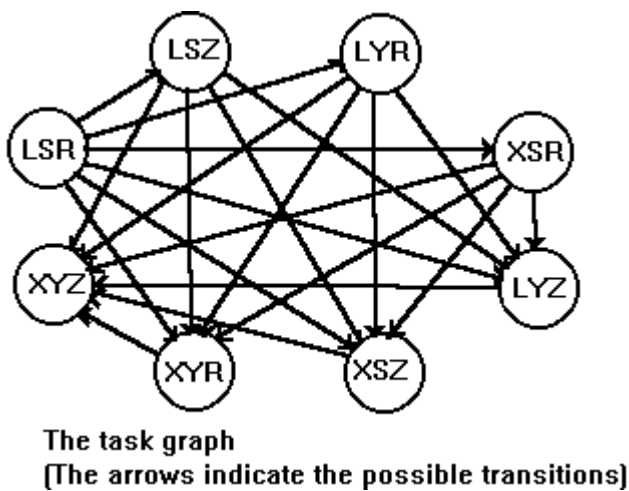
Each subject had a board and a small pack of message cards, as shown in Fig. 3.2.

Fig. 3.2. Group Maze experiment. Player's board and message cards



The experimenter had a map of the group maze in the form of a graph representing the group problem (Fig. 3.3), with arrows indicating the correct group moves (the passable routes through the maze).

Fig. 3.3. Group Maze. The task graph



One can represent the graph in Fig. 3.3 also as a matrix:

The group task as a matrix of the probabilities $p(\text{Succ}())$

	LSR	LSZ	LYR	XSR	LYZ	XSZ	XYR	XYZ
	000	001	010	100	011	101	110	111
	0)	1)	2)	4)	3)	5)	6)	7)
LSR	1	1	1	1	1	1	1	0
LSZ	0	1	0	0	1	1	1	1
LYR	0	0	1	0	1	1	1	1
XSR	0	0	0	1	1	1	1	1
LYZ	0	0	0	0	1	0	0	1
XSZ	0	0	0	0	0	1	0	1
XYR	0	0	0	0	0	0	1	1
XYZ	0	0	0	0	0	0	0	1

(Situations are named by letters, by binary numbers - indicating the points the group gets from the situation - and by decimal numbers corresponding the binary ones. - Notice, particularly, that XSR is number 4 and LYZ number 3.)

The valences of the states were created by telling the players how many points they would win as a group upon reaching each state. (No additional instruction on the final goal was given.) Thus, on every successive move the group would win more points if a state with a higher valence was reached.

The idea was the same as in existing individual maze experiments: to let the subject (here the group) run through the maze several times and record the mistakes it made each time.

According to our theoretical system, a group move or "group response" was achieved when all the members made a simultaneous trial. In terms of the game, this meant an unanimous group decision – *i.e.*, three consecutive "Let's try" messages from the subjects.

The *instructions* were given verbally in five parts: Part 1 after the subject had been asked to keep absolutely silent throughout of the experiment, Part 2 after every player had sent one message, Part 3 when the group was making its first group response, Part 4 after the group had reached the goal for the first time (gone through one "game"), and Part 5 when the group seemed to have learned the solution.

Part 1:

"The aim of this experiment is to study how a group of three individuals solves a common problem. For this game, you will use the board and three pieces you see in front of you. You will be called A, B, and C - as you can see from your boards. You will be A, you B, and you C. On each board there are six positions marked L, S, and R in the bottom row and X, Y, and Z in the top row. Each of you will have two of these positions: A will have positions L and X; B, S and Y; and C, R and Z. To play the game, choose one of these two positions and put one of your pieces on it. A combination of your three choices will then be a trial by the group."

"Use your other two pieces to mark the choices of the other players. This will remind you of your fellow player's moves."

"You can also send messages, using the message cards you see in front of you."

”When you make a move, tell me what your choice is by showing me the appropriate message card. But before going into message-sending in detail, I will tell you about the scoring. After you have all made a trial, I’ll tell you whether the trial has succeeded or not. If it succeeds, the group will get as many points as there are pieces in the top row, and the game will continue from this new combination. If the trial fails, the group will get as many points as there are pieces in the top row and the game will continue from the situation you reached before the trial.”

(Nothing more is said about scoring. The experiment does *not* inform the group what the final goal is. The process of finding out what the real goal is, is an essential part of the game itself.)

”As I said, you can send the messages written on the cards. To do so, first hold up the card with an exclamation mark on it to attract my attention. Then tell me your choice of move by holding up the card with the appropriate message on it. At the same time, if you want, you can send suggestions to one or both of the other players – or to neither. You will find the cards to do so in the pack in front of you. So, to send a message, you can use one, two or three cards. I’ll then read your message out aloud.”

(In passing on a message, the experimenter should always read it in a neutral tone. Suggestions to the other players must always be read in the form ” ... should choose... ” so as to eliminate any variation in the strength of the suggestion.)

”Now you can ask for your turn to send a message. At least, please let me know what your first move choice is – whether to move your piece or keep it where it is. We start with positions L, S, and R. So put your pieces on the bottom row. All right, I’m waiting for your messages.”

Part 2 (after every player has sent his first message):

”Each of you has sent a message, so you know what moves the other players want to make. You can now choose whether to stick to these moves or not. If you think the first trial can be made using this combination, hold up your ‘Let’s try’ card, if not, send the other players further messages with other cards.”

Part 3 (after three successive ”Let’s try” messages):

”Now, as you have all said ‘Let’s try’, the group trial can take place. Thus your pieces are now in positions (*e.g.*) L, Y, and Z. That’s the trial as it stands.”

Using his map of the group problem (Fig. 3.2), the experimenter checks to see whether the trial is in the direction of an arrow or not. Part 3 of the instruction then continues in one of the two forms:

1) (If the trial corresponds to an arrow on the map):

”This trial was successful, so the group has won ... points because there are ... pieces in the top row. The game can now continue from this combination. Move your pieces to positions ... , ..., and ... and send your next messages. Remember – you can send a message with one, two or three cards. You must tell me what your own choice is, but you can also send suggestions to one or both of the other players if you want.”

2) (If the trial does not correspond to an arrow on the map):

”Sorry, but this trial was not successful. The group has won ... because you have ... pieces in the top row. If the trial had succeeded, you would have got ... because there would have been ... pieces in the top row. So the game will continue from the positions ... , ... , and” (After the first trial: ”... from the starting positions L, S, and R.”) ”Now send your next messages. Remember – you can send a message with one, two or three cards. You must tell me what your own choice is, but you can also send suggestions to one or both of the other players if you want.”

Similar instructions are used after every group trial.

Part 4 (at the end of a game, after the group has made a successful trial towards X, Y, and Z):

”You have reached positions X, Y, and Z which gives the group three points. As you can see, this is the maximum score you can get with a single trial, so you will win three points for every trial still left to you. Now I will tell you what I did not tell you before: you had 20 trials in which to reach X, Y, and Z and you have now used ... trials. That means you win ... times three points for the trials left over. So altogether you get ... points for this game.”

”Now we start again from positions L, S, and R. Set your pieces on these circles. The rules are still the same, so please start sending your messages.”

Part 5 (after the group has played two consecutive games with the same moves):

”Before starting again from the initial positions, I’ll ask your opinion about continuing this game. What would you prefer: to go on playing, or to get the same number of points as in the last game for every game left? There are ten games altogether, and you have now played ... games, so you will get ... times ... points for the remaining games if you do not continue. How do you feel about it? Those of you who would like to stop this problem, please hold up your exclamation-mark cards.”

If two players want to stop working on this problem, the experimenter announces it is ended. In its standard form, the whole experiment is now over. If two or three group members want to continue with the problem, the players are asked to start again from the initial positions L, S, and R.

A data protocol was made of all the messages, as well as of the group responses (trials).

This were the instructions used in the main study. However, it seems more reasonable to let the groups play always 6 games. Thus, the Part 5 of the instructions should be changed correspondingly.

(If more difficult tasks are used, the number of games may be larger than 6.)

3.2. Groups studied. Empirical results

There were three experimental studies on the Group Maze made, one by the author and two by Miss Orvokki Vuokko, who made her Master’s thesis 1962 at Tampere University, concerning particularly the characteristics of the leaders in Group Maze test. (That theme is not taken account here. The results are referred in Rainio 1972.)

All the subjects in these three studies were university students.

The number of groups in each study are given in Table 3.1.

Table 3.1. The groups in the experimental studies.

Number of groups:

	Game					
	1	2	3	4	5	6
E1	15	15	15	11	7	5
E2	17	17	17	17	15	14
E3	20	20	19	17	16	13
Σ	52	52	51	45	38	32

Unfortunately, the test was in some cases disrupted before the 6th game, sometimes even earlier, because the group was already solved the problem successfully two times, repeating the same path twice. Thus, the results on the 5th and even more on the 6th game are based on smaller account of cases.

We are focusing our attention here only upon those results which are essential in comparisons we shall make between the estimates produced by simulation and the empirical variable values. (There is a lot of detailed information published in Rainio 1972, concerning the reliability, correlations and common factors of the variables *etc.*)

One of the most important variables is the number of *failures* made by group on each period of the experiment – called a game. Fig. 3.3 shows that the results form a typical learning curve. It may seem rather surprising that the Bush & Mosteller's operator model suits extremely well to these failure results. The same information is given also in Table 3.2, where also the standard deviations are shown.

Fig. 3.3. Failures. Means and standard deviations

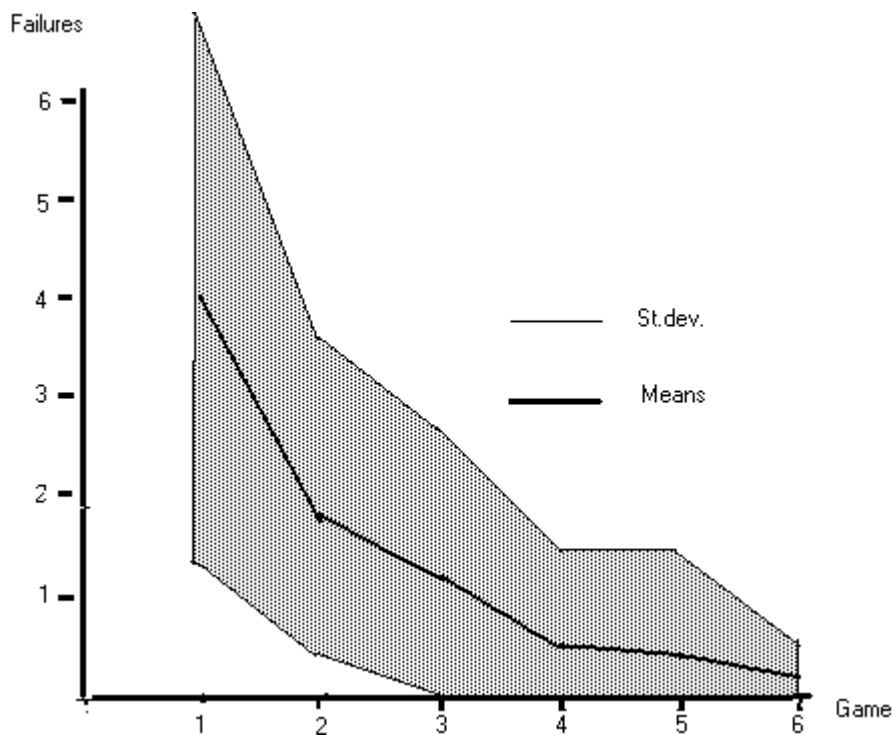


Table 3.2. Failures. Means per game and standard deviations
(Rainio, 1986, p. 185)

Failures/game:						
Means:	4.0	1.7	1.1	.51	.42	.16
St.dev.	2.8	2.0	1.6	.80	1.0	.28
[Trials:	320	201	170	121	100	73]
[Succeeded:	112	112	114	98	84	68]
[Failed:	208	88	56	23	16	5]

Predictions made by Bush & Mosteller's one operator learning model (see Rainio, 1986, p. 172):
 $p_{t+1} = p_t + \alpha(1 - p_t)$ where $\alpha = .3$

Game	p Probability of a trial to be correct	Failures per game	
		Predicted	Empirical
1	.33	4.0	4.0
2	.54	1.7	1.7
3	.68	.94	1.1
4	.78	.57	.51
5	.84	.37	.42
6	.89	.24	.16

The reason of the phenomenon that a mathematical learning model used for predicting trial and error -behavior of individuals is not easily found. It is difficult to understand why it should be so. Mathematically this is a very complicated matter.

We can notice only that *somehow* the result should be a consequence of the complicated process of the cognition and communication of the individuals in the group. We shall deal that process only by simulation, but we can hardly see in it any *explanation* for the found emergent feature of the simple group learning. Why the learning on the group level, as a whole, takes just the form of typical trial and error process?

The standard deviations of the mean values of failures per game are rather large. In the so-called differential psychology, we are used to see this kind of variations as a consequence of individual differences, analogically group differences in our case. In the light of our simulation we shall see that this conclusion is not adequate. Naturally, group differences *may* exist but they are not the explanation of the variance in this case.

Some characteristics of the communication are measured and the results shown in Table 3.3.

Table 3.3. Communication measures.

Messages/game, means:

24.1 14.4 13.0 9.7 10.3 8.6

("Let's try" messages are not taken into account.)

Dominance suggestions/trial, games, means:

.81 .86 .86 .85 .71 .45

(The maximum amount of dominance suggestions/trial is 3.)

Change suggestions/message, games, means:

.167 .140 .133 .110 .088 .078

Making a suggestion to another player before he has sent any message is called a *dominance suggestion*. The first message sender can, thus, make two dominance suggestions, the second sender only one. This can happen only in the beginning of the trial. Therefore the maximum amount of the dominance suggestions per trial is 3.

When a player has sent a message (informed others about his own choice, at least), then another player can, if he will, send to him suggestion to make the other choice. This suggestion is called a *change suggestion*. The frequencies in the Table 3.3 are relatively low.

The successful trials form a *path* from the start, state 0, to the goal, state 7. There are 15 different paths available: 3 paths with one 1-point step to the goal, namely 0-1-7, 0-2-7, and 0-4-7; 3 paths with one 2-point step to the goal, namely 0-3-7, 0-5-7, and 0-6-7. In addition, there are 9 paths with two steps to the goal. The paths are numbered from 1 to 15 in Table 3.4, where the frequencies and the relative frequencies of the paths used by groups are presented.

Table 3.4. Paths in the first game, frequencies and relative frequencies

	Paths															
	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	14)	15)	
	1-7	2-7	3-7	4-7	5-7	6-7	1-3-7	1-5-7	1-6-7	2-3-7	2-5-7	2-6-7	4-3-7	4-5-7	4-6-7	
Paths in the first game:																Σ
Freq.	0	0	12	2	11	15	0	0	3	1	4	1	0	1	1	51
Relat.	0	0	.23	.04	.21	.29	0	0	.06	.02	.08	.02	0	.02	.02	1
Smoothed distribution:																
	.013	.013	.243	.013	.243	.243	.025	.025	.025	.025	.025	.025	.025	.025	.025	1

We can read that the goal was reached through 2-point states with 2 steps in 73% of the cases.

We can hardly see any psychological reason for the difference between the choices of the steps 0-1, 0-2, and 0-4, no more for the difference between the steps 0-3, 0-5, and 0-6. The same holds well for the choices of paths 7-15. Thus, we handle those differences as random variation and smooth the relative frequencies, as shown in Table 3.4.

How stable is the process? Do the groups repeat the exactly same solution they have found or do they transit from one solution to some other, in the game which follows? For examining this stability, the frequencies of path transitions from a game to the next game was coded and are shown in Table 3.5.

Table 3.5 Path transitions

Path transitions, relative frequencies:

Path	Path in the next game															Σ
	1) 1-7	2) 2-7	3) 3-7	4) 4-7	5) 5-7	6) 6-7	7) 1-3-7	8) 1-5-7	9) 1-6-7	10) 2-3-7	11) 2-5-7	12) 2-6-7	13) 4-3-7	14) 4-5-7	15) 4-6-7	
1)	.2	0	0	0	0	.2	.2	.2	0	0	0	.2	0	0	0	1
2)	0	.14	.28	0	.28	0	0	0	0	0	.14	0	.14	0	0	1
3)	.03	.03	.5	.02	.14	.24	.03	0	0	.03	0	0	0	0	0	1
4)	0	0	.75	.25	0	0	0	0	0	0	0	0	0	0	0	1
5)	0	.04	.18	.02	.49	.22	0	0	0	.02	0	0	0	0	.02	1
6)	0	.06	.27	.02	.31	.29	0	0	0	0	0	.02	0	.02	.02	1
7)																0
8)																0
9)	0	0	.25	0	.25	0	0	0	.25	0	0	0	0	0	.25	1
10)	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
11)	0	0	0	0	.66	0	0	0	0	0	.17	0	0	.17	0	1
12)	.25	0	.50	0	0	0	.25	0	0	0	0	0	0	0	0	1
13)																0
14)	0	0	.20	0	.60	0	.20	0	0	0	0	0	0	0	0	1
15)	0	0	.20	0	0	.20	0	0	0	0	0	0	0	.20	.40	1

As a measure of the *path stability* we shall use the probability of repeating the same path in the next game, *i.e.*, the diagonal values in the matrix of transitions. These probabilities are presented in Table 3.6.

Table 3.6. Path stability.

Diagonal frequencies and relative frequencies of the path transition matrix (see Table 3.6):

	1) 1-7	2) 2-7	3) 3-7	4) 4-7	5) 5-7	6) 6-7	7) 1-3-7	8) 1-5-7	9) 1-6-7	10) 2-3-7	11) 2-5-7	12) 2-6-7	13) 4-3-7	14) 4-5-7	15) 4-6-7
Freq.	1	1	29	1	22	15	1	0	1	0	1	0	0	0	2
Sum of the cases on the row:	5	7	58	4	45	52	5	0	4	2	6	4	0	3	5
Relative frequencies:	.20	.14	.50	.25	.49	.30	.20	0	.25	0	.18	0	0	0	.40

Note: In some cases, as already mentioned, the play was stopped when the group was used the same path twice (the experimenter assuming that the group would follow the same pattern). Actually this means that, for making the data comparable with the simulated results, we should add the lacking stable paths with 40 cases. The corrected values of path stability are given later, in the Table 3.16 A.

To summarize the results of path transitions, the paths are classified in types and the transition frequencies are computed according to them. There are 3 types of paths, the first type (A) including the paths 1, 2, and 4, the second type (B) the paths 3, 5, and 6, and the third type (C) the rest, paths 7–15. After the mentioned correction with 8% of stable paths is made, the frequencies are presented in Table 3.7.

Table 3.7. Transitions between the path types.

	Frequencies of transitions				Relative frequencies		
	Path type				Path type		
	A	B	C	Sum	A	B	C
A	3	8	5	16	.19	.50	.31
B	12	147	7	166	.07	.89	.04
C	3	15	12	30	.10	.50	.40

This report of the main results of the Group Maze experiment, failures, communication variables, and the path transitions, should be enough for describing the most essential features of the test. The comparison between these variable values and the simulation products in the following chapters will show, how much of these results can be explained by our theory of cognitive process.

3.3. Simulation program

The Group Maze experiment was carried on in the purpose to produce observable variables connected to a cognitive process so simple that we can hope to be able to build an exact model for it.

It is easy to see that the estimates can be produced only through simulation, because the process is stochastic in a complicated way: The actual occurrences during it are dependent on the occurrences before, the probabilities of the varying according to that what actually has happened (what alternatives have been chosen).

A Turbo-Basic program was written for the simulation. One reason for selecting this programming language was that it is easy to understand: there are no difficulties to see the connection between the theoretical model and the program statements. Also a layman can use it – and possibly make his own variations of it.

The main features of our simulation model are as follows:

1) The choice of messages the ‘stat-subjects’ send is based on a chain of cognitive trials they are assumed to have made. (We shall call the statistical subjects simply subjects, because it hardly causes any misunderstanding in the text. By that we do not mean that they could have any “consciousness”. Subjects we are dealing with are abstract logico-mathematical thing, only.) For every situation we assumed a vector of probabilities to choose a cognitive ‘response set’. A *cognitive response set* is a combination of the three choices expected. A probabilistic choice using this vector is made to determine the subject’s cognitive trial. After that, another probabilistic choice is made to determine whether the cognitive trial succeeds or not. If this trial (in cognition) succeeds, the subject sends a message informing the others of his choice. If the trial fails, the subject makes new cognitive trials until one of them succeeds or else a certain maximum number of them has been made (10 in our simulation). In this last case the subject does not send any message and the others take turn.

2) When a subject decides to send a message, a determination is also made whether he sends suggestion or not. This is based on probabilities to send a ‘dominance suggestion’ or to send a ‘change suggestion’(see Chapter 3.2). If the response set is in accordance with the message sent by the other subject, no suggestion is sent to him.

If a suggestion is sent to the subject from the others, a probabilistic choice is made to determine whether he follows it ('submissive behavior') or not.

3) When all subjects have sent at least one message, the subject sends a 'Let's try' message if his images of the other's future behavior (his 'response set') corresponds to the messages they have sent.

4) A group response (trial of the group) is made in simulation as in the real game: after all subjects have consecutively sent their 'Let's try' messages.

The success of the group response (group trial) is determined as in the real game, according to the 'maze'.

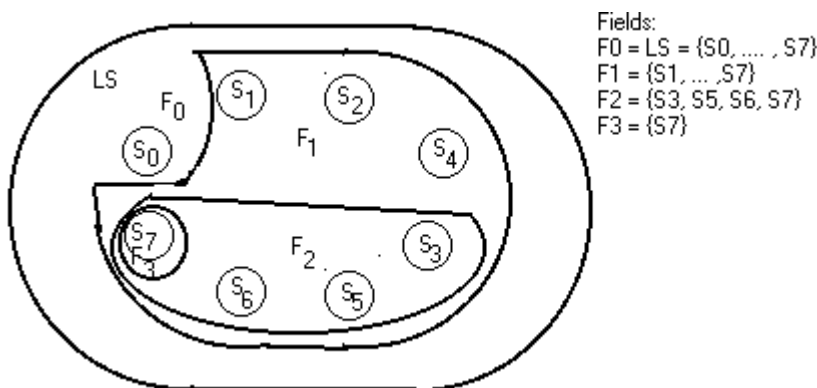
5) If the group response succeeds, a certain reward procedure is carried out and, in the case of a failure, a punishment procedure.

6) The probabilities used for determining the cognitive trials (mentioned in paragraph 1, above) are formed as follows:

We shall assume two cases of the Life Space, the potencies of fields in them varying radically. They are noted by LS_r and LS_d . The first will be called *diffuse Life Space*, the second *differentiated Life Space*.

Three fields – in addition to the Life Space, LS – are assumed (see Fig. 3.4):

Fig. 3.4. Fields of the Life Space. Group Maze. Task



F3 including the 'goal' state (s_7 or 111, *i.e.*, 7 in binary form, corresponding to XYZ in the game),

F2 including all 2-point states ($s_3, s_5,$ and s_6 - in binary forms 011, 101, and 110 - corresponding to LYZ, XSZ, and XYR) and the goal state, s_7 ,

F1 including all states other than s_0 (000, LSR), and

F0 which includes all the states.

Summarizing:

$$F_0 = \{S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$$

$$F_1 = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$$

$$F_2 = \{S_3, S_5, S_6, S_7\}$$

$$F_3 = \{S_7\}$$

For every field, the valences are:

	F	¬F
F	1	0
¬F	1	0

Every field has a potency value. The potency of field F_k is indicated by Po_k . The computation of the cognitive trial probabilities according to the fields was described in Chapter 2.6.3 and 2.6.4. We shall use here the equation presented in that context (Chapter 2.6.4):

$$Pr[\mathbf{x}, \mathbf{y}] = \sum_{j=1}^m Pot_j \cdot Pr[R_j(\mathbf{x}), R_j(\mathbf{y})] / NR_j(\mathbf{y})$$

We shall assume that the potencies of the fields in a completely differentiated Life Space are those given in Table 3.8 on the first row vector:

Table 3.8. Assumed values of the potencies of the fields

	Field			
	F0	F1	F2	F3
Differentiated LS:	0	.07	.23	.70
Diffuse LS:	1	0	0	0

(I shall confess that selection of potency values was totally intuitive. The basic idea was that the potencies might be equal to the squares of the point-values ('rewarding power') they have. This should give the following relative values for them: 0, 1², 2², and 3², *i.e.*, 0, 1, 4, and 9. Thus, the potencies were 0, 1/14 = .07, 4/14 = .29, and 9/14 = .64, so that the sum will be 1. As one can see, only slight changes was necessary to make then during the simulation, to get better goodness-of-fit.)

The reason to use these two components is the following:

We may well assume that, after the subjects have just heard the instruction with the clear point-counting system, the cognitive map, the Life Space, is clearly structured to the valence fields. The expectation that the goal should be easily reached, is high. (Nearly 90 % of the groups actually choose the goal state, s_7 , on the first trial.)

When the actual Life Space is maximally differentiated, *i.e.*, the cognitive trial probabilities are based *only* on the fields, equaling the f-probabilities, then the *field effect* is said to be in its maximum, 1. If the actual Life Space is maximally diffuse, the field effect is 0. Thus, we define the field effect as the *measure of the differentiation* of the Life Space.

(The terms differentiated and diffuse are used here in gestalt-psychological sense.)

But what happens, when the trial toward the goal fails? Psychological understandable is that there occurs some kind of *frustration-like behavior*. Typical for it are, at least in the deep frustration, rather meaningless action outbursts without a clear goal-seeking. One may well argue that it corresponds to such a cognition where the valence structure is disrupted and the actual Life Space homogeneous.

Note: Naturally the frustration is in the play situation very limited. The subject is frustrated only inside the game to which he is committed, so to say. Theoretically we can argue that in the case of a continued frustration during the play the probability to leave the actual Life Space may

increase and cause even the stop of the playing. In that case we may see typical emotional reactions caused by heavy frustration – not in the case of one failed trial.

The frustration-like behavior means in cognition, according to our theory, a radical change of the field effect from 1 in the beginning to nearly 0.

According to the equation above and the values of the potencies of the fields, the cognitive trial probabilities from state to state, based on the fields of the differentiated Life Space, were calculated. They are called shortly *f-probabilities*. In addition to them we have the *r-probabilities* based on the diffuse Life Space.

Each cognitive trial probability is now a weighted sum of two components: a) a r-probability, a random probability (1/8 in our case) and b) a f-probability, the probability computed according to the valence fields of the differentiated Life Space. The weighted sum is the *final cognitive trial probability*. Thus

$$\text{cognitive trial probability } p = w \cdot p_f + (1-w) \cdot p_r$$

where the weight w is the field effect.

Table 3.9 shows the component vectors and the cognitive trial probabilities in the beginning (field effect being 1) as well as the component vectors and the cognitive trial probabilities after the frustration, field effect being now as small as .05.

Table 3.9. Cognitive trial probabilities

(The rows $s_1 - s_7$ in the following matrices are equal to the row s_0 .)

1) In the beginning; field effect =1.

	States							
	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7
r-component	.125	.125	.125	.125	.125	.125	.125	.125
f-component	0	.01	.01	.07	.01	.07	.07	.77
Cognit. trial prob., weighted sums	0	.01	.01	.07	.01	.07	.07	.77

2) After frustration; field effect = .05

	States							
	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7
r-component	.125	.125	.125	.125	.125	.125	.125	.125
f-component	0	.01	.01	.07	.01	.07	.07	.77
Cognt. trial prob., weighted sums	.119	.120	.120	.122	.120	.122	.122	.157

7) The reinforcement procedure, mentioned in paragraph 5, above, is carried out in the following way:

Two operator learning model (Bush & Mosteller's model) is applied to the *field effect*, using different coefficients for reward (α) and punishment (β). In addition to that, the amount of frustration, caused by a failed trial, is assumed to decrease during the process. This effect is caused by letting the β -coefficient itself decrease on every trial by the rule: $\beta_{t+1} = \beta_t - \beta' \cdot \beta_t$, where β' indicates the amount of the decrease of β at every trial.

Bush & Mosteller's model is applied also to the following variables, but the learning coefficients α and β are assumed to be equal:

Success probabilities of cognitive trials (name in the program: CognSuccProb)

Probabilities of submissive behavior (SubmProb)

Probabilities of sending dominance suggestion (DomProb)

Probabilities of sending change suggestion (ChangeProb)

In addition, a transfer effect is assumed, concerning the success probabilities of cognitive trials. In the case of negative transfer, if a trial toward the state s_k is failed, all the probabilities of the success of the trials toward it are decreased, *i.e.*, the column s_k is changed according to the rule:

$p' = p - N_{\text{transf}} \cdot \beta \cdot p$, where p' is the new value of the CognSuccProb-variable, p being the old one, β the learning coefficient, and N_{transf} a special negative transfer -coefficient.

A corresponding change of the probability column is made in the case of success, all the probabilities in the column being increased using a positive transfer coefficient P_{transf} .

A detailed description of the simulation program is given in the Appendix 4 and the total program in Basic language in the Appendix 5.

3.4. Simulation results. Goodness-of-fit

The Group Maze simulation model is very sensitive for some parameter values, particularly for the potencies of the fields and the field effect, because they determine the early failures and successes. Thus, these parameter values are determined carefully by 'range-finding', attempting to fit in the distribution of the very first empirical trials as well as possible. By the 'range-finding' we mean simulations first with largely varying parameter values, intending to get the experimental results *between* the estimates by those simulations (to the 'range'). When the results are in this 'range', the range can be diminished by simulating with parameter sets nearer each other, until the best fit is reached.

When the range-finding method is used, it is clear that the conventional statistical tests to compare the theoretical estimates and the empirical data are not anymore acceptable. Several degrees of freedom are lost, when the empirical results are used for the choice of the best (or nearly best) parameter values in simulation process. To get the correct statistical test for measuring the estimation power of our model, new experimentation is needed. However, we can get an intuitive picture of goodness-of-fit from the comparisons of the empirical and simulated results made in this pioneering study. Anyway we can see how to apply our theoretical framework, in principle, to derive out of it exact mathematical estimates.

3.4.1. Homogeneous groups. Parameters and results

We start with homogeneous groups, *i.e.*, with groups where all the parameter values used in the simulation process are equal for all three subjects ('stat-individuals'). Later we shall examine whether differing parameter values for subjects produce different estimates, when we shall simulate unhomogeneous groups.

The parameter values used in the simulation of homogeneous groups is given in Table 3.10. *Results* of the simulation are presented in Figures 3.5 and 3.6 and in Tables 3.12 - 3.14. There are 200 simulated groups and 52 empirical groups (see Table 3.1).

Table 3.10. Parameter values for GM Simulation. Homogeneous groups

FldEffbeg=1: field effect in the beginning
 alfaFldEff=.1: learning coefficient α for the field effect
 betaFldEffbeg=.95: learning coefficient β for the field effect
 betbetFEff=.1: the rate of change of the betaFldEff
 Ptransf=.15: the rate coefficient of the positive transfer
 NTransf=.3: the rate coefficient of the negative transfer
 wei2=FldEffbeg;wei1=1-wei2; wei1 for TrErProb (r-probabilities), wei2 for f-probabilities

1,1,1,1,1,1,0 Task, success probabilities of the real trials
 0,1,0,1,0,1,1,1
 0,0,1,1,0,1,1,1
 0,0,0,1,0,0,0,1
 0,0,0,1,1,1,1,1
 0,0,0,0,0,1,0,1
 0,0,0,0,0,0,1,1
 0,0,0,0,0,0,1

0,1,1,2,1,2,2,3: Field() which includes the state s_0, s_1, \dots etc.
 0,.07,.23,.70: potencies of fields
 8,7,4,1: nStateInF(), number of states in a field

0,.8,.8, .8,0,.8, .8,.8,0: SubmProb, probabilities of submissiveness
 .25,.25,.25: DomProb, probabilities of sending dominance suggestion
 .2,.2,.2: ChangeProb, probabilities of sending change suggestion
 .2,.2,.2: aTrErr, learning coefficients for r-probabilities
 .8,.8,.8: bCognS, learning coefficients for cognitive success probabilities
 .1,.1,.1: aSubm, learning coefficients for probabilities of submissiveness
 .05,.05,.05: aDom, learning coefficients for probabilities of sending dominance suggestion
 .05,.05,.05: aChange, learning coefficients for probabilities of sending change suggestion

The curve of failures produced by the simulation fits extremely well in the empirical failure data. But they are not only the means which fit. The empirical and estimated standard deviations of the failures do not differ much from each other. One has to notice that the standard deviations of the empirical data are *not* caused by the random variation – as in the case of conventional psychological tests – but *explicable* by the characteristics of our theoretical model.

Fig. 3.5. Failures, empirical and simulated

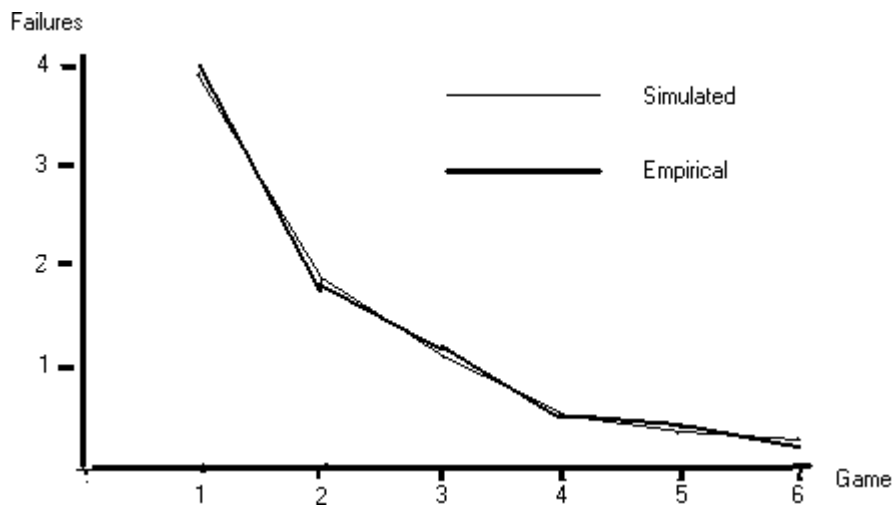


Fig. 3.6. Failures. Standard deviations, empirical and simulated groups

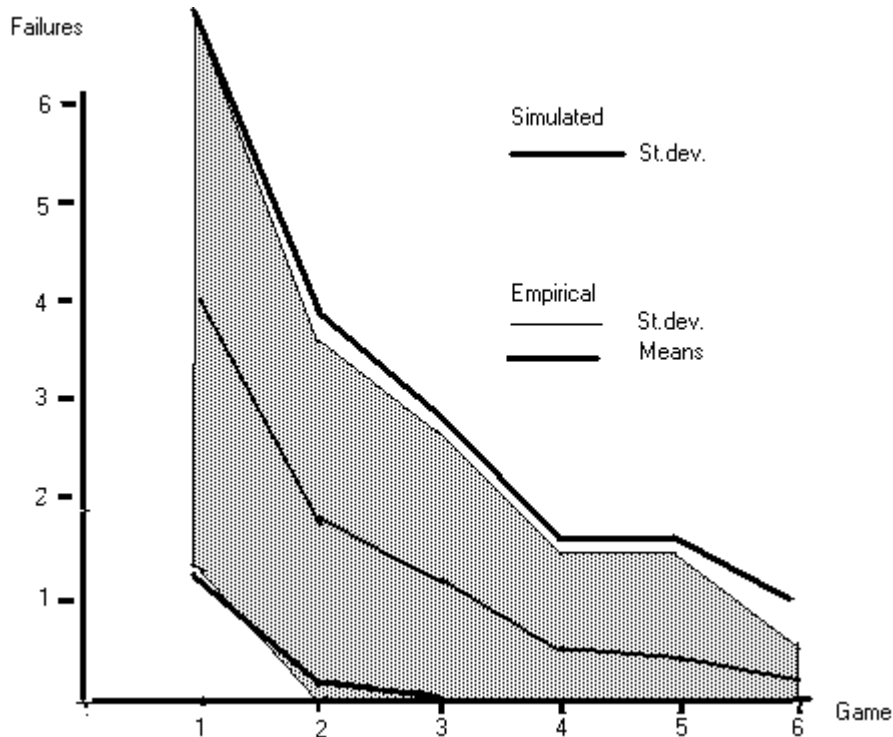


Table 3.12. Failures. Means per game and standard deviations, empirical and simulated

	Failures/game, means:					
<i>Empirical</i>	4.0	1.7	1.1	.51	.42	.16
<i>Simulated</i>	3.95	1.84	1.08	.47	.34	.31
	Standard deviations:					
<i>Empirical</i>	2.8	2.0	1.6	.80	1.0	.28
<i>Simulated</i>	2.86	2.28	1.75	.92	.87	.76

Note: The biggest differences of the empirical and simulated results are in the sixth game, concerning both the failure means and the standard deviations of failures. The reason might be that some plays were stopped before the sixth game, as was mentioned in Chapter 3.2.

Results of the communication variables are presented in Table 3.13. The simulated values fit also in this case well in the empirical data. There exist one notable difference: In the fifth and sixth games

the simulated frequencies of dominance suggestions are somewhat higher than the empirical ones. Actually there seems to be decrease in the empirical values but slight increase in the simulated ones. If it is not occasionally so, we have to think some reason for that. We might well argue that in the real play some subjects notice that the dominance suggestions are not reasonable later in the game, when everybody seems to have already learned the best solution. This kind of ‘higher cognition’, a conscious control of own behavior in this matter, is not taken into account in our model. We handle the tendency to send dominance suggestions on trial and error basis, as an character feature of the individual. In contrast to that, sending the change suggestions is dependent on the situation.

Table 3.13. Messages and suggestions, empirical and simulated

	Messages/game, means:					
<i>Empirical</i>	24.1	14.4	13.0	9.7	10.3	8.6
<i>Simulated</i>	25.1	16.8	12.7	11.1	9.5	9.2
	Dominance suggestions/trial, means:					
<i>Empirical</i>	.80	.95	.98	.99	.80	.77
<i>Simulated</i>	.76	.85	.85	.90	.94	.97
	Change suggestions/message, means:					
<i>Empirical</i>	.18	.13	.13	.11	.10	.10
<i>Simulated</i>	.13	.14	.14	.12	.12	.10
	Submissive behavior/game, means:					
<i>Simulated</i>	5.33	3.85	3.12	2.29	2.14	2.19
<i>(Empirical, not possible to compute.)</i>						

Paths used by the groups in the *first* game are shown in Table 3.14, both empirical and simulated. The paths are numbered from 1 to 15 and from the names of the paths the starting state 0 is left out. Thus, ‘1-7’ indicates path 0-1-7, ‘1-3-7’ the path 0-1-3-7 etc.

Table 3.14. Paths in the first game, empirical and simulated frequencies

		Paths															
		1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	14)	15)	
		1-7	2-7	3-7	4-7	5-7	6-7	1-3-7	1-5-7	1-6-7	2-3-7	2-5-7	2-6-7	4-3-7	4-5-7	4-6-7	
Paths in the first game:																Σ	
Empirical:																	
Freq.		0	0	12	2	11	15	0	0	3	1	4	1	0	1	1	51
Relat.		0	0	.23	.04	.21	.29	0	0	.06	.02	.08	.02	0	.02	.02	1
Smoothed distribution:																	
		.013	.013	.243	.013	.243	.243	.025	.025	.025	.025	.025	.025	.025	.025	.025	1
Simulated:																	
Freq.		9	6	42	3	26	46	7	8	5	10	10	6	7	8	7	200
Relat.		.045	.03	.21	.015	.13	.23	.035	.04	.025	.05	.05	.03	.035	.04	.035	1

It was argued in the Chapter 3-2 that we can not find any reason why the some of the two-step with 1-point path should be chosen more often than another, e.g., 0-4-7 more often than 0-1-7 or 0-2-7. The same argument concerns the two-step with 2-points path, too, as well as the paths 7) - 15). We can clarify the picture by classifying the paths in a new way, i.e., by summing up the classes 1), 2), and 4) to one class A, the classes 3), 5), and 6) to the class B, as well as classes 7)...15) to the new class C. We can say that the results handled in this way are *smoothed for symmetry*. The information in Table 3.14 is represented in this new form in Table 3.15.

Table 3.15. Path types in the first game, empirical and simulated results, smoothed for symmetry

		Path types				
		A	B	C	Σ	
Empirical relative frequencies			.04	.75	.21	1
Simulated	" "	.09	.57	.34		1

Path types: A = paths 1,2, and 4; B = paths 3,5, and 6; C = paths 7 - 15

Looking the path types in Table 3.15 we shall notice that there are somewhat less paths of type B in the simulated estimate than in empirical data.

In examination of the path transitions, we shall use 5 classes of paths for not loosing too much information: type A, paths 3, 5, and 6, and type C. In addition, we shall make the correction for the lack of cases transition by adding 40 cases to the diagonal values, because we assume that the lacking cases are all transitions from path to the same path (stable cases). In the diagonal of the transition matrix (computed from the Table 3.16 in Rainio, 1986, p. 186) there are altogether 74 cases. Adding 40 cases means that every diagonal value needs to be multiplied by $1+40/74 = 1.54$. This *correction for comparison* is shown in Table 3.16.A. After that, the relative frequencies are computed (B) and smoothed for symmetry, as shown in Table 3.16.D.

Table 3.16. Path transitions, empirical and simulated

A) Transitions, empirical frequencies and the corrections for comparison:

	A	3	5	6	C	Σ
A	3+1	5	2	1	5	17
3	5	29+15	8	14	2	73
5	3	8	22+12	10	2	57
6	4	14	16	15+8	3	60
C	3	6	6	3	11+4	33

B) Transitions, empirical relative frequencies:

	A	3	5	6	C	Σ
A	.235	.294	.118	.059	.294	1
3	.068	.603	.110	.192	.027	1
5	.053	.140	.596	.175	.035	1
6	.067	.233	.267	.383	.050	1
C	.090	.182	.182	.090	.455	1

C) Transitions, simulated relative frequencies:

	A	3	5	6	C	Σ
A	.28	.12	.21	.18	.21	1
3	.12	.51	.13	.09	.15	1
5	.13	.13	.48	.12	.14	1
6	.09	.07	.11	.62	.12	1
C	.17	.16	.20	.19	.27	1

D) Transitions, empirical frequencies smoothed for symmetry, compared with simulated results:

	A	3	5	6	C	Σ
A						
<i>Empirical</i>	.23	.16	.16	.16	.29	1
<i>Simulated</i>	.28	.17	.17	.17	.21	1
3						
<i>Empirical</i>	.07	.53	.21	.21	.03	1
<i>Simulated</i>	.12	.54	.11	.11	.13	1
5						
<i>Empirical</i>	.05	.21	.53	.21	.04	1
<i>Simulated</i>	.12	.11	.54	.11	.13	1
6						
<i>Empirical</i>	.07	.21	.21	.53	.06	1
<i>Simulated</i>	.12	.11	.11	.54	.13	1
C						
<i>Empirical</i>	.09	.15	.15	.15	.46	1
<i>Simulated</i>	.17	.18	.18	.18	.27	1

The comparison between the empirical and simulated path transitions shows that the distributions are quite similar, in main features. There are, however, empirically more transitions from a B-type path to another than produced in simulation. Maybe some small change of parameter values in the simulation is needed – or there might be a psychological phenomenon which is not taken into account in our model, *e.g.*, a ‘higher order’ cognitive tendency to examine whether all the 2-point states are exchangeable.

One of the main characteristics of the process is its stability, measured by the path stability, as presented in Table 3.17. Although the comparison between the rows B and D shows some differences, the simulated results fit quite well in this measure of the empirical data.

We define the *total path stability* the ratio: sum of all diagonal frequencies / sum of all transitions. The empirical total path stability is, thus, = 114/258 = .44. Correspondingly, the total path stability by the simulated groups is equal to .373.

Table 3.17. Path stability. Empirical and simulated measures

Diagonal relative frequencies of the path transition matrix:

	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	14)	15)
	1-7	2-7	3-7	4-7	5-7	6-7	1-3-7	1-5-7	1-6-7	2-3-7	2-5-7	2-6-7	4-3-7	4-5-7	4-6-7
A) Empirical relative frequencies (corrected for comparison):	.27	.20	.60	.33	.60	.38	.27	0	.33	0	.60	0	0	0	.38
B) Path stability, smoothed for symmetry (empirical):	.27	.27	.53	.27	.53	.53	.17	.17	.17	.17	.17	.17	.17	.17	.17
C) Simulated relative frequencies:	.28	.21	.51	.15	.48	.62	0	.33	.07	.04	.04	.18	.09	.22	.14
D) Simulated, smoothed for symmetry:	.21	.21	.54	.21	.54	.54	.12	.12	.12	.12	.12	.12	.12	.12	.12

3.4.2. Unhomogeneous groups. Parameter values and results

The simulation with homogeneous groups, where the learning parameters of the subjects were equal, fitted well in the empirical data although we can hardly believe that in each group the real subjects were psychologically equals. We could well start with more natural ‘stat-groups’, the individuals owing different parameter values. The problem is, however, how they differ. Should the characteristics correlate or not and how big the differences should be. To examine all the possibilities is an unlimited task. We shall make, however, one trial to throw some light on the question, how much the process of unhomogeneous group differs from homogeneous. In this example the means of the three values in each parameter are same as in the case of the homogeneous groups.

The parameter values differing from those used in the case of homogeneous groups are presented in Table 3.18.

Table 3.18. Parameter values for GM simulation. Unhomogeneous groups

(Only those parameter values which differ from the data in Table 3.10 are given in the list.)

0,.7,.7, .8,0,.8, .9,.9,0: SubmProb
 .3,.25,.20: DomProb
 .25,.2,.15: ChangeProb
 .25,.2,.15: aTrErr learning coeffic.
 .7,.8,.9: bCognS coeffic
 .08,.1,.12: aSubm coeffic.
 .04,.05,.06: aDom coeffic.
 .04,.05,.06: aChange coeff.

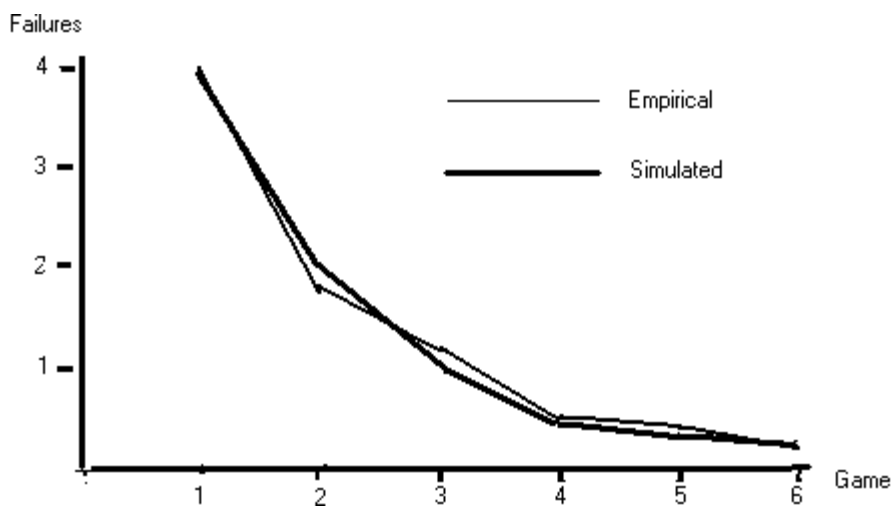
A simulation of 200 unhomogeneous ‘stat-groups’ was made and the results are shown in Table 3.19 - 3.22 and in Figure 3.7.

Table 3.19. Failures, empirical and simulated, unhomogeneous groups

	Failures/game, means:					
<i>Empirical</i>	4.0	1.7	1.1	.51	.42	.16
<i>Simulated</i>	3.97	2.01	1.00	.45	.31	.29

The goodness-of-fit concerning the failures is very high also in this case. The results differ not essentially either from those produced by the simulation of homogeneous groups.

Fig. 3.7. Failures, empirical and simulated, unhomogeneous groups



The figure 3.7 is very illustrative in the matter of goodness.-of-fit.

Four communication measures are given in Table 3.20.

Table 3.20. Messages and suggestions, empirical and simulated, homogeneous and unhomogeneous groups

Messages/game, means:

<i>Empirical</i>	24.1	14.4	13.0	9.7	10.3	8.6
Simulated						
homogen. gr.	25.1	16.8	12.7	11.1	9.5	9.2
unhomog. gr.	25.2	17.3	12.9	10.4	9.5	8.6

Dominance suggestions/trial, means:

<i>Empirical</i>	.80	.95	.98	.99	.80	.77
Simulated						
homogen. gr.	.76	.85	.85	.90	.94	.97
unhomog. gr.	.78	.86	.90	.91	.94	1.0

Change suggestions/message, means:

<i>Empirical</i>	.18	.13	.13	.11	.10	.10
Simulated						
homogen. gr.	.13	.14	.14	.12	.12	.10
unhomog. gr.	.12	.14	.14	.12	.13	.11

Submissive behavior/game, means:

Simulated						
homogen. gr.	5.33	3.85	3.12	2.29	2.14	2.19
unhomog. gr.	5.3	4.0	3.2	2.4	2.2	2.2

(Empirical, not possible to compute.)

The results in two simulations in Table 3.20 do not differ essentially from each other.

Table 3.21. Path types in the first game, empirical and simulated results, smoothed for symmetry

	Path types			Σ
	A	B	C	
<i>Empirical relative frequencies</i>	.04	.75	.21	1
Simulated, unhomogeneous gr.	.08	.60	.32	1

Path types: A = paths 1,2, and 4; B = paths 3,5, and 6; C = paths 7 - 15

In Table 3.21, where the path types in the first game are shown, the simulated values are slightly nearer to the empirical ones than in the case of homogeneous stat-groups. In Table 3.22, the path stability measures are shown and the empirical and simulated values compared in the smoothed form.

Table 3.22. Path stability. Empirical and simulated measures

Diagonal relative frequencies of the path transition matrix:

	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	14)	15)
	1-7	2-7	3-7	4-7	5-7	6-7	1-3-7	1-5-7	1-6-7	2-3-7	2-5-7	2-6-7	4-3-7	4-5-7	4-6-7
A) Simulated frequencies (unhomogeneous groups):	13	2	96	12	101	141	2	4	1	1	3	8	4	5	9
Σ	45	24	192	55	214	248	13	27	30	17	20	27	29	28	31
Rel.fr.	.29	.08	.50	.22	.47	.57	.15	.15	.03	.06	.15	.30	.14	.18	.29
B) Simulated, smoothed for symmetry:	.20	.20	.51	.20	.51	.51	.16	.16	.16	.16	.16	.16	.16	.16	.16
A) Path stability, smoothed for symmetry (<i>empirical</i>):	.27	.27	.53	.27	.53	.53	.17	.17	.17	.17	.17	.17	.17	.17	.17

The Table 3.22 shows that the path stability measures produced by simulation the unhomogeneous groups fit well in the empirical data, still little bit better than in the case of simulating the homogeneous groups.

Summarizing, we have to notice that both simulations reported above are fitted well in the empirical data – surprisingly well, I should say, if we take into account that we have been dealing with a pioneering work on the totally unknown scientific area. To compute statistical significance tests new empirical material is needed, however. Anyway it should be very satisfying to see that our conceptual framework and the main theoretical assumptions are applicable and that mathematical models with good estimation power can be derived from the theory.

3.4.3. Fictive groups. Parameter variations

The mathematical theory has a fascinating characteristics: it allows to examine fictive examples, in our case the cognition and behavior by groups which never existed and never can! We may only change some parameter values radically in our model. It is very tempting to do this to absurdity – particularly because there is no possibility to compare the theoretical estimates with empirical data.

We shall resist the temptation and take only few examples.

It may be interesting to know how the extreme cases look like, *i.e.*, the group problem-solving by subjects with 0-values of learning coefficients as well as by subjects of learning coefficients equaling 1.

The both cases has been simulated (in the first case the field effect also being 0) and the results are shown in Tables 3.23 - 3.24.

Table 3.23. Failures. Fictive groups A and B

Group A: All learning coefficients 0 and field effect 0.

Group B: All learning coefficient 1 and field effect 0.

	Game					
	1	2	3	4	5	6
Group A	7.26	5.89	6.68	6.18	5.76	6.70
Group B	3.11	0	0	0	0	0
Empirical	4.0	1.7	1.1	.51	.42	.16

As it was easily guessed, the failures are not decreased at all in the group A, because no learning could occur. In contrast to that, there are in the group B only few failures in the beginning but no ones later.

In Table 3.24, there are presented the stability of path transitions by the groups A and B, compared with the empirical data. The diagonals of the transition matrices are shown and the total path stability value given. (Total path stability = all diagonal frequencies / all transitions in the transition frequency matrix.)

Table 3.24. Path stability, groups A and B.

Groups A and B as in Table 3.23.

Diagonal relative frequencies of the path transition matrix:

	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	14)	15)
	1-7	2-7	3-7	4-7	5-7	6-7	1-3-7	1-5-7	1-6-7	2-3-7	2-5-7	2-6-7	4-3-7	4-5-7	4-6-7
A) Path stability, group A:	.03	.03	.15	.04	.19	.23	0	.02	.04	.03	.02	.06	.09	0	.03
B) Path stability, group B:	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C) Path stability, smoothed for symmetry (empirical):	.27	.27	.53	.27	.53	.53	.17	.17	.17	.17	.17	.17	.17	.17	.17

Total path stability: Group A: 0.12
 Group B: 1
 Homog.: 0.37
 Unhomog.: 0.37
 Empirical: 0.44

It should be noted that the path stability is very low in group A but complete in Group B. Group B cannot find any alternative solution to the problem.

It is clear that the "groups" A and B do not represent human groups at all. Both are artificial: A is an indeterministic, stochastic automata, B is a deterministic one. It is interesting to notice how the maximum "speed of learning" (coefficients equaling to 1) makes the system inhuman – if we look as inhuman the absolute inability to find more alternatives than one. In other words: in our theoretical model the deterministic process is seen as one of the special forms of behavior, an extreme case.

As examples for the use for estimation by our simulation model, results are produced for three type of groups, where the characteristics of the "leader" and the "followers" are varied:

The data used in the simulation of the three group types are in Table 3.25.

Table 3.25. Leadership simulation. Data

Group A: A "clever" leader and two "stupid" but "submissive" followers.

Group B: A "clever" leader and two "stupid" and "stubborn" followers.

Group C: A "stupid" leader and two normal followers.

Data for all groups:

	Leader	Followers	
Probability of sending dominance suggestions:	.95	0	0
Probability of sending change suggestions:	.95	0	0
Learning coeffic. of submissiveness	0	0	0
Learning coeffic. of sending domin. sugg.	0	0	0
Learning coeffic. of sending change sugg.	0	0	0

Data differing:

Probability of showing submissiveness			
Group A:	.1	.9	.9
Group B:	.1	.1	.1
Group C:	.1	.9	.9
Learning coefficient, trial and error:			
Group A:	.9	0	0
Group B:	.9	0	0
Group C:	.1	.4	.4
Learning coefficient, cognitive success:			
Group A:	.9	.1	.1
Group B:	.9	.1	.1
Group C:	.1	.8	.8

All other data, not shown in Table 3.25, are the same as in the standard version of the simulation program (used in Chapter 3.4.1). In each group type 100 processes were simulated. The results are collected to the Table 3.26.

Table 3.26. Leadership simulation. Results

	Game					
	1	2	3	4	5	6
Failures:						
A	3.7	1.0	.44	.31	.36	.30
B	4.1	1.2	.74	.44	.40	.28
C	3.8	2.1	1.4	.67	.63	.49
Messages:						
A	24	13	11	10	10	10
B	27	15	13	12	11	10
C	27	20	16	12	11	11
Submissive behavior:						
A	8.6	4.5	3.9	3.5	3.2	3.2
B	1.3	0.9	0.7	0.7	0.6	0.5
C	8.2	6.3	4.8	3.9	3.7	3.4
Dominance suggestions/trial:						
A	1.1	1.1	1.2	1.1	0.9	1.0
B	1.1	1.1	1.0	1.2	1.0	0.9
C	1.0	1.1	1.0	1.1	1.2	1.0
Change suggestions/message:						
A	.17	.18	.17	.16	.16	.15
B	.20	.23	.22	.21	.22	.21
C	.12	.13	.13	.12	.11	.12

There are differences between the group types, but not so radical as maybe waited for the big differences in the data. Because the "cleverness" means only the good ability to memorize, the effect of it comes not out in the first game, but later: there are seemingly less failures in the games 2 - 6 by the group type A and B than in C. In addition, the "stubbornness" of the followers increases slightly the amount of failures in the beginning of the process.

Note: In the Group Maze experiment the leaders send the first message significantly more often than the followers (see Vuokko, 1964, and Riita, 1966, and also Rainio, 1972, p. 44). This has not been taken into account in the simulation above. (It could be easily done if with some changes are made in the program.)

3.5. The problem of the roundabout route

A problem of the roundabout route arises when the subject needs to *give up* something already reached for proceeding on the route toward a goal.

Kurt Lewin has analyzed the problem using a bird, a canary, in a cage as an example (Lewin, 1938, pp. 65 - 68):

"A canary P is in a cage E with the wire wall B (Fig. 3.8.A). A person G is beckoning to it. The canary is accustomed to fly to his shoulder. The door R of the cage is open. Nevertheless, this canary will come out only if the person stands at the position 2 or some other place physically not too much opposed to the direction to the door R."

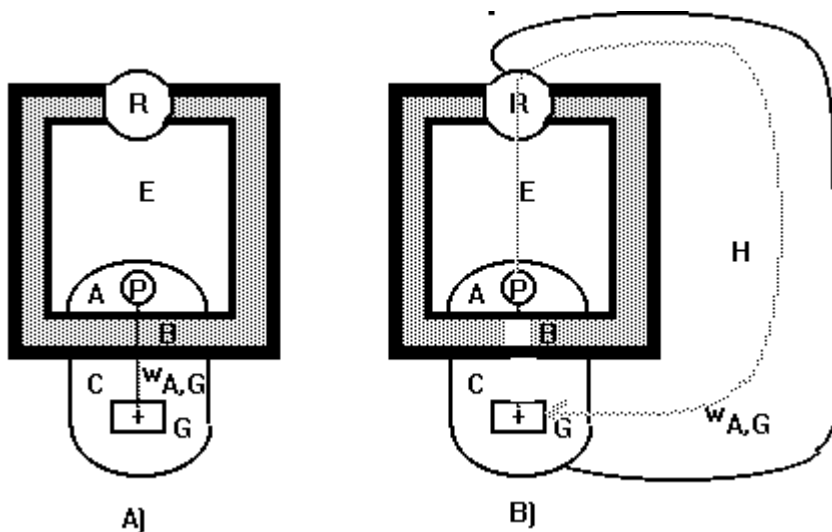
”What are the psychological directions for the canary P if the person G stands physically opposite to the door in region C?”

In the case the door R were closed, the region B blocks totally the locomotion out from E.

”Even if the door R were physically open, the situation would not be different psychologically as long as the fact that R is open is not present cognitively for the animal at that time,” Lewin writes.

”The direction to the door R has for the animal not the character toward but away from its goal G. In the constellation of Fig. 3.8.A we could even say that both directions are opposite.”

Fig. 3.8. A canary in a cage. Kurt Lewin’s example
(Lewin, 1938, p. 68)



P = canary; B = wall of cage; R = open door; G = goal

As long as this situation holds, the canary will not fly to the door to reach the goal.

”The animal will find the solution only if the cognitive structure of the field changes so that $d_{A,R} = d_{A,G}$ ” (the direction from A to R, the door, becomes same as the direction from A to the goal, G). This requires a certain restructuring. The cognitive structure of the situation has to change into that given in Fig. 3.8.B. ”In this case the regions A and C are no longer separated by the Region B, but A and C become part of one connected region. The distinguished path from A to G will not be $w_{A,B,C,G}$ as in Fig. 3.8.A but $w_{A,E,R,H,C,G}$.”

Lewin’s semiformal way to describe the situation is very illustrative, indeed. (Lewin mixes cognitive and physical features in the same picture. This makes the description ”pittoresque” but unfortunately theoretically diffuse.) It might be, thus, interesting to see how the same problem would be described in terms of our theory. This is done in Figures 3.9 and 3.10.

There are two ways to approach the problem:

1) We may assume that the Life Space is changed at moment after moment (Fig. 3.9.) as if the canary was building her cognitive map again and again.

2) We may assume that there begins to exist an enlarged Life Space including now also the means states, when the bird has formed an *insight* of the situation.

The first alternative represents seemingly the more primitive cognition, which we know to be typical for instinctive behavior, as well as trial and error learning.

Considering the cognitive trials, we shall see the following (Fig. 3.10.A):

When no roundabout route has been found, there will be lot of cognitive trials as well as real trials followed them from A to the goal G. They fail, because the success probability of real trials is assumed to be 0. The cognitive trial probability toward G decreases according to the punishment. Thus, the bird makes less and less trials toward G but stays in A. (This does not mean that the bird would not move. There may be lot of "restless movement" inside the cage, but it is not relevant for us. In our analysis we can well think that all that kind of locomotion is inside the state A.)

When the door is found and the bird has – occasionally – fled to G, it is possible that the reconstruction of the Life Space occurs, which we interpret as an *insight* of the situation. (What makes the reconstruction "possible" and when it happens, that is merely a philosophical question. Some assumptions can be set, concerning the time point of the reconstruction, *e.g.*, defining a particular stochastic variable, the value of which increases as a function of time, but without any experimental data it would be merely guessing.)

Fig. 3.9. Canary example. Changes of Life Space

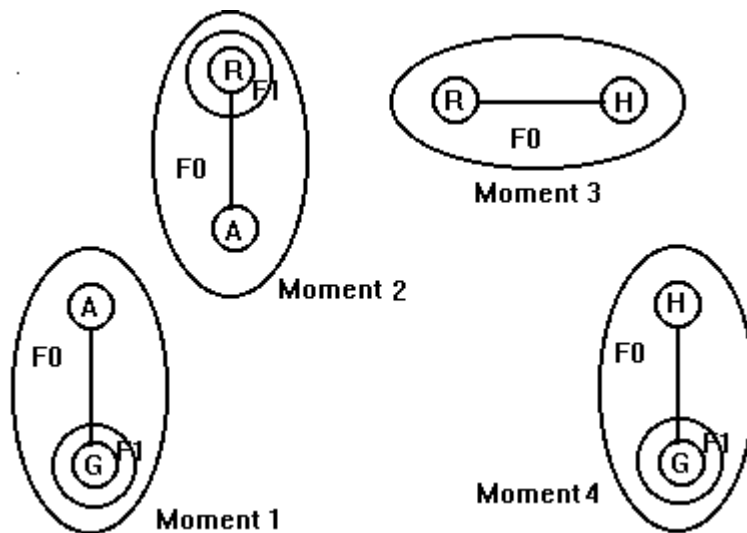
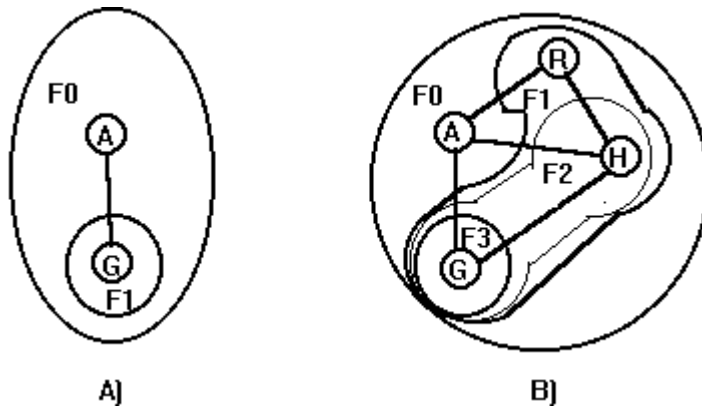


Fig. 3.10. Canary example. Life Space restructuring



Fields and potencies of fields:

$F_0 = \{A, G\}$ $PoF_0 = 0$
 $F_1 = \{G\}$ $PoF_1 = 1$

$F_0 = \{A, R, H, G\}$ $PoF_0 = 0$
 $F_1 = \{R, H, G\}$ $PoF_1 > 0$
 $F_2 = \{H, G\}$ $PoF_2 > 0$
 $F_3 = \{G\}$ $PoF_3 > 0$

In the case of Group Maze, the roundabout route problem means such a task where it is necessary to go from 2-point state to 1-point state for succeeding to reach the goal, 3-point situation. One of this type of tasks is presented in Table 3.27 and called Task 2. There are, thus, 9 possible paths leading to the goal, as shown in the table also.

Table 3.27. Roundabout route. Task 2 in matrix form
 Probabilities of the success of real trials (= task)

	State							
State:	0	1	2	3	4	5	6	7
0	1	0	0	1	0	1	1	0
1	0	1	0	0	0	0	0	1
2	0	0	1	0	0	0	0	1
3	0	1	1	1	1	0	0	0
4	0	0	0	0	1	0	0	1
5	0	1	1	0	1	1	0	0
6	0	1	1	0	1	0	1	0
7	0	0	0	0	0	0	0	1

The possible paths:

1	2	3	4	5	6	7	8	9
3-1-7	3-2-7	3-4-7	5-1-7	5-2-7	5-4-7	6-1-7	6-2-7	6-4-7

Is this kind of roundabout route problem solvable by the group with the exactly same characteristics we assumed in our simulation. We shall test it simply by changing the task according to matrix in Table 3.27. (The program needs to be changed in that block which numbers the paths and computes the path frequencies and the transitions of paths. This is presented in Appendix 6 in details.)

The problem turns out to be solvable – and not even very ”difficult”. Naturally more failures are waited now, and this comes out in the results presented in Table 3.28. In the simulation A, there were 200 groups simulated, data being the same as in the case of simulated homogeneous group in Chapter 3.4.1 (the standard version). In the simulation B, the field effect was complete, *i.e.*, setting the change coefficients of the field effect equaling 0, it was kept during the whole process equal to 1. - 100 cases were simulated.

Table 3.28. Roundabout route task. Failures

	Game					
	1	2	3	4	5	6
Simulation A:	6.14	2.85	1.64	1.11	.69	.52
Simulation B:	4.90	3.22	2.26	1.87	1.39	1.05

Information of the paths and path stability is given in Table 3.29.

Table 3.29. Roundabout route task. Paths and their stability

Paths, relative frequencies in the first game

	1	2	3	4	5	6	7	8	9
	3-1-7	3-2-7	3-4-7	5-1-7	5-2-7	5-4-7	6-1-7	6-2-7	6-4-7
Simul. A:	.125	.11	.115	.10	.13	.09	.13	.085	.115
Simul. B:	.09	.09	.19	.06	.16	.07	.17	.07	.10

Path transitions, relative frequencies, diagonal:

Simul. A:	.58	.62	.63	.63	.59	.57	.48	.53	.65
Simul. B:	.18	.11	.25	.18	.27	.08	.20	.08	.10

Total path stability: A: .587 B: .176

It was simply quite wrong to simulate the cognition and the behavior in Task 2 with the exactly same program model and with the same data as in Task 1. This concerns particularly the assumption of the structure of the Actual Life Space. Following Lewin’s thinking about the restructuring the Life Space, we seemingly have to abandon the assumption of the continuously stable field structure of the LS. (Actually, we have already abandoned it in assuming the frustration effect.)

We based our assumption of the structure of the LS on the fact that the subjects were instructed to take into account the point rewards in the play. This was supposed to mean that the field of the goal state (3 points) had a very high potency, the field of two-points state (+goal) the second highest potency *etc.*

How has the roundabout route task to be changed so that it should correspond to the psychological situation?

It is reasonable to think that, once the subject has found the route (once or more times) he may recognize that the one-point states are the *means* to reach the goal, *i.e.*, he may see the *sense* of those states as the means. If this is a case, the 1-point states form an *instrumental field*, beginning to

exist in the Actual Life Space as soon as the sense, idea, of the means character of these states has been detected during the play. (A possible alternative would be that, e.g., only one of the 1-point states is seen as the means. We do not, however, handle all cases here.)

Practically, in programming, we make the following simplified assumption:

We suppose that the first game is enough to the subjects to get the idea of the mean character of the 1-point states. When the group comes in the second game to a 2-point state (s_3 , s_5 , or s_6) the actual LS is changed in such a way that the fields and their potencies are now as follows:

$F_0 = \{s_0, \dots, s_7\}$, i.e., all the states, and $PoF_0 = 0$;

$F_1 = \{s_1, s_2, s_4\}$, i.e., all the 1-point states, and $PoF_1 = 1$.

These fields produce the vectors of the cognitive trial probabilities as:

	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7
s_0	0	.333	.333	0	.334	0	0	0
...								
s_5	as above							
s_6	as above							
...								

We assume that this change of the actual LS is occasional: in the other states than s_1 , s_2 , and s_4 the original actual LS is restored.

When this change, described above, is made to the program, a simulation (with continuous field effect 1) gives the failure results shown in Table 3.30.

Table 3.30. Roundabout route task. Simulation with an instrumental field. Failures

	Game					
	1	2	3	4	5	6
	4.99	1.40	1.11	.82	.83	.67

(Total path stability = .156)

One can see that the solution is obviously improved. This is easy to understand, because we have actually *forced* the group to choose some 1-point state after it has reached a 2-point state.

We do not have any experimental results produced using Group Maze with Task 2. Therefore the further analysis would be more or less speculation.

The assumption of the change of the Actual Life Space during the play is so radical that it leads us to a general discussion about our Group Maze model.

3.6. Discussion

3.6.1. Changes of the Actual Life Space

In creating our model for Group Maze process, we separated two Life Spaces from each other:

$$\begin{aligned} \text{LS1:} \quad & F_0 = \{s_0, \dots, s_7\} \text{ and } \text{PoF}_0 = 0 \\ & F_1 = \{s_1, \dots, s_7\} \text{ and } \text{PoF}_1 = .07 \\ & F_2 = \{s_3, s_5, s_6, s_7\} \text{ and } \text{PoF}_2 = .23 \\ & F_3 = \{s_7\} \quad \text{and } \text{PoF}_3 = .70 \end{aligned}$$

$$\text{LS2:} \quad F_0 = \{s_0, \dots, s_7\} \text{ and } \text{PoF}_0 = 1$$

We assumed that the first one of these Life Spaces was the actual one in the very beginning of the play (as a consequence of the instruction).

When the first trial toward the goal s_7 failed, a radical change of the Actual Life Space was assumed. It was a mixture by 5% of the LS1 and 95% by the LS2 and, thus, could be described using the potencies:

$$\text{PoF}_0 = .9500, \text{PoF}_1 = .0035, \text{PoF}_2 = .0115, \text{ and } \text{PoF}_3 = .0350$$

When the field effect was then slightly changed after each real trial, we could say that the Actual Life Space was changed during the play. Thus, the change of the LS in the beginning of the second game in the roundabout route task was not anything new, in principle. What was new was its *ad hoc* character (the change was based neither upon the instruction nor upon the laws of learning).

After this analysis we can look the trial and error cognition from another perspective. Suppose that there exists the Actual Life Space LS2, *i.e.*, the field F_0 only, including all the states and having the potency equal to 1. Suppose further that the subject, staying, *e.g.*, in s_1 , has learned to choose s_3 with a rather high probability, say .90. We can handle this situation also in such a way that we assume the Actual LS to change occasionally so that there exist *two* fields with potencies as follows:

$$F_0 = \{s_0, \dots, s_7\} \text{ and } \text{PoF}_0 = .114$$

$$F_1 = \{s_3\} \text{ and } \text{PoF}_1 = .886$$

(The probability of the cognitive trial from s_1 to s_3 is, thus, = $.114 \cdot 1/8 + .886 \cdot 1 = .9$ as above.)

Note: In assuming these two fields only, we have actually made also the assumption that the other cognitive trial probabilities equal to each other. If the probability of one alternative is very high, this is approximately true, indeed.

In trial and error process, when the subject who has, *e.g.*, reached s_3 makes his new choice s_7 , we may well think that the Actual LS changes again. If the cognitive trial probability from s_3 to s_7 is, say, .94, the new fields and their potencies must be now:

$$F_0 = \{s_0, \dots, s_7\} \text{ and } \text{PoF}_0 = .07$$

$$F_1 = \{s_7\} \text{ and } \text{PoF}_1 = .93$$

To generalize, we can say that it is typical for the cognition produced by *trial and error* learning that the whole Life Space changes successively after each trial. The learning by *insight*, in contrast, is understood, according to our theory, as forming a relatively stable Life Space which can be continuously held as a guidance to the goal – and changed only exceptionally. This is in accordance to the well-known phenomena in developmental psychology: the primitive psyche is like closed in a capsule which is thrown from situation to another. (See, *e.g.*, Heinz Werner's Comparative Psychology of Mental Development.)

This vision might be a useful hypothesis for studies where, *e.g.*, children and adults may be compared in Group Maze experiment.

3.6.2. Simplifications made in building the model for Group Maze simulation

A model applied to a certain process is always somehow simplified, if it is compared to the theoretical framework as a whole. Something is seen relevant and taken into account, something irrelevant and left out. In addition to that, some very little varying features of the process are described by constants, although in some other context the use of variables should be more relevant. – Some simplifications may lead to fatal errors, particularly in those cases where a model formed for one process is applied to another without analyzing whether the simplifications are in the new case relevant or not. Therefore it is necessary to make the simplifications used as explicit as possible.

We have made several simplifications in building the model for Group Maze. Some of them need to be mentioned here:

1) The cognitive map was assumed to be the same for every individual and correspond to the structure of the real game in every detail, *i.e.*, the probability $\text{Pr}[\text{RCCor}(i, S_r, S_c, t)] = 1$, where S_r indicates the real state in the game and S_c the cognitive map, including that the cognitive starting state of trials corresponds exactly to the actual real state in the game.

Although we can think that there may appear some absentminded or demential persons among the subjects, who totally forget the situation, this simplification seems not to be fatal in the case of normal people – as those in our experiment.

The cognitive map was assumed to be complete, *i.e.*, all the time *all* combinations of moves exist cognitively. This needs not to be the case always: we may well imagine individuals who, *e.g.*, concern to their own choice between upper and lower row, only, not taking notice of any moves of the other players. There are, however, no clear signs of this in our results, but this is a phenomenon which seems really worthy to be examined carefully.

2) The theoretical concept of *action threshold* was not taken into account, *i.e.*, we have put it equal to 1. There seems to be no reason to abandon this simplification. (Maybe this concept is not needed even in the theoretical framework itself. On the other hand, it seems, intuitively, very natural that the human beings may differ just in their decision-making characteristics. Thus the action threshold would be important, because it is the measure of its speed.)

3) Always, when we use the exact value 1 for some probability, we make, in principle, a simplification. Sometimes the value of probability may be, practically, so near 1 that this simplification is justified. So seems to be the case when we have used 1 as the beginning values of the probabilities of the success of the cognitive trials.

4) The programming practice forces us sometimes to use a technical variable which may be interpreted theoretically important, too. This was the case when we needed in our program a limiting number of cognitive trials one subject get make before the turn of the others (N_{trials} in the program). This is mainly a technical problem. Actually the subjects seem to think, to make their cognitive trials, simultaneously, but this is impossible to carry through technically in our program. Both letting a subject to continue very long time his cognitive trials and cutting the process shortly, are unrealistic. But what would be the "right" value of N_{trials} from the theoretical point of view that is a complicated problem. Any simplification we are forced to do is questionable.

Perhaps, instead of assuming the N_{trials} constant ($= 10$), we could let it vary around some mean value, but because we do not have any idea of the amount of this variation, this would be only a "cosmetic" change.

5) A very general way to simplify the model is to handle probabilities *unconditional* instead of taking their conditionality into account. One example of this is the assumption of dominance suggestion probabilities in our model. We apply to them the learning process, but otherwise we

handle them as unconditional. During the game process, there might well exist a kind of saturation process which decreases these probabilities. Thus, the dominance suggestion probabilities need perhaps to be taken as conditional, the duration of the game being the condition.

The simplification we have made concerning the dominance suggestion probabilities seems not to lead to any big errors in simulation.

Naturally, the important assumption that the subjects commit themselves to the problem situation following the rules of it, is a rough simplification *per se*. It may be surprising to notice that, actually, this commitment occurs so regularly. Otherwise we could hardly find any invariances to study experimentally.

3.6.3. Further studies on Group Maze

The Group Maze can be varied in many ways to study different aspects of cognition in group problem-solving: the size of the group and its communication network may be modified in numerous ways, a large number of different tasks can be studied, the interacting group can be varied for special purposes by choosing the members systematically, modifying the Instructions, *etc.* Some of these possibilities are discussed below. (A more detailed description about the variations of Group Maze experiment is represented in Rainio, 1972, pp. 47 - 54.)

1) *Varying the size of the group:*

Group Maze can be run for groups of two or four, as well as three. A five-person group would probably be difficult, because the graph of the group task would include at least 32 points (vertices).

2) *Content of communication:*

In the standard form of the Group Maze, the number of messages available to the players for communication is extremely limited. Only task-oriented communication is possible. No shades of persuasiveness are permitted them in their suggestion messages – *i.e.*, a message is sent or not sent without any quantitative variation of such variables as the conviction of the sender, the strength of the suggestion, *etc.* It is, however, quite obvious that the Group Maze experiment offers a very wide variety of possibilities to analyze the effects of content variables of the communication systematically, *e.g.*:

– Use of scaled expressions: Instead of one standard message for a player to announce his own choice, a set of scaled messages could be used: "I feel I should choose...", "I am almost sure I should choose...", *etc.* – Suggestions could be scaled in a similar way: "Perhaps *b* could choose...", "*b* must choose...", "It is absolutely necessary for *b* to choose...", *etc.* – The use of the scaled suggestions, in particular, would enable us to study the effect of the intensity of the suggestions on the probability its being followed.

– Evaluations: For special purposes, a Group Maze can be arranged so that standard evaluation messages are available to the players – cards such as "Good", "Very bad", "Wise!" *etc.*

– Message cards to influence the climate or feeling of the group, such as: "Relax!", "Courage!" *etc.*, could be included.

– Attempts to influence working style -messages such as: "Hurry up!", "Slower, please!", *etc.*

– Role-setting messages: Possibilities of role-sending are numerous and diverse, *e.g.*: "Let's decide that *b* will tell us what to do" and "Let's decide that *b* should always send the first message", *etc.*

– Conditional messages: We could use message cards such as "If *a* chooses... , then I shall choose...", "If nobody alters his choice, I shall choose..." *etc.*

3) *Variations of the problem:*

– Problem series: Group Maze experiment would be arranged as a series of successive problems. This would give us an opportunity to study the transfer phenomenon in the learning

process. After having learned to solve one problem the group may be immediately faced with another. The new problem can differ from the old one to a varying extent.

– Problem difficulty: If the task graph is a directed one, the maximum number of arrow combinations indicating the possible moves from one state to another is 2 to the potency $(p^n - 1) \cdot p^n$, where p indicates the number of alternatives and n the number of members in the group. Thus, if $p = 2$ and $n = 3$, the number of possible arrow combinations is 2^{56} . So in practice, there seems to be no limit whatever to the creation of new problems! – The difficulty of a problem is 0 , if it is possible to move from any state to any other state whatever. The most useful way to estimate the difficulty of a problem would be to simulate it and to calculate the relative frequency of errors to be expected. It would be interesting to vary the difficulty of the group problem systematically.

One variation of the task, the roundabout-way problem we examined already in Chapter 3.5.

4) *Group formation:*

Special problems connected with group problem-solving can be analyzed by carefully selecting the group members in advance and forming special types of groups for the Group Maze experiment, *e.g.*, we can put together leaders and non-leaders according to some sociometric measurement, task-leaders and non-leaders according to the results in other experiments, flexible and stubborn individuals, intelligent and less intelligent, normal and psychotic, introvert and extrovert, *etc.*

5) *Groups with prepared subjects:*

Prepared subjects can be told to show a generally high or low degree of submissiveness, or discriminately high or low submissiveness, *i.e.*, to follow certain types of suggestions but not others, or to act submissively at the start of the experiment but not later, *etc.* – The main purpose of using prepared subjects in Group Maze experiment is to eliminate certain sources of variation, and thus focus the study more closely on the theoretical phenomena we are interested in.

6) *Special designs:*

We could compare the results of a normal three-person group and of a "one-person group" – all other things being equal, "one-person group" simply meaning a single individual trying to solve the problem of moving the three pieces by himself. The subject could be asked to think aloud as far as possible, so as to give the experimenter data on his "inside communication". We can assume that a group with an extremely good ability to communicate would show features fairly similar to an individual.

The instructions can be varied in many ways, *e.g.*, we can try different scoring systems and study their effect on the problem-solving (the cognitive field structure).

As stated above, the Group Maze experiment offers a great variety of research designs for analyzing details of the process of group problem-solving and, thus, for testing the hypotheses of human cognition in that.

It could be fascinating to build systematically particular simulation models to derive estimates by them and test them then experimentally. This way of the psychological study would be comparable with "physics" – as Kurt Lewin once dreamed.

Chapter 4. Heider's balance theory in a mathematical form and its simulation

4.1. Heider's conceptual system

The close relationship of our theory of cognitive process with Lewin's conceptual system has already been shown. In attempting to find links between our theoretical constructs and observable data, it is logical to use works that have originated inside the Lewinian framework. Obviously Heider's "naive analysis of action" such as he has developed it in his main work "The Psychology of Interpersonal Relations" is one of those, probably the most outstanding among them. Because Heider emphasizes the close relation between his concepts and the simple situations in everyday life, it is promising - from the point of view of operationalization - to use just his analysis as a bridge from theory to observations.

There is another advantage in choosing Heider's work as a tool for operationalization. Heider has clearly succeeded in his efforts to find out the limited essential set of concepts necessary for deriving theoretically coherent descriptions for psychological phenomena in their great variety. Fortunately, Heider's basic concepts seem to be very near the concepts in our system.

Two steps need to be taken in formalizing the Heiderian system:

1) The "semiformal" presentation Heider himself uses in several contexts needs to be carried out in a thoroughly systematic way.

2) To be able to make quantified predictions, we have to analyze Heider's hypotheses (concerning particularly balance tendencies) in terms of probabilities.

In addition, we have to make Heider's concepts more precise by taking the time variable explicitly into account.

The most essential concepts by Heider are defined in our terms in an earlier work by the author of this text (Rainio, 1966, pp. 188 – 200) including: causing, can, can cause, cannot cause, trying, trying to cause, wanting, wishing, ought and perceiving. Those exact definitions open a possibility to handle Heider's statements (particularly those concerning attribution and balance) in such a way that estimations for probability distributions can be done.

In this chapter we shall examine only Heider's balance theory where his presentation concerns a *process*, instead of being just description of psychological situation. Also in this case we shall apply our theoretical framework in a very simple form, just to show how an estimation model can be built.

In his cognitive balance theory Heider uses the concepts of sentiment relation (between the subject and some cognitive object) and of unit relation (between two cognitive elements). Heider's definition of unit relation is somewhat diffuse, intuitive, we should say. According to him, the unit relation between two cognitive elements means that they are somehow "belonging together".

In our system the basic concept of the analysis is (as has been stated in Chapter 2) a *cognitive state* of the subject. This cognitive state has one or more characteristics, relevant to the case to be analyzed. Thus, we shall handle Heider's sentiment and unit relations as *believes that certain thought categorizations are true*.

If – in terms of Heider – the subject i likes person j , this is represented in our system:

$C(i, \text{Bel}(j, \text{Lkb}), t', t)$

i.e., subject i is in a cognitive state $\text{Bel}(j, \text{Lkb})$ at the cognitive time point t' at the real time point t , where state $\text{Bel}(j, \text{Lkb})$ means the content of this cognitive state: j is believed by i "to be likeable".

In other words: in i 's cognition the belief that j is likeable holds true. There exists, thus, an \in -relation: $j \in \text{Lkb}$.

The *unit* relation simply means that a belief is *conditional*, the condition being the truth of another belief. In terms of probabilities this is:

$$\text{PosU}(x,y) \equiv \Pr[C(i,\text{Bel}(x \in F),t') \mid C(i,\text{Bel}(y \in F),t')] = 1$$

$$\text{and } \Pr[C(i,\text{Bel}(y \in F),t') \mid C(i,\text{Bel}(x \in F),t')] = 1$$

This is the case of the *full* (or maximum) *positive unit relation*.

Note: We could define a *measure* of the positive unit relation in terms of the probabilities above, but in that case, seemingly, the measure need not to be symmetric, *i.e.*, it would be possible that $\Pr[\text{PosU}(x,y)] \neq \Pr[\text{PosU}(y,x)]$.

To be exact, the measures would be:

$$\Pr[\text{PosU}(x,y)] = \Pr[C(i,\text{Bel}(x \in F),t') \mid C(i,\text{Bel}(y \in F),t')] \text{ and}$$

$$\Pr[\text{PosU}(y,x)] = \Pr[C(i,\text{Bel}(y \in F),t') \mid C(i,\text{Bel}(x \in F),t')]$$

Heider handles not asymmetric cases of unit relations.

For the sake of simplicity, we assume in the following that the unit relation is symmetric. (We do not here follow the fascinating temptation to build a totally new theoretical system on Heider's psychology.)

Heider's essential assumption concerning the cognitive balance in the case of three elements is simple:

The union of three cognitive elements is *in balance* if *all* of the relations between them are *positive* or if there exist *two negative* relations and *one positive*.

(This balance principle can be enlarged to concern more than three elements. The rule then would be that the whole system is in balance if all possible element *triplets* are in balance, as Cartwright and Harary present. The rule concerning only one triplet is enough for our purpose.)

Before we describe Heider's balance theory in terms of our framework, it is illustrative to examine an example which also serves as an empirical data source for our simulation.

4.2. Experiment by Esch

One item from an experiment by Esch seems to be an illustrative example of cognition in an unbalanced situation to start with. Heider refers the experimental set-up as follows (Heider, 1958, p. 176):

"Subjects were given short description of social situations and were asked to write the most probable outcome, that is, 'what would happen nine times out of ten when something like this occurs'. One situation was following:

Bob thinks Jim very stupid and a first class bore. One day Bob reads some poetry he likes so well that he takes the trouble to track down the author in order to shake his hand. He finds that Jim wrote the poems.

"The 101 subjects consisted of high-school and college students and other adults."

Heider refers the results:

"In the situation presented to the subjects the poetry was liked whereas its author was not. Such a combination of positive and negative entities produce an unbalanced situation. The subjects resolved the disturbance in the following ways: (1) Forty-six per cent changed the negative author to a positive person, *e.g.*, 'He grudgingly changes his mind about Jim'. In this way both entities became positive and balance was achieved. (2) Twenty-nine per cent changed the value of the

poetry, *e.g.*, ‘He decides the poems are lousy’. In this way balance was achieved by transforming the unit into one that was consistently negative. (3) Five per cent challenged the unit formation itself, *e.g.*, ‘Bob would probably question Jim’s authorship of the poems’. (4) Two subjects altered the unit by differentiating the author in such a way that the unit comprised only the positive part of the author and the admired poetry, *e.g.*, ‘He then thinks Jim is smart in some lines but dumb in others’. (5) The rest of the subjects did not resolve the disharmony, but some were definitely aware that the situation presented a conflict - ‘Bob is confused and does not know what to do. He finally briefly mentions his liking of the poems to Jim without much warmth’.”

4.3. Tendency to reach cognitive balance

4.3.1. Actual Life Spaces in Esch’s experiment

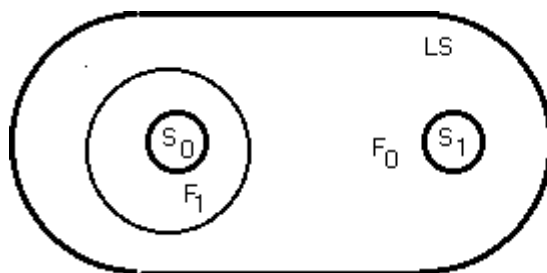
We shall use in our analysis and simulation the concepts represented in Chapter 2, namely: Actual Life Space, cognitive state, cognitive field and ”real state”. (This last one appearing in a new meaning, but analogous to the real state in behavior. It will mean a combination of ”true believes” or ”long-term memory believes”.)

How to describe, firstly, the Actual Life Space by the subject (Bob) in the situation before getting the information about the unit relation (= Jim wrote the poems)?

(Although in the experiment by Esch the question was about the cognition by the test subjects, we describe the situation *as if* the subjects were identifying themselves to Bob. We shall talk shortly about ”Bob” meaning actually ”Bob seen by the eye of the experimental subject”.)

The situation before the information about the unit relation is presented in Fig. 4.1: Bob thinks the likeability of Jim. There are two relevant states: s_0 = Jim is not likeable, and s_1 = Jim is likeable.

Fig. 4.1. Esch’s experiment. Situation before unit information



In addition there are two fields:

$$F_0 = \{s_0, s_1\}; \text{Po}F_0 = 0 \text{ and } F_1 = \{s_0\}; \text{Po}F_1 = 1$$

The valences are assumed as usual:

	F	¬F
F	1	0
¬F	1	0

Thus, according to the equation for computing the cognitive trial probabilities, we get:

$$\Pr[\text{CTr}(s_0, s_0), t', t] = 0 + \text{PoF1} \cdot 1/1 = 1$$

That means that Bob will keep his sentiment that Jim is not likeable. Situation is in balance.

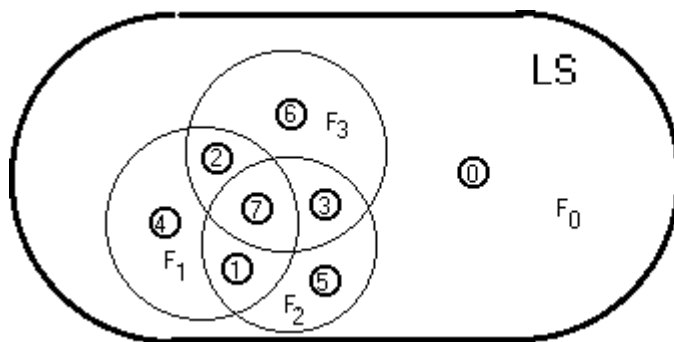
The Actual Life Space describing the situation where Bob thinks the poems are likeable has the same structure as above, but the two Actual Life Spaces exist not in the same time. The relevant states are now:

- s_0 = the poems are likeable to Bob,
- s_1 = the poems are not likeable to Bob.

The fields have now the same potencies and valences as above, the probability of Bob staying in the state 0 equaling 1.

Situation *after the information about the unit relation* (the writer of the poems) is given is quite different. The information forces now Bob to take into account at the same time the dislikeability of Jim, the likeability of poems, and the unit relation (the statement that Jim wrote the poems). Logically there exist 8 different cognitive states. They are in \in -relation to 4 fields as shown in Fig. 4.2.

Fig. 4.2. Situation after the information about the unit relation



The fields and their meaning as categories are following:

- $F0 = LS = \{s_0, \dots, s_7\}$ the whole Actual Life Space
- $F1 = \{s_1, s_2, s_4, s_7\}$ Jim is dislikeable
- $F2 = \{s_1, s_3, s_5, s_7\}$ poems are likeable
- $F3 = \{s_2, s_3, s_6, s_7\}$ unit relation (Jim wrote the poems)

Staying in some cognitive state on an intersection of fields means that the subject's cognition has components which are in accordance to those fields, *i.e.*, correspond to the \in -relations. These components we shall call *actual* believes. Thus, *e.g.*, staying in the state s_1 means that there exist three actual believes, namely: Bob dislikes Jim, Bob likes the poems but does not believe that Jim wrote the poems; staying in the state s_4 means that the subject dislikes Jim, dislike poems and does not believe Jim wrote the poems *etc.*

There are *signs* of the fields corresponding to the positive or negative quality of the belief. They are as follows:

- F1 is negative, F2 positive, and F3 positive.

The cognitive state is a balanced one, if – according to Heider’s rule – all cognition components are positive or two of them are negative. Thus, there are 4 balanced states:

- s_0 , because $s_0 \in \neg F1$, $s_0 \in \neg F2$, and $s_0 \in \neg F3$, so that the corresponding signs are + , - , and -
 - s_1 , because $s_1 \in F1$, $s_1 \in F2$, and $s_1 \in \neg F3$, so that the corresponding signs are - , + , and - ,
 - s_2 , because $s_2 \in F1$, $s_2 \in \neg F2$, and $s_2 \in F3$, so that the corresponding signs are - , - , and + ,
 - s_3 , because $s_3 \in \neg F1$, $s_3 \in F2$, and $s_3 \in F3$, so that the corresponding signs are + , + , and + .
- All other states are unbalanced.

4.3.2. Assumptions

We base our theoretical model on 9 assumptions:

1) The cognitive process is a series of cognitive trials, the probabilities of which are derived from the potencies of the fields according to the rule given in Chapter 2, the potencies being

$$Po(F0) = 0, Po(F1) = 1/3, Po(F2) = 1/3, \text{ and } Po(F3) = 1/3$$

Thus, the cognitive trial probabilities are:

	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7
s_x	0	.167	.167	.167	.083	.083	.083	.250

Note: The cognitive trial probabilities have the form of *transition probabilities*:

$$Pr[CTr(i, s_x, s_y, t', t)]$$

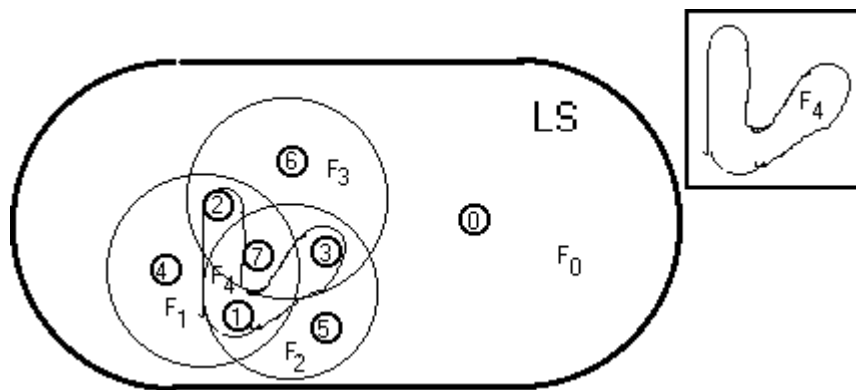
which indicates the probability to make a trial from a state s_x toward s_y . However, in our case, all rows in the transition matrix are equal. Thus, s_x can be whichever of the cognitive states (as well as a state outside the Actual Life Space). – We are used to describe the Life Space as a graph, the lines of it combining the states. There should be such lines drawn in Fig. 4.1 and 4.2, but they are, however, left out for simplicity.

2) The cognitive trial succeeds if the state is balanced, but failed, if it is unbalanced. (*Balance rule.*)

3) If the cognitive trial fails, no changes occur.

4) When all of the balanced states, s_1 , s_2 , and s_3 are chosen at least one time, a *restructuring* of the Actual Life Space occurs in such a form that a new field, *balance field* is created. (We call this occurrence *experience of balances.*) This field includes the balanced states mentioned above (see Fig. 4.3).

Fig. 4.3. Restructuring of the Life Space after experience of balances



Creating the field changes radically the vector of the potencies of the fields. We assume it as following:

F0	F1	F2	F3	F4
0	.1	.1	.1	.7

5) If the cognitive trial succeeds, the actual believes corresponding to it are compared with the *long-term memory believes* which are determined by their probabilities. If the actual belief is same as the LTM-belief, they are said to be *congruent*. Thus, on each succeeded cognitive trial there will be three *tests of congruencies*.

6) If *all* the actual believes of the cognitive trial are congruent with the LTM-believes, then the cognitive trial is *rewarding*, if not, it is *punishing*. (*Congruence rule.*)

7) If the cognitive trial is punishing, then the probability of the *incongruent* LTM-belief is changed according to the mathematical law:

$$p_{t+1} = p_t - \beta \cdot p_t \cdot (1 - p_t),$$

where p_{t+1} is the new value of probability p_t and β a coefficient between 0 and 1.

8) The cognitive trial is rewarding only if *all* the actual believes are congruent with the long-term-memory believes. This can happen only if one of the outcomes of the Monte-Carlo choices is opposite to that LTM-belief which defines the field. Let us take an example: The outcome of probability choice using $\text{Pr}[\text{Bob dislikes Jim}]$ is "yes" or "no", as well as the outcome using $\text{Pr}[\text{Bob likes the poems}]$ can be "yes" or "no", *etc.* As mentioned above, a state can be balanced and all the believes congruent at the same time only if one of these outcomes will be "no". If the state is, *e.g.*, s_1 , the outcome of Monte-Carlo choice of unit-relation needs to be "no" in the case of the total congruence. In the state s_3 , the outcome of the choice of the LTM-belief "Bob dislikes Jim" needs to be "no" in the case of the total congruence, *etc.*

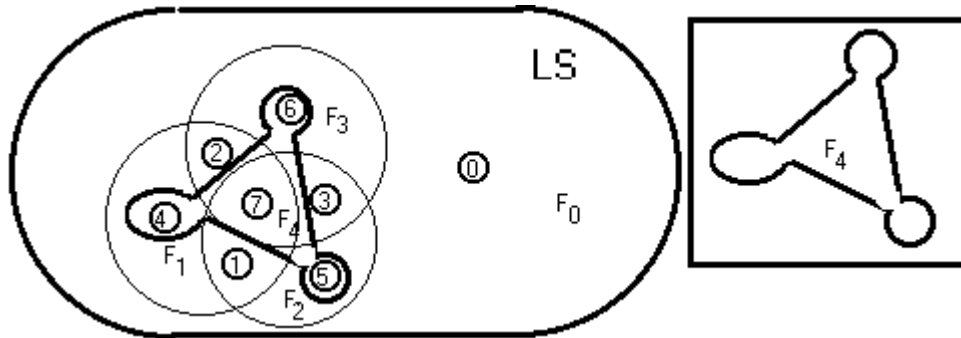
We shall call that LTM-belief the outcome of which is "no" the *changed LTM-belief*.

When the total congruence is reached the first time, the Actual Life Space is *restructured* by taking in the use the *opposite sign* of the changed belief.

The restructuring, the change of one sign, has several consequences:

a) The balanced states are now different. F4, the field of balanced states includes now s_4 , s_5 , s_6 , and s_7 (instead of the earlier s_1 , s_2 , and s_3). See Fig. 4.4.

Fig. 4.4. Restructuring the Life Space after a rewarding trial



A new field of balanced states, F4, is created. $F4 = \{s_4, s_5, s_6, s_7\}$

b) The new ϵ -relations in F4 change the cognitive trial probabilities.

c) The probabilities of the LTM-beliefs are assumed to equal now to the values they had in the beginning.

9) After the restructuring, there exists one balanced state with total congruence, s_7 . This state is *rewarding*, i.e., when this state is reached the LTM-belief probabilities are changed according to the law

$$p_{t+1} = p_t + \alpha \cdot p_t \cdot (1 - p_t)$$

where p_{t+1} stands for the new value of p , p_t is the earlier value of p and α a coefficient between 0 and 1.

Note: The coefficient α needs to be rather high. Then the LTM-belief probabilities will be strengthened more than weakened on the following trials. Thus, the process takes the direction toward a more and more strong balance.

One may argue that we have made too many assumptions to estimate only four frequencies in the experiment by Esch! - It is true that for the estimation purpose, only, much simpler models can be used. Our main purpose is different: we have attempted to show how to describe the process of belief formation in terms of our basic theory of cognitive processes. Is our way to build the application model the only one, about that we shall discuss later in Chapter 4.5.

4.4. Simulation of the experiment by Esch

4.4.1. Program

The Turbo-Basic program used in the simulation is given in detail in Appendix 7. Here it is described in the form of a flow list:

1. Set the number of simulations (Nsimul), of cognitive trials (Ntrials), of cognitive states, of cognitive fields, and of believes
2. *Start the simulation process*
 - 2.1. Read the data:
 - \in -relations of the states
 - belief probabilities
 - signs of the believes
 - potencies of the fields
 - alfa and beta coefficients
 - 2.2. Set the congruencies according to \in -relations
 - 2.3. Go to subroutine BALANCES:
 - 2.3.1. Compute the sum of the negative signs of the believes corresponding to each cognitive state; if it equals to 0 or 2, name the state as a balanced one
 - 2.3.2. Determine the success probabilities of the cognitive trials: if the goal state of the trial is balanced, the probability equals to 1, otherwise 0
 - 2.4. Go to subroutine NUMSTATEFIELDS:
 - For each field, compute the number of states included in it
 - 2.5. Go to subroutine DETTRIALPROB:
 - Compute the cognitive trial probabilities toward each state summing the values: potency of the field / number of states in the field
 - 2.6. *Start the trial*
 - Choose the actual state by Monte-Carlo -method using cognitive trial probabilities
 - 2.7. Determine the success of the trial
 - 2.8. Test the success of the trial. If the trial fails, go to 2.15
 - 2.9. Test the balance experience: if all the balance states s_1 , s_2 , and s_3 are chosen at least once,
 - then form a balance field by
 - 2.9.1 changing the potencies:
 - Pot(0) = 0, Pot(1) = .1, Pot(2) = .1, Pot(3) = .1, and Pot(4) = .7
 - 2.9.2. going to subroutine NUMSTATEFIELDS, and
 - 2.9.3. going to subroutine DETTRIALPROB
 - 2.10. Determine the actual believes by Monte-Carlo -method using the belief probabilities
 - 2.11. Test the congruencies between the actual believes and the LTM-believes. If the congruence for belief k exists, set Cobel\$(k) = "yes", otherwise "no"
 - 2.12. Test the Cobel\$()-congruence values
 - 2.12.1. If all of them are "yes", then the *total congruence* exists and the trial is also rewarding. Go to 2.13
 - 2.12.2. If there is at least one Cobel\$() = "no", then the trial is punishing.
 - Go to subroutine PUNISH:
 - If Cobel\$(k) of the belief k is "no", decrease the corresponding belief by the equation $p' = p - \text{beta}(k) \cdot p \cdot (1-p)$, where p' is the new value of p and beta(k) the learning coefficient
 - Go to 2.15
 - 2.13. Go to subroutine REWARD:
 - Determine the new LTM-belief probabilities:
 - if the LTM-belief k and actual belief k are in congruence, then increase the probability using the equation: $p' = p + \text{alfa}(k) \cdot p \cdot (1-p)$, where p' is the new value of p and alfa the learning coefficient

- if the congruence does not exist, then decrease the probability of the LTM-belief k by the following equation: $p' = p - \alpha(k) \cdot p \cdot (1-p)$, where p' is the new value of p and α the learning coefficient

2.14. Go to subroutine RESTRUCTURING:

If the actual belief is in congruence with a *changed LTM-belief*, then

- change the sign (and the name) of that LTM-belief
- carry out the subroutine BALANCE and, thus, determine new \in -relations concerning the new balance field
- carry out the subroutine NUMSTATEFIELDS and, thus determine the new numbers of states in each field
- set the LTM-belief probabilities same as in the beginning of the simulation

2.15. Test the number of trials carried out. If it is $< N_{trials}$, go to 2.6

2.16. Test the number of simulations carried out. If it is $< N_{simul}$, go to 2

3. End the simulations and show the results

4.4.2. Results

Thousand simulations were run by the program described above and by the data given in Table 4.1. In that table are also shown the distribution of the belief combinations after 37 trials. These frequencies produced by simulation are compared with the results of the experiment by Esch. The results are extremely close to each other.

Table 4.1. Simulation of Heider's balance theory. Results

Turbo-Basic Program: Heiders.bas

Number of simulations: 1000

Number of trials: 37

Belief probabilities in the beginning:

Disliking Jim	.97
Liking poems	.98
Unit relation	.992

Coefficients for changing the belief probability in the case of punishment and reward:

	Punishment (Incongruence)	Reward (Balance and total congruence)
Disliking Jim	.25	.8
Liking poems	.25	.8
Unit relation	.06	.8

<i>Results:</i>	<i>Simulation</i> %	<i>Esch's experiment</i> %
Liking Jim:	45.9	46
Disliking poems:	28.4	29
Unit relation broken:	5.4	5
No change of the believes:	20.3	20

Note: It is obvious that the distributions do not differ significantly from each other. However, it would be misleading to compute some conventional statistical test to show it, because lot of "range

finding” has been done to discover the suitable variable values for simulation. We have to leave the problem of the correct estimation power open.

4.5. Discussion

Our simulation model and the variable values used in the running it are not the only possible to produce the frequency distributions with high goodness-of-fit, not at all. We shall examine here some variations.

1. We often meet the problem: Is the distribution of the results a consequence of different individual characteristics of the subjects (*i.e.*, consequence of heterogeneity of the subject group) or does the process produce variation among a homogeneous group of subjects?

Very rarely we have any mathematical equipment to solve the problem. In our actual case both alternatives produce the same results.

Intuitively we may assume that the probabilities determining the human cognition or behavior never equal exactly 1. Our experience tells as well that 100 individuals hardly have exactly the same characteristics in any experiment. The truth lies between. We have to look the variable values we used in the main version of our simulation as *mean* values but to remember that very little can be said of the actual variation of the characteristics of the subjects around these means.

We can assume that the experimental group in Esch’s test consisted of several *types* of subjects. Thus, we could characterize our main version so that it handled only one type of subjects, Type_m, with the following LTM-belief probabilities:

	Disliking Jim	Liking poems	Unit-relation
Type _m	.97	.98	.992

Instead of this assumption, we can well suppose that the group actually consisted of 4 subgroups, or types:

	Disliking Jim	Liking poems	Unit-relation
Type1	.94	1	1
Type2	1	.94	1
Type3	1	1	.94

Actually, three runs were made, one for each type and applying beta-values equaling to .25.

After a run with 32 trials there were about 80 % those who made a change of one belief and found, thus, the cognitive balance and the congruence, naturally in that way that subjects of Type1 changed their belief 1, those of Type2 the belief 2, and those of Type3 the belief 3. (500 simulations were run and the exact values after 32 trials were 80.6 %, 80.7 %, and 80.05 %.)

If we suppose that the Esch’s group consisted of 58 subjects of Type1, 36 subjects of Type2, and 6 subjects of Type3, and if 80 % of each group found the balance, while 20 % did not, then we get through the simulation the following distribution of the believes after 32 trials:

Liking Jim, liking poems, and unit-relation:	$0.806 \cdot 58 = 46.7$
Disliking Jim, disliking poems, and unit-relation:	$0.807 \cdot 36 = 29.0$
Liking Jim, liking poems, and no unit-relation:	$0.8005 \cdot 6 = 4.8$
No solution after 32 trials:	
Disliking Jim, liking poems, and unit-relation:	19.5

The simulated distribution fits well to the empirical one.

We can hardly think, however, that the subject group included just this 3 extreme types. Nor we can assume that all subjects were of the same mean type, $Type_m$.

We have to leave the problem of the distribution of the individual characteristics open. The practical assumption that we can handle the subject group homogeneous, with mean variable values, is anyway a reasonable approximation.

2. The assumption that the balance field is formed as a consequence of "balance experience" (see Chapter 4.4.1, the point 2.9 on the flow list) is not necessary from the estimation point of view. We may well run the simulation without that (*i.e.*, using the field potencies: 0, 1/3, 1/3, 1/3, and 0) and get practically the same results (some trials later in the run). Thus, is the assumption of forming the balance field something unnecessary and needs therefore to be cut off by "Occam's razor" ? – Maybe not, because we have to assume later during the run the restructuring the Life Space and need anyway to use a balance field. Is it reasonable to think that this kind of cognitive structuring occurs in some form already at the early stage of the process? At least it suits well to the psychological intuition.

3. The mean probability values of the LTM-believes used in the main version (.97, .98, and .992) are not absolutely fixed. We may well use higher probabilities, but then higher values of betas also - or *vice versa*.

We may even increase the LTM-belief probabilities and decrease the beta-values, but then the balance through the change of one belief would occur much later.

4. Is the triplet (the combination of 3 believes) the only possible object of our simulation? – Not necessarily. We may well build a program for handling more than 3 believes, keeping in mind that the balance state of the whole means the situation where *all* the possible triplets are in balance. *E.g.*, in the case of 4 believes, 8 final balance states are possible (see Table 4. 2)

Table 4.2. Balance states in the case of 4 believes

Believes 1&y are sentiments, other, x&y; $x \neq 1$, are unit relations.

Sign 1 indicates a positive sentiment or an existing unit relation (+), while sign 0 indicates a negative sentiment or lack of the unit relation (-).

There are 63 different cognitive states, according to the different relational combinations between the elements.

Balanced cognitive states are the following:

State	x&y					
	1&2	1&3	1&4	2&3	2&4	3&4
S ₇	0	0	0	1	1	1
S ₁₂	0	0	1	1	0	0
S ₁₈	0	1	0	0	1	0
S ₂₃	0	1	1	0	0	1
S ₃₃	1	0	0	0	0	1
S ₄₂	1	0	1	0	1	0
S ₅₂	1	1	0	1	0	0
S ₆₃	1	1	1	1	1	1

Other states are unbalanced.

5. We have neglected in our analysis the 2 subjects which solved the problem by altering the unit by "differentiating the author of the poems in such a way that the unit comprised only the positive part of the author and the admired poetry", as Heider writes.

How to understand this exceptional cognition process from the point of view of our theory?

We have emphasized already the *commitment* of the subjects to the cognitive task as a condition for that they follow the rules of the applied model. Any time the subject may "cancel" this commitment by leaving the Actual Life Space. After that event our description is no more relevant.

The probability of leaving ALS is obviously not equal to 0, not in *any* case to which we apply our theory. This is true as long as we assume the *free will* of a person possible. The only way to take it into account in our theory is to assume that there exists at every step in time a (low) probability of a cognitive trial toward some x , this x being some state outside the ALS. We could easily add this to our simulation model. We could even approximate the value of that probability in Esch's experiment:

The probability that a subject makes a trial toward x during 37 trials is .02. We may simply see it as a product $37 \cdot p_1$, where p_1 indicates the probability to choose x at *one* trial. This p_1 equals then to 0.00054. Thus, for getting our model "complete", we had – on every cognitive trial – to make a Monte-Carlo choice, whether the x is chosen (the ALS left) or not, using this probability, 0.00054.

We could have done it, but obviously it is meaningless to include in a stochastic model such very rare occurrences. If there were some logical reason to argue that some particular situation during the process creates a high tendency to leave ALS, then the case would be different, but then also there would be much higher (conditional) probability for such an event. It is difficult to see any particular reason for this in the case of Esch's experiment. We merely see the cognitive process of those 2 subjects as a consequence of "free will". (More of this philosophically very important topics in the following chapter.)

Our model had turned out to be, "unfortunately", rather flexible, so that it is difficult to answer to the question, which variable values are the "right ones". It may seem to be a weakness from the estimation point of view, but it should be noted that our modest goal of the study has been only to show that, deriving rules from our theoretical framework, we can logically build a cognition model that *can* be made to *fit* also to empirical data.

Chapter 5. Further analyses

5.1. Chains of cognitive trials. Reality testing. Fancying

In the previous chapters, there was merely described how to formalize the main features of cognitive process. To show the testability of the theory we had to make certain simplifications. Maybe this has led to such misunderstanding that the theory itself were very limited. One has to note, however, that the studies reported in the previous chapters have to be taken as *elementary* examples of the application of the theory. Very little has been said about the rich variations of the human cognitive process. In this chapter we try to show at least how to build a more general view by abandoning one simplification after another.

The simplification concerning the *action threshold* is already mentioned. But in the same context we used another simplification which seems to be relevant in the case of Group Maze but is rather rough: We assumed *no chain* of cognitive trials but supposed that the starting state was always that one which corresponded to the real state in the game – formally:

$$(\forall t') \text{ Ctr}(i, x', y', t', t) \rightarrow x' = \text{Outc}(\mathbf{p}(\text{RCCor}(x, t))),$$

e.g., when the real state in the Group Maze game was s_0 , LSR, then all cognitive trials – until some of them succeeded – had s_0' as the starting state.

We can enlarge our model by assuming a *chain* of cognitive trials allowed before the subject makes his decision to act, formally:

Cognitive trial chain, CoChain:

$$\text{CoChain}(i, s_x', s_n', t_0', t_m')$$

$$\equiv \{(\text{Ctr}(i, s_x', s_{x+1}', t_0') \& \text{Outc}(\text{Succ}'(\text{Ctr}(i, s_x', s_{x+1}', t_0')))=\text{succ}'), \\ (\text{Ctr}(i, s_{x+1}', s_{x+2}', t_1') \& \text{Outc}(\text{Succ}'(\text{Ctr}(i, s_{x+1}', s_{x+2}', t_1')))=\text{succ}'), \\ \dots, \\ (\text{Ctr}(i, s_{n-1}', s_n', t_m') \& \text{Outc}(\text{Succ}'(\text{Ctr}(i, s_{n-1}', s_n', t_m')))=\text{succ}')\}$$

The cognitive trial chain from s_x' to s_n' in m steps in cognitive time is the series of succeeded cognitive trials where the goal state of a trial is the starting state of the following one.

Note that the number of steps in cognitive time, m , needs not to be the same as the number of states in the chain, n , but can also be bigger, because there might be failed cognitive trials, too, which do not appear in the chain.

Note also that the series $x, x+1, \dots, n-1, n$ indicates the *ordinal* numbers of the states in the chain; they do not refer to the indices of the states in the cognitive map, *e.g.*, s_5 may appear in the chain as s_{x+k} and again as s_{x+k+r} (if there exists a *loop* in the chain).

How to come, during a cognitive chain, to the state of *decision*?

We need here the concept of *reality testing*.

Let's examine two examples of cognitive maps given in Table 5.1.

Example A represents the case where a *reality testing* occurs, if action threshold > 1 . Because all probabilities $\text{Pr}[\text{Ctr}(i, s_x', s_0', t', t)] = 1$, the length of the chain is always 2 cognitive trials, *i.e.*, after 2 steps the subject comes back to the starting state s_0' .

We assume, further, that cognition-reality correspondence probability between the cognitive state s_0' and the real state s_r is at every point in cognitive time equal to 1, *i.e.*, formally:

$$(\forall t) \text{ Pr}[\text{CRCor}(s_0', s_r, t', t)] = 1.$$

This means that, after the cognitive chain of 2 cognitive trials, the subject always comes to the cognitive state s_0' and tests which is the corresponding real state. According to the probability given and equaling 1, the outcome is s_r .

Note: Testing the real state ("testing reality") is not the same occurrence as action. If action threshold = n, there will be n times those tests before the decision to act is made, *i.e.*, the *real trial* occurs.

Table 5.1. Reality testing

Cognitive map in the form of a cognitive trial probability matrix
 $\mathbf{p}(\text{Ctr}(s_x', s_y', t'), t)$

Example A (decision-making for an action):

	States					Σ
	s_0'	s_1'	s_2'	s_3'		
s_0'		.05	.25	.30	.40	1
s_1'	1		0	0	0	1
s_2'	1	1		0	0	1
s_3'	1	1	0		0	1

Example B (fancying):

	States							
	s_0'	s_1'	s_2'	s_3'	...	s_n'	...	Σ
s_0'		.05	.25	.30	.40	...	0	...
s_1'	1		.10	.30	.4020	...
s_2'	1	1		.10	.2030	...
s_3'	1	1	.01	.06	.1030	...
...								

The example B is quite different. The probability values of cognitive trials show that the cognitive process occurs as a long chain and never comes back to the cognitive state s_0' . Thus, no test of reality nor real trial occurs during that chain. It is reasonable to call such a process *fancying* (or dreaming).

There seem to be two possible reasons to stop the chain of fancying:

1) Leaving the fantasy life space, *i.e.*, giving up the commitment and then restructuring the life space.

2) Environmental change which forces restructuring the life space.

Our examples above are intended to describe extreme cases, a very strict decision-making and a loose fantasy. We may well suppose that the cognitive process usually has characteristics of both extremities.

Note: There seems to be an interesting structural analogy between some types of group discussion and the individual cognitive process. "Small talk", totally free discussion in an informal group proceeds from topics to topics (from state to state) rather randomly, without determination - so that the "reality testing" ("how did we come to this odd question") does not

succeed, because very soon nobody can anymore remember what was the actual topics. In problem-solving groups, the main theme is frequently emphasized in the discussion, "the reality tested".

5.2. Some features of thinking. "Anchor states"

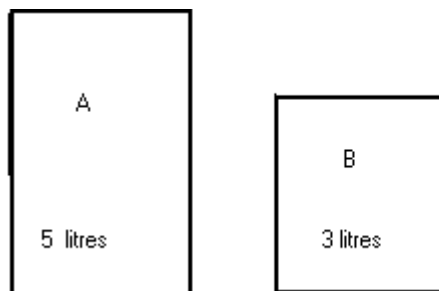
We have separated strictly the cognitive and real states from each other. The cognitive process has been seen as an "inside" process of the individual, not observable, but leading to action, to real trials, which are observable (by individuals who understand the psychological relevance, however).

If we leave out the requirement that "real trials" need to lead to "outside actions", observable acts, we may find an interesting structural analogy between "cognition – reality" -relations and "cognition, stage 1 – cognition, stage 2" -relations, *i.e.*, some cognitive processes deal with some other cognitive processes as if the latter ones were "reality" but "inside" the person, not observable.

Let us again start with an example.

In a well-known item, used in many intelligence tests, the following task is given to the subject (see Fig. 5.1):

Fig. 5.1. Cans



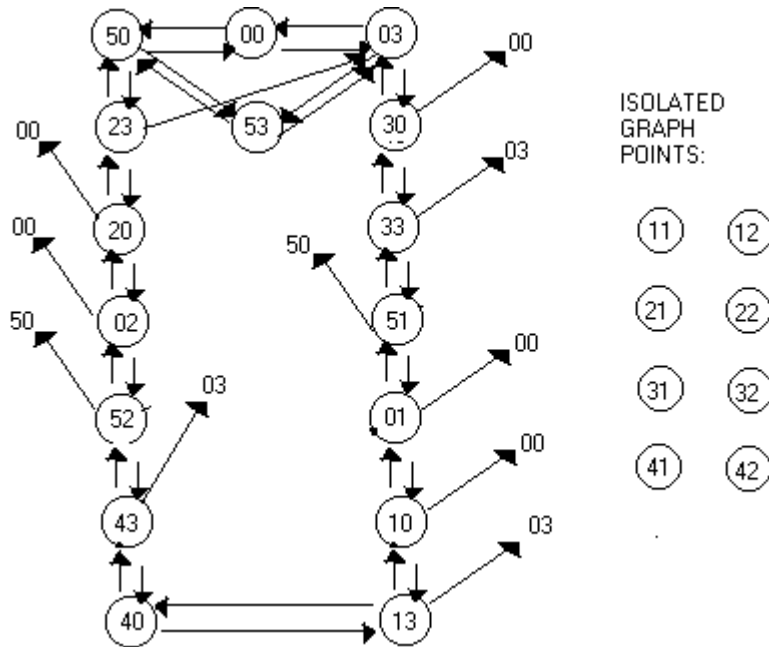
"You are near a fountain where you have in your use so much water you need. There are two cans, A and B, A holding 5 litters and B 3 litters, exactly. Now your task is to measure exactly 4 litters of water according to the following rules:

- you can fulfill a can
- you can empty a can
- you can pour water from one can to another but only all you have in that can or you need to fulfill the other can

Think in your mind how the task has to be solved and when you have found the way how to do that, tell me the solution."

The task is given as a graph in Fig. 5.2. (Note that there are 8 isolated states which are not possible to reach. They are, thus, not relevant.

Fig. 5.2. Cans. Task graph



If we let the subject make notes by paper and pen, the task is an easy one. The subject may write down the pouring acts as pairs of numbers telling the situation – how many litters there are in the cans, *e.g.*, starting from 00

5-0, 2-3, 0-3, 3-0, 3-3, 5-1, 0-1, 1-0, 1-3, 4-0.

(Notice the third one "0-3", because it is an "error".)

Another possibility:

0-3, 3-0, 3-3, 5-1, 0-1, 1-0, 1-3, 4-0.

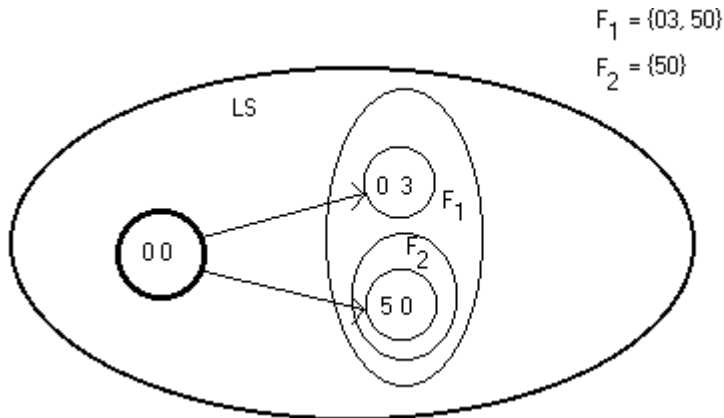
A third possibility:

5-0, 2-3, 2-0, 0-2, 5-2, 4-3, 4-0.

(There are two "right" solutions, one beginning from 5-0 and leading then in 6 steps to 4-0, and the other beginning from 0-3 and leading then in 7 steps to 4-0.)

For illustration purpose, we show two examples of cognitive maps to be formed by the subject during the process in Figures 5.3 and 5.4. We may assume some potency values of the fields and compute – according to them – the probabilities of the cognitive trials in those cases.

Fig. 5.3. Cans. Cognitive map. Example 1: Anchor state 00



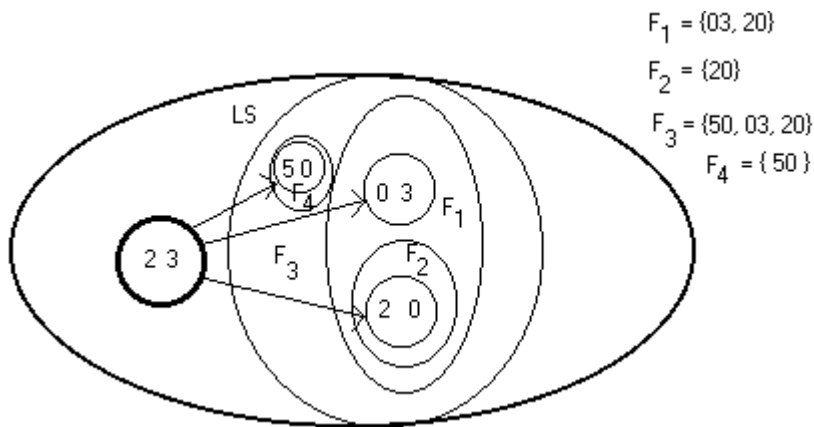
$F_1 = \text{"Possible, new"}; F_2 = \text{"More water"}$

$PoF_1 = .80; PoF_2 = .20$

$Pr[Ctr(s_{00}', s_{03}')] = .80 \cdot \frac{1}{2} + .20 \cdot 0 = .40$

$Pr[Ctr(s_{00}', s_{50}')] = .80 \cdot \frac{1}{2} + .20 \cdot 1/1 = .60$

Fig. 5.4. Cans. Cognitive map. Example 2: Anchor state 23



$F_1 = \text{"New"}; F_2 = \text{"More interesting"}; F_3 = \text{"Possible"}; F_4 = \text{"Stupid"} \text{ (negative valence!)}$

$PoF_1 = .50; PoF_2 = .20; PoF_3 = .10; PoF_4 = .20$

$Pr[Ctr(s_{23}', s_{20}')] = .50 \cdot \frac{1}{2} + .20 \cdot 1/1 + .10 \cdot 1/3 + .20 \cdot 1/3 = .25 + .20 + .033 + .067 = .55$

$Pr[Ctr(s_{23}', s_{03}')] = .50 \cdot \frac{1}{2} + 0 + .10 \cdot 1/3 + .20 \cdot 1/3 = .25 + .033 + .067 = .35$

$Pr[Ctr(s_{23}', s_{50}')] = 0 + 0 + .10 \cdot 1/3 + 0 = .033$

$Pr[Ctr(s_{23}', s_{23}')] = 0 + 0 + 0 + .20 \cdot 1/3 = .067$

So far as making the notes is allowed to the subject, we see writing the pair of numbers as real trials (which succeeds) and every time when this is done, the cognitive process starts from this new situation.

Now, it is interesting to examine what actually happens, if writing the notes is *not* allowed.

Through introspection we can easily notice the analogy between really writing the notes and "keeping in mind" the situation reached in imagining the acts of pouring the water. We can well speak about a kind of "*inner note writing*". The essential difference is that the "inner notes" may easily vanish, while the real ones do not. Otherwise the process is very same in both cases: the subject does a cognitive trial imagining fulfilling, emptying or pouring the water and if it succeeds (is possible and seems reasonable) he somehow *fixes* it as a situation from which he starts new cognitive trials. That fixed situation which he then try to keep in mind, we shall call an "*anchor state*" of cognition. It is an image of a state that *could be real*. It is "inside" our mind, and, thus, cognitive, but it has in our theoretical framework, in relation to cognitive trials, the same function as a real state.

As far as I can see, *thinking* is a cognitive process which uses *anchor states* in one way or other. Reaching an anchor state means restructuring the actual life space *as if* a real state had been reached, *e.g.*, fulfilling can A with 5 liters of water means in our cognition same as perceiving the same action *really* done at real fountain, with real water and real can. For the solution of the can-problem, the difference is totally unessential, as well as the difference between "keeping in mind" and "book-keeping".

One way of thinking is to *simulate* cognitively the real processes, *i.e.*, to search the anchor states in the same time order as the real process is assumed to occur. Examples of this were already presented above. But there is at least one other way, too, which shows the radical difference between the trial and error type of problem-solving and the inference type one:

In our can-example, one can well start the solution of the task from its *end* searching anchor states in a reverse order:

1) "How one could come to the state 4-0? The earlier state need to be 1-3." (An anchor state: 1-3)

2) "The earlier state was necessarily 1-0!" (Anchor state 1-0)

3) "It might be that now the earlier state were 1-3, but as well it could be 0-1; yes, I feel it was 0-1." (Anchor state 0-1)

4) "How could we come to this? What was the earlier state? 1-0 ? No, it is a wrong way! It needs to be 5-1." (An anchor state 5-1)

5) "How to come to this? 0-1? No! Perhaps from 3 litters in B 2 litters were poured to A. How it was possible? Oh, yes, if there were 3 litters in A. Thus, the earlier state was 3-3. (An anchor state 3-3)

6) "But this was reached clearly by fulfilling B, pouring it to A and fulfilling B again! OK, the beginning is 0-3 and then 3-3. We start in that way."

This kind of inference seems, in the case of can-example, more difficult than simulating the actions from the beginning. But it is one way to solve problems and tests can be build on that principle, too. The aim of this example was to emphasize the role of anchor states in both cases of thinking.

It has been stressed above that from the problem-solving point of view it is rather indifferent whether there are written notes about the steps available during the solution process or only "anchor states" in cognition. This is quite in accordance with the findings in studies of chess-playing. *E.g.*, Pertti Saariluoma writes (Saariluoma, 1999, pp. 91-92):

"It is cognitively very interesting what chess players' seeing is. Chess players 'seeing' cannot be object perception or attending. Chess players 'seeing' is not modality specific but chess

players may equally well imagine visually or auditory presented chess positions equally well to normal positions. The contents of ‘seeing’ are thus quite independent of the stimulus content and the arguments against taking ‘seeing’ as object perception are very evident.”

”Consequently, the ambiguous term ‘see’ is replaced by the classic term apperception (Kant, Leibniz, Stout, Wundt). Apperception refers to conceptual perception or construction of representational contents. It assimilates the perceptual stimulus and conceptual memory information into a semantically self-consistent representation that is characteristic of human mind.”

”... Apperception determines which semantic elements of conceptual memory and of the stimulus information can be and should be assimilated into one single representation.”

From the epistemological point of view the relation between the ”anchor states” and ”real states” is fascinating.

5.3. Content of cognitive state

The cognitive state, formally s' in $C(i,s',t')$, indicated in our Group Maze study simply one position in game, a combination of positions of three pieces. In the simulation of Esch’s experiment the cognitive state was combination of three believes, a simple thing, too. The cognitive state in our every-day life seems to contain much more, a tremendous amount of details. Even perceiving the environment at one moment consists from millions or billions of bits – if one wants to use here some technical information measure. How can we ever describe it as one state of our cognition? Does the simplifying go here too far?

In the introductory chapter we already emphasized the *holistic* way of our analysis. From that point of view all we perceive or cognize in a state is *one gestalt*, although more or less differentiated. This gestalt has *one* meaning, in principle, although it hardly can be interpreted by one or even by few words – sometimes a whole story were needed for that.

We have been talking about *relevant* features of situation. In interpreting the meaning of the cognitive state the relevance indicate that we sometimes have to talk of different states in connection of one gestalt, namely in cases when it is differentiated in differing relevant ways. Sometimes the parts of the gestalt have no relevance. In other words, if the meaning of the whole changes, then we have to change the cognitive state: If somebody who admires a beautiful forest lets his mind to wander so that he starts to compute the price of it, then the change of the cognitive state is obviously necessary – although the ”perceived view” is physically unchanged.

So far we have not examined *emotions*, although they are an important aspect in the consciousness as taken a whole. We have been dealing with *cognitive* phenomena, so that the emotions are not included to our topics. One should notice, however, that we can define the cognitive state in that way that the emotions are taken into account in the content of it. As well we could define the fields so that the states classified together for their emotional content were represented as elements of the same field. In that case, the hypotheses for the transition process need seemingly not to be the same as in analysis of the purely cognitive, on knowledge-based process, only.

5.4. Perception. Forming cognitive maps

According to modern, standard standpoint, the perception is an *active* process. Ludwig Klages, a famous gestalt psychologist, already described it as a process of ”*actual genesis*”, *i.e.*, the process starts from interpreting the sensation as undifferentiated whole and continues then by differentiating it step by step. How could this be described in our theoretical framework?

Following the principle of actual genesis we can represent the process of perception in a form of ”flow list” using probabilities of correspondences between cognition and reality:

We have now to examine those probabilities more closely, particularly the probability of the success of real trial in connection with the CRCor-probabilities. In the earlier chapters we have not handled them together, *i.e.*, in dealing with the games it was always reasonable to simplify the system such a way that a certain cognitive state x' corresponded to one real state x by the probability equaling 1. Thus, no Monte Carlo method to determine the outcome was needed.

Now, in analyzing perception, this correspondence becomes an essential characteristics. The vector of CRCor-probabilities needs to be used and determination of the outcome by Monte Carlo method made. This procedure occurs *between* the determination of the real trial alternative and the determination of the success of it, *i.e.*, we assume that the determination of the success of the real trial happens *after* the cognitive decision-making and the determination of the real trial (corresponding to it).

The "flow list" of the perception process is the following:

1. $C(i, w_0', t', t)$

The subject i is in a cognitive state of "awareness" at time t' and t , *i.e.*, active to cognize the sensory environment as a whole.

2. CRCor- (cognition-reality-correspondence) -outcome is determined using the CRCor-probability vector:

$Outc(\mathbf{p}(CRCor(i, w_0', t', t))) = W_0$ and $Tr(i, W_0, t)$

where W_0 indicates perceiving the whole (undifferentiated).

3. The success of perceiving W_0 is determined:

If the trial succeeds then

$Be(i, Perc(W_0), t)$

We shall use an abbreviated form of that:

$Perc(i, W_0, t)$

4. Supposing that perception W_0 has succeeded, the more differentiated cognitive map is determined:

$Outc(\mathbf{p}(RCCor(i, W_0, t+1)) \mid Succ(Perc(i, W_0, t))) = C(i, DM_1', t', t+1)$

where DM_1' indicates the cognitive map differentiated on the first level.

5. The CRCor-outcome is determined:

$Outc(\mathbf{p}(CRCor(i, DM_1', t', t+1))) = DW_1$

and $Tr(i, Perc(DW_1), t+1)$

where DW_1 now indicates the corresponding differentiated whole perceived by i at the new real time point $t+1$.

6. The success of trying to perceive DW_1 is determined. In the case of success

$Perc(i, DW_1, t+1)$

7. The RCCor-outcome is determined:

$Outc(\mathbf{p}(RCCor(i, DW_1, t+2)) \mid Perc(i, DW_1, t+1)) = C(i, DM_2', t', t+2)$

where DM_2' indicates the cognitive map on the 2nd level of differentiation.

Etc.

Note: It is reasonable to think that the differentiation of a cognitive map does not indicate only the differentiation of the state structure of the map (= graph) but also the differentiation of the *meaning* of the states: they have a meaning as a part of the whole but also an independent meaning.

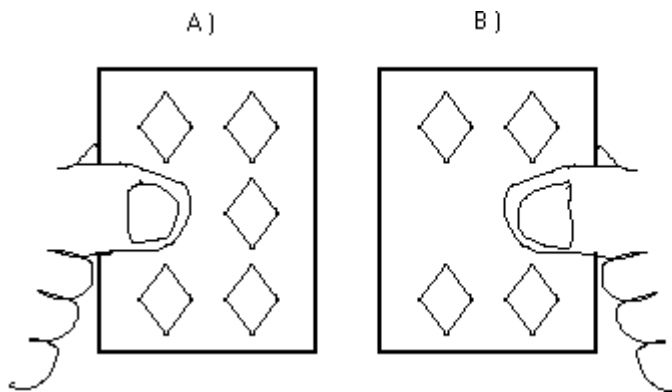
Usually the perception process seems to be very quick and automatic. We may think that there are in every-day life very often repeated perception processes where the perception – after a long learning history – succeeds by a very high conditional probability:

$$\Pr[\text{CRCor}(i, \text{DM}_2', t', t+2) \mid C(i, \text{DM}_1', t', t+1)] = 1$$

There may rise, however, problems in perception, when something unusual is perceived during the differentiation process, *i.e.*, when some new parts of the perceived whole do not match to the meaning of it. Let's take an example:

A magician shows to an audience in his right hand a play-card which is seen the diamond-6 (see Fig. 5.5.A). He turns it around and the same card shown now in his left hand is diamond-4 (see Fig. 5.5.B)!

Fig. 5.5. The miraculous cards



How this is represented in our conceptual framework?

On certain level of differentiation, say, on the level n , the cognitive map DM_n' has the meaning "Diamond-6 is seen", because the parts suit well to that whole. The outcome using the CRCor-probability vector is

$$\text{Outc}(\mathbf{p}(\text{CRCor}(i, \text{DM}_n', t', t_n))) = \text{Perc}(i, \text{DW}_n, t_{n+1})$$

where DW_n is the object "Card diamond-6". (The outcome was "success".)

When Fig. 5.5.B is seen, the cognitive map DM_n' has now the meaning "the same card, diamond-6, is seen as diamond-4". Outcome is now

$$\text{Perc}(i, \text{DW}_n, t_{n+2}), \text{ where } \text{DW}_n \text{ is the object "Card diamond-6 and diamond-4".}$$

The probability of success of this kind of perception is 0 and it fails. According to our conceptual framework this leads back to the cognitive state DM_n' and to a new real trial, *i.e.*, to a trial to perceive differentiating the whole in a new way, $\text{DW}_{n,2}$. It might be very difficult to find a *suitable* cognitive map. The subject has to "cancel" his process of differentiating and come back to earlier level, where the perception has the meaning of "some kind of card" and try again and again cognitive maps which can create a meaningful perception. This continues until the subject creates finally a cognitive map by meaning "a new type of card with 5 diamonds but in such an order that in one case all 5 are shown and in another case only 4, because in the first case a finger covers the

empty place and in the other one diamond figure is covered". It is a rather complicated meaning. It is not a wonder that it is usually not created at once by any of the subjects in the audience.

It is interesting to notice that actually such a stimulus content as "a card with 5 diamonds and an empty area" is *not* seen at all – not in the case A, because then the magicians finger blocks the perceiving the empty place, nor in the case B, because only 4 diamonds are visible. Psychologically this is a trivial thing: naturally it is a *gestalt* that has been seen, a gestalt with qualities of "a card with 5 diamonds but in such an order that..." *etc.* The very known epistemological question arises, however: Do the human beings perceive the *reality "as such"* at all, only gestalts formed by their cognitions – as our card-case illustrates very clearly?

In other words, there arises again the question: What actually means the "object" – or the "real state" we have handled as given? Has it been always a radical simplification? This is actually an old question. After Immanuel Kants philosophy we can not accept any kind of naive realism here.

Analogously with the perception process we can create a model for an "inner perception", *i.e.*, a model for forming higher order cognitive maps which suit to the lower order cognitive "material" differentiating it in a new way. We need only substitute the "first-level cognition", C', for "reality", R. Thus, we may examine the following process, quite analogous to perception:

$$1. C''(i, W_0'', t')$$

On the second-level cognition (indicated by two dots), *i* starts to think or imaging something.

$$2. \text{Outc}(\mathbf{p}(C''C''\text{Cor}(W_0'', t')))) = C'(i, W_0', t')$$

Imagining takes the form of an undifferentiated whole W_0' .

(3. Determining the success' of the imagining W_0' is not needed because W_0' is undifferentiated "something".)

4. Determining the (higher order) cognitive map which could be used to differentiate the lower order (diffuse) cognitive matter:

$$\text{Outc}(\mathbf{p}(C'C''\text{Cor}(W_0', t'+1)))) = C''(i, DW_1'', t'+1)$$

5. Determining the form which the imagined matter takes, when the higher order way of differentiation (the cognitive map DW_1') is applied to it:

$$\text{Outc}(\mathbf{p}(C''C''\text{Cor}(DW_1'', t'+2)))) = C''(i, DW_1', t'+2)$$

6. Determining the success of the trial to differentiate the cognitive matter according to the cognitive map DW_1' :

$$\text{Outc}(\mathbf{p}(\text{Succ}(C'(i, DW_1', t'+2)))) = \text{succ}' \text{ or } \neg\text{succ}'$$

If the trial succeeds, the cognitive matter (an imagined whole) is now "seen in a new way" – *i.e.*, it gets its meaning from the higher order cognitive map. It is a "new way to understand". One consequence of this new differentiation may be that the matter will now be *named* in some way.

If the trial fails, a new cognitive map may be selected or formed and its success of suiting to differentiate the matter will be tested.

The differentiation process can continue in that way step by step until a detailed but in same time well-structured cognitive map is formed and experienced successful.

Thus, we can continue our formal representation:

$$1. C'''(i, W_0''', t')$$

$$2. \text{Outc}(\mathbf{p}(C'''C''\text{Cor}(W_0''', t')))) = C''(i, W_0'', t')$$

$$3. \text{Outc}(\mathbf{p}(\text{Succ}(C''(i, W_0'', t'))))) = \text{succ}''$$

$$4. \text{Outc}(\mathbf{p}(C''C'''\text{Cor}(W_0'', t'+1)))) = C'''(i, DW_1''', t'+1)$$

etc.

We do not go further in this analysis. The model is mentioned here merely for philosophical reasons: the "relativity of reality" is emphasized. Maybe the "objects" which are called "physical

things” should be taken as ”more real” than images, but not ”absolutely real”. Our attempt to build exactly formalized theoretical framework may clarify the types of problems we have met.

5.5. Communication

If the real state s in the behavior formula $Be(i, s, t)$ indicates expressing a message, $s=Expr(Me)$, or receiving a message, $s=Rec(Me)$, then this way of behavior is called a *communicative act*.

Our aim here is to analyze what kind of consequences communicative acts have in the cognitive process. Only an overview is given here. (There is a larger analysis from social-psychological point of view in Rainio, 1966, pp. 98 - 115.)

In the following, the former described analysis of perception will be applied to the communication.

It is necessary to make a distinction between the concept *message matter* (abbreviated: $MMess$), indicating the message as undifferentiated matter without any meaning, and *message content* (abbr.: $DMess$), indicating the message differentiated according to its meaning (assumed by i , the receiver).

Thus, accordingly, we separate the *receiving* a message (= perceiving the message matter) and *understanding* it from each other. The understanding means giving (some) meaning to the message. So, the flow of the process is, *mutatis mutandis*, same as of the perception, formalized in the following way:

Receiving a message and giving a meaning to it (*understanding* it):

1. $C(i, W_0', t', t)$
2. $Outc(\mathbf{p}(CRCor(W_0', t', t))) = Perc(MMess)$
3. $Outc(\mathbf{p}(Succ(Tr(i, Perc(MMess), t)))) = succ$
4. $Outc(\mathbf{p}(RCCor(MMess, t+1))) = C(i, DMess_0', t', t+1)$

where $DMess_0'$ indicates the message differentiated according to the meaning of it.

5. $Outc(\mathbf{p}(CRCor(i, DMess_0', t', t+1))) = DMess_0'$
6. $Outc(\mathbf{p}(Succ(Tr(i, Und(DMess_0), t+1)))) = succ$

where $Und(DMess_0)$ indicates receiving the message in a differentiated form, *i.e.*, in a meaningful, understandable form.

7. If the outcome is successful, then: $Be(i, Und(DMess_0), t+1)$, *i.e.*, the subject i has received a message and understands it in the differentiated form.

If the trial to understand the message in the differentiated form $DMess_0$ fails, a new form is tried (starting from the point 4), until some form succeeds (is experienced by i to correspond to the message matter and to be meaningful).

Sending a message is formalized in the following way:

1. $C(i, DMess', t', t)$

A message is imagined by i in a differentiated form, as a message content.

2. $Outc(\mathbf{p}(CRCor(DMess', t))) = MMess$

The corresponding real state is the utterance of the message content in the form of message matter, $MMess$.

3. $Tr(i, Utt(MMess), t)$

i tries to utter the message matter.

4. $Outc(\mathbf{p}(Succ(Tr(i, Utt(MMess), t)))) = succ$

The utterance succeeds.

5. $Be(i, Utt(MMess), t)$ which can be abbreviated

Send(i , MMess, t)
 i sends the message (*i.e.*, the message matter).

Note particularly:

The message matter in the communication need not to be the "same" to j , the receiver, and to i , the sender. If the sender i tries to receive his *own* utterance and understand it, he most probably understands it in the way he himself differentiated it, *i.e.*, believes to be the meaning of it. To the receiver, j , the utterance exists in the form of message matter and he tries to use *his* own cognitive maps in differentiating the message matter – in giving to it some meaning.

In the everyday life, there are situations where we do not need to understand the message in the "same" way as the sender intended it to be understood, not at all (*e.g.*, in the so-called "small-talk"). Naturally, in the cases where the errors would be fatal, we do continue to send new message matter (usually in both directions) in the hope that the actual message could be in that way easier to differentiate and to be understood. The communicators are not satisfied until they *believe* that the message content is understood in that meaning the sender intended to give to it. Philosophically it is important to emphasize that there is absolutely no way to guarantee that the intention by i , the sender, is understood "in the same way" by j , the receiver, *i.e.*, that $DMess_i = DMess_j$. We do not even have any objective way to compare them. We have to be satisfied if the subjects feel that they *believe* they have understood the message "in the same way".

But what we now *do* have: we can, in principle, create a *measure* for understanding between individuals on the basis of our probability model. Using well-controlled experiments we may define behavior consequences as results from "right" understanding or misunderstanding certain varying messages and estimate the error distribution according to our probability model.

The *content of a message* may vary in enormous many ways. The following formal categorization might open possibilities to create probability functions for mathematization some main characteristics of the communication.

We make the distinction between *utterance* and *statement*. The former is defined totally undifferentiated *per se* with no reference to truth, *i.e.*, it represents to a receiver a pure matter of perception. The latter, a *statement*, is a content of message with a meaning to state that something has to be believed true (or false). In this case there can exist an *addressee* to whom the message was intended.

A *simple statement* contains a description of the behavior of a referred subject (j_r) at a referred time point (t_r), formally:

Stm (j_r , a_r , t_r)

The j_r may be an individual or "nature" (N). In the former case, the receiver try to understand a_r as an *act with a meaning*, in the latter case it is a "state of things" or "state of facts" not *intended* to be done. Note, however, that in primitive thinking also the "behavior of Nature", the "case of things", is often personalized, seen not only to *occur* but also to *mean* something: A flash of lightning is not only an electric discharge but an "act of God", intended to scare or to warn people.

In receiving a message containing a statement, an essential aspect is to *believe* it or not, measured by the probability

$Pr[i, Bel(Stm(j_r, a_r, t_r)), t]$

If the outcome of a Monte Carlo choice using this belief probability is "believing", then it has a consequence that the receiver i *expects* the referred subject j_r to behave in the way a_r at the referred time point t_r . We may assume that this expectation follows the laws of forgetting, but as long as it exists and in the situations where it is relevant, it makes the cognitive process by i *conditional*. (This was the case in the Group Maze experiment, analyzed in Chapter 3.)

In addition to simple statements the messages may contain *conditional statements*. As an important example of those it should be mentioned the *sanction statement*: There is in it two

elements included in the statement: the sanctioned behavior and the sanction (sanctioning behavior), formally:

$$\text{Stm}(\text{Be}(i_r, a_r, t_r) \rightarrow \text{Be}_{\text{sanc}}(k_r, b_r, t_{r+n}))$$

It is stated in above: If the subject i_r behaves in the way a_r at time point t_r , then a subject k_r will behave in the way b_r at some time point t_{r+n} , later than t_r . To be a case of sanction, it is needed, in addition, that the behavior by k_r is rewarding to i (a positive sanction) or punishing (a negative sanction).

If the sanction message is believed by i at the time point t , it makes his cognition conditional, *i.e.*, it has an effect upon i 's choice of a_r .

Notice that the "behavior of Nature", the estimated "case of things" in some time point in future, is also seen as a sanction behavior, if the implication between a_r and b_r is believed to be true. The personalized expressions "the Nature rewards" or "the Nature punishes" for certain human actions includes often truth. (The primitive characteristics of thinking comes out not until the subject believes that Nature *intends* to sanction him.)

Summarizing, we can write the message sending in the case of a simple message formally in the following way:

$$\text{Send}(i, \text{Mess}(\text{Addr}, \text{Stm}(\text{Be}(j_r, a_r, t_r))), t)$$

A subject i sends a message to an addressee Addr stating that a subject j_r behaves in the way a_r at time point t_r , this sending act happening at time point t .

Even this simple formula includes many interesting cases of communication, if analyzed:

The addressee may be another subject than j . He may be i himself or a group or a large audience or none. Note the case of "inner speech", when $i = \text{Addr}$.

The referred subject j_r may be Nature, as mentioned above, but as well another individual or i himself. If $i = j_r$ but $i \neq \text{Addr}$, we have a case where i tells to another subject his own behavior – his own intended behavior, if $t_r > t$ or recalling the past, if $t_r < t$. If $i = \text{Addr} = j_r$, then i recalls in his "inner speech" his own past.

If $j_r = \text{Addr}$ but $i \neq j_r$ and $t_r > t$, then i foretells to j_r the future by j_r .

Etc.

Thus, our formula of simple statement contains implicitly a whole system of categorizing messages and message sending situations.

Chapter 6. Philosophical perspectives

Quantum-mechanical theory in modern physics has shown how we need to accept a theory without being able to answer to the questions what the variables, used in the theory, *really* are. What are "really" the "particles", what are the "waves"? What "really" happens at the moment of the reduction of the "wave-packet" ? – The pragmatically oriented physicists answer that it is not a task of physics to solve such "philosophical" problems. The main thing from the point of view of physics is that the mathematical models are correct and give the right estimates for the measurements. – On the same basis we may answer pragmatically to the ontological and epistemological questions concerning our theory of cognitive processes: What the cognitive states "really" are? How occurs the decision-making and starting the observable behavior in "reality"? Is it not pragmatically enough that our theory gives the right estimates to the measurements, *i.e.*, to the observations of the behavior traits like the frequencies of errors in Group Maze -game, for instance?

But the human mind can not avoid the philosophical "why": Why the mathematical laws of processes are such as they are? – We seek interpretation of the formal description in terms of those concepts which are for us *meaningful enough*. We are not satisfied until we can have a *holistic* view over things, see the world as a well-differentiated whole. The mathematics *per se* does not fulfill these requirements, because the language of mathematics does not refer to the *qualia* in the content of our consciousness. To be a whole and, thus, understandable in a deep meaning, our world-view needs to be based not only on the logical structure of processes but also, and perhaps mainly, on perceptions and images, because they "carry" the *qualia*.

For instance, from theoretical point of view we can be satisfied with stating the mathematical law of changing some values of choice probabilities: $p_{t+1} = p_t + \alpha(1 - p_t)$. However, just interpreting it, calling it a *learning* law, connects it to a large net of meaningful images, gives a "rich content" to the formula. It is then *understood* as a part of a holistic world-view, of the reality. The "learning" might have for us a rather diffuse meaning – it is not so "exact" as the mathematical law - but it is something which "really happens". I should say: it is connected to the *qualia* in our experience.

In this chapter we shall examine some philosophical views our theoretical framework opens, although it would not be necessary, pragmatically.

6.1. Psycho-physical problem. Eccles' theory

When the connection between cognition and behavior was examined, we assumed simply that there exist an action threshold (AT), which determines the time of behavior decision, and the CRCor-probability vector which determines the direction of the behavior trial. If the mathematical model works, *i.e.*, if our estimate, when certain AT- and CRCor -values are used, have a goodness-of-fit high enough, we can be pragmatically satisfied without asking anything more.

But, in trying to build a holistic, meaningful view over the "reality", we are not satisfied. We meet here an extremely important question: How is it *understandable* that a *cognitive* decision state is followed by a certain behavioral, *physical* act? Or – as Penrose puts it into words: "How is it that a consciousness, by the action of its will, actually *influences* the (apparently physically determined) motion of material objects?" (Penrose, 1989, p. 405)

This is the old philosophical question called the *psycho-physical problem*. Most of the philosophers see it still unsolved. The great nobelist in quantum physics Wolfgang Pauli found it to be the most important problem of our time.

Without going to the long history of the psycho-physical problem in philosophy we note that there exists in our days a strong tendency to look the relation between the brain and consciousness from *materialistic* point of view – be it emergentistic or reductionistic materialism. The basis seems to be as G. M. Edelman writes:

”Any adequate global theory of brain function must include a scientific model of consciousness, but to be scientifically acceptable it also must avoid the Cartesian dilemma. In other words, it must be uncompromisingly physical.” (Edelman, 1989, p. 10)

Irrespective of the fact that the materialistic basis is more or less clear there are lot of nuances in the discussion and many different ”theories” could be named, *e.g.*, identity theory by Feigl, Sperry’s emergent interactionism, Smart’s physicalism, emergentistic materialism by Bunge, *etc.*

The famous neuroscientist, nobelist Sir John C. Eccles has referred – in his book ”*How the Self Controls Its Brain*” – the works by Changeux, Crick and Koch, Dennett, Edelman, Hodgson, Penrose, Searle, Sperry, and Stapp criticizing some of them very sharply (Eccles, 1994, pp. 27 - 53). Instead of going to this debate, we shall focus our attention on Eccles’ own mind/brain theory which he calls ”dualistic-interactionistic” – being fully conscious of the heavy critics directed in our days toward dualism, understood in Cartesian way. Eccles emphasizes that his dualism is *not* a Cartesian one, because he does not use the concept of mind in the sense of a substance. He writes:

”The concept of substance leads to a materialist aspect of mind. I speak instead of the spiritual existence of the self without mentioning any ‘substance’ properties. The great problem is ‘how the self controls its brain’. This is dualistic, but not in terms of two substances.” (Eccles, 1994, p. 38.)

As far as I can see, calling Eccles’ theory dualistic is actually unnecessary and misleading. By that label Eccles seems to dissociate his view from materialism as clearly as possible.

Without stating that the mind/brain theory by Eccles is absolutely true, we shall refer it here because it gives an opportunity to understand clearly what our cognition-reality-correspondence could mean in ”reality”.

The theory by Eccles explains the process in the synapses of the brain-neurons, the process which is called *exocytosis*. It is briefly introduced in the following.

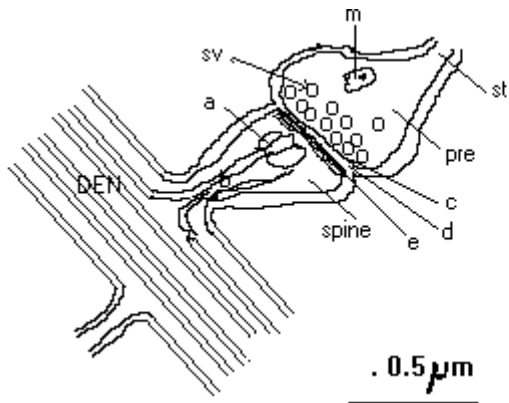
As an introduction to the problem ”how mental events could influence neural events”, Eccles writes:

”It has long been recognized that, if non-material mental events, such as the intention to carry out an action, are to have an effective action on neural events in the brain, it has to be at the most subtle and plastic level of these events. Attention has to be focused on the biological units of the brain, the neurons or nerve cells, and on the manner of their communication at specialized sites of close contact, the synapses. An introduction to conventional synaptic theory leads on to an account of the manner of operation of the ultimate synaptic units. These units are the synaptic boutons that, when excited by an all-or-nothing nerve impulse, deliver the total contents of a single synaptic vesicle, not regularly, but probabilistically. *This quantal emission of synaptic transmitter molecules* (about 5000 - 10000) is the ultimate functional unit of the transmission process from one neuron to another. This refined physiological analysis leads on to an account of the ultrastructure of the synapse, which gives clues as to the manner of its unitary probabilistic operation. The essential feature is that the effective structure of each synapse is a *paracrystalline presynaptic vesicular grid*, which acts probabilistically in quantal release.” (Eccles, 1994, p. 55.)

Then Eccles tries to ask to the specified question ”how a non-material mental event, such as an intention to move, can influence the subtle probabilistic operation of synaptic boutons” (p. 55).

A rough description of the structure of the synapse is needed here and given in the Fig. 6.1.

Fig. 6.1. The detailed structure of a spine synapse on a dendrite



DEN = dendrite

st = axon terminating in a synaptic bouton or presynaptic terminal (pre)

sv = synaptic vesicles

c = presynaptic vesicular grid

d = synaptic cleft (c. 200 Å = 0.00002 mm)

e = postsynaptic membrane

a = spine apparatus

b = spine stalk

m = mitochondrion

(Eccles, 1994, Fig. 7.1.b, originally in Gray, 1982.)

The *exocytosis* Eccles describes in the following way:

”A nerve impulse propagating into a bouton causes a large influx of Ca^{2+} ions. The input of four Ca^{2+} ions activates a synaptic vesicle via calmodulin and may cause momentarily to open a channel through the contacting presynaptic membrane, so that its total transmitter content is liberated into the synaptic cleft in a process called *exocytosis*.” (Eccles, 1994, p. 132.)

The liberation is caused by a *trigger* which is of quantal size and behaves purely probabilistically.

About the probability of exocytosis Eccles writes, *inter alia*:

”... the statistical technique of deconvolution analysis enables the determination of the probability of release by a nerve impulse of a single synaptic vesicle, an exocytosis. As in simpler situations this probability of release is always less than one, and is in fact very low for the hippocampus with average mean values of 0.27, 0.24, and 0.16 for the three completely reliable experiments on the hippocampus.” (Eccles, 1994, p. 134)

Concerning this probability of exocytosis Eccles presents his hypothesis:

”... the presynaptic vesicular grids are ideally fitted to be the targets for the non-material mental events such as intention to carry out some movement. It is *not* proposed that the mental

events initiate activity at a synapse by an excitatory action either on the presynaptic or postsynaptic elements of a synapse. On the contrary, the hypothesis is that the mental events merely alter the probability of a vesicular emission that is triggered by a presynaptic impulse.” (Eccles, 1994, p. 73)

The effect of the ”change in probability of emission of a single vesicle is however too small for modifying the patterns of neuronal activity even in small areas of the brain (Eccles, 1994, p. 74)”.

But the synaptic boutons are clustered. There are *bundles of dendrites* called *dendrons* according to Eccles. In each dendron there are about 100 apical dendrites and over 100000 spine synapses on the bundled apical dendrites of a dendron. When these dendrons are taken into account, the magnitude of the effect of mental event is increased and big enough for modifying the patterns of neural activity. The hypothesis by Eccles does not anymore concern the change of probability by a single exocytosis but the effect of a mental event upon a whole dendron (or – as far as I can see – upon a group of dendrons):

”*The hypotheses is that the probability field of the mental intention is widely distributed not only to the synapses on that neuron but also to the synapses of a multitude of other neurons with similar functions of the dendron.*” (Eccles, 1994, p. 74)

This has been said by Eccles by other words:

”We put forward the hypothesis that mental intention becomes neurally effective momentarily increasing the probabilities for exocytoses in a whole dendron and, in this way, couples the large number of probability amplitudes to produce coherent action.”

”Our hypothesis offers a natural explanation for voluntary movements caused by mental intentions without violating physical conservation laws.” (Eccles, 1994, p. 163)

The last expression – ”without violating physical conservation laws” – is philosophically extremely interesting. Because there is assumed an effect on *probabilities*, no hypothesis of an ”extra substance” is made. As far as it is known in physics, there is no physical way to exercise an effect on those probabilities determining the single quantum phenomena. (Thus, the relation between the mental events and brain functions is not *interaction* – as Eccles presents it – but one-way action.) Because those probabilities belong essentially to our world-view, designed by the modern quantum physics, we can ask whether the Eccles’ theory is even dualistic. It extends our conception of reality essentially in the way that it shows how the mental and physical events are linked together, but it does not *add* substantially anything (supposing that we have seen mental events as real as the physical ones).

Eccles has an explanation to the phenomenon that the mental events seem to ”find” just the right dendrons to put them in operation for certain activity. The explanation is *learning*:

”One has to recognize that, in a lifetime of learning, the intention to carry out a particular movement would be directed to those particular dendrons of the neocortex that are appropriate for bringing about the required actions. We believe that the proposed hypothesis accounts for action across the mind-brain interface.” (Eccles, 1994, p. 164)

As an evidence for the fact that mental events can influence the brain Eccles sees several results of ”silent thinking” experiments, mentioning *e.g.*, the findings by Roland:

”... when the human subject was attending to a finger on which a just-detectable touch stimulus was to be applied, there was an increase in the rCBF (regional cerebral blood flow) over the finger touch area of the postcentral gyrus of the cerebral cortex as well as in the midprefrontal area. These increases must have resulted from the mental attention because *no touch was applied during the recording*. Thus, ... mental act of attention can activate appropriate regions of the cerebral cortex.” (Eccles, 1994, p. 78)

Same kind of evidence give the experiments by *pure ideation* (by Ingvar, Roland and Friberg, Raichle, Posner, Corbetta, Pardo et al.; see Eccles, 1994, pp. 168 - 171).

The evidence of the activation of the human brain during pure ideation and silent thinking puts the old problem of *free-will* in a new light. Eccles writes:

”There has long been a question about voluntary movement: materialists have asserted that a *mental intention* cannot bring about a voluntary movement because that is contrary to the conservation laws. However, Searle (1986) writes well on free-will and correctly believes that it is tied to consciousness despite the conservation laws. But now with the work of Beck and myself (Beck and Eccles, 1992, and Chapter 9 in Eccles, 1994) there is no such proscription. In fact we are presented with the wonderful exuberance of relating to our brains with full freedom, for action and imagination.” (Eccles, 1994, p. 172.)

6.2. Eccles’ theory formalized

It is easy to see that in terms of our theory of cognitive process and behavior the theory by Eccles can be represented formally.

In our framework, cognition and behavior are connected through the following probabilities:

1. $\Pr[C(i,a',t')]$, the probability that the subject i is in his cognition in the state a' at the cognitive time point t' .

2. $\Pr[\text{CRCor}(a',a,t)]$, the probability that the state a corresponds to the cognitive state a' at time point t .

3. $\Pr[\text{Be}(i,a,t)]$, the probability that i behaves in the way a at time point t .

According to the way Eccles uses the language, we can well call the cognitive state a' a ”mental state” and show it by writing $M(i,a',t')$ instead of $C(i,a',t')$.

According to Eccles, the exocytosis has its own probability in one step in time, the step in time being the short period during which *one* vesicular emission is triggered:

$\Pr[\text{Be}(S,e,t)]$ where S indicates the synapse, e is the occurrence of exocytosis and t is the step in time.

While the exocytosis is an all-or-none occurrence, then

$$\Pr[\text{Be}(S, \text{non-}e,t)] = 1 - \Pr[\text{Be}(S,e,t)]$$

Let us indicate by Be_B the event of exocytosis on the *basic level of activation*, *i.e.*, when no mental event is influencing the synapse, and, correspondingly, by Be_W the occurrence then, when a mental event influences it. (W indicates here ”will”.) The hypothesis by Eccles means thus:

$$\Pr[\text{Be}_W(S,e,t)] > \Pr[\text{Be}_B(S,e,t)]$$

$$\text{But } \Pr[\text{Be}_W(S,e,t)] = \Pr[\text{Be}(S,e,t) \mid \mathbf{I}(t), M(i,a',t)]$$

and

$$\Pr[\text{Be}_B(S,e,t)] = \Pr[\text{Be}(S,e,t) \mid \mathbf{I}(t)]$$

Note: One condition for the exocytosis is that a presynaptic impulse comes to the presynaptic terminal in the time step t . This condition, $\mathbf{I}(t)$, has been taken with to the formulas.

Now, the hypothesis by Eccles concerning the probability of an exocytosis is formally as follows:

$$\Pr[\text{Be}(S,e,t) \mid \mathbf{I}(t), M(i,a',t)] > \Pr[\text{Be}(S,e,t) \mid \mathbf{I}(t)]$$

Note: If there are no presynaptic impulses coming to the presynaptic terminal, *i.e.*, if $\neg\mathbf{I}(t)$, then the mental event cannot have any influence, because the probability with that condition is 0.

The probability above concerns the occurrence that the impulse *goes over the synaptic cleft*. If there is no impulse ready for that at time point t , then the change of the probability of the *triggering* the exocytosis, going over the synaptic left, has no consequence, *i.e.*, the mental event does not have any effect. Thus, if the neural activity in brain is vanished, no mental events can influence brain (although there may still be those events - there are no logical reasons against that). This is just what seems to happen in the case of death.

The formula is applied to all the synapses which are relevant to the case of a' , *i.e.*, the synapses which the mental state a' influences. – For each synapse there may exist several mental states which can influence its exocytosis.

The exocytosis in one synapse is not enough to produce significant change of activation in brain – as has been already noticed above. More synapses are needed, a whole dendron or a group of dendrons. Thus, the cognition-reality-correspondence probability gets the following form:

$$\Pr[\text{CRCor}(a', D_a, t)]$$

where a' is the mental state and D_a is the state of increased neural activity in the dendron or in the group of dendrons influenced by a' .

It should be noted that nothing has been said above about the *kind* of neural activity produced. The increased brain activity may cause particular motoric action or create a cognitive process to interpret the meaning of the neural impulse flow coming from the receptors – or it may start a thinking or fancying process. Anyway a' indicates the mental state by Self and D the state by Brain, *i.e.*, the state of the combination of neural impulses.

There are rough measures for the cognition-brain-correspondence already available: the results of the experiments of "silent thinking" and "pure idealization". The subject of debate is merely the question whether the correspondence is based just on the change of the probabilities of the exocytoses as Eccles has assumed or not. It does not matter from the point of view of our *holistic* framework, but philosophically it is most important, because the role of materialism in our scientific thinking depends on it.

6.3. Free will. Commitment

Concerning the old philosophical problem of free will, there is a clear distinction between all materialistic mind/brain theories, including the identity theories, and the view by Eccles. According to the materialistic theories all processes are determined by causal chains and there exists no possibility for any event not caused by another event. The "free will" is, thus, seen illusory (see *e.g.*, Dennett, 1991). Note that the probabilistic (or statistical) causation does not make any difference: the probabilistic choice makes the process indeterministic but this is not same as to select from the group of available alternatives one – intending to start a meaningful process by that choice.

According to the theory by Eccles, the free will is self-evident. The appearance of mental event is, in principle, not deterministically caused. In our framework this comes out formally in that way that there exists a category of cognitive states which are not determined by any probabilistic choice; for them there does not exist any probability vector to use. These correspond to the mental states produced by free will.

In Chapter 6.2 we made cognitive and mental states identical. It is, however, necessary to make a distinction between the probabilistically determined cognitive (or mental) states and undetermined, spontaneous cognitive (or mental) states. The first ones exist in the chains of cognitive trials, *e.g.*, as y' in $\text{Ctr}(i, x', y', t')$ and are probabilistically determined by the vector

$$p(C(i, y', t'+1) | C(i, x', t'))$$

(In directed graphs these cognitive states appear as points (vertices) through which there are going one or more paths.)

As has been presented in our framework, the cognitive trials occur in an actual life space, graph structure and probabilities of it determining stochastically the cognitive process. *But what determines the actual life space ?* – We have solved this problem by assuming a probability vector

$$p(\text{RCCor}(a, M', t', t))$$

to be used in determining the "cognitive map" and the initial state from which the cognitive process starts. This simplification suits well to limited applications as to some simple game situations. However, this can not be the right *start* of conscious activity. To be exact, the probability vector of reality-cognition-correspondence is already a *consequence* of a cognitive selection act. The initiating act includes the selection of the situation on which the attention (the activity of consciousness) will be focused and which is differentiated to the form of a cognitive map (actual life space). This primary selection and focusing process we shall call *commitment*. The consequences of commitment come then out as "following the rules" of the life space. This "following rules", being probabilistically determined by the qualities and laws of the life space, continue as long as the commitment holds. Thus, everything which happens in a cognitive process in a certain actual life space occurs conditionally, the condition being the commitment. Thus, we needed, properly, to write all probabilities we have handled, in the form

$$\text{Pr}[(...) \mid \text{Commitment}_X]$$

indicating that the probability values are valid only as far as a certain commitment X continues.

In our framework, *free will appears in making a commitment, focusing the consciousness as a whole to something*. The commitment is not an outcome of any probability choice. We are not able to make any estimation concerning its form or its occurrence point in time. It is outside the models we have built.

I believe, according to Eccles, that the commitment, the selection of actual life space, is a consequence of an act by Self, of an original, spontaneous mental event. It is something like the process we usually call *intuition*. It is a free choice between two alternatives, to commit or not to commit. I believe, too, that all the time when the commitment is valid – and the Self controls its cognition and behavior according to the model with which it is committed – the Self has the possibility to act consciously for ending the commitment. Our experiences in everyday life support strongly this view.

In certain situations we can experience the commitment very problematic, just because the use of our free-will forces us to be *responsible* for our choice. If the consequences of the available alternatives are difficult to see, the choice by free-will may be experienced so painful that one try to *avoid* it. How to avoid the free-will in the choice situation? (Note: If there exists no free-will, there exists, logically, no problem to avoid it!) – There is a way to make commitment without owing the responsibility for it – a way to commit *as if* no free-will was used: one can *draw a lot* to determine his behavior alternative! What does it actually mean? It means that the actual problem-situation is solved as in trial-and-error behavior: by a probability choice. And that is a choice *without* responsibility. (It is just that kind of choice which is assumed by materialistically oriented theory-makers to guide all human cognition and behavior.) In a clever way the subject has made himself *free from his free-will* letting himself to follow another will, a random event – or mysterious Fate. – Did it succeed? – Naturally the subject has not succeeded to avoid the commitment: he has actually used his free-will to commit himself to draw a lot and follow its result. This is a conscious commitment and the subject is responsible for its consequences, because he has used his free-will to choose this curious way of decision-making.

It is not the free-will which is illusory. In our example, the presumption *not to use the free-will has been illusory*.

It is true that our behavior seems to consist of tremendous amount of routine activities, carried out rather automatically, caused by situations which do not leave much freedom. But it is as well true, that we can – not only in principle but practically – to start or stop those actions any time we *consciously will*. (Some reflexes need to be excluded.)

Lets take one example: you may normally blink your eyes 30 times per minute. Thus, the blinking behavior is determined probabilistically: the probability of one blink in a second may be 0.5. Using Monte Carlo method we can derive an estimate distribution for longer period of time. But if somebody *wants* to blink just at certain moment and as many times he decides, he may – in normal case – do that (or decide not to do). And these acts are *fully unpredictable* – there exists absolutely no model to estimate the time point or the frequency of blinks, not even the probabilities of them. The behavior depends totally on the conscious free commitment by the subject – the commitment to do or not to do, and only he himself knows what and when.

One may criticize this view, particularly from social-psychological point of view, stating that the experience of free will is illusory, because there may exist "unconscious factors", such as unknown social pressure which actually leads to a certain outcome also in those cases we feel to be free to decide. – This view is a misunderstanding. Certainly there are such factors as social pressure, but it comes in the picture afterwards, after the subjects very initial commitment to take some action plan into consideration. One is free to think through or not to think through the plan. Just *after* the *commitment to think* is made, all kind of relevant factors begin to influence the way of thinking, included social pressure.

6.4. Dynamic properties of the world; a comment

Actually, our conceptual framework concerns not only the cognition but, *mutatis mutandis*, it opens the possibility to a general description of occurrences. We can classify systems according to their dynamic properties, *i.e.*, according to the transition probabilities and their conditions.

There seems to be in the world five types of occurrences characterized by different forms of transition probabilities, as follows:

1) *Macro-physical* (or molecular) *systems* are determined by the probability type:

$$(\forall t) \Pr[\text{Be}(m, y, t+1) \mid \text{Be}(m, x, t), \text{Ech}(x,y,t)] = 1 \text{ and} \\ (\forall t) \Pr[\text{Be}(m, y, t+1) \mid \text{Be}(m, x, t), \neg\text{Ech}(x,y,t)] = 0$$

i.e., at every point in time, t , the probability of a macro-physical system, m , of behaving in the way y at the following point in time, $t+1$, with the condition that it behaves in the way x at the time point t and with the condition that there exists a environmental change of x to y , equals 1, but 0 with the condition that there does not exist that environment change.

In other words: There is no transition from state to another state by a macro-physical system possible without an environmental reason. The occurrence is, thus, *deterministic*, *i.e.*, the probabilities can equal the values 1 or 0, only.

Note the way we use the concept of "state". *E.g.*, a falling thing is – as long as it falls – in a "state of free fall", although its speed of falling increases, because this increase of the speed is a staying, invariant characteristics of the state.

2) *Quantum system* is determined by the probability type:

$$(\forall t) 0 < \Pr[\text{Be}(q, y, t+1) \mid \text{Be}(q, x, t)] < 1$$

The "behavior" transition probabilities are given by the intensity function $|\psi(x)|^2$ of the wave function $\psi(x)$. Usually this functions are theoretically handled as continuous, but nothing prevents us to use discrete variables, both for the local positions and for the time, when the

observations are concerned. Actually, this has been always done in practice, because the finite numbers are used in the measurements. (There are two good reasons to criticize the use of infinite mathematics in the context of quantum phenomena: 1) Heisenberg's inaccuracy principle, and 2) the fact that the number of *particles* can never be infinite and, thus, their distribution has to be described using *discrete* variable.)

The state of the quantum system at the time point $t+1$ is determined by a transition probability vector at the preceding time point t . This is in accordance with that, how, *e.g.*, von Weizsäcker describes the function ψ . It is a list of information which is a consequence of *one* event and gives the probabilities for the possible new events. (See v. Weizsäcker, 1985, p. 525.)

Philosophically it is interesting to notice that the actual state of the system is the only condition for the probabilities, *i.e.*, there seems to be no environmental change effecting as a condition. (Note: After the wave-function is reduced, *i.e.*, after a particle is actualized, the system need to be handled in the same way as a molecular one, which means that the states of the particle shall be determined by the probabilities type 1 in our presentation.)

As an example we examine the dissolution process of a radioactive atom. There exists two (relevant) states of the system:

s_1 = no radiation, s_2 = one quantum (an electron) has been radiated, *i.e.*, the dissolution has occurred.

The transition probability vector $|\psi(x)|^2$ from time point t to the time point $t+1$ is:

State	State s_1	State s_2
s_1	p_{11}	p_{12}

If we assume that the mean dissolution time is, *e.g.*, 0.2 sec. and one step in time in our system 0.2 sec., the transition probability vector equals:

The transition probability matrix which describes the system is	s_1	s_2
s_1	0.5	0.5
s_2	0	1

because no transition from state 2 to state 1 is possible.

The *state vector* at the starting point in time, t , is:

s_1	1
s_2	0

When we multiply this state vector by the transpose of the transition probability matrix, we get the new state vector for the succeeding time point $t+1$:

s_1	.5
s_2	.5

By multiplying this state vector by the transpose of the transition probability matrix, we get the state vector for the time point $t+2$:

s_1	.25
s_2	.75

etc.

The step in time needs not to 0.2 sec. We can choose it freely. If we assume it to be, *e.g.*, $1/69 \cdot 0.2$ sec., then the transition probability matrix were:

	s_1	s_2
s_1	.99	.01
s_2	0	1

In this case, the state vector at time point 69 is 0.5 0.5, and at time point 138 it will be .25 .75. (The step in time get not be, however, smaller than Planck's time.)

3) *Bio-systems*, living organisms, seem to behave according to the probabilities

$$0 < \Pr[\text{Be}(b,y,t+1) \mid \text{Be}(b,x,t), \text{Ech}, \text{Reinf}] < 1$$

which means that they have a common property with the quantum systems, namely they are probabilistically determined, behavior probability being > 0 and < 1 . But they differ from quantum behavior in that they are in interaction with the environment. The other difference is that the probabilities determining the behavior of living organisms follow the learning laws – the reinforcement (Reinf) appears as one condition for those probabilities.

4) *Cognitive systems* are described by the transition probabilities:

$$0 < \Pr[\text{Ctr}(i, y', t+1') \mid \text{C}(i,x',t'), \text{Comm}, \text{Reinf}^*] < 1$$

Note: There can not be any Ech-condition, and the particular cognitive time, t' , is used. This means that the cognitive system is *not in direct interaction* with its environment (but indirectly through the bio-system).

The cognitive systems are assumed to follow their own learning laws (Reinf^{*}).

The cognitive states are not observable, they exist for the "inner observation", for the consciousness by the individual i , only. For him they are separable according to their meanings, intentional characteristics.

One important condition for the cognitive transition probabilities is the *commitment* (Comm), as mentioned above in Chapter 6.3. (If there is no actual commitment by the individual, there exists no cognition and we have to examine the behavior of the individual as the behavior of a bio-system.)

5) *Self*: We do not define any probability for the *commitment*-event, because we see it *acausal*, spontaneous mental phenomenon, a consequence of the free will. This event we can formulate:

$$\text{Comm}(S, \text{CM}', t', t)$$

where $S = \text{self}$, $\text{CM}' = \text{cognitive map}$ (including the starting state x'), $t' = \text{the cognitive time point of the beginning of the cognitive process}$, and $t = \text{time point of the commitment in the real time}$.

The commitment-event, the choice of a cognitive map, is not determined by any probability. If we have to say something of the cause of the commitment, we call it *transcendental* – according to Eccles' view. Of course, this sounds totally unscientific, because our logic in science does not approve acausal phenomenon. The other alternative, the view-point of emergentistic materialism, is, however, not more satisfactory, because it is impossible to approve that a state with meaning, intentional characteristics, could "emerge" from the materia.

Because all the probabilities determining the events are in our framework conditional, interesting *ontological* problems arise: How those conditions are fulfilled?

In the case of a macro-physical system, there has to exist an environment, consisted of other macro-physical systems. But there can not exist any such environment for the *first* macro-physical system. We have to think that the beginning of macro-physical systems is the *actualization* of particles, the reduction of wave-packets.

But the problem of the origin of quantum systems is an open one. What causes that there has to exist those probability waves according to which particles are actualized? As far as I can see, this is an open question in modern physics and in cosmology: the "cause" of Big Bang.

We can call Big Bang acausal or assume that the cause is transcendental. Thus, both the physics and the science of mind (psychology) seem to be in the same epistemological position.

6.5. Conclusion

Because the aim of this book was to show how the deep wishes by Kurt Lewin could be fulfilled and, particularly, how the framework concerning the "psychological" forces could be created in an exact form, the philosophical considerations have been unessential. However, as far as I can see, the philosophical discussion has shown the complex problematics of the consciousness and the relatively limited possibilities we have in creating scientific theories about human cognition and behavior. Paradoxically, if we had not tried so seriously to build an exactly formulated framework and to use it in simulations of very simplified applications, we had not seen so clearly the limits we will meet soon, when the problem field becomes more complicated.

I hope that the *holistic view* that has been the very basis of this analysis, could guide the researchers of human mind and give them understanding to avoid reductionistic errors; in our days, such a danger is increasing very fast following the quick - but in the same time somewhat blind - development of technology.

Bibliography

- Abelson, R. P. (1968): Simulation of Social Behavior. In G. Lindzey and E. Aronson (eds.): *The Handbook of Social Psychology*. Second edition. Reading, Mass.: Addison-Wesley.
- Beck, F. and Eccles, J. C. (1992): Quantum aspects of brain activity and the role of consciousness. *Proc. Nat. Acad. Sci.* **89**, 11357.
- Bush, R. R. and Mosteller, F. (1955): *Stochastic Models for Learning*. New York: Wiley.
- Bush, R. R. and Mosteller, F. (1959): A Comparison of Eight Models. In R. R. Bush and W. K. Estes (eds.): *Studies in Mathematical Learning Theory*, Stanford Mathematical Studies in the Social Science, III. Stanford: Stanford Univ. Press.
- Dennett, D. C. (1991): *Consciousness Explained*. Brockman Inc.
- Eccles, John C. (1994): *How the Self Controls Its Brain*. Heidelberg: Springer Verlag.
- Heider, F. (1958): *The Psychology of Interpersonal Relations*. New York: Wiley.
- Hilgard, E. R. and Bower, G. H. (1966): *Theories of Learning*. 3. ed. New York: Appleton-Century-Crofts.
- Kemeny, J. G., Snell, J. L., and Thompson, G. L. (1957): *Introduction to Finite Mathematics*. Englewood Cliffs, N. J., USA: Prentice-Hall, Inc.
- Lewin, K. (1938): The Conceptual Representation and the Measurement of Psychological Forces. *Contributions to Psychological Theory*, Vol. I, No. 4. Durham, N.C., USA: Duke Univ. Press
- Penrose, R. (1989): *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*. Oxford: Oxford University Press.
- Rainio, K. (1972): Group Maze: Experiments and Simulations in Problem-Solving by Groups. *Commentationes Scientiarum Socialium*, 3. Helsinki: Societas Scientiarum Fennica.
- Rainio, K. (1983): Corrected Lewinian System in Terms of Probabilistic Theory. In C. Micko and U. Schulz (eds.): *Formalization of Psychological Theories*. Proceedings of the 13th European Mathematical Psychology Group Meeting Bielefeld. September 1982. Materialien XXXX. Univ. Bielefeld, Schwerpunkt Mathematisierungf.
- Rainio, K. (1986): Stochastic Field Theory of Behavior. *Commentationes Scientiarum Socialium*, 34. Helsinki: Societas Scientiarum Fennica.
- Riita, T. (1966): *Johtajakäyttäytymisestä ryhmän probleemanratkaisutilanteessa*. (Unpublished manuscript, Tampere University, Finland.)
- Saariluoma, P. (1999): Human chess players' problem solving. In *Pelit, tietokone ja ihminen; Games, Computers and People*. Suomen Tekoälyseuran julkaisuja - symposiosarja - No 15, pp. 91-99. Helsinki.
- Suppes, P. (1967): *Introduction to Logic*. 10th printing. Princeton, New Jersey: D. van Nostrand Company, Inc.
- Vuokko, O. (1964): *Johtajuuden eriytyneisyyden vaikutus kommunikaatioon ja tuloksiin pienryhmän probleemanratkaisutilanteissa*. (Unpublished manuscript, Tampere University, Finland.)
- von Weizsäcker, C. F. (1985): *Aufbau der Physik*. Varl Hansen Verlag.

APPENDICES

Appendix 1. The Monte Carlo method

The Monte Carlo method is used for drawing lots to determine which alternative (or event) will be actually chosen when the probabilities of the alternatives (or events) are given.

If an event E occurs by the probability p, we take a random number r so that $0 < r \leq 1$ (or we generate it by a special computing method). We compare r with p. If $r \leq p$, then the *outcome* is E, *i.e.*, the event E occurs, but if $r > p$, then the outcome is non-E, *i.e.*, the event E does not occur.

E.g., suppose that the probability $p = .37$. If the random number r happens to be .35, the outcome is E, the event occurs. If the random number is, say, .58, the outcome is non-E, the event does not occur.

If there are more alternatives (or events), we need to compute the *cumulative sums* of the probabilities. *E.g.*, suppose there are 5 events, A, B, C, D, and E with the following occurrence probabilities:

A	B	C	D	E	Σ
.17	.12	.23	.29	.19	1

The vector of the cumulative sums is now the vector:

.17	.29	.52	.81	1.0
-----	-----	-----	-----	-----

We take a random number r, $0 < r \leq 1$, and compare it to the elements of the cumulative sum vector, one after one, beginning from the smallest element. First case, where r is smaller or equal to the element, the outcome is that element, *e.g.*, if $r = .54$, the outcome is D; if $r = .28$, the outcome is B, *etc.*

Appendix 2. The assumptions concerning the success of real trials when the environmental change is taken into account

9. $\text{Succ}(\text{Tr}(i,a,b,t)) \wedge (\forall x)(\neg\text{ECh}(b,x,t)) \rightarrow \text{Be}(i,b,t+1)$
10. $\neg\text{Succ}(\text{Tr}(i,a,b,t)) \wedge (\forall x)(\neg\text{ECh}(a,x,t)) \rightarrow \text{Be}(i,a,t+1)$
11. $\text{Succ}(\text{Tr}(i,a,b,t)) \wedge (\exists c)(\text{ECh}(b,c,t)) \rightarrow \text{Be}(i,c,t+1)$
12. $\neg\text{Succ}(\text{Tr}(i,a,b,t)) \wedge (\exists c)(\text{ECh}(a,c,t)) \rightarrow \text{Be}(i,c,t+1)$

Appendix 3. Learning operators

A) Learning operator in case of reward, general form:

The symbol k indicates the chosen alternative. α = the learning coefficient. $0 \leq \alpha \leq 1$.

	Q_{rew}						
	1	2	3	...	k	...	n
1)	$1-\alpha$	0	0	...	0	...	0
2)	0	$1-\alpha$	0	...	0	...	0
3)	0	0	$1-\alpha$...	0	...	0
...	...						
k)	α	α	α	...	1	...	α
...	...						
n)	0	0	0	...	0	...	$1-\alpha$

B) Learning operator in case of punishment, assumption I, general form:

The symbol k indicates the chosen alternative. β = the learning coefficient. $0 \leq \beta \leq 1$.

	Q_{pun}							p
	1	2	3	...	k	...	n	
1)	$1+q$	0	0	...	0	...	0	p_1
2)	0	$1+q$	0	...	0	...	0	p_2
3)	0	0	$1+q$...	0	...	0	p_3
...
k)	$-q$	$-q$	$-q$...	1	...	$-q$	p_k
...
n)	0	0	0	...	0	...	0	p_n

In the matrix $q = \beta p_k / (1 - p_k)$.

C) Learning operator in case of punishment, assumption II, general form:

The symbol k indicates the chosen alternative. β = the learning coefficient. $0 \leq \beta \leq 1$.

	Q_{pun}						
	1	2	3	...	k	...	n
1)	1	0	0	...	$\beta(n-1)$...	0
2)	0	1	0	...	$\beta(n-1)$...	0
3)	0	0	1	...	$\beta(n-1)$...	0
...	...						
k)	0	0	0	...	$1-\beta$...	0
...	...						
n)	0	0	0	...	$\beta(n-1)$...	1

D) Multiplying a vector by a matrix. A numeric example.

Multiplication $\mathbf{Q} \times \mathbf{p}$, the result vector being \mathbf{r} .

	\mathbf{Q}				\mathbf{p}	\mathbf{r}
	1	2	3	4		
1)	.1	.2	.3	.4	.1	.29
2)	.5	.4	.3	.2	.3	.31
3)	0	.1	.2	.3	.2	.19
4)	.4	.3	.2	.1	.4	.21

The calculations: $r_1 = .1 \cdot .1 + .2 \cdot .3 + .3 \cdot .2 + .4 \cdot .4 = .01 + .06 + .06 + .16 = .29$

$r_2 = .5 \cdot .1 + .4 \cdot .3 + .3 \cdot .2 + .2 \cdot .4 = .05 + .12 + .06 + .08 = .31$

$r_3 = 0 \cdot .1 + .1 \cdot .3 + .2 \cdot .2 + .3 \cdot .4 = 0 + .03 + .04 + .12 = .19$

$r_4 = .4 \cdot .1 + .3 \cdot .3 + .2 \cdot .2 + .1 \cdot .4 = .04 + .09 + .04 + .04 = .21$

Appendix 4. Flow List of Turbo-Basic program for Group Maze simulation

1. Start the PROCESS (= series of 6 games). Read the data and set the beginning values of variables.
2. Compute the matrix of the probabilities of cognitive trials (CognTrProb) as a weighted sum of the (homogeneous) TrErProb-matrix – with the elements 1/8 – and the field probability matrix (CombFProb), constituted by 8 row vectors of values 0, .01, .01, .07, .01, .07, .07, .77.
3. Start the GAME. Set RealState = 0.
4. Start REAL TRIAL. Set SuggMade%(), Lets%(), and Nlets equal to = 0 and ShownSubm\$(), ShownDS\$(), ShownCS\$(), Mess\$, and Subm\$ as ”no”.
5. Start COGNITIVE TRIAL. Determine the trial-maker *i* by a Monte Carlo choice using the vector Probi() and set the actual cognitive trial probabilities ActTrProb() = CognTrProb().
 - 5.1. Compute the new Probi() vector.
 - 5.2. Set NofCognFails (number of failed cognitive trials by *i*) equal to 0.
6. Carry out the process leading to a COMMUNICATION DECISION:
 - 6.1. Compute how many subjects have already sent their message on this real trial (NofCho).
 - 6.2. Fail test: If NofCognFails \geq NofCognTr, *i.e.*, there are failed cognitive trials more than allowed, then go to ... (OutofDecision).
 - 6.3. TESTSUGG: Compute the number of suggestions sent to *i* (NofSugg).
 - 6.4. Test NofSugg, NofFails, and NofCho.
 - 6.4.1. If NofSugg = 0, NofFails = 0 and NofCho = NofI, *i.e.*, 3, then carry out the subroutine DetAlt, *i.e.*, set ChosenAlt to correspond to the choices in the messages of all 3 subjects. (ChosenAlt = the state toward which the cognitive trial by *i* is directed. It is numbered from 0 to 7.) – Go to 6.6 (DETSUCC).
 - 6.4.2. If NofSugg > 0, *i.e.*, there are one or two suggestions sent to *i*, carry out subroutine SubmTest, *i.e.*, determine whether *i* will choose submissiveness or not: The submissiveness is determined by Monte Carlo method using the probability vector SubmProb(). – However, if there are two uncongruous suggestions, the nonsubmissiveness is chosen.
 - 6.4.3. If the outcome in 6.4.2 was submissiveness, then set the ChosenAlt the same as the alternative in the suggestion, set the message (the message sender + the content of the message, *i.e.*, the alternative chosen by *i*), and go to ... (show the message).
 - 6.4.4. If the outcome is nonsubmissiveness, eliminate the list ovat suggestions sent and determine by Monte Carlo method the direction of cognitive trial (ChosenAlt) using the matrix ActTrProb().
 - 6.5. Show the cognitive trial by *i*, *i.e.*, ChosenAlt.
 - 6.6. DETSUCC: Determine the success of the cognitive trial (ChosenAlt) by Monte Carlo method using the probability matrix CognSuccProb().
 - 6.7. Test the success of the cognitive trial:
 - 6.7.1. If the cognitive trial fails, then add 1 to NofCognFails and go to 6.2.
 - 6.8. Set CognRS equal to ChosenAlt in binary form and determine according to it the images by *i* of the choices by all subjects (CognPos(), the upper or lower rows, 1 or 0).
 - 6.9. Set Choice = CognPos(*i*). (Thus, ”Choice ” is *i*’s own choice.)
 - 6.10. Compare the CognPos() values with the ChoiceMade values. (The last ones are the choices informed by the subjects in their last messages.)
 - 6.10.1. If somebody has not yet sent his message (and, thus, *i* do not know his choice), carry out subroutine DomSuggSending: Determine by Monte Carlo method using

the probability vector DomSuggProb(i,j) whether *i* sends a suggestion to the other (*j*) or not. If *i* sends the suggestion, the content of it is CognPos(j).

6.10.2. If some other subject, *j*, has sent already his message, but his choice differs from the image by *i*, *i.e.*, is different of CognPos(j), then carry out subroutine ChangeSuggSending: Determine by Monte Carlo method using the probability vector ChangeSuggProb(i,j) whether *i* sends to *j* the suggestion to change his choice.

6.10.3. If there are now suggestions by *i*, write the message by *i* combining his own choice and his suggestions and go to (MessPrint).

6.11. Test the Choice by *i*.

6.11.1. If Choice is the same as the alternative which *i* have reported in his earlier message (and he has no suggestions to send), then test whether everybody has a message or not.

sent

6.11.1.1. If NofCho < Nofl, *i.e.*, 3, set Dec\$ = "no", which indicates that *i* will not send a message now.

6.11.1.2. If NofCho = Nofl, *i.e.*, 3, then *i*'s message will be "Lets", let's try.

6.11.2. If Choice is not the same as the alternative which *i* have reported in his earlier message, then set Dec\$ = "yes", which indicates that *i* decides to send a message, the content of it being Choice.

6.12. MessPrint:

If Dec\$ = "yes" carry out the following procedure:

6.12.1. Set ChoiceMade(i) = CognPos(i), *i.e.*, save the cognitive choice made by *i*.

6.12.2. If the message by *i* is something else than "Lets", eliminate all earlier "Lets" - messages, *i.e.*, set the elements in vector Lets%() to equal to 0.

6.12.3. Add the name of the sender to the message.

6.12.4. Show the message.

If Dec\$="no", then show the information "nM" and *i*, which means that *i* do not send any message, but the the others are now in turn.

6.13. OUT OF DECISION

5. Set realtrial = ChosenAlt.

6. Determine whether the real trial succeeds or not – using the task matrix.

7. Carry out the subroutine reinf – the learning reinforcement:

7.1. If the real trial succeeded, increase, but if it failed, decrease the following probabilities:

CognSuccProb() – also the learning transfer – TrErProb(), SubmProb(), DomProb(), and ChangeProb().

7.2. If the real trial succeeded, increase, but if it failed, decrease the field effect (FldEff), which determines the weights for summing the TrErProb() and CombFProb() matrices to get the CognTrProb() matrix.

8. Decrease the coefficient betaFldEff according to the equation $\beta_{t+1} = \beta_t - k \cdot \beta_t$, where $0 < k < 1$.

9. Test the real trial:

9.1. If the real trial was 7 (*i.e.*, all pieces on the upper row) and succeeded, go to 10 (GAME OVER)

9.2. Compute CognTrProb() matrix and go to 4 (start the new real trial)

10. GAME OVER: Test the number of games carried out. If it is < maxgames then go to 3.

11. End of the PROCESS: If the number of processes is < maxproc then go to 1.

12. End of the SIMULATION: Show and save the results.

(The program is available on diskette as a turbo-basic program or in an .exe program form, for demonstration or for quick computing.

Please ask: kullervo.rainio@pp.inet.fi)

Appendix 5

Group Maze -simulation; basic program

```
print"Remember that you can break the run by pressing e"
print:print:print"*****"
print"Group Maze Simulation Program"
print"29.9.1997 - 2.4.2000 Kullervo Rainio, emer. prof."
print"Helsinki University"
print"GMS-STD.BAS, with files for data and main results"
print"with standard deviations of failures"
print"*****"
print
Altmax=7:nFmax=3:NofAlt=8:Nofl=3
maxgame=6:maxtr=20:maxpaths=15
input"How many processes (10) (max. 200)";maxproc
print"Remember that you can break the run by pressing e"
input"Pause after each game, seconds (10)";seconds
FldEffbeg=1
alfaFldEff=.1
betaFldEffbeg=.95
betbetFEff=.1
rem The rate of change of the betaFldEff
input"Do you want to read the data from file (y/n)";datfromfil$
if datfromfil$="y" then goto Overtransf
PTransf=.15
NTransf=.3
wei2=FldEffbeg:wei1=1-wei2
Overtransf:
input"Do you want to save the data to the file (y/n)";dattofil$

rem 8 states, 4 fields, wei1 for TrErProb, wei2 for CombFProb

data 1,1,1,1,1,1,1,0
data 0,1,0,1,0,1,1,1
data 0,0,1,1,0,1,1,1
data 0,0,0,1,0,0,0,1
data 0,0,0,1,1,1,1,1
data 0,0,0,0,0,1,0,1
data 0,0,0,0,0,0,1,1
data 0,0,0,0,0,0,0,1
rem Task-data above

data 0,1,1,2,1,2,2,3:rem Field() for every state
data 0,.07,.23,.70:rem potencies of fields
data 8,7,4,1: rem nStateInF(), number of states in a field

data 0,.8,.8,.8,0,.8,.8,.8,0:rem SubmProb
```

```

data .25,.25,.25:rem DomProb
data .2,.2,.2:rem ChangeProb
data .2,.2,.2:rem aTrErr learning coeffic.
data .8,.8,.8:rem bCognS coeffic.
data .1,.1,.1:rem aSubm coeffic.
data .05,.05,.05:rem aDom coeffic.
data .05,.05,.05:rem aChange coeff.

```

```

dim Task%(Altmax,Altmax)
dim Fld(Altmax),Pot(nFmax),TrErProb(NofI,Altmax,Altmax)
dim CognTrProb(NofI,Altmax,Altmax)
dim nStateInF(nFmax),CombFProb(Altmax,Altmax)
dim Corr%(maxproc,maxgame),Fail%(maxproc,maxgame)
dim stdevfail(maxproc):rem standard deviation of failures
dim SeqTr%(maxproc,maxgame,maxtr),SumSeq%(maxgame,maxtr,Altmax)
dim pr(Altmax),trial%(maxproc,maxgame,maxtr)
dim path%(maxproc,maxgame,maxtr),FreqPath%(maxgame,maxpaths)
dim Pathform%(maxproc,maxgame),Transpath%(maxpaths,maxpaths)
dim Ntrials%(maxproc,maxgame),SumFlr%(maxgame)
dim NMess%(maxgame),NSubm%(maxgame),NDom%(maxgame)
dim NChange%(maxgame),Ntrgame%(maxgame)

```

```

NofPos=2:NofCognTr=10:AcThr=1
Positmax=NofPos-1
BetaI=.95:rem coeffic. used with Probi()

```

```

randomize timer

```

```

dim ChoiceMade%(NofI),SuggMade%(NofI,NofI),aTrErr(NofI)
dim CognSuccProb(NofI,Altmax,Altmax),bCognS(NofI)
dim SubmProb(NofI,NofI),Equal$(NofI),aSubm(NofI)
dim DomProb(NofI),ChangeProb(NofI)
dim aDom(NofI),aChange(NofI)
dim Dom$(NofI,NofI),Change$(NofI,NofI)
dim Ch$(NofI,NofPos),Sugg$(NofI,NofI),Lets%(NofI)
dim Aux%(20),CognPos%(NofI)
dim Probi(NofI),SenderName$(NofI)
dim ShownSubm$(NofI,NofI),ShownDSS$(NofI,NofI)
dim ShownCSS$(NofI,NofI),ActTrProb(Altmax,Altmax)

```

```

Ch$(1,0)="1":Ch$(1,1)="X":Ch$(2,0)="s":Ch$(2,1)="Y"
Ch$(3,0)="r":Ch$(3,1)="Z"
for k=1 to NofI
if k=1 then SenderName$(k)=" A:"
if k=2 then SenderName$(k)=" B:"
if k=3 then SenderName$(k)=" C:"
next k

```

```

rem GROUP LEARNING PROCESS STARTS

```

```

for proc=1 to maxproc
print:print"*** PROCESS ";proc;" STARTS ***"
if proc>1 then restore

rem Reading Task%(), i.e., the success of real trial prob.
for k=0 to Altmax:for m=0 to Altmax
    read Task%(k,m)
next m,k

for h=1 to Nofl
for k=0 to Altmax:for m=0 to Altmax
    CognSuccProb(h,k,m)=1
    if m=k then CognSuccProb(h,k,m)=0
next m,k,h

for k=0 to Altmax:read Fld(k):next k
for k=0 to nFmax:read Pot(k):next k
rem print"nStateInF"
for k=0 to nFmax:read nStateInF(k)
rem    print nStateInF(k);
next k:rem print

for k=1 to Nofl
    for m=1 to Nofl:read SubmProb(k,m):next m,k
for k=1 to Nofl:read DomProb(k):next k
for k=1 to Nofl:read ChangeProb(k):next k
for k=1 to Nofl:read aTrErr(k):next k
for k=1 to Nofl:read bCognS(k):next k
for k=1 to Nofl:read aSubm(k):next k
for k=1 to Nofl:read aDom(k):next k
for k=1 to Nofl:read aChange(k):next k

if datfromfil$="y" then gosub datin
if dattofil$="y" and proc=1 then gosub datsav

FldEff=FldEffbeg:wei2=FldEff:wei1=1-wei2
betaFldEff=betaFldEffbeg

for k=0 to Altmax
    for m=0 to Altmax
        prob=0
            for n=1 to nFmax
                rem 0-field not needed to be calculated
                if Fld(m)<n then FProb=0 else FProb=1
                rem pF,F and pnonF,F are 1,pnonF,nonF and pF,nonF are 0
                rem and if Fld(m)=n then m is inside field n
                p=FProb/nStateInF(n)
                p=p*Pot(n)
                prob=prob+p
            next n
        next m
    next k

```

```

        next n
        CombFProb(k,m)=prob
    next m
rem  aux(0)=CombFProb(k,0):rem testing the sum the vector starts
rem  for kk=1 to Altmax:aux(kk)=aux(kk-1)+CombFProb(k,kk):next kk
rem  for mm=0 to Altmax:rem print using" .###";CombFProb(k,mm);:rem next mm
rem  print using" #.###";aux(Altmax)
rem 4 rows above are for testing
next k

rem print"Combined field-prob. calculated"

for h=1 to Nofl:for k=0 to Altmax:for m=0 to Altmax
    TrErProb(h,k,m)=1/NofAlt
next m,k,h

for game=1 to maxgame
rem GAME
RealState=0
print"RealState=";RealState

for tr=1 to maxtr
rem REALTRIAL

for h=1 to Nofl:for k=0 to Altmax:for m=0 to Altmax
    CognTrProb(h,k,m)=wei1*TrErProb(h,k,m)+wei2*CombFProb(k,m)
rem    print using" .###";CognTrProb(h,k,m);
next m:rem print
next k:rem print
next h
rem print"CognTrProbs calculated"

CognSit=RealState
for k=1 to Nofl:for m=1 to Nofl
    SuggMade%(k,m)=99
    Sugg$(k,m)="n"
    ShownSubm$(k,m)="*"
    ShownDS$(k,m)="*"
    ShownCSS$(k,m)="*"
    next m
    ChoiceMade%(k)=99
    Lets%(k)=0
next k
Nlets=0:Mess$="":Subm$=""

COGNTRIAL:

for k=1 to Nofl:ProbI(k)=1/Nofl:next k

NEWI:

```

```
rem Determining new i and the cognitive trials by i follow
SubmChoice%=99:Choice%=99:Choice$="":Mess$="":Dec$=""
```

```
NofCho=0
for k=1 to NofI
  if ChoiceMade%(k)<99 then NofCho=NofCho+1
next k
rem print"NofCho=";NofCho:rem Testing
NofCognFails=0
```

```
if NofCho=0 then
  for k=1 to NofI:ProbI(k)=1/NofI:next k
  else
  b=BetaI*ProbI(i):c=b/(NofI-1)
  for k=1 to NofI
    if k=i then
      ProbI(k)=ProbI(k)-b
    else
      ProbI(k)=ProbI(k)+c
    end if
  next k
end if
```

```
rem Testing
rem print" ProbI's"
rem for k=1 to NofI:rem print using ".##";ProbI(k);
rem next k:rem print
```

```
call MonteCVec1(ProbI(),NofI,choice)
rem print"new I=";choice:rem Testing
i=choice
```

```
rem print"ActTrProbs"
for k=0 to Altmax:for m=0 to Altmax
  ActTrProb(k,m)=CognTrProb(i,k,m)
rem   print using ".###";ActTrProb(k,m);
next m:rem print
next k
```

```
rem input x:rem Testing
```

```
rem print"Suggestions sent to i:":rem Testing
rem for k=1 to NofI
rem   print" From";k;" to";i;" ";Sugg$(k,i);
rem next k
```

```
DECISION:
FailTest:
e$=inkey$:if e$="e" then goto OUTPROGR
rem print"NofCognFails";NofCognFails
```

```

SuggMess$=""
if NofCognFails<NofCognTr then
    goto TESTSUGG
else
    Dec$="n":goto OutofDecision
end if

TESTSUGG:
NofSugg=0
for k=1 to NofI
    if sugg$(k,i)="y" then NofSugg=NofSugg+1
next k
rem print"Test NofSugg";NofSugg:rem Testing

if NofSugg=0 and NofCognFails=0 and NofCho=NofI then
    rem All intended moves are known, thus i tests
    rem directly the success of the intended trial
rem    print"DetAlt":rem Testing
    gosub DetAlt:goto DETSUCC
end if

if NofSugg>0 then gosub SubmTest else Subm$="n"
if Subm$="y" then
    Choice%=SubmChoice%:Dec$="y"
    Choice$=Ch$(i,Choice%):Mess$=Choice$
rem    print"Submissive"
        incr NSubm%(game)

    ChoiceMade%(i)=Choice%
end if
if Subm$="y" then
    for k=1 to NofI:sugg$(k,i)="n":Lets%(k)=0:next k
    goto MessPrint
end if

call MonteCMX(ActTrProb(),Altmax,Altmax,CognSit,ChosenAlt)
DETSUCC:
rem    print ChosenAlt;"*";

rem input x

Prob=CognSuccProb(i,CognSit,ChosenAlt)
call MonteCp(Prob,answ$)
succ$=answ$
rem print"Test succ$";succ$
if succ$="n" then NofCognFails=NofCognFails+1:goto FailTest
NofCognSucc=NofCognSucc+1
if NofCognSucc<AcThr then goto FailTest
NofCognSucc=0
CognRS=ChosenAlt

```

```

rem print"Test CognRS";CognRS

rem input x

call DecimBinary(CognRS,bin)
BinRS=bin
rem print"BinRS";BinRS
call CognitPosit(CognPos%(),Nofl,BinRS)
rem for k=1 to Nofl
rem print"Test CognitPos";CognPos%(k);rem next k:rem print
Choice%=CognPos%(i)

Comparing:
rem print"Comparing"
for k=1 to Nofl
  if k=i then goto NextEq
  if CognPos%(k)=ChoiceMade%(k) then Equal$(k)="y" else Equal$(k)="n"
  NextEq:
next k
rem input x:rem Testing

Sugges$="":AllEquals$="y":SuggMess$=""
for k=1 to Nofl
  if k=i or Equal$(k)="y" then goto NextSugg
  c%=ChoiceMade%(k)
  if c%=99 then
    gosub DomSuggSending:goto NextSugg
  else
    gosub ChangeSuggSending
  end if
  AllEquals$="n"
rem print"AllEq";AllEquals$
  NextSugg:
next k
rem input x

if Sugges$="y" then Dec$="y":Mess$=Ch$(i,Choice%)+ " "+SuggMess$:goto MessPrint
if Choice%=ChoiceMade%(i) then
  if NofCho<Nofl then
    Dec$="n"
  else
    if AllEquals$="y" then
      Dec$="y":Lets%(i)=1:Mess$="Lets"
    end if
  end if
else
  Dec$="y"
rem print"Choice=";Choice%:rem print"Cogn.Pos.=";CognPos%(i)
rem input x

```

```

    ChoiceMade%(i)=CognPos%(i):Mess$=Ch$(i,Choice%)
end if

```

```

MessPrint:

```

```

    if Dec$="y" then
        ChoiceMade%(i)=CognPos%(i)
        if Mess$="Lets" then
            goto OVERMESS
        else
            incr NMess%(game)
            for k=1 to Nofl:Lets%(k)=0:next
            rem One Lets message cancels them all
            end if
            OVERMESS:
            Mess$=SenderName$(i)+Mess$
rem    print Mess$;" ";
            for k=1 to Nofl:Sugg$(k,i)="n":next k
            rem Earlier suggestions to i not valid anymore
            else
rem    print"nM";i;"/";
            end if

```

```

OutofDecision:

```

```

rem input x

```

```

rem gosub Testmx
rem input"Continue y/n";a$
rem if a$="n" then goto OUTPROGR

```

```

NLets=0:for k=1 to Nofl
    if Lets%(k)>0 then NLets=NLets+1
next k

```

```

rem print" Nlets=";NLets;
if NLets=Nofl then goto REAL

```

```

goto NEWI

```

```

REAL:

```

```

print"State";RealState;
gosub DetAlt
realtr=ChosenAlt:rem real trial in the decimal form
print" realtr=";realtr;
SeqTr%(proc,game,tr)=realtr
p=Task%(RealState,realtr)
rem print" Task p=";p;
call MonteCp(p,answ$)
if answ$="y" then succtr$="y" else succtr$="n"

```

```

print" succtr ";succtr$:rem Testing
rem gosub Testmx
rem input x
SeqTr%(proc,game,tr)=realtr
if succtr$="y" then
    pnum=pnum+1
    path%(proc,game,pnum)=realtr
end if
gosub reinf
rem input x

if succtr$="y" then gosub rewardtr else gosub punishtr

p=betaFldEff:betaFldEff=p-betbetFEff*p:rem decreasing betaFldEff

rem input"Test prob. matrices y/n";a$
rem if a$="y" then gosub Testprobs

if succtr$="y" and realtr=Altmax then Ntrials%(proc,game)=tr:tr=maxtr
print"tr number= ";tr;
print" game number= ";game
rem for k=1 to seconds*200:a=0:next k:rem pause

next tr

rem REAL TRIAL FINISHED

GAMEOVER:
pnum=0
print:print"GAME= ";game;" OVER":print
print"Trials:";
for k=1 to maxtr:print SeqTr%(proc,game,k);:next k:print
rem for k=1 to seconds*2000:a=0:rem next k:rem pause
delay seconds
rem input"Continue y/n";a$
rem if a$="n" then goto OUTPROGR

next game:rem Game finished

print"Failures/game:"
for k=1 to maxgame:print Fail%(proc,k);:next k:print
print"Trial sequences:"
for k=1 to maxgame:for m=1 to maxtr
    print SeqTr%(proc,k,m);
    next m:print
next k
rem input x
next proc

```

```

print:print" ***RESULTS***"

input"Do you want the results to be written to a file (y/n)";rf$
if rf$="y" then
    print"Write the name of the file (e.g. gmsxxxxx.res"
    print"or d:gm\gmsxxxxx.res)"
    if datfromfil$="y" then
        print"The name of the file from which the data were"
        print"read was ";fildatnam$
    end if
    if dattofil$="y" then
        print"The name of the file into which the data were"
        print"written was ";filwrinam$
    end if
FILNAMRES:
input"Write the name of the file";filres$
input"Is the name right (y/n)";answ$
    if answ$="n" then
        print"Again!":goto FILNAMRES
    end if
    open filres$ for output as #3
    write #3,maxproc
end if

print"Failures"
for k=1 to maxproc:for m=1 to maxgame
    print Fail%(k,m);
    SumFlr%(m)=SumFlr%(m)+Fail%(k,m)
next m:print

next k
print"Means:";
for k=1 to maxgame
    s=SumFlr%(k)/maxproc
    print using" ##.###";s;
    if rf$="y" then write #3,s
    sumvar=0
    for m=1 to maxproc
        var=(s-Fail%(m,k))^2
        sumvar=sumvar+var
    next m
    stdevfail(k)=sqr(sumvar/maxproc)
rem stdevfail=standard deviation of failures
next k:print
print"Standard deviations:"
for k=1 to maxgame:print using" ##.###";stdevfail(k);:next k
print:print"To continue press Enter":input x

print"Paths:"
for k=1 to maxproc:for m=1 to maxgame:for mm=1 to 3
    print path%(k,m,mm);

```

```

        if rf$="y" then write #3,path%(k,n,mm)
        next mm:print" / ";
    next m:print
next k
print"Do you want to see the probab. matrices"
input"(concerning the last process only) (y/n)";a$
if a$="y" then gosub Testprobs

gosub Trialsums

gosub Pathtrans

print"Messages/game:"
for k=1 to maxgame:print NMess%(k);:next k:print
print"Messages/game, means:"
for k=1 to maxgame
    print using" ###.#";NMess%(k)/maxproc;
    if rf$="y" then write #3,NMess%(k)/maxproc
next k:print

print"Submissive choices/game, means:"
for k=1 to maxgame:print using" ##.###";NSubm%(k)/maxproc;
    if rf$="y" then write #3,NSubm%(k)/maxproc
next k:print

print"Dominance suggestions/trial, means in each game:"
for k=1 to maxproc
    for m=1 to maxgame
        Ntrgame%(m)=Ntrgame%(m)+Ntrials%(k,m)
    next m,k
for k=1 to maxgame
    print using" ##.###";NDom%(k)/Ntrgame%(k);
    if rf$="y" then write #3,NDom%(k)/Ntrgame%(k)
next k:print

print"Change suggestions/game, means:"
for k=1 to maxgame:print using" ##.###";NChange%(k)/maxproc;
    if rf$="y" then write #3,NChange%(k)/maxproc
next k:print

print"Change suggestions/message, means in each game:"
for k=1 to maxgame
    print using" #.####";NChange%(k)/NMess%(k);
    if rf$="y" then write #3,NChange%(k)/NMess%(k)
next k:print
input"To continue, press Enter";x

OUTPROGR:
if rf$="y" then close 3

```

```

print"END":end

reinf:rem reinforcement starts
      if succtr$="y" then lamb=1 else lamb=0
rem print"lamb=";lamb
rem print"Test, CognSuccProb, old"
      for h=1 to Nofl
        p=CognSuccProb(h,RealState,realtr)
rem print using" .###";p;
        CognSuccProb(h,RealState,realtr)=p+bCognS(h)*(lamb-p)
        for k=0 to Altmax
          if k=RealState then goto REOVER
          p=CognSuccProb(h,k,realtr)
          if lamb=0 then
            CognSuccProb(h,k,realtr)=p+NTransf*bCognS(h)*(lamb-p)
          else
            CognSuccProb(h,k,realtr)=p+PTransf*bCognS(h)*p*(lamb-p)
          end if
          if k=realtr then CognSuccProb(h,k,k)=0
REOVER:
          next k
rem print" CognS...new"
rem for k=0 to Altmax:rem for m=0 to Altmax
rem print using" .#####";CognSuccProb(h,k,m);
rem next m:rem print
rem next k:rem print
      next h

      for k=1 to Nofl:for m=1 to Nofl
        if ShownSubm$(k,m)="y" then
          p=SubmProb(k,m)
          SubmProb(k,m)=p+aSubm(k)*(lamb-p)
        end if
      next m,k
rem print"Test SubmProb"
rem for k=1 to Nofl:for m=1 to Nofl
rem print using" .##";SubmProb(k,m);
rem next m:rem print
rem next k

      for k=1 to Nofl:for m=1 to Nofl
        if ShownDSS$(k,m)="y" then
          p=DomProb(k)
          DomProb(k)=p+aDom(k)*(lamb-p)
          m=Nofl
        end if
      next m,k
      for k=1 to Nofl:for m=1 to Nofl
        if ShownCSS$(k,m)="y" then

```

```

                p=ChangeProb(k)
                ChangeProb(k)=p+aChange(k)*(lamb-p)
                m=Nofl
            end if
        next m,k
rem Testing
rem print"New ChangeProbs"
rem for k=1 to Nofl
rem print using" .##";ChangeProb(k);
rem next k:rem print

return:rem reinf finished

rewardtr:
Corr%(proc,game)=Corr%(proc,game)+1
for h=1 to Nofl
    for k=0 to Altmax:pr(k)=TrErProb(h,RealState,k)
    next k
    call VecReward0(pr(),Altmax,realtr,aTrErr(h))
    for k=0 to Altmax
rem        print using" .##";pr(k);
        TrErProb(h,RealState,k)=pr(k)
    next k:rem print
next h
rem print"RealState=";RealState;
RealState=realtr
rem print" realtr=";realtr
rem wei1=1-OrdEff:rem wei2=1-wei1
rem print"wei1=OrdEff="; wei1
p=FldEff:p=p+alfaFldEff*(1-p):FldEff=p
wei2=p:wei1=1-p
print"FldEff";:print using" .#####";p

return:rem rewardtr finished, RealState changed

punishtr:
Fail%(proc,game)=Fail%(proc,game)+1
    print"real trial";realtr
for h=1 to Nofl
    for k=0 to Altmax:pr(k)=TrErProb(h,RealState,k)
rem        print using" .##";pr(k);
    next k:rem print

    call VecPunish0(pr(),Altmax,realtr,aTrErr(h))
    for k=0 to Altmax
rem        print using" .##";pr(k);
        TrErProb(h,RealState,k)=pr(k)
    next k:rem print
next h
rem wei1=FailEff:rem wei2=1-wei1

```

```

rem print"wei1=FailEff=";wei1
p=FldEff:p=p-betaFldEff*p:FldEff=p
print"FldEff=";:print using" .###";p
wei2=p:wei1=1-p

return

Testmx:
print"TESTING MATRICES:"
print"ChoiceMade%:"
      for k=1 to NofI:print ChoiceMade%(k);:next k:print
print"SuggMade%:"
      for k=1 to NofI:for m=1 to NofI:print SuggMade%(k,m);:next m
      print
      next k

print"Sugg$"
      for k=1 to NofI:for m=1 to NofI:print Sugg$(k,m);:next m
      print
      next k

print"Lets%"
      for k=1 to NofI:print Lets%(k);:next k:print

print"CognPos%"
      for k=1 to NofI:print CognPos%(k);:next k:print
print"ShownSubm$=";
      for k=1 to NofI:for m=1 to NofI
      print ShownSubm$(k,m);
      next m:print"-";
      next k:print
print"ShownDS$=";
      for k=1 to NofI:for m=1 to NofI
      print ShownDS$(k,m);
      next m:print"-";
      next k:print
print"ShownCS$=";
      for k=1 to NofI:for m=1 to NofI
      print ShownCS$(k,m);
      next m:print"-";
      next k:print

print"To continue press Enter":input x
return:rem testmx

rem Subroutines VecReward0 and VecPunish0

Sub VecReward0(Vector(1),rowmax,Target,Alfa)
  local k
  for k=0 to rowmax

```

```

if k=Target then Vector(k)=Vector(k)+Alfa*(1-Vector(k)):goto OVER1
Vector(k)=Vector(k)-Alfa*Vector(k)
OVER1:
next k
end sub:rem VecReward0

```

```

Sub VecPunish0(Vector(1),rowmax,Target, Beta)
rem      print"subr vecpun"
local k,x
x=(1+(Beta*Vector(Target))/(1-Vector(Target)))
for k=0 to rowmax
if k=Target then Vector(k)=Vector(k)*(1-Beta):goto OVER2
Vector(k)=Vector(k)*x
OVER2:
rem      print using"##"; Vector(k);
next k
rem      print
end sub:rem VecPunish0

```

```

sub MonteCVec1(vec(1),veclength,choice)
local k,r
aux(1)=vec(1)
for k=2 to veclength
aux(k)=aux(k-1)+vec(k)
rem      print using"##";aux(k);
next k
rem      print
r=rnd(1)
rem      print"r=";rem print using"#####";r
for k=1 to veclength:if r<aux(k) then choice=k:k=veclength
next k
end sub:rem Monte Carlo choice from a prob. vector (from 1)

```

```

SubmTest:
if NofSugg>1 then goto Multisugg
if NofSugg=1 then goto Onesugg
subm$="n":goto TESTOVER

```

```

Multisugg:
sum=0
for k=1 to NofI
if k=i then goto Outs1
sum=sum+SuggMade%(k,i)
Outs1:
next k
if sum>0 and sum<NofI-1 then Comp$="n" else Comp$="y"
rem      print"Comp ";Comp$;" sum=";sum:rem Testing
if Comp$="y" then

```

```

p=0:x=0
for k=1 to Nofl
if k=i then goto Outs2
p=SubmProb(i,k)
  if x>0 then
    sdr2=k
    p=p+SubmProb(i,k)*(1-SubmProb(i,sdr1))
rem          print"p=";rem print using" .###";p
  else
    sdr1=k
    end if
x=x+1
    Outs2:
next k
rem print"p=";rem print using" .##";p
r=rnd(1)
rem print"r=";rem print using" .##";r
rem print"sdr= ";sdr1;sdr2
if r<p then
  Subm$="y"
  ShownSubm$(i,sdr1)="y":ShownSubm$(i,sdr2)="y"
  SubmChoice%=SuggMade%(sdr1,i)
  else
  Subm$="n"
  ShownSubm$(i,sdr1)="n":ShownSubm$(i,sdr2)="n"
end if
  else
  Subm$="n"
end if
rem print"Subm ";Subm$
rem Multisugg over
goto TESTOVER

Onesugg:
Subm$="n"
for k=1 to Nofl
if k=i then goto Ones1
if Sugg$(k,i)="y" then
rem          p=SubmProb(i,k)
rem          print"p=";rem print using" .##";p
rem          r=rnd(1)
rem          print"r=";r:rem Testaus
if r<p then
  Subm$="y":SubmChoice%=SuggMade%(k,i)
  ShownSubm$(i,k)="y"
  k=Nofl
  else
  Subm$="n"
  ShownSubm$(i,k)="n"
  k=Nofl

```

```

end if
Ones1:
next k

TESTOVER:
rem print"TESTOVER"
rem print"Subm ";Subm$
rem print"ShownSubm=" ";
rem for k=1 to NofI:rem print ShownSubm$(i,k);:rem next k:rem print

return

```

```

sub MonteCp(prob,answ$)
  local r
  r=rnd(1)
rem      print"Test rnd";r
  if r<prob then answ$="y" else answ$="n"
rem      print"Test, answ$";answ$
end sub:rem Monte Carlo choice with one prob.

```

```

sub MonteCMX(mx(2),rowmax,colmax,row,choice)
  local k,r
  aux(0)=mx(row,0)
  for k=1 to colmax:aux(k)=aux(k-1)+mx(row,k):next k
  r=rnd(1)
rem  print"Test rnd";r
  for k=0 to colmax:if r<aux(k) then choice=k:k=colmax
  next k
end sub:rem Monte Carlo choice from a 2-dim. matrix
rem vectors from 0 to colmax

```

```

sub MonteCVec0(vec(1),colmax,choice)
  local k,r
  dim dynamic aux(colmax)
  aux(0)=vec(0)
  for k=1 to colmax:if r<aux(k) then choice=k:k=colmax
  next k
end sub:rem Monte Carlo choice from a prob. vector (from 0)

```

```

sub DecimBinary(decim,bin)
  local z,x1,x2,x3
  z=decim:if z>7 then print"Too big bin":stop
  x1=int(z/4):x2=int((z-x1*4)/2):x3=z-x1*4-x2*2
  z=x1*100+x2*10+x3
  bin=z
rem      print"Test bin";bin
end sub:rem Changing decimal number to binary

```

```

sub CognitPosit(CognPos%(1),Nofl,BinRS)
  local x,y,z,v
  y=BinRS
  for k=Nofl to 1 step -1
    v=v+1
    x=10^(k-1):z=int(y/x):CognPos%(v)=z:y=y-x*z
  next k
end sub:rem Cognitive positions

```

```

DomSuggSending:
  prob=DomProb(i)
rem  print"DomProb";prob
  call MonteCp(prob,answ$)
  if answ$="y" then
    incr NDom%(game)
    Dom$(i,k)="y":Sugges$="y":SuggMade%(i,k)=CognPos%(k)
    Sugg$(i,k)="y":ShownDSS$(i,k)="y"
    SuggMess$=SuggMess$+Ch$(k,CognPos%(k))
    goto OutDSS
  else
    Dom$(i,k)="n":SuggMade%(i,k)=99:Sugg$(i,k)="n"
    ShownDSS$(i,k)="n"
  end if
  OutDSS:
return:rem Sending dominance suggestion

```

```

ChangeSuggSending:
  prob=ChangeProb(i)
rem  print"ChangeProb";prob
  call MonteCp(prob,answ$)
  if answ$="y" then
    incr NChange%(game)
    Change$(i,k)="y":Sugges$="y":SuggMade%(i,k)=CognPos%(k)
    Sugg$(i,k)="y":ShownCSS$(i,k)="y"
    SuggMess$=SuggMess$+Ch$(k,CognPos%(k))
    goto OutCSS
  else
    Change$(i,k)="n":SuggMade%(i,k)=99:Sugg$(i,k)="n"
    ShownCSS$(i,k)="n"
  end if
  OutCSS:
return:rem Sending change suggestion

```

```

DetAlt:
  x=0
  for k=1 to Nofl:y=2^(Nofl-k)
    x=x+ChoiceMade%(k)*y
  next k
  ChosenAlt=x

```

```
return:rem sub DetAlt finished
```

```
Testprobs:  
print"TrEr"  
for h=1 to Nofl  
print"h=";h  
for k=0 to Altmax:for m=0 to Altmax  
print using" .##";TrErProb(h,k,m);  
next m:print  
next k  
print"To continue press Enter":input x  
next h
```

```
print"CognSucc"  
for h=1 to Nofl  
print"h=";h  
for k=0 to Altmax:for m=0 to Altmax  
print using" .##";CognSuccProb(h,k,m);  
next m:print  
next k  
print"To continue press Enter":input x  
next h
```

```
print"Subm"  
for k=1 to Nofl:for m=1 to Nofl  
print using" .##";SubmProb(k,m);  
next m:print" * ";  
next k
```

```
print"Dom";  
for k=1 to Nofl:print using" .##";Domprob(k);  
next k:print" Change ";  
for k=1 to Nofl:print using" .##";ChangeProb(k);  
next k:print
```

```
print"Subm";  
for k=1 to Nofl:for m=1 to Nofl  
print using" .##";SubmProb(k,m);  
next m:print" * ";  
next k:print  
print"To continue press Enter":input x  
return:rem Testprobs finished
```

```
Trialsums:  
print"Trial frequencies per game over processes:"  
for k=1 to maxproc:for m=1 to maxgame  
mx=Ntrials%(k,m)  
for n=1 to mx  
f% = SeqTr%(k,m,n)
```

```

        incr SumSeq%(m,n,f%)
    next n
next m,k
for k=1 to maxgame:print"Game ";k
print"  0 1 2 3 4 5 6 7"
for m=1 to 10
print m;") ";
for n=0 to Altmax
    print SumSeq%(k,m,n);
    if rf$="y" then write #3,SumSeq%(k,m,n)
next n:print
next m
print"To continue press Enter":input x
next k
print"Looking the frequencies of trials over"

return:rem subr Trialsums finished

Pathtrans:
for k=1 to maxproc:for m=1 to maxgame
p1=path%(k,m,1):p2=path%(k,m,2):p3=path%(k,m,3)
if p1=1 then
    if p2=7 then Pathform%(k,m)=1:incr FreqPath%(m,1)
    if p2=3 then Pathform%(k,m)=7:incr FreqPath%(m,7)
    if p2=5 then Pathform%(k,m)=8:incr FreqPath%(m,8)
    if p2=6 then Pathform%(k,m)=9:incr FreqPath%(m,9)
end if
if p1=2 then
    if p2=7 then Pathform%(k,m)=2:incr FreqPath%(m,2)
    if p2=3 then Pathform%(k,m)=10:incr FreqPath%(m,10)
    if p2=5 then Pathform%(k,m)=11:incr FreqPath%(m,11)
    if p2=6 then Pathform%(k,m)=12:incr FreqPath%(m,12)
end if
if p1=3 then Pathform%(k,m)=3:incr FreqPath%(m,3)
if p1=4 then
    if p2=7 then Pathform%(k,m)=4:incr FreqPath%(m,4)
    if p2=3 then Pathform%(k,m)=13:incr FreqPath%(m,13)
    if p2=5 then Pathform%(k,m)=14:incr FreqPath%(m,14)
    if p2=6 then Pathform%(k,m)=15:incr FreqPath%(m,15)
end if
if p1=5 then Pathform%(k,m)=5:incr FreqPath%(m,5)
if p1=6 then Pathform%(k,m)=6:incr FreqPath%(m,6)
next m,k

print"Paths in the first game:"
print" 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15"
for k=1 to maxpaths
    print FreqPath%(1,k);" ";
    if rf$="y" then write #3,FreqPath%(1,k)
next k:print

```

```

print"Path transitions from game to game"
for k=1 to maxproc:for m=2 to maxgame
incr Transpath%(Pathform%(k,m-1),Pathform%(k,m))
next m,k

print" ";
for k=1 to 9:print k;");:next k
for k=10 to maxpaths:print k;:next k:print
for k=1 to maxpaths:print k;") ";
for m=1 to maxpaths
    print Transpath%(k,m);" ";
    if rf$="y" then write #3,Transpath%(k,m)
next m:print
next k
print"To continue press Enter":input x

return:rem Subroutine for showing path frequencies finished

datin:rem Reading the data from the file
if proc>1 then goto STRAIT

FILNAMGIV:
print"Write the name of the file (eg. d:gm\GMSxxxxx.DAT"
print"d being the drive, gm the path)"
input"File name";fildatnam$
input"Is the name right (y/n)";answ$
if answ$="n" then print"Again!":goto FILNAMGIV

STRAIT:
open fildatnam$ for input as #1
print"File ";fildatnam$;" is opened for data reading"
input #1,x:FldEffbeg=x:print" FldEffbeg=";x;
input #1,x:alfaFldEff=x:print" alfaFldEffbeg=";x;
input #1,x:betaFldEff=x:print"betaFldEffbeg=";x;
input #1,x:betbetFEff=x:print" betbetFEff=";x;
input #1,x:PTransf=x:print"PTransf=";x;
input #1,x:NTransf=x:print"NTransf=";x;
print"Task:"
for k=0 to Altmax:for m=0 to Altmax
input #1,x:Task%(k,m)=x:print using" #.#";x;
next m:print
next k
if proc=1 then input"To continue press Enter";y
print"Potencies of the fields"
for k=0 to nFmax:input #1,x:Pot(k)=x:print using" .##";x;
next k:print
print"SubmProb, DomProb, ChangeProb"
for k=1 to NofI:for m=1 to NofI
input #1,x:SubmProb(k,m)=x:print using" .##";x;

```

```

next m:print" ";
next k:print
for k=1 to Nofl
input #1,x:DomProb(k)=x:print using" .##";x;
next k:print
for k=1 to Nofl
input #1,x:ChangeProb(k)=x:print using" .##";x;
next k:print
print"aTrErr, bCognS, aSubm, aDom, aChange:"
for k=1 to Nofl
input #1,x:aTrErr(k)=x:print using" .##";x;
next k:print
for k=1 to Nofl
input #1,x:bCognS(k)=x:print using" .##";x;
next k:print
for k=1 to Nofl
input #1,x:aSubm(k)=x:print using" .##";x;
next k:print
for k=1 to Nofl
input #1,x:aDom(k)=x:print using" .##";x;
next k:print
for k=1 to Nofl
input #1,x:aChange(k)=x:print using" .##";x;
next k:print
print"Data read from file ";fildatnam$
close 1
print" *** PROCESS ";proc;" ***"
return:rem data from the file finished

```

datsav:rem Saving the data to a file

FILWRIT:

```

print"Write the name of the file (e.g. d:gm\GMSxxxxx.DAT"
print"d being the drive, gm the path)"
print"The name of the data file read was ";fildatnam$
input"File name";filwrinam$
print"Is ";filwrinam$;" the right name?"
input"y/n";answ$
if answ$="n" then print"Again!":goto FILWRIT

```

```

open filwrinam$ for output as #2
print"File ";filwrinam$;" opened for writing the data into it"
x=FldEffbeg:write #2,x:print" FldEffbeg=";x;
x=alfaFldEff:write #2,x:print" alfaFldEff=";x
x=betaFldEffbeg:write #2,x:print"betaFldEffbeg=";x;
x=betbetFEff:write #2,x:print" betbetFEff=";x
x=PTransf:write #2,x:print"PTransf=";x;
x=NTransf:write #2,x:print"NTransf=";x
print"Task:"
for k=0 to Altmax:for m=0 to Altmax

```

```

x=Task%(k,m):write #2,x:print using" #.#";x;
next m:print
next k
input"To continue press Enter";y
print"Potencies of the fields"
for k=0 to nFmax
x=Pot(k):write #2,x:print using" .##";x;
next k:print
print"SubmProb, DomProb, ChangeProb"
for k=1 to NofI:for m=1 to NofI
x=SubmProb(k,m):write #2,x:print using" .##";x;
next m:print" ";
next k:print
for k=1 to NofI
x=DomProb(k):write #2,x:print using" .##";x;
next k:print
for k=1 to NofI
x=ChangeProb(k):write #2,x:print using" .##";x;
next k:print
print"aTrErr, bCognS, aSubm, aDom, aChange:"
for k=1 to NofI
x=aTrErr(k):write #2,x:print using" .##";x;
next k:print
for k=1 to NofI
x=bCognS(k):write #2,x:print using" .##";x;
next k:print
for k=1 to NofI
x=aSubm(k):write #2,x:print using" .##";x;
next k:print
for k=1 to NofI
x=aDom(k):write #2,x:print using" .##";x;
next k:print
for k=1 to NofI
x=aChange(k):write #2,x:print using" .##";x;
next k:print
print"Data saved to the file ";filwrinam$
close 2
return:rem reading the data to the file finished

```

Appendix 6

Program block for task 2 in GM

*The program block in GMS-STD.bas to be rewritten:
(All the rows between Pathtrans and next m,k)*

```
Pathtrans:
for k=1 to maxproc:for m=1 to maxgame
p1=path%(k,m,1):p2=path%(k,m,2):p3=path%(k,m,3)
.....
if p1=6 then Pathform%(k,m)=6:incr FreqPath%(m,6)
next m,k
```

The program block for task 2 needs to be:

```
Pathtrans:
for k=1 to maxproc:for m=1 to maxgame
p1=path%(k,m,1):p2=path%(k,m,2):p3=path%(k,m,3)
if p1=3 then
    if p2=1 then Pathform%(k,m)=1:incr FreqPath%(m,1)
    if p2=2 then Pathform%(k,m)=2:incr FreqPath%(m,2)
    if p2=4 then Pathform%(k,m)=3:incr FreqPath%(m,3)
end if
if p1=5 then
    if p2=1 then Pathform%(k,m)=4:incr FreqPath%(m,4)
    if p2=2 then Pathform%(k,m)=5:incr FreqPath%(m,5)
    if p2=4 then Pathform%(k,m)=6:incr FreqPath%(m,6)
end if
if p1=6 then
    if p2=1 then Pathform%(k,m)=7:incr FreqPath%(m,7)
    if p2=2 then Pathform%(k,m)=8:incr FreqPath%(m,8)
    if p2=4 then Pathform%(k,m)=9:incr FreqPath%(m,9)
end if
next m,k
```

Note: The paths are numbered as follows:

1	2	3	4	5	6	7	8	9
3-1-7	3-2-7	3-4-7	5-1-7	5-2-7	5-4-7	6-1-7	6-2-7	6-4-7

Appendix 7

Simulation of Heider's balance theory; experiment by Esch; basic program

```
rem Simulation of the Heider's balance theory
rem Kullervo Rainio, emer.prof., Helsinki University
rem 16.4. - 8.5.2000
print"Heiders.bas"
print"Simulation of Heider's balance theory"

randomize timer
NSim=500:rem number of simulations
NTrials=37:rem number of cognitive trials (50)
NStates=8:rem number of cognitive states
NFields=5:rem number of cognitive fields
      maxstates=NStates-1:maxfields=NFields-1
NBel=3:rem number of the believes

data 1,0,0,0,0:rem epsilon of s0
data 1,1,1,0,1:rem epsilon of s1
data 1,1,0,1,1:rem s2
data 1,0,1,1,1:rem s3
data 1,1,0,0,0:rem s4
data 1,0,1,0,0:rem s5
data 1,0,0,1,0:rem s6
data 1,1,1,1,0:rem s7
data 0,1,1,1,1:rem valences of the fields

data .97,.98,.992:rem belief probabilities
data 1,0,0:rem characteristics of the believes (1=posit., 0=negat.)
data 0,.333,.333,.334,0:rem potencies of the fields
data .8,.8,.8:rem alfas
data .25,.25,.06:rem betas

dim Epsilon%(maxstates,maxfields):rem 1 if state is in the field
dim Valence%(maxfields):rem valences of the fields
dim namef$(NBel,1):rem Verbal interpretation of the believes
dim actname$(NBel):rem actual names of the believes
dim TrialProb(maxstates):rem cognitive trial probabilities
dim SuccProb(maxstates):rem success of cogn. trial prob's
dim ActBel%(NBel):rem actual believes
dim ChangedBel%(NSim,NTrials):rem belief changed
dim SumChBel%(NTrials,NBel):rem sum of the belief changes
dim CumSumChBel%(NTrials,NBel):rem cumulative sums of the bel.ch.
dim BeliefProb(NBel):rem belief probabilities
dim BegBelProb(NBel):rem belief probs in the beginning
dim alfa(NBel),beta(NBel):rem learning coeff. for bel.prob.
dim Sign%(NBel):rem signs of the believes (0=posit., 1=negat.)
dim Cobel$(Nbel):rem y=congruent belief, y=incongr.
```

```

dim Pot(maxfields):rem potencies of the fields
dim Congr%(maxstates,NBel):rem congruences betw. states and believes
dim Bal%(maxstates):rem the balances of the states
dim NumberFail(NSim)

namef$(1,0)="Disliking Jim"
namef$(1,1)="Liking Jim"
namef$(2,0)="Liking poems"
namef$(2,1)="Disliking poems"
namef$(3,0)="Unit-relation"
namef$(3,1)="No unit"

for simul=1 to NSim
print "SIMULATION: ";simul

for k=1 to NBel:actname$(k)=namef$(k,0):next k

for k=0 to maxstates
    for m=0 to maxfields
        read Epsilon%(k,m)
    next m
next k
for k=0 to maxfields:read Valence%(k):next k:rem reading valences

for k=1 to NBel:read BeliefProb(k)
    BegBelProb(k)=BeliefProb(k)
next k
for k=1 to NBel:read Sign%(k):next k

rem print "Potencies:"
for k=0 to maxfields:read Pot(k)
rem    print using " #.##### ";Pot(k);
next k:rem print:rem reading potencies
for k=1 to NBel:read alfa(k):next k
for k=1 to NBel:read beta(k):next k
for k=0 to maxstates
    for m=1 to NBel
        Congr%(k,m)=Epsilon%(k,m)
    next m,k
next m,k

gosub BALANCES
gosub NUMSTATEFIELDS
gosub DETTRIALPROB

balf$="n"
    for k=1 to 3:Balfield%(k)=0:next k
state=0
for trial=1 to Ntrials
rem gosub TESTS
call MonteCVec(TrialProb(),maxstates,choice)

```

```

state=choice
if balf$="y" then goto OVERBALF
if state=1 or state=2 or state=3 then Balfield%(state)=1
sum=0:for k=1 to 3:sum=sum+Balfield%(k):next k
if sum>2 then balf$="y"
if balf$="y" then
    baltrial=trial
rem    print"baltrial=";baltrial:input x
    Pot(1)=.1:Pot(2)=.1:Pot(3)=.1:Pot(4)=.7
    gosub NUMSTATEFIELDS
    gosub DETTRIALPROB
end if
rem balance field formed and trial probabilities changed

OVERBALF:
rem print"-----"
rem print"State=";state;" / ";
if Bal%(state)=1 then succ$="y" else succ$="n"
rem print"succ=";succ$
if succ$="n" then goto FAILURE
rem determining the actual believes

Pun$="n"
rem print"BeliefProbs: ";
rem for k=1 to NBel
rem    print using" #.##";BeliefProb(k);
rem next k:rem print
rem print"r and Actual believes:";
for k=1 to NBel
    prob=BeliefProb(k)
    call MonteCp(prob,answer$)
    if answer$="y" then ActBel%(k)=1 else ActBel%(k)=0
rem print ActBel%(k);
    if Congr%(state,k)=ActBel%(k) then Cobel$(k)="y" else Cobel$(k)="n"
    if Cobel$(k)="n" then Pun$="y"
next k: rem congruent believes determined
rem if at least one Cobel$ is no, then Pun$ is yes
rem print:rem print"Congruences: ";
rem    for k=1 to NBel:rem print Cobel$(k);:rem next k:rem print
rem print"Pun=";Pun$

if Pun$="n" then
    gosub REWARD
else
    gosub PUNISH
end if
rem print"Reinforcements made"

if Pun$="n" then gosub RESTRUCTURING

```

```

goto OVERTRIAL

FAILURE:
rem print"Failure"
incr NF

OVERTRIAL:
rem input"To continue, press ENTER";x

next trial

NumberFail(simul)=NF:NF=0
rem print"Number of failures=";NumberFail(simul)
restore

print"Believes in the end of simul. ";simul;": "
for k=1 to NBel:print actname$(k);" / ";:next k:print
rem print"To continue, press enter":rem input x

next simul

print"Changed believes:"
for k=1 to NSim:for m=1 to NTrials:p=ChangedBel%(k,m)
    if p>0 then print"sim ";k;" trial ";m;" Bel ";p
next m,k
input"To continue, press ENTER";x
print"RESULTS:"
for k=1 to NTrials
    for m=1 to NSim:bel=ChangedBel%(m,k)
        incr SumChBel%(k,bel)
next m,k
print"Sums of belief changes: "
for k=1 to NBel
    print namef$(k,1);": ";
    for m=1 to NTrials
        print SumChBel%(m,k);
    next m:print
next k:print
for k=1 to NBel
    print namef$(k,1)
    for m=1 to NTrials
        CumSumChBel%(m,k)=CumSumChBel%(m-1,k)+SumChBel%(m,k)
        print CumSumChBel%(m,k);
    next m:print
next k:print
rem results shown

print"END"
end

```

```

sub MonteCVec(vec(0),colmax,choice)
    local k,r
rem    print"MonteC: ";
    aux(0)=vec(0)
    for k=1 to colmax
        aux(k)=aux(k-1)+vec(k)
rem    print using " #.###";aux(k);
    next k
    r=rnd(1):rem print:rem print using " #.###";r
    for k=0 to colmax:if r<aux(k) then choice=k:k=colmax
    next k
rem    print"MonteC choice: ";choice
end sub:rem Monte Carlo choice from a vector
    rem starting from k=0

```

```

sub MonteCp(prob,answ$)
    local r
    r=rnd(1)
rem    print using " #.###";r;
    if r<prob then answ$="y" else answ$="n"
end sub:rem Monte Carlo choice with one probability

```

BALANCES:

```

rem print"Balances=";
for k=0 to maxstates
    s=0:for m=1 to NBel
        if Epsilon%(k,m)=1 then
            s=s+Sign%(m)
        else
            if Sign%(m)=0 then s=s+1
        end if
    next m
    if s/2=int(s/2) then Bal%(k)=1 else Bal%(k)=2
rem    print Bal%(k);
next k:rem print:rem print"SuccProb="
for k=0 to maxstates
    if Bal%(k)=1 then SuccProb(k)=1 else SuccProb(k)=0
rem    print SuccProb(k);
next k:rem print
return:rem balances and SuccProbs determined

```

NUMSTATEFIELDS:

```

rem print"NStatesF:"
for k=1 to maxfields:s=0
    for m=1 to maxstates
        if Epsilon%(m,k)=1 then incr s
    next m: NStatesF(k)=s
rem    print s;
next k:rem print
return:rem number of states in the fields determined

```

DETTRIALPROB:

```
rem print"TrialProb's: ";
for m=0 to maxstates:s=0
    for k=1 to maxfields
        if Epsilon%(m,k)=1 then s=s+Pot(k)*Valence%(k)/NStatesF(k)
    next k
    TrialProb(m)=s
rem    print using" #.###";s;
next m:rem print
return:rem trial probabilities determined
```

REWARD:

```
rem print"REWARD"
rem    print"Belief Probs: ";
    for k=1 to NBel
        if ActBel%(k)=1 then
            p=BeliefProb(k)
rem        print"old p";:print using" #.###";p;
            p=p+alfa(k)*p*(1-p)
rem        print using" #.###";p;
        else
            p=p-alfa(k)*p*(1-p)
rem        print"opp.";:print using" #.###";p;
        end if
        BeliefProb(k)=p
    next k:rem print
return:rem rewards made
```

PUNISH:

```
rem    print"Punish    ";
    for k=1 to NBel
        if Cobel$(k)="n" then
            p=BeliefProb(k)
rem        print"old p";:print using" #.###";p;
            p=p-beta(k)*p*(1-p)
rem        print k;:print using" #.###";p;
            BeliefProb(k)=p
        end if
    next k:rem print
return:rem punishment made
```

RESTRUCTURING:

```
rem print"RESTRUCTURING"
restr$="n"
for kk=1 to NBel
    if ActBel%(kk)=0 then
        restr$="y"
```

```

        Sign%(kk)=1-Sign%(kk)
        ChangedBel%(simul,trial)=kk
        actname$(kk)=namef$(kk,1)
        gosub BALANCES
        for m=0 to maxstates
            if Bal%(m)=1 then Epsilon%(m,4)=1 else Epsilon%(m,4)=0
        next m
        gosub NUMSTATEFIELDS
        gosub DETTRIALPROB
    end if
next kk
if restr$="y" then
rem     print"BeliefProbs:"
        for k=1 to NBel
            BeliefProb(k)=BegBelProb(k)
rem     print using " .##";BeliefProb(k)
        next k
    end if

rem input"To continue, press ENTER";x
return:rem restructuring done

TESTS:
for k=1 to NBel:print actname$(k);
        print using " #.##";BeliefProb(k);
        print " S:";Sign%(k);
next k:print
for k=0 to maxstates
        print using " #.##";Pot(k);
        print " Bal ";Bal%(k);
next k:print
input x
return

```